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A refined theory with stretching effect for the flexure analysis of laminated composite plates

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Abstract. This work presents a static flexure analysis of laminated composite plates by utilizing a higher order shear deformation theory in which the stretching effect is incorporated. The axial displacement field utilizes sinusoidal function in terms of thickness coordinate to consider the transverse shear deformation influence. The cosine function in thickness coordinate is employed in transverse displacement to introduce the influence of transverse normal strain. The highlight of the present method is that, in addition to incorporating the thickness stretching effect ($\varepsilon_z \neq 0$), the displacement field is constructed with only 5 unknowns, as against 6 or more in other higher order shear and normal deformation theory. Governing equations of the present theory are determined by employing the principle of virtual work. The closed-form solutions of simply supported cross-ply and angle-ply laminated composite plates have been obtained using Navier solution. The numerical results of present method are compared with those of the classical plate theory (CPT), first order shear deformation theory (FSDT), higher order shear deformation theory (HSDT) of Reddy, higher order shear and normal deformation theory (HSNDT) and exact three dimensional elasticity theory wherever applicable. The results predicted by present theory are in good agreement with those of higher order shear deformation theory and the elasticity theory. It can be concluded that the proposed method is accurate and simple in solving the static bending response of laminated composite plates.

Keywords: shear deformation; stretching effect; static flexure; laminated plate

1. Introduction

Fiber reinforced composite are widely employed in various engineering industries such as the aerospace, automotive, marine and other structural applications due to superior mechanical properties of these materials. In the past three decades, investigations on laminated composite plates have attracted considerable attention, and a variety of laminated theories has been developed. The classical plate theory (CPT), which ignores the transverse shear influences, gives reasonable results for thin plates. However, the errors in deflections and stresses are quite significant for

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moderately thick plates when determined utilizing CPT. To overcome the deficiency of the CPT, many shear deformation plate theories which consider the transverse shear deformation influences have been introduced. Mindlin (1951) has developed the first order shear deformation theory (FSDT) which is based on a linear variation of in-plane displacements through the thickness. A shear correction coefficient is needed for FSDT to compensate the error induced the constant shear strain supposition within the thickness. Thus, FSDT is not convenient for employ because of the difficulty in computation of the correct value of the shear correction factor (Sadoune *et al*. 2014, Meksi *et al*. 2015, Bellifa *et al*. 2016). The higher-order shear deformation plate theories (HSDT) have been introduced to avoid the use of shear correction factor. These theories consider a Taylor series expansion of higher order terms to define the displacement vector, which was developed and discussed by different researchers (Hidebrand *et al*. 1949, Nelson and Lorch 1974, Librescu 1975, Lo *et al*. 1977a, b, Levinson 1980, Murthy 1981, Reddy 1984, Bhimaradi and Stevens 1984, Kant 1982). A number of HSDTs are also proposed for investigating functionally graded material (Bachir Bouiadjra *et al*. 2012, Bourada *et al*. 2012, Bouderba *et al*. 2013, Bachir Bouiadjra *et al*. 2013, Tounsi *et al*. 2013, Saidi *et al*. 2013, Ait Amar Meziane *et al*. 2014, Belabed *et al*. 2014, Zidi *et al*. 2014, Bakora and Tounsi 2015, Nguyen *et al*. 2015, Larbi Chaht *et al*. 2015, Ait Yahia *et al*. 2015, Sallai *et al*. 2015, Tagrara *et al*. 2015, Tebboune *et al*. 2015, Belkorissat *et al*. 2015, Ait Atmane *et al*. 2015, Mahi *et al*. 2015, Al-Basyouni *et al*. 2015, Bennai *et al*. 2015, Attia *et al*. 2015, Mantari and Granados 2015, Bounouara *et al*. 2016, Bouderba *et al*. 2016, Boukhari *et al*. 2016). Various investigators have proposed a number of HSDTs to examine the mechanical response of laminated composite plates. Soldatos (1988) proposed hyperbolic shear deformation theory for the flexure analysis of laminated composite plates. An analytical solution is presented by Kant and Swaminathan (2002) for the bending analysis of laminated composite and sandwich plates using a higher order refined theory. Akavci (2007) developed a novel hyperbolic theory in terms of tangent and secant functions for the analysis of plates. Brischetto *et al*. (2009) studied the bending response of unsymmetrically laminated sandwich flat panels with a soft core. Zhen and Wanji (2010) proposed developed C°-type higher-order theory for static analysis of laminated composite and sandwich plates under thermo-mechanical loads. Pandit *et al*. (2010) and Chalak *et al*. (2012) developed finite element models based on an improved higher order zigzag plate theory for the bending and vibration analysis of soft core sandwich plates. Global–local theories are proposed by Kapuria and Nath (2013) for bending and vibration behaviors of laminated and sandwich plates. Grover *et al*. (2013), Sahoo and Singh (2013) developed a novel inverse hyperbolic shear deformation theory for the laminated composite and sandwich plates. Draiche *et al*. (2014) studied the free vibration response of rectangular composite plates with patch mass using a trigonometric four variable plate theory. Sayyad and Ghugal (2014a) proposed a trigonometric shear deformation theory taking into account transverse shear deformation effect as well as transverse normal strain effect or static flexure of cross-ply laminated composite and sandwich plates. Nedri *et al*. (2014) investigated the free vibration response of laminated composite plates resting on elastic foundations by using a refined hyperbolic shear deformation theory. Chattibi *et al*. (2015) studied the thermo-mechanical effects on the bending of antisymmetric cross-ply composite plates using a four variable sinusoidal theory. Since the HSDTs are based on supposition of quadratic, cubic or higher-order variations of axial displacements within the thickness, their governing equations are much more complicated than those of FSDT. Hence, there is a scope to propose an accurate theory which is simple to use.

