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Application of mesh-free smoothed particle hydrodynamics (SPH) for study of soil behavior

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Abstract. The finite element method (FEM), discrete element method (DEM), and Discontinuous deformation analysis (DDA) are among the standard numerical techniques applied in computational geo-mechanics. However, in some cases there no possibility for modelling by traditional finite analytical techniques or other mesh-based techniques. The solution presented in the current study as a completely Lagrangian and mesh-free technique is smoothed particle hydrodynamics (SPH). This method was basically applied for simulation of fluid flow by dividing the fluid into several particles. However, several researchers attempted to simulate soil-water interaction, landslides, and failure of soil by SPH method. In fact, this method is able to deal with behavior and interaction of different states of materials (liquid and solid) and multiphase soil models and their large deformations. Soil indicates different behaviors when interacting with water, structure, instrumentations, or different layers. Thus, study into these interactions using the mesh based grids has been facilitated by mesh-less SPH technique in this work. It has been revealed that the fast development, computational sophistication, and emerge of mesh-less particle modeling techniques offer solutions for problems which are not modeled by the traditional mesh-based techniques. Also it has been found that the smoothed particle hydrodynamic provides advanced techniques for simulation of soil materials as compared to the current traditional numerical methods. Besides, findings indicate that the advantages of applying this method are its high power, simplicity of concept, relative simplicity in combination of modern physics, and particularly its potential in study of large deformations and failures.

Keywords: SPH (smoothed particle hydrodynamics); mesh-free methods; numerical modelling; soil; interaction; large deformation; failure

1. Introduction

By now, discrete element method (DEM), proposed by Cundall and Struck in 1971, has been the most popular mesh-free numerical method in geotechnical engineering (Cundall and Struck 1979) . This method facilitated the simulation of granular materials, and the main advantage of this method is that it can study large deformations and failure problems and involves a relatively easy

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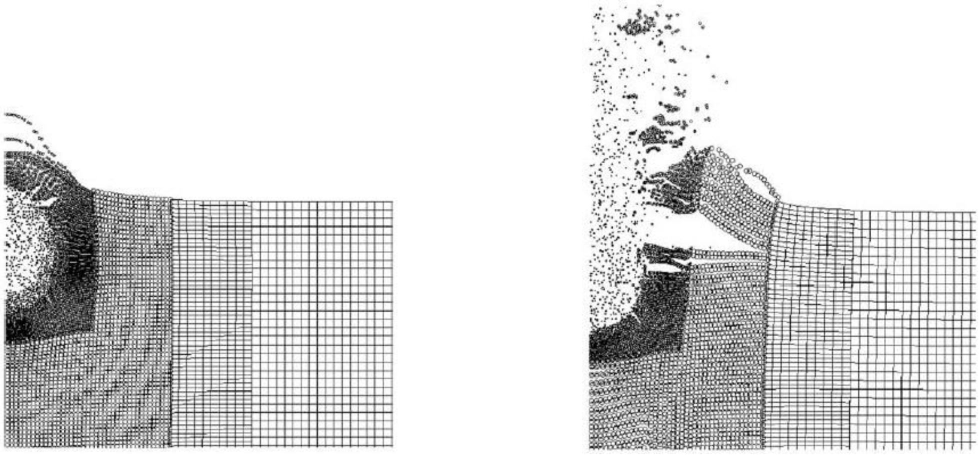
and simple implementation in a computer code. However, DEM cannot be applied in the advanced constitutive soil models. Discontinuous deformation analysis (DDA) method, proposed by Shi in 1988, is another numerical model with a wide range of applications in geotechnical engineering; which is majorly used for rock engineering, and stress displacement problems. One of the limitations of DDA is that the stress and strain within a block of a model do not change at all. This approximation is the major problem of this method while dealing with significant stress variations within a block. To deal with limitations of traditional numerical modelling, mesh-free methods such as mesh-free Galerkin element method (EFG) (Belytschko *et al*. 1994), material point method (MPM) (Sulsky *et al*. 1994), particle in cell method (PIC) (Harlow *et al*. 1964), mesh-free Local Petrov-Galerkin method (Atluri and Zhu 1998), the finite point method (Oñate and Idelsohn 1998), etc. can be applied for simulation of large soil deformations and its interactions. For instance, Murakami *et al*. (2005) applied EFG for simulation of water-coupled problems; Beuth *et al*. (2011) developed MPM method for simulation of pseudo-static simulation of slope failure considering soil-structure interaction; and Cuomo *et al*. (2013) applied FEM coupled Lagrangian Integration Points (FEMLIP) for study of failure mechanism of vertical excavation. Despite their advantages, these methods need a very complicated time-consuming computer code and larger amount of computation source as compared to FEM, as they simultaneously need interpolation points and a background mesh for integration of governing equations. As a more general method, SPH was proposed by Lucy (1977) and Gingold and Monaghan (1977) for practical applications of astrophysics. In fact, this method has been basically developed for simulation of fluid flows by dividing the fluid into several distinct particles. SPH is a completely Lagrangian meshless approach. The computation domain of SPH is indicated by the particles. Typically, the mass of particles is a constant parameter in SPH. Each particle contain all information related to field variables such as compression, pressure, velocity, etc. and moves with a particle velocity in the Lagrangian framework. Therefore, the particles themselves can be considered as geometric net of computation domains. Recently, application of mesh-free methods prevail in studying of soil behavior. The most significant of these methods is disappearance of mesh dependence problems. It is worth to be noted that the first versions of SPH formulations lacked tensile stabilities, and consistency. However, in recent years many alterations have improved the accuracy of the simulation. The main aim of the current article is investigation into recent applications of SPH for solving problems dealing with large deformations, soil flow, soil-structure interaction, etc.

2. Simulation of soil structure by means of SPH in collaboration with FEM

Among all researches which have been conducted into various applications of SPH, researches which applied both SPH, and FEM are among the most applicable ones. Wang *et al*. (2004) introduced a coupled complete numerical method for simulating response of underground structure undergoing the explosion loading. Coupling with FEM, SPH can cope with many problems recognized by other coupled techniques. Through this method, the large displacement and deformation in soil environment is modeled for the near-field load by SPH mesh, while FEM mesh is modeled for far-field and mean-field soil environments and RC structure. This combination of FEM and SPH has two distinct advantages for representation of various physical phenomena. As well as its computational significance, the proposed hybrid method can be applied for enhancing the reliability of simulation results using different models, particularly for three-phase soil and RHT concrete model. The numerical example shows that the proposed method is capable



(a) t = 2.44 ms (b) t = 16.3 ms

Fig. 1 Computed formation of crater in soil

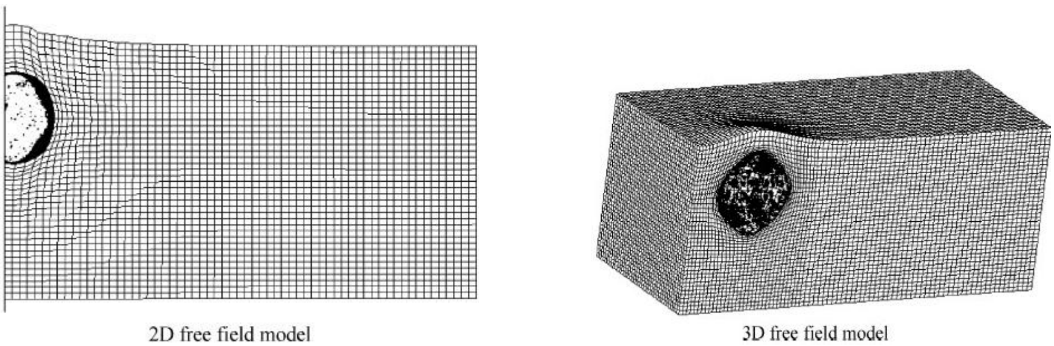


Fig. 2 Computed craters in soil

realistic reconstruction of physical phenomena in a constantly numerical method. The proposed numerical model has been validated with empirical predictions and indicated appropriate results. Fig. 1 presents computed formation of crater in soil at different times.

Lu *et al*. (2005) presented a complete coupled numerical model for response simulation of an underground buried structure by an emphasis on 2D and 3D modeling. Under medium explosive charge conditions, soil and RC structure are placed in a single modelling system. This model is an efficient combination of SPH and FEM in which SPH is utilized for its higher potentials for study of large deformations, while FEM is used for modelling the buried structure and soil area with relatively low deformations. The computational efficiency and accuracy of 2D model was studied in comparison to 3D model and it was shown that 2D provides more satisfactory predictions for propagation of explosion wave in soil environment. The results of this work also revealed that the 2D model can provide reasonably accurate outputs for crater size, blast loading on the structure, and critical response in the front wall. Fig. 2 is a comparison between 2D and 3D states for computation of craters in soil. The residual response of the structure indicates a considerable difference for 2D and 3D models. Using the simulation results, the properties of shock environment in the structure were also discussed in terms of shock response spectrum.

Xu and Liu (2008) proposed a numerical model using a hybrid SPH-FEM method for analysis of structure response to explosive charges. This model covers some key issues such as SPH coupling with FEM, selection of appropriate constitutive model, and simulation of explosive charges. The analytical work of this research was conducted for two numerical models, one for free ground explosion and another for structure response under explosive charges, for modelling all modelling steps from compressional wave propagation to structure response. Based on the performed simulations, it can be concluded that the model can be applied for the reasonably accurate explosion of structure.

3. Application of SPH in simulation of soil and water interaction

Bui *et al*. (2007) developed a SPH model for simulating the mutual water-soil behavior. In this work, water was modeled as a viscose fluid with low compressibility, while soil was considered as an ideal elasto-plastic material. The Mohr-Coulomb failure criterion was used for stress description for a soil unit in plastic flow regime. In this work, the dry soil was modelled using a fuzzy model, while the saturated soil was separately modelled by water/soil phase. The mutual effect of soil/water was taken into account in the computations using the water pore pressure and seepage force. Simulation of soil boring process was conducted using a water jet as a challenging example for studying application of SPH technique. In this regard, borings were performed in two different soil samples: dry soil and fully saturated soil. The performed simulations can be observed in Figs. 3 and 4. Effect of pore water pressure and seepage force can be properly represented by SPH simulation. The numerical results obtained by this article show that the sharp discontinuity of the soil failure can be simulated without any problem. Although the numerical results were not quantitatively compared with laboratory data due to the difficulty of the test, the calculations are stable and the obtained results are acceptable throughout the paper. The results are very promising; however, it is required to improve structural modeling of the soil, specifically saturated soil.