In the present article, an analytical solution of the static flexural analysis of laminated composite plates subjected to uniformly distributed, uniformly varying and concentrated loads is proposed by using a simple quasi-3D HSDT. Just five independent unknowns are employed in the present theory against six independent unknowns or more independent unknowns employed in the corresponding shear and normal deformations theories. The performance of the present formulation is verified by comparing results with other quasi-3D HSDTs and 2D HSDTs available in literature and exact solution given by Pagano (1970) wherever applicable.

2. Theoretical formulation

Consider a rectangular plate of total thickness h made up of n orthotropic layers with the coordinate system as shown in Fig. 1.

2.1 The displacement field

The displacement field of the present work is built on the basis of the following assumptions: (1) The transverse displacement (w) is composed with three parts, namely: bending, shear and stretching components; (2) the inplane displacement u in x-direction and v in y-direction each consists of three components (extension, bending and shear); (3) the bending parts of the inplane displacements are analogous to those used in CPT; and (4) the shear parts of the inplane displacements are assumed to be trigonometric in nature with respect to thickness coordinate in such a way that the shear stresses vanish on the top and bottom surfaces of the plate. Based on these assumptions, the following displacement field relations can be obtained (Bousahla *et al.* 2014, Fekrar *et al.* 2014, Hebali *et al.* 2014, Hamidi *et al.* 2015, Meradjah *et al.* 2015, Bourada *et al.* 2015, Bennoun *et al.* 2016)

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y} \\ w(x,y,z) &= w_b(x,y) + w_s(x,y) + g(z)\,\varphi(x,y) \end{aligned} \tag{1}$$

where u_0 and v_0 denote the displacements along the x and y coordinate directions of a point on the mid-plane of the plate; w_b and w_s are the bending and shear components of the transverse displacement, respectively; and the additional displacement φ accounts for the effect of normal stress (stretching effect). The shape functions $f(z)$ and $g(z)$ are given as follows

$$f(z) = z - \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \tag{2}$$

and

$$g(z) = 1 - f'(z) \tag{3}$$

The non-zero strains associated with the displacement field in Eq. (1) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \quad \varepsilon_z = g'(z)\,\varepsilon_z^0 \tag{4}$$

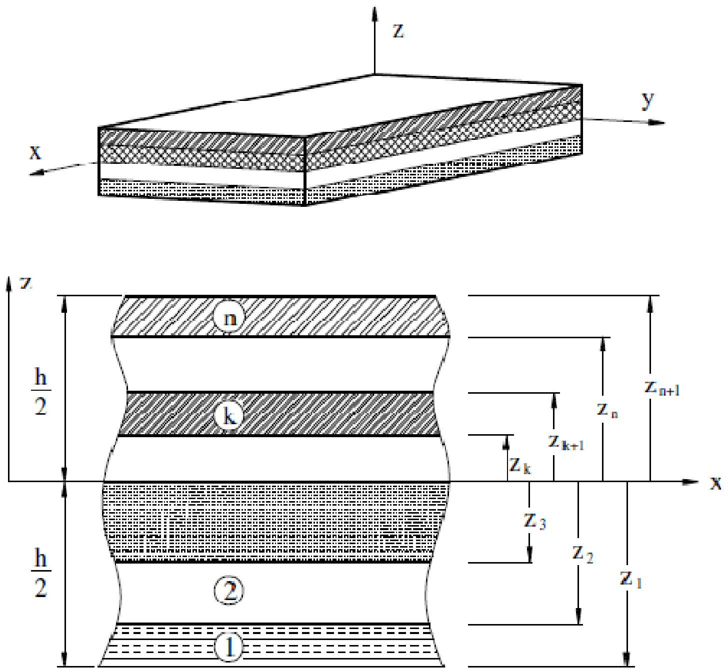

Fig. 1 Coordinate system and layer numbering used for a typical laminated plate

and

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_s}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial y^2} \\ -2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \tag{5}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial y} + \dfrac{\partial \varphi}{\partial y} \\ \dfrac{\partial w_s}{\partial x} + \dfrac{\partial \varphi}{\partial x} \end{Bmatrix}, \quad \varepsilon_z^0 = \varphi$$

and

$$g'(z) = \frac{dg(z)}{dz} \tag{6}$$

2.2 Constitutive relations

Each lamina in the laminated plate is supposed to be in a three-dimensional stress state so that the constitutive relations in the k^{th} layer can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}^{(k)} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{13} & 0 & 0 & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{23} & 0 & 0 & \overline{Q}_{26} \\ \overline{Q}_{13} & \overline{Q}_{23} & \overline{Q}_{33} & 0 & 0 & \overline{Q}_{36} \\ 0 & 0 & 0 & \overline{Q}_{44} & \overline{Q}_{45} & 0 \\ 0 & 0 & 0 & \overline{Q}_{45} & \overline{Q}_{55} & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{36} & 0 & 0 & \overline{Q}_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}^{(k)} \tag{7}$$