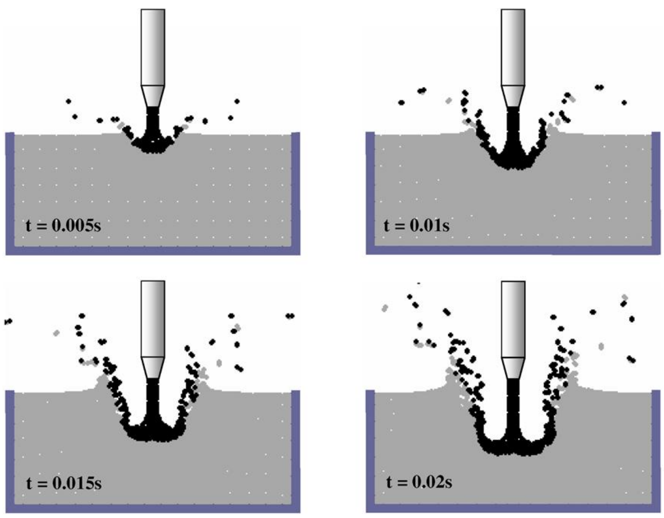


Fig. 3 Erosion during excavation by a water jet on dry soil model by SPH simulation. Water jet has speed of vjet = 25 m/s

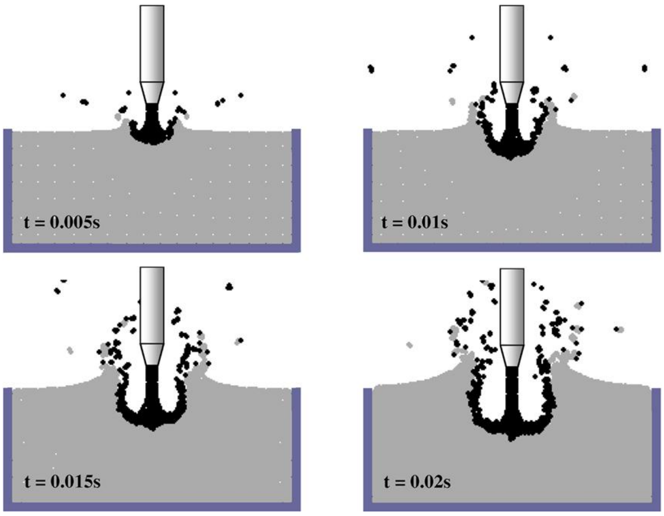


Fig. 4 Erosion during excavation by a water jet on saturated soil model by SPH simulation. Water jet has speed of vjet = 25 m/s

Bui *et al*. (2008b) concluded that the stress-strain SPH model is a powerful and reliable method in elasto-plastic state for simulating the progressive slope failure induced by quick rise of water level after storm and heavy rainfall events. In this system, slope stability is studied by analyzing the in-situ measurements, empirical models, and results of numerical simulations. Figs. 5 to 8 indicate simulation results. To update the numerical models for support systems of the slope failure, it is required to run a numerical model which can tolerate the maximum soil displacement after slope failure, where the probability of multi-stage slope failure induced by heavy rainfalls is probable. Furthermore, using this method, it is possible to prediction safety factor of the slope and maximum post-failure displacement.

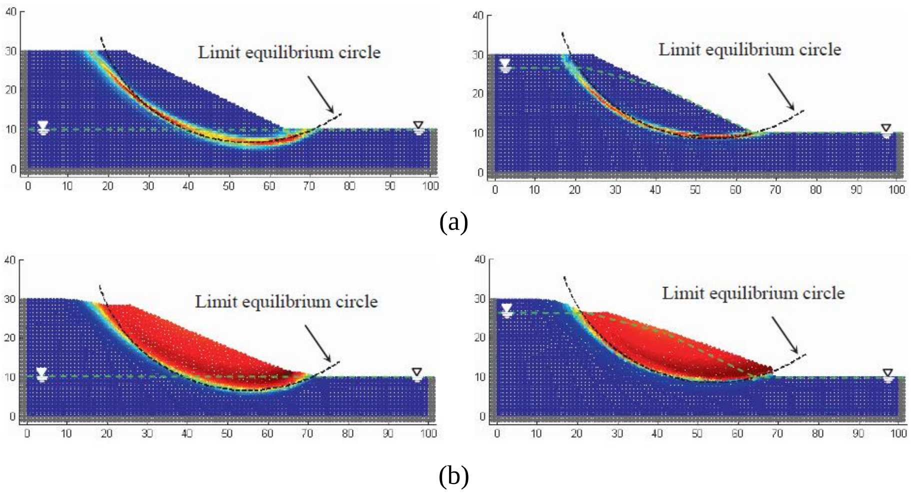


Fig. 5 Slope failure simulation by SPH: (a) Contour plot of accumulated plastic shear strain; (b) Contour plot of total displacement at the final deformation. Color is auto adaptive

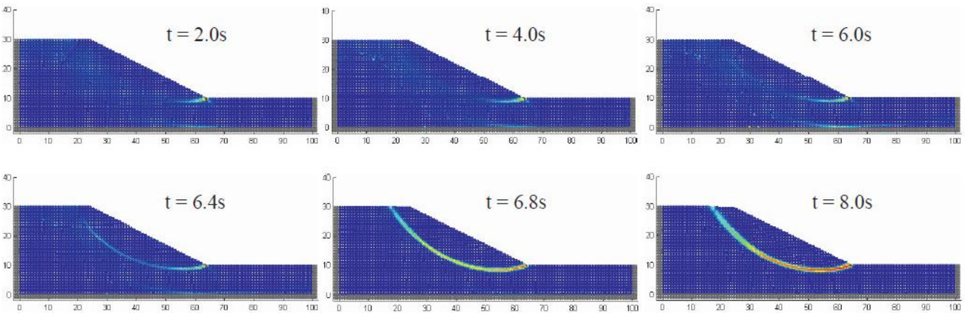


Fig. 6 Formation of critical slip surface during the step loading of water table level

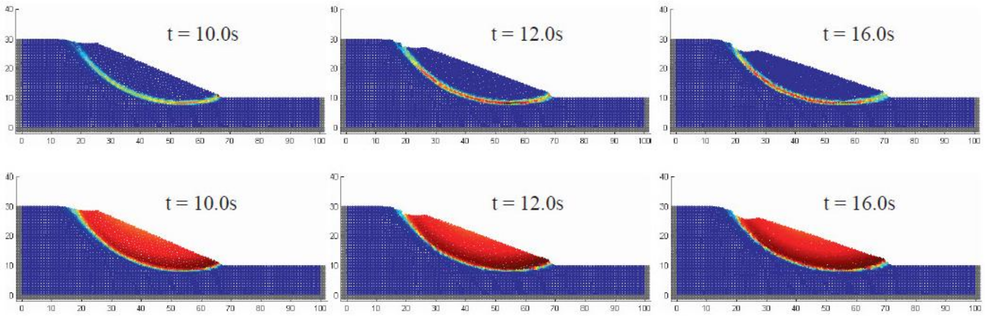


Fig. 7 Failure mechanism of the slope due to the rapid increase in water table via contour plots of plastic shear strain (upper images) and total displacement (lower images)

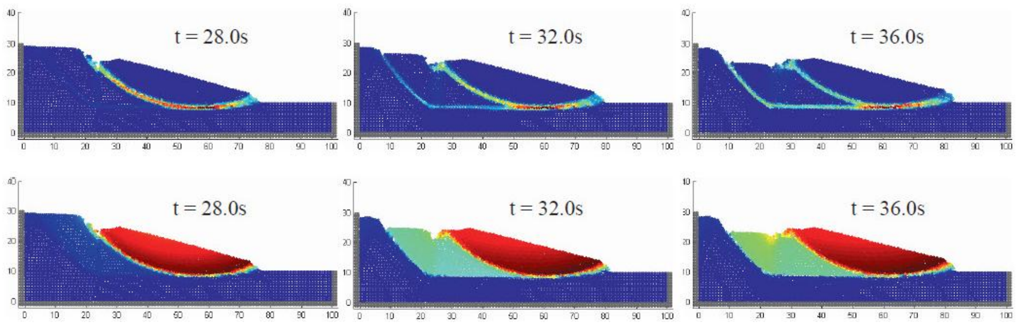


Fig. 8 Multi-stage failure process (second stage failure) of the slope via contour plots of plastic shear strain (upper images) and total displacement (lower images)

Pastor *et al*. (2008b) proposed a landslide modeling method for catastrophic landslide which can create impact waves in reservoirs, fjords and other water bodies. The analytical stages of these work consists of: (a) landslide triggering; (b) landslide propagation; (c) wave introduction to water (near-field); and (d) far-field propagation. Then, using the integrated model in displacements and pore water pressure and a proper generalized plasticity constitutive model, soil behavior was described. The propagation is analyzed with a depth-integrated model combined with fluidized soil rheology. In the next step, the interaction between soil and water was modelled using a set of level s*et al*gorithms which maintains the track of relationships among solid, air, and water phases. Wave propagation along the fjords or reservoirs, which is typically modeled using the depth-integrated model presents an acceptable agreement between cost and accuracy. These settings must be able to accommodate the excess propagation over dry terrains and flooding. In the case of reaching to a barrier, model upgrade to a 3D state would be inevitable. In addition to shortcomings of each sub-model, transfer of data from one model to another is an important problem in modelling process, as during the model selection it should be careful that the “upstream” model is able to present all information required by the downstream model. In this work, also the results of a depth-integrated model for a double layer soil was presented. All models described in this paper has their own advantages and shortcomings and there remains yet much work conducted in these modelling phases.