where $\overline{Q}_{ij}^{k}$ are the transformed material constants, given by

$$\begin{aligned}
\overline{Q}_{11}^{k} &= Q_{11}\cos^4\theta_k + 2(Q_{12}+2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{22}\sin^4\theta_k \\
\overline{Q}_{12}^{k} &= (Q_{11}+Q_{22}-4Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{12}(\sin^4\theta_k+\cos^4\theta_k) \\
\overline{Q}_{13}^{k} &= Q_{13}\cos^2\theta_k + Q_{23}\sin^2\theta_k \\
\overline{Q}_{16}^{k} &= Q_{11}\cos^3\theta_k\sin\theta_k + Q_{12}(\cos\theta_k\sin^3\theta_k-\cos^3\theta_k\sin\theta_k) - Q_{22}\cos^3\theta_k\sin\theta_k \\
&\quad -2Q_{66}\cos\theta_k\sin\theta_k(\cos^2\theta_k-\sin^2\theta_k) \\
\overline{Q}_{26}^{k} &= Q_{11}\cos\theta_k\sin^3\theta_k + Q_{12}(\cos^3\theta_k\sin\theta_k-\cos\theta_k\sin^3\theta_k) - Q_{22}\cos\theta_k\sin^3\theta_k \\
&\quad +2Q_{66}\cos\theta_k\sin\theta_k(\cos^2\theta_k-\sin^2\theta_k) \\
\overline{Q}_{22}^{k} &= Q_{11}\sin^4\theta_k + 2(Q_{12}+2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{22}\cos^4\theta_k \\
\overline{Q}_{23}^{k} &= Q_{13}\sin^2\theta_k + Q_{23}\cos^2\theta_k \\
\overline{Q}_{33}^{k} &= Q_{33} \\
\overline{Q}_{36}^{k} &= (Q_{13}-Q_{23})\cos\theta_k\sin\theta_k \\
\overline{Q}_{44}^{k} &= Q_{44}\cos^2\theta_k + Q_{55}\sin^2\theta_k \\
\overline{Q}_{45}^{k} &= (Q_{55}-Q_{44})\cos\theta_k\sin\theta_k \\
\overline{Q}_{55}^{k} &= Q_{55}\cos^2\theta_k + Q_{44}\sin^2\theta_k \\
\overline{Q}_{66}^{k} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{66}(\sin^4\theta_k+\cos^4\theta_k)
\end{aligned} \tag{8}$$

where θ_k is the angle of material axes with the reference coordinate axes of each layer and Q_{ij} are the plane stress-reduced stiffnesses, and are known in terms of the engineering constants in the material axes of the layer.

$$\begin{aligned}
Q_{11} &= \frac{E_1(1-\nu_{23}\nu_{32})}{\Delta};\ Q_{12} = \frac{E_1(\nu_{21}+\nu_{31}\nu_{23})}{\Delta};\ Q_{13} = \frac{E_1(\nu_{31}+\nu_{21}\nu_{32})}{\Delta}; \\
Q_{22} &= \frac{E_2(1-\nu_{13}\nu_{31})}{\Delta};\ Q_{23} = \frac{E_2(\nu_{32}+\nu_{12}\nu_{31})}{\Delta};\ Q_{33} = \frac{E_3(1-\nu_{12}\nu_{21})}{\Delta};
\end{aligned} \tag{9}$$

$$Q_{66}=G_{12};\ Q_{55}=G_{13};\ Q_{44}=G_{23};\ \Delta=1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{31}\nu_{13}-2\nu_{21}\nu_{32}\nu \tag{9}$$

In which, E_1, E_2, E_3 are the Young's moduli in the x, y and z directions respectively, G_{23}, G_{13}, G_{12} are the shear moduli and ν_{ij} are the Poisson's ratios for transverse strain in j-direction when stressed in the i-direction. Poisson's ratios and Young's moduli are related as

$$\nu_{ij}E_j=\nu_{ji}E_i \qquad (i,j=1,2,3) \tag{10}$$

2.3 Governing equations

The principle of virtual work (PVW) is employed for the static flexure problem of any plate. Also it can be utilized to examine the considered laminated plates. The principle is written as

$$\delta U+\delta V=0 \tag{11}$$

with δU is the virtual strain energy, δV is the external virtual works induced to an external load applied to the plate. They can be expressed as

$$\delta U=\int_V \left(\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\sigma_z\delta\varepsilon_z+\tau_{xy}\delta\gamma_{xy}+\tau_{yz}\delta\gamma_{yz}+\tau_{xz}\delta\gamma_{xz}\right)dV \tag{12}$$

$$\delta V=-\int_\Omega q\delta w\,d\Omega \tag{13}$$

where Ω is the top surface and q is the distributed transverse load.

Substituting Eqs. (1), (4) and (7) into Eq. (11) and integrating through the thickness of the plate, Eq (11) can be rewritten as

$$\begin{aligned}\int_\Omega \Big[&N_x\delta\varepsilon_x^0+N_y\delta\varepsilon_y^0+N_z\delta\varepsilon_z^0+N_{xy}\delta\gamma_{xy}^0+M_x^b\delta k_x^b+M_y^b\delta k_y^b+M_{xy}^b\delta k_{xy}^b\\&+M_x^s\delta k_x^s+M_y^s\delta k_y^s+M_{xy}^s\delta k_{xy}^s+S_{yz}^s\delta\gamma_{yz}^0+S_{xz}^s\delta\gamma_{xz}^0-q\delta w\Big]d\Omega=0\end{aligned} \tag{14}$$

where the stress resultants (N, M^b, M^s, S^s and N_z) are as follows

$$\begin{gathered}\left(N_i,M_i^b,M_i^s\right)=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}(1,z,f)\sigma_i dz,\quad (i=x,y,xy),\quad S_i^s=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\tau_i g(z)dz,\quad (i=xz,yz)\\ \text{and}\qquad N_z=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\sigma_z g'(z)dz\end{gathered} \tag{15}$$

The governing equations of equilibrium can be obtained from Eq. (14) by integrating the displacement gradients by parts and setting the coefficients δu_0, δv_0, δw_b, δw_s and $\delta\varphi$ to zero separately. Thus one can obtain the equilibrium equations associated with the present simple quasi-3D theory

$$\begin{aligned}
&\delta u_0: \ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0 \\
&\delta v_0: \ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0 \\
&\delta w_b: \ \frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x \partial y}+\frac{\partial^2 M_y^b}{\partial y^2}+q=0 \\
&\delta w_s: \ \frac{\partial^2 M_x^s}{\partial x^2}+2\frac{\partial^2 M_{xy}^s}{\partial x \partial y}+\frac{\partial^2 M_y^s}{\partial y^2}+\frac{\partial S_{xz}^s}{\partial x}+\frac{\partial S_{yz}^s}{\partial y}+q=0 \\
&\delta \varphi: \ \frac{\partial S_{xz}^s}{\partial x}+\frac{\partial S_{yz}^s}{\partial y}-N_z=0
\end{aligned} \tag{16}$$

By substituting Eq. (4) into Eq. (7) and the subsequent results into Eq. (15), the stress resultants are readily obtained as

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} + \begin{bmatrix} L \\ L^a \\ R \end{bmatrix} \varepsilon_z^0, \quad S = A^s \gamma , \tag{17a}$$

$$N_z = R_{33}^a \varphi + L_{13}\varepsilon_x^0 + L_{23}\varepsilon_y^0 + L_{13}^a k_x^b + L_{23}^a k_y^b + R_{13} k_x^s + R_{23} k_y^s, \tag{17b}$$

where

$$N = \{N_x, N_y, N_{xy}\}, \quad M^b = \{M_x^b, M_y^b, M_{xy}^b\}, \quad M^s = \{M_x^s, M_y^s, M_{xy}^s\}, \tag{18a}$$

$$\varepsilon = \{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\}, \quad k^b = \{k_x^b, k_y^b, k_{xy}^b\}, \quad k^s = \{k_x^s, k_y^s, k_{xy}^s\}, \tag{18b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}, \tag{18c}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}, \tag{18d}$$

$$S = \{S_{xz}^s, S_{yz}^s\}, \quad \gamma = \{\gamma_{xz}, \gamma_{yz}\}, \quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \quad L = \begin{Bmatrix} L_{13} \\ L_{23} \\ 0 \end{Bmatrix}, \quad L^a = \begin{Bmatrix} L_{13}^a \\ L_{23}^a \\ 0 \end{Bmatrix}, \quad R = \begin{Bmatrix} R_{13} \\ R_{23} \\ 0 \end{Bmatrix} \tag{18e}$$

Here the stiffness coefficients are defined as

$$\{A_{ij}, B_{ij}, D_{ij}\} = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)} \{1, z, z^2\} dz, \qquad i, j = 1, 2, 6 \tag{19a}$$

$$\{B_{ij}^s, D_{ij}^s, H_{ij}^s\} = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)} f(z) \{1, z, f(z)\} dz, \qquad i, j = 1, 2, 6 \tag{19b}$$

$$\{L_{i3}, L_{i3}^a, R_{i3}\} = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \overline{Q}_{i3}^{(k)} g'(z) \{1, z, f(z)\} dz, \qquad i = 1, 2 \tag{19c}$$

$$R_{33}^a = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \overline{Q}_{33}^{(k)} [g'(z)]^2 dz \tag{19d}$$

$$A_{ij}^s = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)} [g(z)]^2 dz, \qquad i, j = 4, 5 \tag{19e}$$

3. Illustrative examples

In order to demonstrate the accuracy of the present formulation, the following numerical examples on laminated composites plates subjected to different loading types are presented and discussed.

Example 1: A laminated composite square plate with simply supported boundary conditions and subjected to sinusoidal loading $q = q_0 \sin(\pi x/a)\sin(\pi y/b)$ on the top surface of the plate is proposed where, q_0, is the magnitude of the sinusoidal loading at the centre. The laminate configuration considered in this example is presented in Fig. 2.

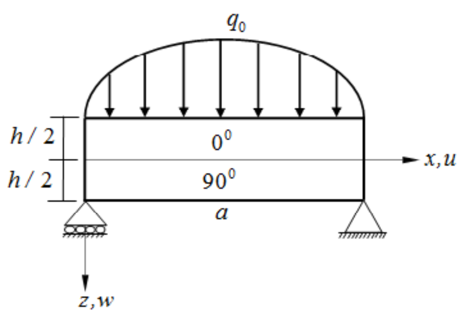

Fig. 2 Simply supported laminated plates under sinusoidal loading

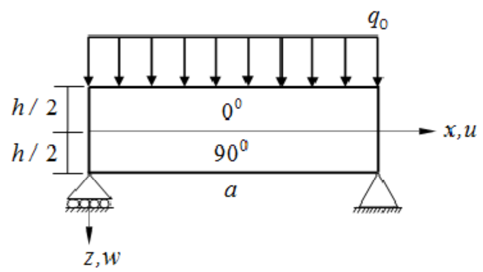

Fig. 3 Simply supported laminated plates under uniformly distributed loading

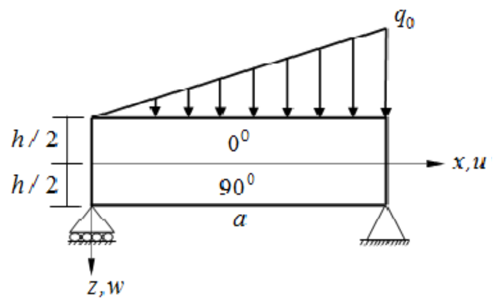

Fig. 4 Simply supported laminated plates under linearly varying load

Example 2: A laminated composite plate with simply supported boundary conditions and subjected to uniformly distributed transverse load is considered (Fig. 3). The loading is represented by $q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn}\sin(m\pi\, x/a)\sin(n\pi\, y/b)$ on the top surface of the plate where m and n are positive integers and q_{mn} is the coefficient of Fourier expansion of load as expressed below

$$q_{mn}=\frac{16q_0}{mn\pi^2} \tag{20}$$

Example 3: A laminated composite plate with simply supported boundary conditions and subjected to linearly varying load on the top surface of the plate is proposed (Fig. 4). The load is given by $q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn}\sin(m\pi\, x/a)\sin(n\pi\, y/b)$ with the coefficient of Fourier expansion q_{mn} of the load as follows

$$q_{mn}=-\frac{8q_0}{mn\pi^2}\cos(m\pi) \tag{21}$$

4. Numerical results and discussion

In this section, various numerical examples are presented and discussed for checking the efficacy of the present formulation in predicting the vibration response of simply supported antisymmetric cross-ply and angle-ply laminates.