Lenaerts and Dutré (2009) showed that how it is possible to simulate sand fluidized model in a SPH framework. Furthermore, using the porous particles for sand body, the fluids and granolas can be considered as completely integrated. Sand and fluid simulation as a continuous body enables the animations involving greater values of sand interacting with fluid as compared to the previous

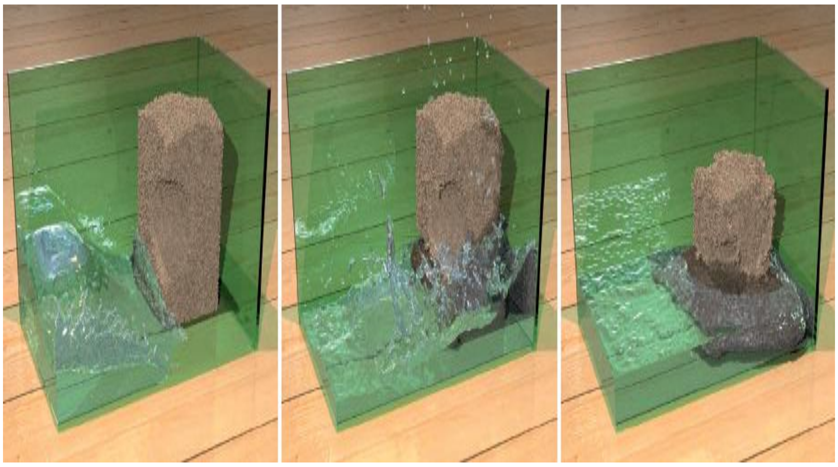


Fig. 9 A moist sand column stands rigidly until a column of water is released. The water percolates through the sand and erodes pieces of the structure

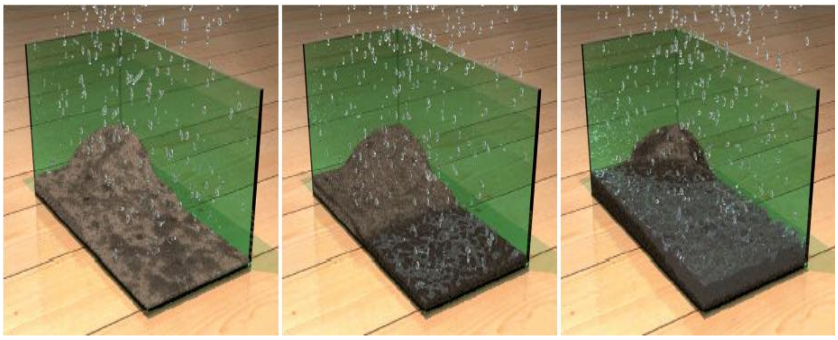
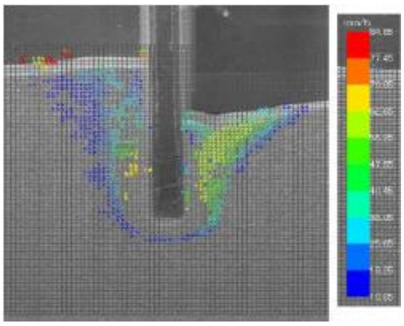
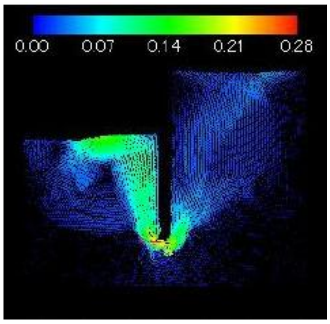


Fig. 10 A rain shower on dry soil. As the soil becomes saturated a layer of water forms on top and the soil turns to mud



(a) Model test (unit: mm/hr)



(b) Simulation (unit: m/s)

Fig. 11 Comparison in velocity distributions around sheetpile without air bubbles: (a) PIV image of ground velocity for model test; (b) water velocity for SPH simulation

methods (Figs. 9 and 10). The conducted method enables effects such as fluid in sand percolation, rigid granular structures, and soil deformation to mud.

Sakai *et al*. (2009) studied evolution of air bubbles in pore water developed by percolation-induced failure, regarding the fat that air bubbles enhance soil failure and erosion risk. They also carried out the required tests for modelling, developed a new numerical simulation method for computing the flowage deformation and interactions of solid-water-air bubbles using the SPH method, and then validate simulation results using the empirical data. Analysis of PIV image illustrated that dynamics of air bubbles in the terrain results in its failure. Although performance analysis was carried out qualitatively, little verification exists for some of its data. Fig. 11 compares SPH simulation and PIV image. In this paper, the validity, utility, and application of SPH was clearly shown in 3D problems for prediction of flood events. Besides, the developed preventive methods of the catastrophic events were presented in their paper. For the future works, this method might be developed for failure simulation of percolation induced by bubbles for entropy and enthalpy problems.

Ulrich *et al*. (2011) analyzed the SPH model for water/soil interaction computations in multiphase soil by coarsening the dynamic particles. This procedure is assigned for hydrodynamic harbor and ocean engineering problems by an emphasis on sediment scouring. This type of simulation is typically applied for large-scale computations, fluid-soil interactions, and irregular geometries which results in generation of a large number of particles. To have reasonable solution time even for the full-scale simulations, the strategies effective on enhancing the computation performances are required. A dynamic particle refinement/ coarsening strategy is presented based on the varying mass of the particles mass and their distances and the particle properties were updated according to their present location. The classic preservation equations of SPH are completed by the source conditions to ensure density and momentum preservation along the transport from a given particle to another, while the particle mass and volume are not necessarily conserved. To multiphase samples studied through this research indicated that simulation using by coarsening is consistent for unified mass distribution. The test materials show that the basic flow phenomena are reproduced by coarsening simulation process even for strongly connective flows. The further works in this work are about the assessment of limitations of this method in gradient steepness of mass to work. Moreover, this method is also applied for 3D simulations.

Using the SPH, Dakssa and Harahap[22] predicted velocity of landslides in runoff release along a saturated slope, such as debris flow and avalanches. Runoff occurs whenever the rainfall intensity in higher than infiltration rate of water in soil slope. Rain water triggering the landslide has an accurate balance of infiltration and runoff, as accumulation of water pore pressure due to water infiltration generally results in soil slope instability. Although generation of negative pore water pressure helps soil slope stability, creation of positive pore water pressure interrupts its stability. Such conditions mainly result in soil slope failure within or after heavy or long-term rainfall events, particularly in tropical or semi-tropical areas. The governing equation utilized in this research is Navier-Stokes (NS) equation. As each particle is within the computational range, the important parameters of the flow such as velocity and pressure can be easily computed. The boundary conditions can be considered either as permeable (in porous environment) or impermeable (in hard strata/rock) conditions. Thus, in the present work, the upslope of the soil is considered as impermeable and a barrier to water infiltration. The SPH codes were developed in FORTRAN for running the developed simulations. Specific reports are available for simulation process; for instance, the real-time imaging shows that SPH design is able to record important aspects of runoff process.

Using the SPH simulation, Stefanova *et al*. (2012) presented soil/water interaction by the single-phase soil model using the Mohr-Coulomb failure criterion and suspension approach for the boundary layer between liquid and granular materials. The mesh free methods such as SPH are capable of dealing with behavior and interaction of different materials (liquid and solid) as well as the large deformations and are efficient for multiphase soil modelling. Effect of the flow induced by the lowered gravity base foundation (GBF) was investigated for offshore wind turbines at soil surface to propose the maximum lowering speed. This approach, however, does not take into account compactness, pore ratio, as well as other soil properties which affect stress and strain computations of the soil using a proper constitutive equation. The recent case can still be studied as a research subject. Moreover, the jet grouting was simulated using the SPH method to predict the developed grout zone versus the eroded zone. Despite the soil behavior problems, SPH has several advantages over the mesh-based methods which should be carefully studied. The ability of this method for studying the large deformations as well as the gradient velocity and density are highly important for simulation process. For the future works, it is recommended to apply a soil constitutive model and the multiphase model since they allow more sophisticated analyses and full exploitation of SPH.

Bui and Fukagawa (2013) studied SPH method for modeling the problems in submerged or saturated soils. This work indicated that modelling these soils involves high errors, if the pore pressure gradient of the water is obtained using the standard SPH formula. For the saturated soil problems, which are common in computational geomechanics, two phase system is applied instead of the single phase system, where the interaction between soil and water is incorporated in the modelling by addition of pore water pressure to the effective stress. This approach was applied in this work to model the saturated soil problem using the SPH method. In this regard, the SPH approximation for pore water pressure gradients should be done in a way exactly similar to that of effective stress state. It was shown that the standard SPH formulation should be improved for studying the problems of submerged soil. Throughout this work, by studying the numerical errors induced by standard SPH formula, a new SPH formula, which works in both dry and saturated soil conditions, was proposed. Furthermore, it was shown that the new formulation can automatically satisfy the dynamic boundary conditions in the submerged soil/water interface. Therefore, it can be applied for a wide range of geotechnical problem, particularly for problems where a complicated submerged soil surface exists. As an application of new SPH formula, effect of dilation angel on failure mechanism of two-side inclined embankment was studied in this work and the failure surface was compared with that of traditional limit equilibrium (modified Bishop) method. The limit equilibrium predicts a failure surface more or less similar to that of SPH for soil with high dilation, while it does not meet the required ability to estimate the failure surface for soil with low dilation. The slip area and failure zone is decreased by an increase in dilatation angle.

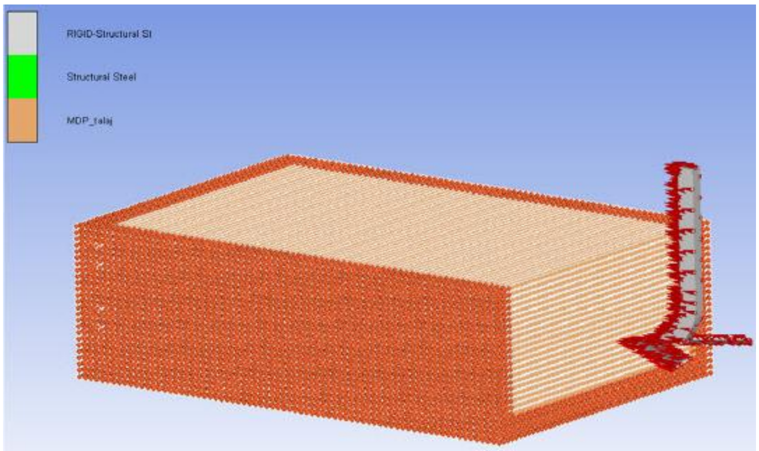


Fig. 12 Cultivator tool simulation model

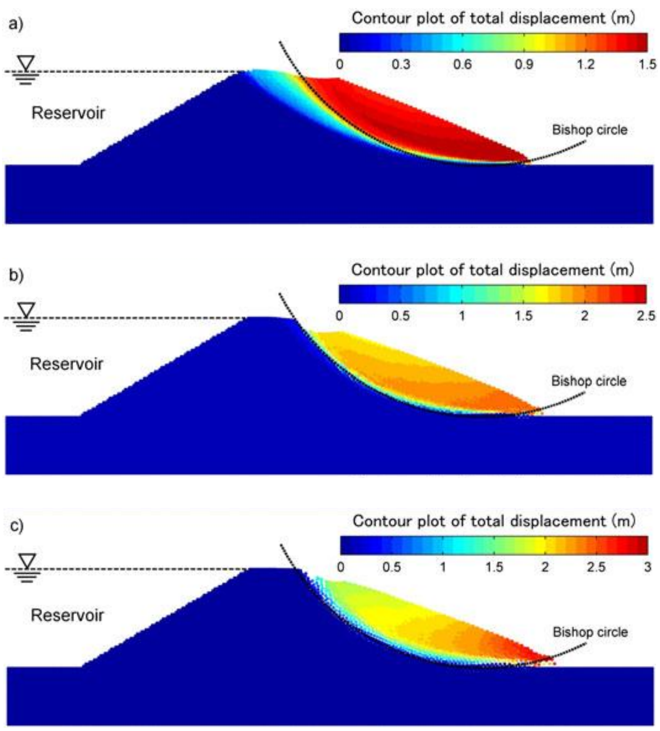


Fig. 13 Final deformation pattern of the slope after failure: (a) non-associated flow rule, $\boldsymbol{\psi} = 0$; (b) non-associated flow rule, $\psi = 0.5\boldsymbol{\varphi}$; and (c) associated flow rule, $\psi = \boldsymbol{\varphi}$

Fig. 28 shows final deformation pattern of the slope after failure using different values of dilatancy angle (Ψ). One of the major advantages of SPH method as compared to the traditional method is its compatibility for simulation of discrete soil failure, and consequently, simulation of the entire slope failure process. To increase applicability of SPH for post-failure simulation behavior, it is better to apply a critical case simulation since it allows a more realistic improvement. Furthermore, to improve physical realism and application range of the method, the coupling behavior of soil and water should be considered. The proposed SPH formula would be valid mostly for such cases. Finally, the proposed formula can be considered as a basic relation for further SPH advancements of the saturated soils.

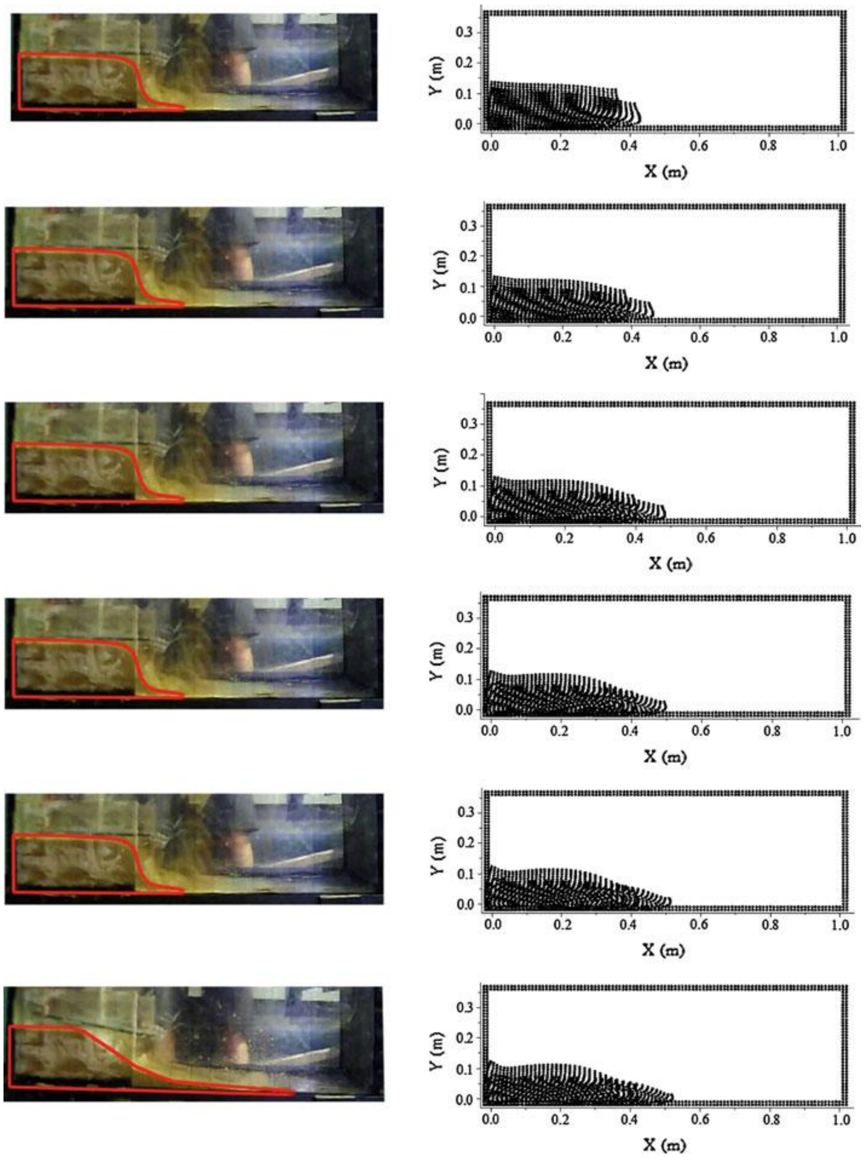


Fig. 14 Model test configurations and SPH simulated configurations

Dakssa and Harahap (2013) studied applications of SPH, mesh-free numerical methods for runoff trigger along a soil slope in terms of fast-moving landslides such as debris flows and avalanche. The fast movement of water along the soil slop creates a severe hydraulic scour as well as intense washing of protective coating of the slope and result in its instability. The results show that Navier-Stokes equation can be effectively applied for simulation of runoff along the soil slope. SPH is able to study water induced deformation, which is not an easy task in FEM and FEM considering the mesh distortion and other computational problems. The important flow parameters such as velocity and pressure can be calculated in by considering each particle in its given computational range. In this regard, the boundary can be considered as either permeable (for instance in the porous media) or impermeable (for instance in hard layer/rock). Therefore, the soil in the upstream slope was considered as impermeable, as if it can prevent rainfall infiltration. To run the proposed simulation, the source SPH code of the flow was written in FORTRAN. The results and specific reports of this simulation indicate that the SPH design can effectively image the important aspects of runoff. Comparing the results obtained using the standard hydraulic equation of open Manning channels with those of SPH reveals that SPH predicts a more reasonable velocity for flow. It must be noted that model validation through the tests is a necessary step.

To deal with shortcomings of the traditional analyses in liquefied soils, Huang *et al*. (2013) investigated coupled SPH model in water-soil system to analyze flow in the liquefied soils. Through the proposed SPH, water and soil are simulated as different layers in a way that the permeability, porosity, and the mutual forces can be mixed to model porous environment saturated with water. The simple shear test was simulated using the SPH method by designing an elastic model for studying its application in solid phase materials. Next, application of the proposed SPH model was proved for simulation of the interactive forces between soil and water through the falling-head permeability test. The developed coupled SPH method indicated appropriate simulation for both simple shear and falling-head permeability tests. Using a fit-for-purpose experimental apparatus, the physical model flow model tests were designed for liquefying sand. To complete physical tests, the numerical simulation was conducted based on coupled water-soil SPH. The numerical results are in good agreement with physical model results in the observed configuration and velocity vectors.

Fig. 32. The failure of an embankment in north of Sweden was used to extend the coupled soil-water SPH method. The site investigations show that the coupled SPH method simulated the embankment failure efficiently. Estimations of the horizontal displacement and velocity can also be applied for further improvement of the seismic safety of the structures.

Zhang and Maeda (2014) proposed their test model and applied it for infiltration mechanism and failure rate of slope and levee under artificial rainfall considering the coupling between soil, air, water, and air. A new SPH was proposed in this work for numerical simulations. The test model was simulated using the proposed SPH model. For different rainfall intensities were considered for two base layers (undrained and drained conditions) for model test. For the proposed SPH model, water pressure was computed using the state equation. For the studied soil, the behavior was explained using the constitutive model for the unsaturated soil in which saturation degree is considered as state parameter. Using the SPH method, friction force was introduced for simulation of interaction force between different phases. SPH simulation of infiltration process and failure mechanism is compared with the observed results from model test and it was revealed that the proposed SPH model can be useful tool for stability analysis of slope and levee under heavy rainfall conditions.

4. Simulation of geo-materials failure by SPH method

Bui *et al*. (2008a) studied large deformations and simulations of geomaterials failure in a SPH framework for the problem of soil and structure interaction. The results of this work show that the sharp post-failure discontinuity of soil can be properly simulated by SPH. Moreover, development shear band along the failure was appropriately presented through the intervals of accumulated plastic strain in this work. The Drucker-Prager model with plastic flow law irrelevant to SPH code was applied to explain elastoplastic behavior of the soil. Besides, the solid structure was simulated as an ideal elastic-plastic material using the Von-Mises criterion. The interaction between soil and solid structure was modeled using the coupling conditions related to Lenard-Jones distraction force between two phases. Afterward, this model was applied to simulate slope failure and slope failure with pile support.

Failure and displacement processes are presented in Figs. 9 and 10, respectively. The numerical results revealed that gross discontinuities failure of the geomaterials can be properly simulated by SPH, the soil-structure algorithm in the SPH framework can effectively work, and bending mechanism of the reinforced pile and stress distribution in the solid structure are appropriately simulated by SPH. The obtained numerical results throughout their work are qualitatively correct. This paper suggests that SPH can be effectively applied in geomechanical models of soil-structure interaction.