The following lamina properties are employed:

Material 1: (Pagano 1970)

$$E_1 = 25E_2,\ E_3 = E_2,\ G_{12} = G_{13} = 0.5E_2,\ G_{23} = 0.2E_2 \ \text{ and } \ \nu_{12} = \nu_{13} = \nu_{23} = 0.25 \tag{22}$$

Material 2: (Ren 1990)

$$E_1 = 40E_2,\ E_3 = E_2,\ G_{12} = G_{13} = 0.5E_2,\ G_{23} = 0.6E_2 \ \text{ and } \ \nu_{12} = \nu_{13} = \nu_{23} = 0.25 \tag{23}$$

In addition, the following dimensionless displacements and stresses have been employed throughout the tables and figures

$$\begin{gathered}\bar{w}\left(\frac{a}{2},\frac{b}{2},\frac{z}{h}\right) = \frac{100h^3E_3}{qa^4}w, \qquad \left(\bar{\sigma}_x,\bar{\sigma}_y\right)\left(\frac{a}{2},\frac{b}{2},\frac{z}{h}\right) = \frac{h^2}{qa^2}\left(\sigma_x,\sigma_y\right), \\ \bar{\tau}_{xy}\left(\frac{a}{2},\frac{b}{2},\frac{z}{h}\right) = \frac{h^2}{qa^2}\tau_{xy}, \qquad \bar{\tau}_{xz}\left(0,\frac{b}{2},\frac{z}{h}\right) = \frac{h}{qa}\tau_{xz}, \qquad \bar{\tau}_{yz}\left(\frac{b}{2},0,\frac{z}{h}\right) = \frac{h}{qa}\tau_{yz}\end{gathered} \tag{24}$$

The results determined for displacement and stresses are illustrated in Tables 1 to 6 and graphically in Figs. 5 to 7. The results determined by the proposed theory for displacements and stresses are compared with those of classical plate theory (CPT), first order shear deformation theory (FSDT) of Mindlin (1951), higher order shear deformation theory (HSDT) of Reddy (1984), trigonometric shear and normal shear deformation theory (TSNDT) of Sayyad and Ghugal (2014a, b) and exact theory by Pagano (1970).

Table 1 Comparison of transverse displacement and stresses for simply supported two-layer (0/90) square laminated plate subjected to single sine load

a/h	Theory	Model	$\bar{w}$ ($z = 0$)	$\bar{\sigma}_x$ ($z = -h/2$)	$\bar{\sigma}_y$ ($z = -h/2$)	$\bar{\tau}_{xy}$ ($z = -h/2$)	$\bar{\tau}_{xz}$ ($z = 0$)	$\bar{\tau}_{yz}$ ($z = 0$)
4	Present	TSDT	1.9424	0.9063	0.0964	0.0562	0.3189	0.3189
	Ref[(a)]	TSDT	1.9424	0.9063	0.0964	0.0562	0.3189	0.3189
	Reddy	HSDT	1.9985	0.9060	0.0891	0.0577	0.3128	0.3128
	Mindlin	FSDT	1.9682	0.7157	0.0843	0.0525	0.2274	0.2274
	Kirchhoff	CPT	1.0636	0.7157	0.0843	0.0525	---	---
	Pagano	Elasticity	2.0670	0.8410	0.1090	0.0591	0.3210	0.3130
10	Present	TSDT	1.2089	0.7471	0.0876	0.0530	0.3261	0.3261
	Ref[(a)]	TSDT	1.2089	0.7471	0.0876	0.0530	0.3261	0.3261
	Reddy	HSDT	1.2161	0.7468	0.0851	0.0533	0.3190	0.3190
	Mindlin	FSDT	1.2083	0.7157	0.0843	0.0525	0.2274	0.2274
	Kirchhoff	CPT	1.0636	0.7157	0.0843	0.0525	---	---
	Pagano	Elasticity	1.2250	0.7302	0.0886	0.0535	0.3310	0.3310

(a) Results taken from reference Sayyad and Ghugal (2014a)

Example 1: A simply supported two-layer antisymmetric cross ply (0/90) square laminate under sinusoidal transverse load is examined. Material set 1 is employed. The comparison of results of transverse displacement and stresses for slenderness ratios 4 and 10 is demonstrated in Table 1. The maximum deflections predicted by present model are in good agreement with those of exact solution (Pagano 1970) and other solutions of Reddy and Sayyad and Ghugal (2014a) for (0/90) cross-ply laminated plate whereas CPT underestimates the results for all slenderness ratios. The axial normal stress $\bar{\sigma}_x$ determined by the present formulation is in excellent agreement with that of Sayyad and Ghugal (2014a) and in tune with exact solution whereas FSDT and CPT underestimate this stress for all slenderness ratios when compared with the values of other refined theories. Both the present theory and the theory proposed by Sayyad and Ghugal (2014a), give the same values of the axial normal stress $\bar{\sigma}_y$ and shear stress $\bar{\tau}_{xy}$. These results are also in good agreement with those of exact solution (Pagano 1970). Table 1 also shows the comparison of transverse shear stresses ($\bar{\tau}_{xz}$ and $\bar{\tau}_{yz}$) for the two layered (0°/90°) anti-symmetric cross-ply

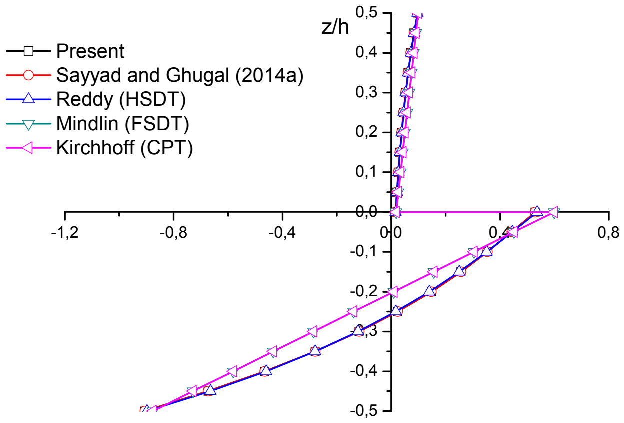

Fig. 5 Through thickness distribution of the axial normal stress $\bar{\sigma}_x$ of (0/90) cross-ply laminated plate under sinusoidal loading for $h/a = 0.25$

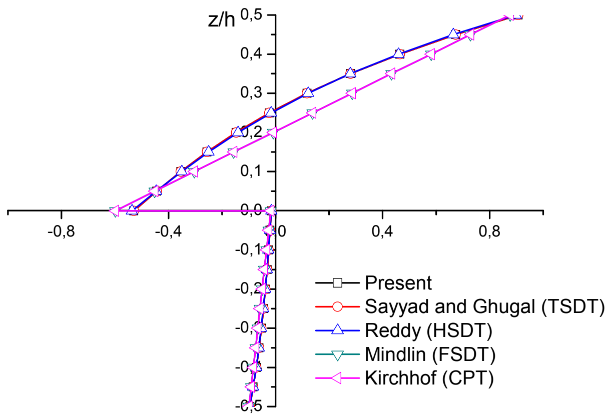

Fig. 6 Through thickness distribution of the axial normal stress $\bar{\sigma}_y$ of (0/90) cross-ply laminated plate under sinusoidal loading for $h/a = 0.25$

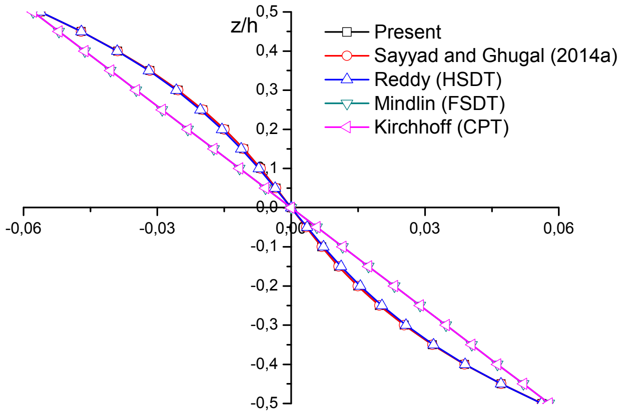

Fig. 7 Through thickness distribution of the shear stress $\bar{\tau}_{xy}$ of (0/90) cross-ply laminated plate under sinusoidal loading for h/a = 0.25

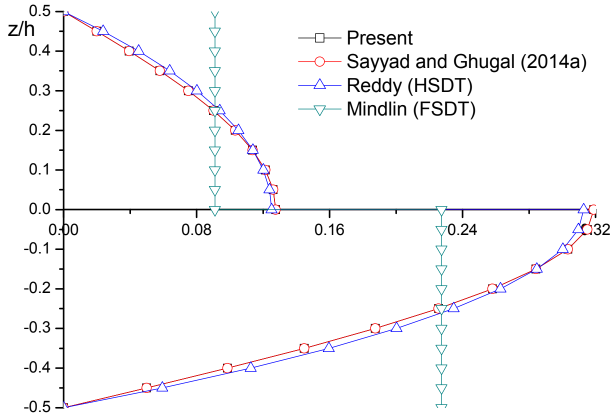

Fig. 8 Through thickness distribution of the transverse shear stress $\bar{\tau}_{zx}$ of (0/90) cross-ply laminated plate under sinusoidal loading for h/a = 0.25

laminated plates under a sinusoidal loading. The proposed theory predicts more accurate transverse shear stresses than those provided by other refined theories as compared to exact values. The variation of stresses ($\bar{\sigma}_x$, $\bar{\sigma}_y$, $\bar{\tau}_{xy}$ and $\bar{\tau}_{xz}$) of (0/90) cross-ply laminated plates through thickness is shown in Figs. 5 to 8 using different models.

Example 2: A simply supported two-layer antisymmetric cross ply (0°/90°) square laminate under uniformly distributed load is considered in this example. Layers are of equal thickness and made up of Material 1. Table 2 shows the numerical results of deflection and stresses for the (0°/90°) laminated plate. From Table 2 it is seen that the deflection and stresses predicted by present formulation and the methods of Reddy, Sayyad and Ghugal (2014a) as well as the exact solution of Pagano (1970) are in excellent agreement with each other whereas CPT underestimates the results of deflection and stresses compared to those of present theory and HSDT. In addition, it can be seen, that FSDT underestimates also the axial stresses for all slenderness ratios as compared to the results of other theories.

Table 2 Comparison of transverse displacement and stresses for simply supported two-layer (0/90) square laminated plate subjected to uniformly distributed load

a/h	Theory	Model	$\bar{w}$ ($z = 0$)	$\bar{\sigma}_x$ ($z = -h/2$)	$\bar{\sigma}_y$ ($z = -h/2$)	$\bar{\tau}_{xy}$ ($z = -h/2$)	$\bar{\tau}_{xz}$ ($z = 0$)	$\bar{\tau}_{yz}$ ($z = 0$)
4	Present	TSDT	3.0006	1.2687	0.1401	0.1073	0.5893	0.5893
	Ref(a)	TSDT	2.9983	1.2603	0.1394	0.1104	0.5966	0.5966
	Reddy	HSDT	3.0706	1.2691	0.1314	0.1070	0.6034	0.6034
	Mindlin	FSDT	3.0082	1.0636	0.1258	0.0992	0.4775	0.4775
	Kirchhoff	CPT	1.6955	1.0763	0.1269	0.0934	---	---
	Pagano	Elasticity	3.1580	1.1840	0.1590	---	0.647	0.591
10	Present	TSDT	1.9079	1.1089	0.1310	0.0962	0.6488	0.6488
	Ref(a)	TSDT	1.9070	1.1057	0.1307	0.0978	0.6669	0.6669
	Reddy	HSDT	1.9173	0.1049	0.1274	0.0977	0.6591	0.6591
	Mindlin	FSDT	1.9050	0.0533	0.1265	0.0961	0.4849	0.4849
	Kirchhoff	CPT	1.6955	0.0763	0.1269	0.0934	---	---
	Pagano	Elasticity	1.9320	0.0860	0.1300	---	0.702	0.744