As the numerical modelling can play a key role in understanding and prediction of the complicated failure processes and provides useful inputs for a resistance design to failure, Das and

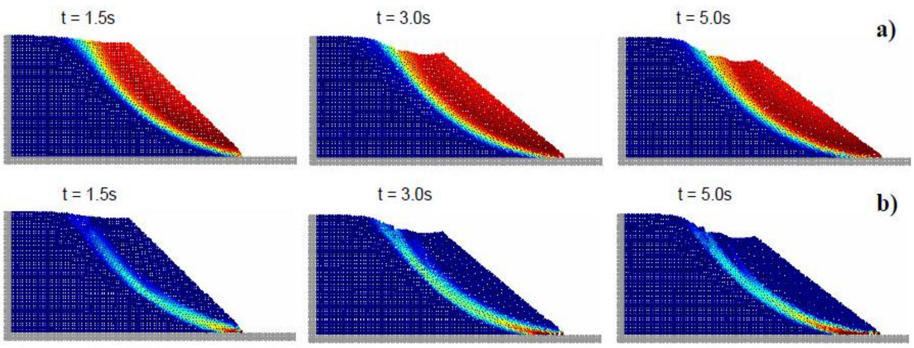


Fig. 15 Process of slope failure: (a) Auto adapted contour plot of total displacement; (b) Auto adapted contour plot of accumulated deviatoric plastic strain

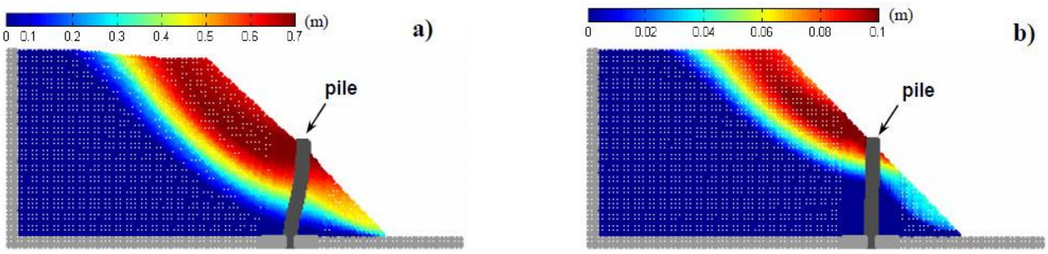


Fig. 16 Contour plot of total displacement of the slope with reinforcing pile.
(a) Ep = 0.15 GPa; (b) Ep = 15GPa

Cleary (2013) applied the mesh-free SPH method for modelling the gravity dams under fluctuating dynamic earthquake loads. The structure response of Koyna Dam was analyzed for the case that base of the dam is subject to the high intensity periodic ground excitations. The modified form of the Grady–Kipp continuum damage model can effectively predict growth of dynamic crack propagation at full-scale for the concrete dam structure. The prediction of SPH method from the crack initiation and propagation pattern was found to be in agreement with FEM results and the laboratory outputs of the physical models. Understanding the key mechanisms of the failure and the results of fragmentation processes in gravity dams provides a ground for constant improvement of the design characteristics required for creation of structures resistant to failure. SPH seems to be an effective approach which can be applied in dam design and provide the information related to the important failure sites, failure patterns, and post-failure dynamic response of the dams. Some designs can be applied quickly to detect the optimum configuration of the dam structure under different loading conditions. The dynamic response and prediction of failure patterns reveals the high potentials of SPH for modeling the failure in dam and similar large structures. Fig. 30 depicts a fracture pattern for FEM and SPH, while Fig. 31 compares results of laboratory fracture pattern and SPH simulation.

Huang and Dai (2013) presented an overall view of SPH applications for solving a wide range of large deformation and failure problems known as "geo-disasters" such as dam break, slope

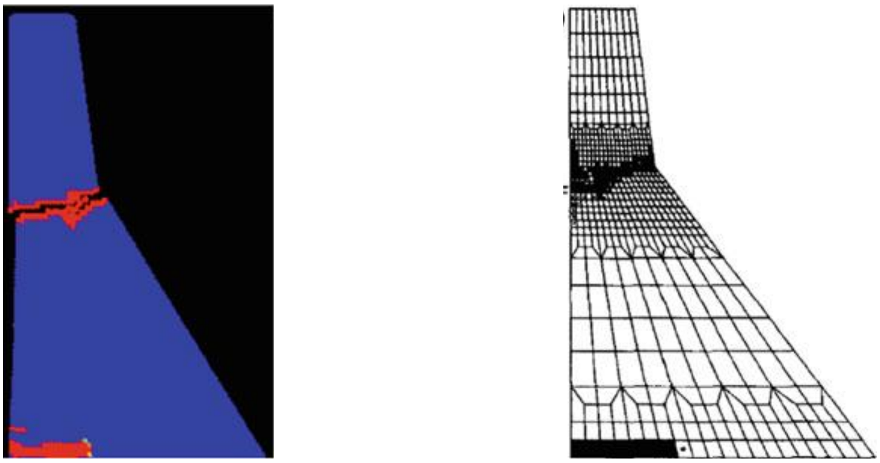


Fig. 17 Comparison of the: (a) SPH predicted fracture pattern with; (b) FEM

Fig. 18 Comparison of the SPH predicted fracture pattern with the experimental failure characteristics of a physical model of the dam

failure, soil liquefaction, seepage damage, dynamic erosion, underground explosion, and rock failure. However, the most important and successful application of SPH might be in dam break. The other practical applications (e.g., soil liquefaction and underground explosion) must be applied cautiously until conducting further research on them. It is proved that the SPH involves considerable advantages over the numerical methods based on the traditional meshing system when dealing with geo-disasters concerning the remarkably large deformations, free surface, interface movement, and deformable boundaries. The coupled SPH-FEM model is widely applied for geo-disasters because of its promising outputs which show an improvement in computational output and expansion of SPH method. Some Eulerian method such as VOF and LS are incorporated in SPH. Moreover, some advanced SPH methods such as incompressible SPH, refined SPH, and discrete SPH enhance stability and accuracy of the typical SPH. Despite appropriate performance of this method in many cases, SPH still suffers from some shortcomings; i.e.: (1) inaccuracy induced by particle coverage near boundaries; (2) elastic instability during study of problems with material strength; and (3) energy-zero modes, when their field variables and their derivatives are computed in one location. Furthermore, many of simulations performed in the previous works are based on a 2D symmetric modelling which might result in inaccuracy or even error. Here, a 3D analysis is required for more realistic simulations. Nevertheless, it must be noted that SPH is a compacted computational numerical methods, and since 3D simulation required a large number of particles from the entire SPH zone, this method needs large computer memory and longer computational time. The simple computational algorithm is a preferred method for improving the computational yield, as it provides parallel computation options and can save the time needed by the computer. These options are new technical methods which facilitate using the multiple computational sources in a simultaneous manner. To fully understand these processes, many mechanical phenomena such as crack initiation and propagation are required to be interpreted in a wide range of scales. The future investigates for coupled molecular-surface simulations of the phenomena existing in the molecular dynamic and expanding a wide range of multiple-scale analyses in geotechnical engineering problems need SPH method.

5. Simulation of soil-material interactions

Fredj and Dinovitzer (2010a, b) developed application and verification of 3D continuous modeling techniques for assessing the performance of pipeline systems subjected to the massive soil displacements. Understanding the interactive impact of soil-pipeline during the large ground displacements plays a key role for pipeline design. Both laboratory study and computational analyses are important in soil-pipeline investigations. A hybrid framework of continuous soil mechanics and advanced finite element methods (such as ALE and SPH) was proposed for modelling the pipeline-soil interaction. Theses developed numerical models can be applied for prediction of wrinkling and post-formation behavior of pipeline considering the confinement impact of the applied soil. The main objective of this work is to develop a comprehensive wrinkle integrity assessment process in the form of two papers. Paper one develops a 3D continuous model using the MM-ALE (Multi-material Arbitrary Eulerian Lagrangian) and SPH methods based on LS-DYNA and considers a wide range of soils and soil displacement states. The obtained results were compared with laboratory data obtained from large scale tests for assessment of the numerical analytical methods, and a good agreement was achieved. The SPH and ALE methods applied for the interaction of pipeline-soil interaction indicated simulation accuracy of the observed phenomena in terms of pipeline response and soil deformation (Figs. 16 to 19).

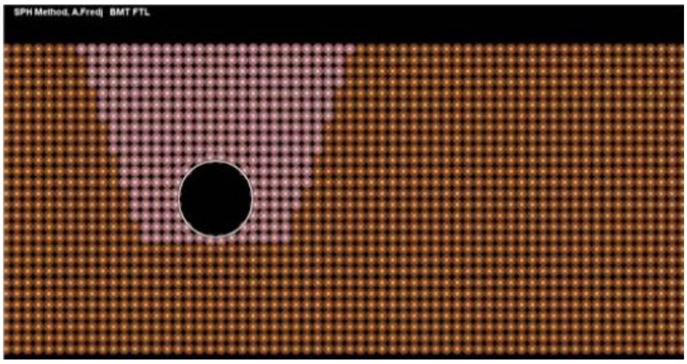


Fig. 19 Illustration of the SPH FE model including the pipe

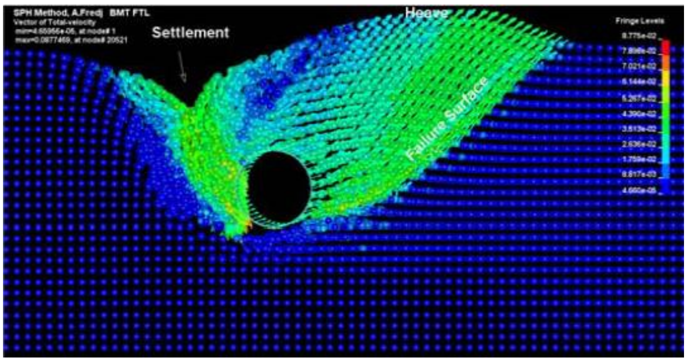


Fig. 20 SPH-FEM simulation-vector velocity

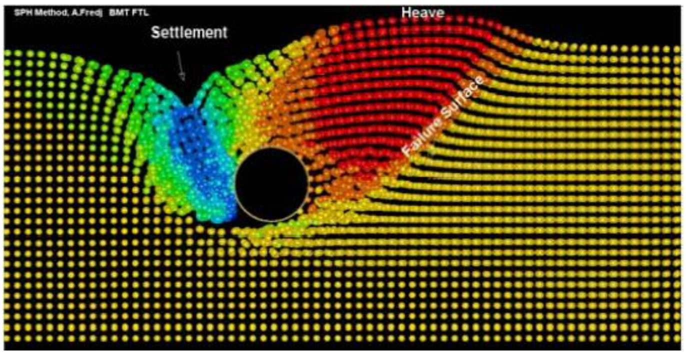


Fig. 21 SPH-FEM simulation-vertical displacement

The second paper discusses soil restrain impact on response of pipeline-soil, e.g., pipeline wrinkle and buckle and strain demand. The results were compared with the outputs of FE model for pipeline wrinkle in the air. It was found that soil effects restraining displacement of the pipeline and increases amplitude of wrinkle, in-service deformation, and consequently the fatigue damage. It is worth to mention that the results applied for this paper are from the same pipeline material and geometry, soil material, and loading system.