(a) Results taken from reference Sayyad and Ghugal (2014a)

Table 3 Comparison of transverse displacement and stresses for simply supported two-layer (0/90) square laminated plate subjected to linearly varying load

a/h	Theory	Model	$\bar{w}$ ($z = 0$)	$\bar{\sigma}_x$ ($z = -h/2$)	$\bar{\sigma}_y$ ($z = -h/2$)	$\bar{\tau}_{xy}$ ($z = -h/2$)	$\bar{\tau}_{xz}$ ($z = 0$)	$\bar{\tau}_{yz}$ ($z = 0$)
4	Present	TSDT	1.5003	0.6343	0.0700	0.0536	0.2947	0.2947
	Ref(a)	TSDT	1.4992	0.6301	0.0697	0.0552	0.2983	0.2983
	Reddy	HSDT	1.5353	0.6345	0.0657	0.0535	0.3017	0.3017
	Mindlin	FSDT	1.5041	0.5318	0.0629	0.0496	0.2387	0.2387
	Kirchhoff	CPT	0.8478	0.5381	0.0635	0.0467	---	---
	Pagano	Elasticity	1.5790	0.5920	0.0795	---	0.3235	0.3235
10	Present	TSDT	0.9540	0.5545	0.0655	0.0481	0.3244	0.3244
	Ref(a)	TSDT	0.9535	0.5524	0.0653	0.0489	0.3334	0.3334
	Reddy	HSDT	0.9587	0.5524	0.0637	0.0488	0.3295	0.3295
	Mindlin	FSDT	0.9525	0.5267	0.0632	0.0480	0.2424	0.2424
	Kirchhoff	CPT	0.8478	0.5381	0.0635	0.0467	---	---
	Pagano	Elasticity	0.9660	0.35430	0.0650	---	0.3510	0.3510

(a) Results taken from reference Sayyad and Ghugal (2014a)

Example 3: A simply supported two-layer antisymmetric cross ply (0°/90°) square laminate under linearly varying load is studied in this example. Comparison of deflection and stresses for the (0°/90°) laminated plate is demonstrated in Table 3. Material set 1 is utilized. The deflection

Table 4 Comparison of transverse displacement and stresses for simply supported four-layer (0/90/0/90) square laminated plate subjected to single sine load

a/h	Theory	Model	$\bar{w}$ $(z=0)$	$\bar{\sigma}_x$ $(z=-h/2)$	$\bar{\sigma}_y$ $(z=-h/2)$	$\bar{\tau}_{xy}$ $(z=-h/2)$	$\bar{\tau}_{xz}$ $(z=0)$
	Present	TSDT	1.5827	0.4057	0.0351	0.1398	0.1398
4	Ref(*)	SSNDT	1.5827	0.4057	0.0351	0.1398	0.1398
	Zenkour (2007)	Exact	1.9581	0.6146	0.0457	0.2325	0.2410
	Present	TSDT	0.6847	0.4531	0.0266	0.1433	0.1433
10	Ref(*)	SSNDT	0.6847	0.4531	0.0266	0.1433	0.1433
	Zenkour (2007)	Exact	0.7624	0.4942	0.0292	0.2713	0.2714
	Present	TSDT	0.5512	0.4598	0.0254	0.1439	0.1439
20	Ref(*)	SSNDT	0.5512	0.4598	0.0254	0.1439	0.1439
	Zenkour (2007)	Exact	0.5717	0.4706	0.0260	0.2781	0.2781
	Present	TSDT	0.5136	0.4617	0.0251	0.1440	0.1440
50	Ref(*)	SSNDT	0.5136	0.4617	0.0251	0.1440	0.1440
	Zenkour (2007)	Exact	0.5169	0.4636	0.0251	0.2800	0.2800
	Present	TSDT	0.5083	0.4620	0.0250	0.1440	0.1440
100	Ref(*)	SSNDT	0.5083	0.4636	0.0552	0.1440	0.1440
	Zenkour (2007)	Exact	0.5091	0.4626	0.0250	0.2803	0.2803

(a) Results taken from reference Sayyad and Ghugal (2014a)

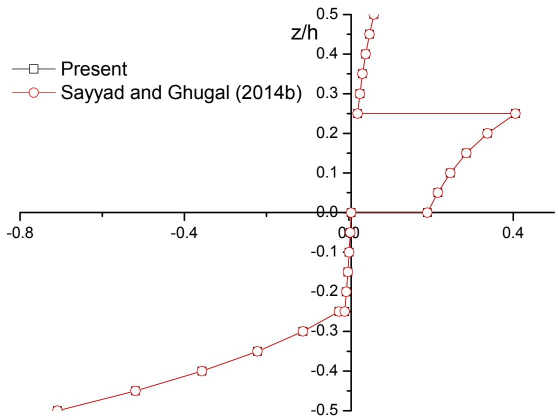

Fig. 9 Through thickness distribution of the axial normal stress $\bar{\sigma}_x$ of (0/90/0/90) cross-ply laminated plate under sinusoidal loading for $h/a = 0.25$

and stresses predicted by present method are in close agreement with Reddy's theory and the solution of Sayyad and Ghugal (2014a) whereas FSDT and CPT underestimate the same for all slenderness ratios.