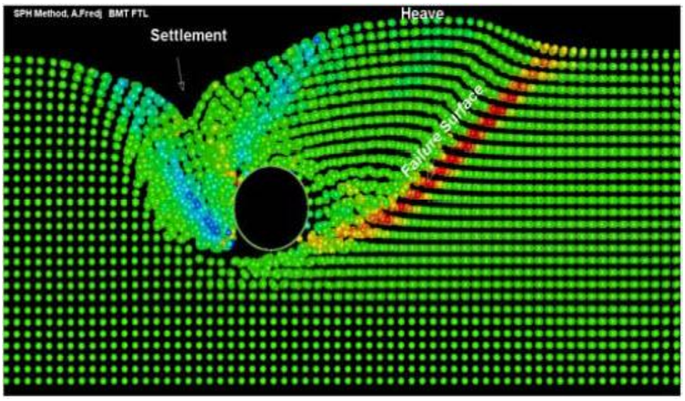
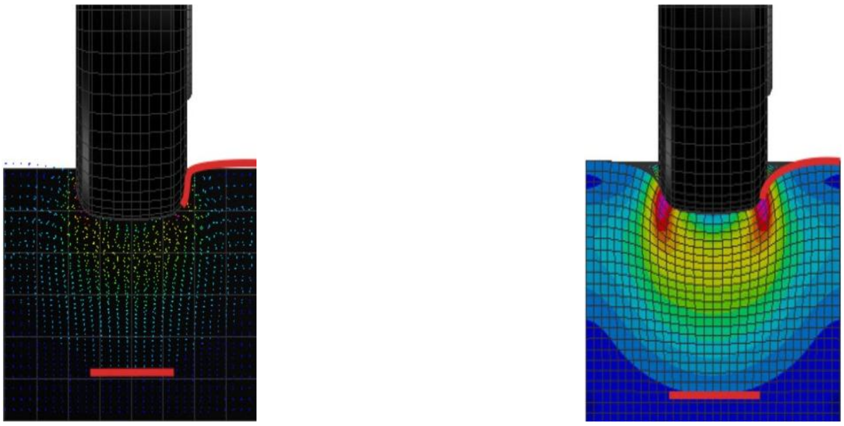


Fig. 22 SPH-FEM simulation: Shear strain with failure surface

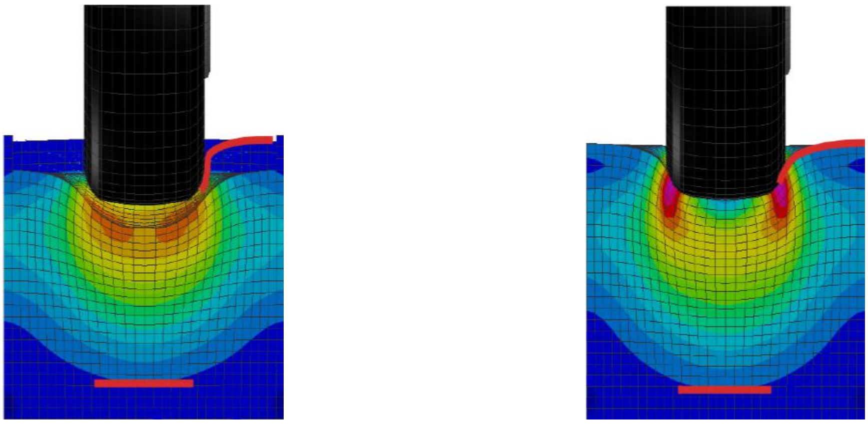


(a) (b)

Fig. 23 Deformation of (a) SPH; and (b) FEA soil plots

However, the work presented in these papers cannot cover a large number of important factors for soil dynamic under pipeline loading effect such as trench design (e.g., fill versus native soil material and trench geometry), soil displacement direction along the pipeline, diversity of mechanical soil properties, soil moisture and pore water pressure, and pipeline characteristics (D/t, material properties, etc.). Application of this model can be extended for prediction of post-wrinkle deformation and measuring the wrinkled pipeline performance. The information of the applied load the pipeline can be used for assessing the wrinkling behavior by taking into account the effect of filling soil and embankment depth in long-term integrity of the wrinkled pipeline (e.g., growth or cyclic deformation).

Lescoe (Lescoe 2010, Lescoe *et al*. 2010) developed and contrasted new models for soil-tire interaction. They applied the rigid tire model for an extensive sensitivity study. The model was previously studied using the finite element analysis (FEA) for soft soil (dense sand), mesh size change, soil particle size, and boundary limitations. Furthermore, SPH parameters are determined for complete or partial replacement of EFA in the soil model. Through the extensive vertical displacement test, it was found that the optimum agreement between the results and calculation



(a) (b)

Fig. 24 Deformation of (a) combined SPH/FEA; and (b) FEA soil plots

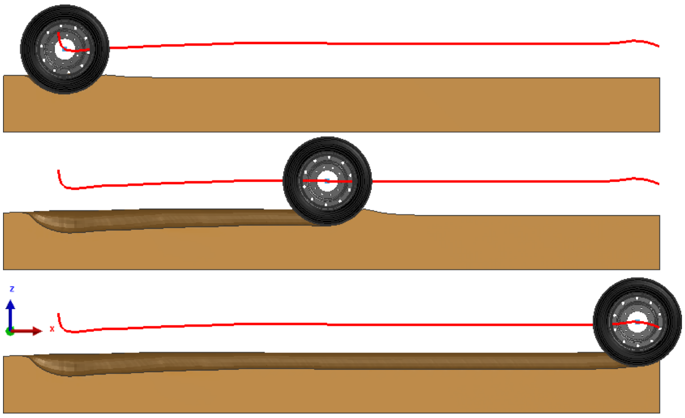


Fig. 25 Cutaway of the FEA soil at the beginning, middle, and end of the simulation with the trajectory of the center point shown in red

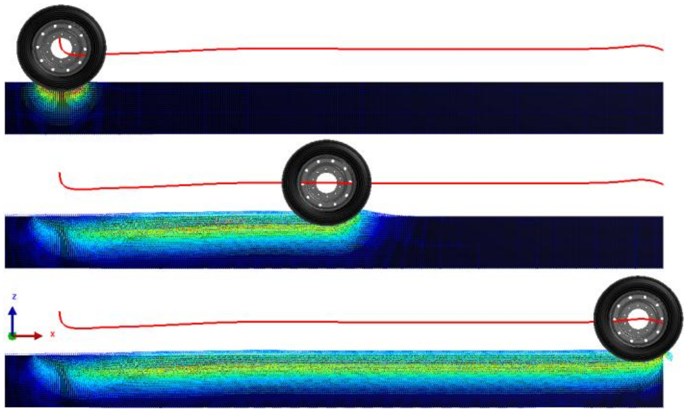


Fig. 26 Cutaway of the SPH soil shown at the beginning, middle, and end of the simulation with the trajectory of the center point shown in red

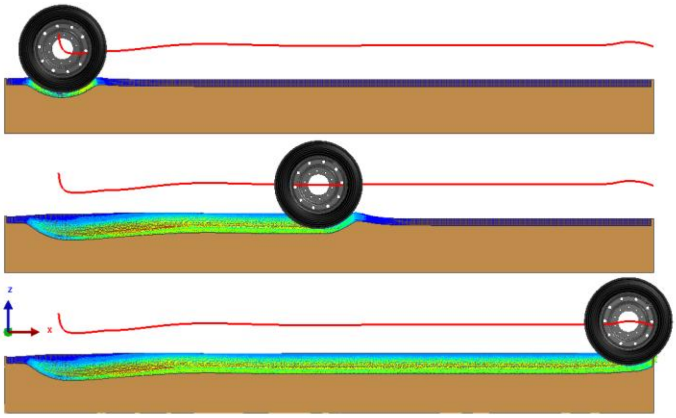


Fig. 27 Cutaway of the combined FEA/SPH soil shown at the beginning, middle, and end of the simulation with the trajectory of the center point shown in red

time can be obtained by the smaller and main element of the soil or letting the edges freely move in *Z*-direction (similar to vertical movement). Besides, the vertical displacement using the SPH particles show that SPH can be used for a material resistance model and, then, applied completely or partially instead of FEA elements in model of applied soil. Then, the rolling resistance test was carried out to study the differences between FEA, SPH, and FEA/SPH soil models. Substituting the FEA elements with SPH elements as a separating variable revealed that the deeper amount of SPH particles leads to the increased rolling resistance; while, the increased density of SPH particles have a small impact on rolling resistance. The physical tests are required to carry out to study the minimum soil modeling in the ideal sate and tire-soil interaction physics. Figs. 20 to 24 illustrate a comparison among FEA, SPH, and SPH/FEA simulations.

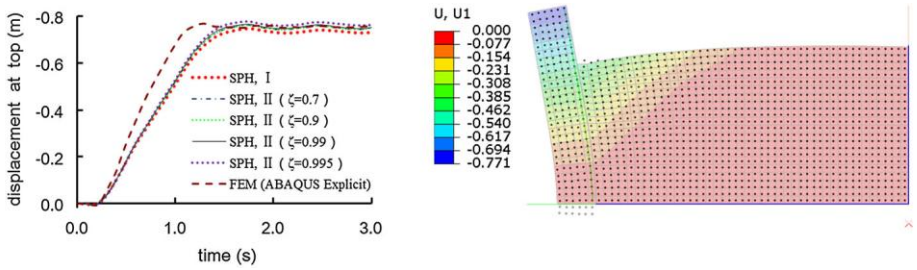
Fredj and Dinovitzer (2012) studied the mechanical behavior of buried pipelines, with respect to the active landslides. The details of pipeline interaction analysis were completed using the continuous 3D SPH. This paper explains LS-DYNA numerical modeling previously improved by the authors and applied for a site-specific problem. To represent performance of modelling process, the transitional slide and validation of the additional numerical model is completed and discussed in this paper. This analysis shows that there is a good agreement between modeling results and laboratory data. The results also show that the reaction between force-displacement for pipeline-soil is highly sensitive to the interface friction of pipeline and soil materials. The higher friction values can also result in the increased strain of the pipeline. Hence, it is highly recommended determining the pipe coating friction coefficient. The friction coefficient is highly controlled by the pipe coating and soil properties. Using the pipe coating with low wrap friction reduces friction and shear transfer from the terrains around the pipe. The results of applying the 3D continuous modelling indicate appropriate presentation validity, so that these results are utilized to develop general trend of pipeline response to slope movements. This paper explains the present and potential advancements in design of simple engineering instruments for assessment of the required bearing capacity or deformation of buried pipeline exposed to different types of slope kinetics. The comparison between the agreed levels of the presented SPH modeling with the empirical data, it was found that this method can be considered as a sustainable engineering instrument. Doing so, as the first step in design of an instrument, it would be possible to consider integrity of pipeline and geotechnical members and study pipeline response to the slope movements and have a better mitigative decision. However, the method proposed in this work cannot study many problems with potential of affecting the soil movement induced by pipeline loading such as the structural parameters including wall diameter to thickness D/T, internal pressure and material grade, sensitivity of the analysis results to variations of soil properties, soil movement mechanism and its amount, the ratio of pipeline burial depth to its diameter (H/D), and the inclination angle of the slope movement. It is suggested to complete the model and improve its shortcomings in order to have a more complete understanding of the effect of these parameters and other also parameters, so that to evaluate their relative importance and value in design and estimation based on the strain values.

Pang *et al*. (2012) studied soil variables to highlight interaction mechanism of pile/soil in pile-driven process. Pile driven-operation process is a very important tool for flood control. The anti-flood spiral pile is a very light flood control facility with high work output and low labor intensity. The constitutive model of the soil was designed using the SPH method. The pile drive process was simulated using the dynamic software. Through tracing the SPH particles, the interaction and analysis of soil particles movement with time and at different depths induced by friction, extrusion, and shear effect of pile was carried out. This work provide an interesting process for design of spiral piles and the reduced secondary damage induced to the dam. Using the SPH particles for design of constitutive model of the soil, it is possible to extract the soil parameters by particle tracing and clarify the interaction mechanism of pile/soil in pile drive process; which in turn covers lack of theoretical analyses and empirical methods.

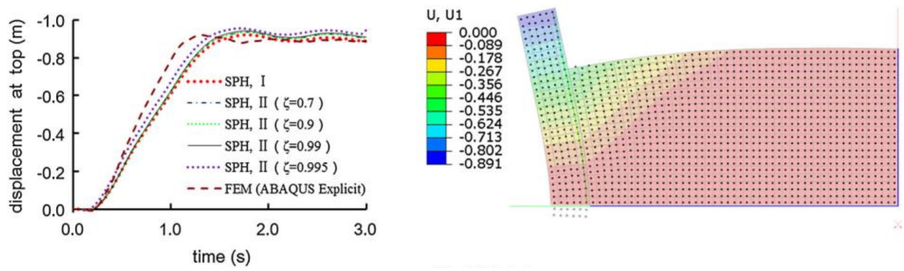
Nguyen *et al*. (2013) proposed a new numerical post-failure model for simulation of modular-block retaining wall system. An advantage of this model is the complete degree of freedom for blocks of retaining wall, which cannot be simulated using the traditional numerical methods such as FEM. In this method, soil is modelled using an elastoplastic constitutive model, while the wall block is considered as completely rigid with complete rotation freedom. The linear contact model was proposed to simulate the interaction between block and soil. The conducted test also confirms validity of the numerical framework. The tests showed that the proposed numerical framework can effectively simulate the modular-block retaining wall system (Fig. 29). The large and post-failure behavior of the geomaterials can also be easily simulated using this model. To extend applications of the numerical methods in geotechnical engineering, the tangential model was generalized in this work.

Bui *et al*. (2013, 2014) predicted large deformations and post-failure behavior of soil and blocks of segmental wall considering the importance of segmental retaining wall (SRW) systems in large geotechnical projects for providing stability of the excavations and slope coating. As the SRWs are flexible, this system can tolerate the partial movement and settlement without any damage or cracking. The proposed numerical framework in this work is a hybrid continuous/discrete approach which can model soil as an elastoplastic material and segments of retaining wall as independent rigid objects associated with both transitional and rotational degree of freedom. The smooth contact model was proposed for simulating the mutual effects of block-soil and block-block. The 2D collapse tests of the reinforced wall were carried out to evaluate the numerical results. It was observed that the proposed method can efficiently simulate the post-failure behavioral simulation of the SRW blocks. A comparison between the proposed method and the tests provides acceptable results, implying that the new method is a promising framework of the numerical modelling for SRW systems. The main advantage of this method is that it can simulate the complete degree of freedom for block displacement in the retaining wall in a continuous framework which was could not be simulated using the traditional methods such as FEM. The large deformations and post-failure behavior of geomaterials can also be efficiently simulated using the present framework. To develop practical applications of the numerical methods, further implementations such as connection with geosythesis reinforcement, seismic modeling of the earthquakes, and the bound between blocks are required to be taken into account. The complete development of 3D codes can have significant benefits for having a wide insight about the governing mechanisms of the segmental retaining wall systems.

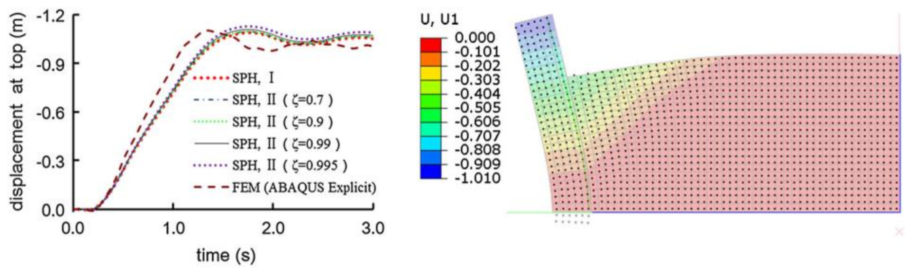
Wang *et al*. (2013a, b) presented a simulation for friction contact between soil and rigid structure and/or deformation in SPH framework. Two algorithms are implemented in SPH code to describe contact behavior. First algorithm (marked with I label) is based on the ideal plastic contact, while the second algorithm (marked with II label) is based on the partial diffusion assumption. Both algorithms properly deal with boundary failure problem induced by SPH, as the particles placed in boundary can have accurate acceleration; which is critical for contact detection. Movement and rotation of the rigid structure is considered in the calculations, so that the process simulation of pile embankment or displacement of a retaining wall is facilitated for geotechnical



(a) $\delta = 30°$, $t = 3$s



(b) $\delta = 15°$, $t = 3$s



$\delta = 0°$, $t = 3$s

Fig. 28 Comparison of the deformation of SPH and FEM engineering analyses. Furthermore, the deformation modeling capability for structure during the frictional contact simulation has expanded practical aspects of SPH. The numerical tests show that algorithm I constantly has stable outputs, while stability of algorithm II is controlled by ζ. The values excessively larger or smaller than ζ can result in fluctuation or even failure of SPH computations. In general, to ensure stability and accuracy, ζ is required to be selected from a range of 0.9 to 0.99. Some practical applications of this algorithm are performed for simulation of frictional contact behavior of sample slide shear, slide friction along a slope, rigid retaining wall, deformable rigid wall, and soil collapse and flow in a completely smooth or completely rough terrain and the obtained results were compared with those of analytical solutions or FEM. Fig. 33. The good agreement between results of SPH and analytical solutions obtained for these applications implies that the presented contact algorithms are powerful and can help expansion of SPH applications for geotechnical problems such as numerical simulation of deep penetration of spudcan foundations, modelling the silent pile jacking, shield tunneling, and excavation and penetration of open and suction caisson. For further development of SPH applications in geomechanics, it is required to extend these contact algorithms to more general positions such as simulation of frictional contact between two soft materials (for simulation of sand pile construction in which the frictional contact occurs with large deformations in different sites) in a simple, efficient, and accurate method.

Wang *et al*. (2008a) analyzed the cause of boundary deficiency in SPH simulation for friction contact. Simulation of the friction contact, which is very common in geotechnical engineering, some physical quantities are discontinuous in both contact interface and cannot meet the convergence needs of interpolation function of SPH integral. Therefore, the main defect of contact surface must be treated as the boundary. Doing so results a boundary deficiency problem in friction contact simulation. To solve this problem, using the mathematical derivation, a method was discussed to modify boundary deficiency of the friction contact in terms of theoretical aspects. In this method, the developed friction contact algorithm is divided into several subdomains based on the computation range and considering the existing contact boundaries. Then, using the contact forces, as the bridges of these subdomains, the problem was solved and the correction coefficient was obtained by comparing the SPH results of contact particles by calculating them using Newton's second law. In the same time, in numerical computation aspects, an optimized value was proposed for the correction coefficient and a thorough research was performed on Kernel function of cubic spline and quantic spline kernel, whit correction coefficient of 0.2 and [2.0, 2.16], respectively. Finally, the proposed numerical model was tested to analyze the proposed method. Using the contact algorithm method and treatment of the hidden boundary deficiency enjoys the high efficiency and accuracy as well as a simple programming process.

6. Simulation of landslides by SPH method

Considering the importance of predicting the pathway, velocity, and depth of flow-like catastrophic landslides, to reduce risk level and doing protective actions, Pastor *et al.* (2008a) proposed a model which involves all connections between pore pressure and solid skeleton inside the avalanching mass. The mathematical depth-integrated model is derived from a velocity-pressure Biot-Zienkiewicz model which is applied in soil dynamic. The depth-integrated model in combination with SPH method provides an attractive modelling framework for flow-like catastrophic landslide. The computation time fir this model is less than that of Eulerian classic

FEM, as the applied network grid is a structured terrain mesh separated for terrain topography description. The model is capable to measure pore water pressure dissipation in the avalanching mass. The validity of the proposed is tests by some criteria. Finally, landslide propagation was simulated for some flow-like catastrophic landslides occurred in May 1998 in Campania in south of Italy. The obtained results highlight compatibility of the model proposed for simulation of propagation state of complicated phenomena and the related roles performed by rheological model for an efficient simulation with suitable deviation distance, velocity, and height of propagated masses. Fig. 11 is exhibits a comparison between analytical and SPH results for different times.

Using the visual-SPH software developed in Visual Basic 6.0 environment analyzed the fluidized displacement based on SPH methods. They, first, verified accuracy and validity of the model by simulating a benchmark problem of dam-break and then analyzed fluidized movement of Tangjiashan landslide using this software (Fig. 25). Simulation of drivers plots of displacement versus time, reproduce of all flow steps for Tangjiashan landslide, and run-out are in complete agreement with observations of flow-like sliding of this landslide. The software developed in this work enables visual analysis simulation of the landslide fluidized movement. Moreover, it can accurately analyze impact force, velocity, and other basic dynamic behaviors during the sliding and can determine the necessary parameters of landslide such as run-out and coverage. Therefore, SPH-visual provides a reliable scientific basis for designing catastrophe-preventive structures and site selection for reconstruction.