Example 4: A simply supported four-layer antisymmetric cross ply (0°/90°/0°/90°) square laminate under sinusoidal transverse load is investigated in this example for various slenderness

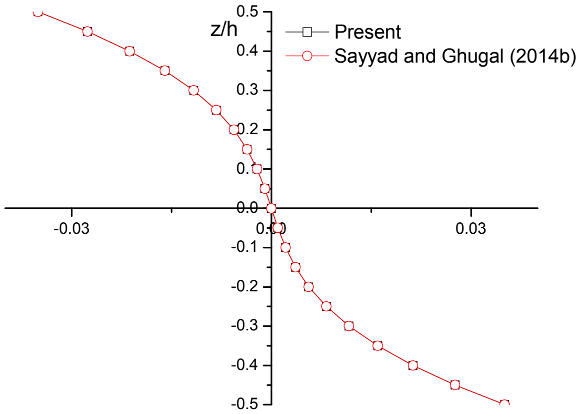

Fig. 10 Through thickness distribution of the shear stress $\bar{\tau}_{xy}$ of (0/90/0/90) cross-ply laminated plate under sinusoidal loading for h/a = 0.25

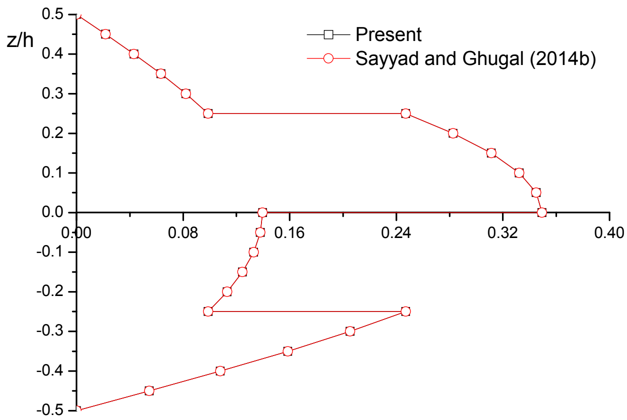

Fig. 11 Through thickness distribution of the transverse shear stress $\bar{\tau}_{zx}$ of (0/90/0/90) cross-ply laminated plate under sinusoidal loading for h/a = 0.25

ratios. Comparison of deflection and stresses for the (0°/90°/0°/90°) laminated plate is shown in Table 4. Material set 1 is utilized. From Table 4 it is observed that the deflection and stresses predicted by present theory and the method proposed by Sayyad and Ghugal (2014b) are in excellent agreement with each other. The results are also compared with those of the exact result obtained by Zenkour (2007). The variation of stresses ($\bar{\sigma}_x$, $\bar{\tau}_{xy}$ and $\bar{\tau}_{xz}$) of (0°/90°/0°/90°) cross-ply laminated plates through thickness is shown in Figs. 9 to 11 using both the present theory and the model proposed by Sayyad and Ghugal (2014b).

Example 5: A simply supported two-layer antisymmetric angle-ply (45°/-45°) laminated plate under sinusoidal transverse load is examined in this example. Material set 2 is employed. The numerical results of non-dimensional transverse displacement for the square and rectangular plates are given in Table 5. In the case of thick plates, there is a significant difference between the results predicted by utilizing the various models and the values indicated by Ren (1990). The small

Table 5 Comparison of transverse displacement for simply supported two-layer (45°/-45°) square and rectangular laminated plate subjected to single sine load

a/h	Source	$\overline{w}$	
		Square plate ($a = b$)	Rectangular plate ($b = 3a$)
4	Present	0.9766	3.0278
	Ren (1990)	1.4471	3.9653
	HSDT	1.0203	3.1560
	FSDT	1.1576	3.3814
10	Present	0.5508	2.2173
	Ren (1990)	0.6427	2.3953
	HSDT	0.5581	2.2439
	FSDT	0.5773	2.2784
100	Present	0.4643	2.0593
	Ren (1990)	0.4685	2.0686
	HSDT	0.4676	2.0671
	FSDT	0.4678	2.0674
	CPT	0.4667	2.0653

Table 6 Effect of thickness stretching on non-dimensional transverse displacements for simply supported two-layer (30°/-30°) square and rectangular laminated plate subjected to single sine load

a/h	Source	$\overline{w}$			
		Square plate (a/h)		Rectangular plate ($b = 3a$)	
		$\varepsilon_z \neq 0$	$\varepsilon_z = 0$	$\varepsilon_z \neq 0$	$\varepsilon_z = 0$
4	Present	1.0172	1.0432	2.3085	2.3386
10	Present	0.5807	0.5864	1.4754	1.4818
100	Present	0.4924	0.4966	1.3112	1.3153

difference observed between the results predicted by the present theory and HSDT is due to the effect of thickness stretching which is omitted this latter (HSDT).

Example 6: The effect of thickness stretching on non-dimensional transverse displacements of a simply supported two-layer antisymmetric angle-ply (30°/-30°) laminated plate under sinusoidal transverse load is performed in this example. Material set 2 is employed. The numerical results for the square and rectangular plates are listed in Table 6. It can be seen again that the results with the thickness stretching effect ($\varepsilon_z \neq 0$) are lower than those without it ($\varepsilon_z = 0$) and especially for thick plates. It confirms again that this influence is considerable and should be considered in investigation of thick plates.

5. Conclusions

This work presents a bending analysis for antisymmetric laminated composite plates by employing a simple quasi-3D trigonometric theory subjected to various loading conditions. The governing equations are obtained by utilizing the principle of virtual works. Results demonstrate that the present theory is able to produce more accurate results than the FSDT and other HSDTs with higher number of unknowns.

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