Huang *et al*. (2013a) analyzed the run-out of flow-like landslide using the SPH method. In comparison to the traditional methods, the proposed SPH method is a combination of Bingham flow model and Navier-Stokes equations in the framework of computational flow dynamics. The results obtained for simple dam break and the force induced by effect of grain flow simulated using the SPH method were compared with results derived from the reference papers. For both cases, simulation results are in good agreement with laboratory tests and observations, so that evaluating the accuracy of SPH model shows that it can be applied for simulation of deformation in a large soil mass. Fig. 26 shows results of SPH simulation.

The run and analysis of the landslides including Tangjiashan, Wangjiayan, and Donghekou, where their flow-like movement was similar to the one induced by Wenchuan earthquake, was carried out in this work using the practical programs of SPH method for real flow-like landslides. For all three simulation cases, the implemented model indicated a high level of similarity with

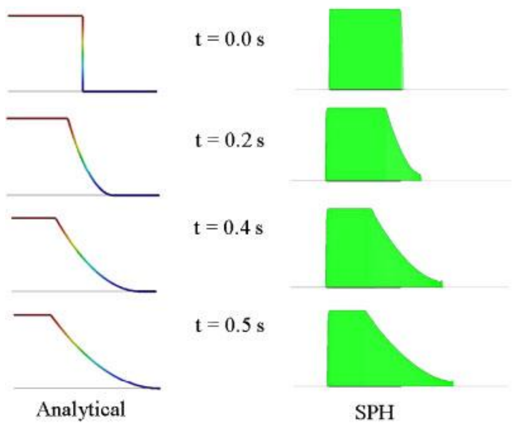
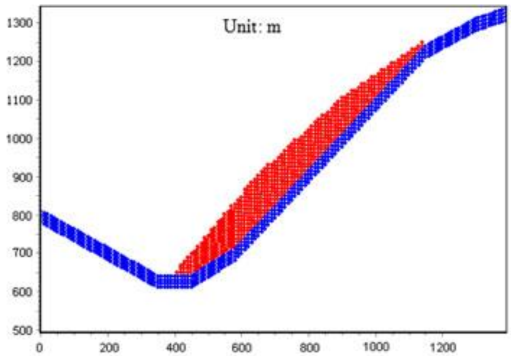
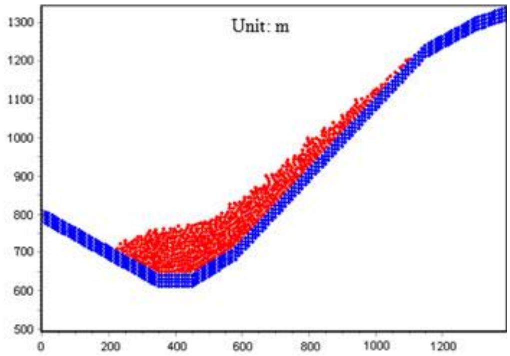


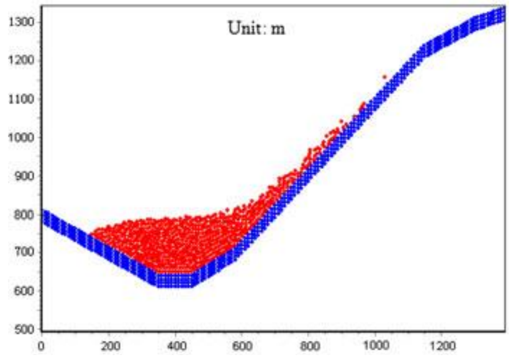
Fig. 29 1D dam break problem: dry bed. Analytical (left) and computed results (right)



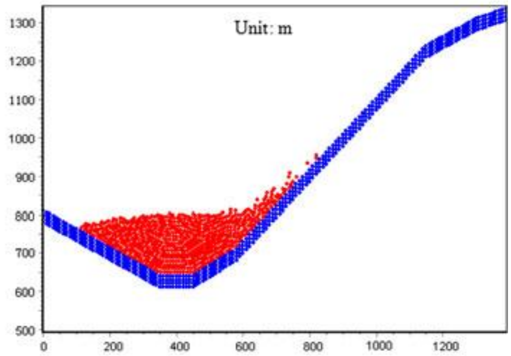
(a) $t = 0$ s



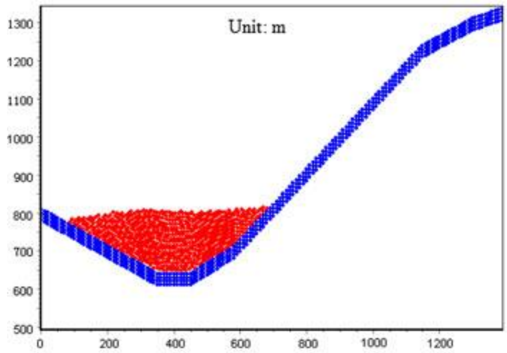
(b) $t = 5$ s



(c) $t = 10$ s



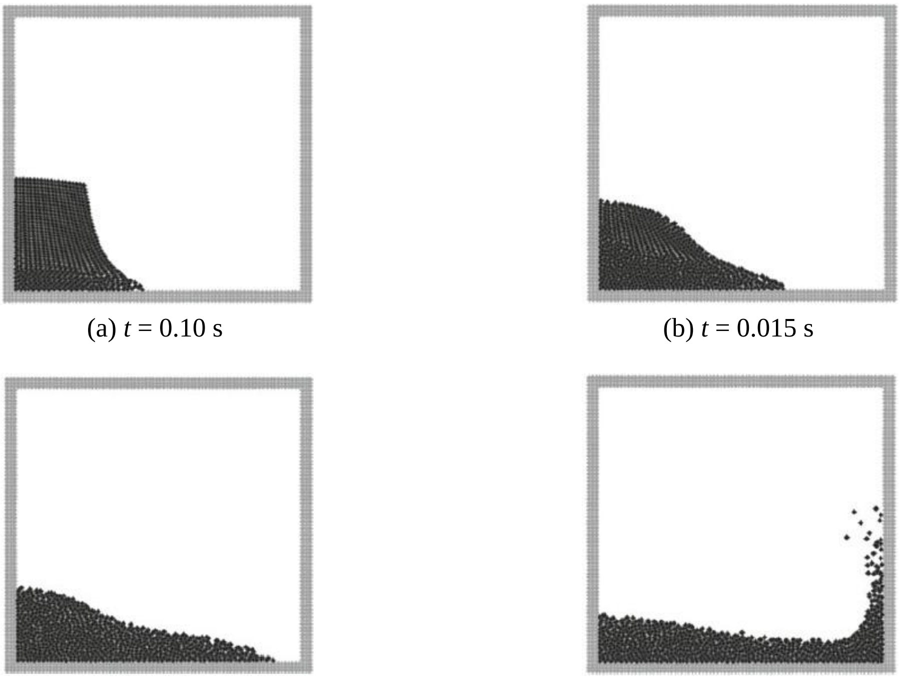
(d) $t = 15$ s



(e) $t = 20$ s

Fig. 30 Simulated run-out process for the Tangjiashan landslide

landslide configurations. Therefore, applying SPH method can be considered as an effective instrument for studying the flow mechanism of landslide. Some index parameters of landslides such as run-out and velocity and other basic dynamic behaviors can be extracted from SPH model. Nevertheless, SPH is rather vague in some aspects. The further works provide further advancements for better simulation of the landslide materials. The results obtained from simulation can allow evolution of geological hazards, selection of suitable sites for post-even reconstruction, and help prevention of the effective damage of landslides in various engineering structures.



(a) t = 0.10 s (b) t = 0.015 s

(c) t = 0.20 s (d) t = 0.30 s

Fig. 31 SPH simulation results

Pastor *et al*. (2013b) presented a scheme of different alternative mathematical and numerical models which can be applied for initiation and progress mechanism of the fast catastrophic landslides and other related problems such as the waves caused by landslides. The mathematical and numerical models are essential tools to predict behaviors of geostructures and their interaction with environment. The fast catastrophic landslide is a special case which cannot be accurately modeled by the conventional methods. Furthermore, many of FEM codes cannot be applied for propagation of the mobilized mass. In this paper, two alternative numerical models were presented for description of triggering and landslide propagation. Based on landslide triggering, a coupled SPH model which prevents intrinsic elastic instability of classic SPH formulation was proposed. The propagation is analyzed using a depth-integrated SPH model which consists of a vertical DEM mesh for providing higher accuracy of pore water pressure. Finally, it was indicated that how the viscoplastic models of Perzyna type can reconstruct rheological behavior of Bingham and friction model.

Pastor *et al*. (2008a) modelled landslide propagation using a depth-integrated soil–pore fluid mixture with a 1D model developed for pressure development in the soil mass. Using this model it is possible to compute variations of height and base level permeation with higher computational accuracy. Many rheological models applied in depth-integrated models are either heuristic (e.g., Voellmy model) ore extracted from the general rheological 3D models. In this paper, another solutions is proposed based on the viscoelasticity Perzyna. This approach perform the numerical modelling based on the SPH method, which is reinforced by addition of a 1D DEM to every SPH node to improve properties of pore water in the avalanching soil. The proposed model has all limitations of the depth-integrated model, as representation of effective stress is not considered in the computations of landslide body and the underlying terrain. The model cannot reconstruct movement mechanism since it applies deferential descriptions for material slides. The presented (mathematical, rheological, and numerical) set of the model suffers from some shortcomings induced by its employed presumptions. The main limitations of this model are: (A) the depth-integration is difficult or even impossible for monitoring the variations with depth, unless special techniques like those presented in this work (1D integration of DEM in all SPH nodes) is applied. The other models such as 3D SPH, MPM, FEMLIP, etc. can provide this information. Nevertheless, the depth-integrated models are useful for long run-out distances. A compromise solution can transfer the information to a 3D model to the vicinity of barriers or sites a more accurate structure of flow is required. (B) The mathematical model is based on Biot–Zienkiewicz u-PW equations and is suitable for problems in which the relative displacement of liquid and solid phases is small.

Sanchez *et al*. (2013) mentioned that depth-integrated model is applied for many flow slides and debris flows in which propagation distance is large. Nevertheless, there is not much information available for landslides with very short propagation distance (with the same order of their length). The main feasibility problem of this method is lack of similar models for triggering and propagation phases. In general, the main problem is using different materials for launching a constitutive equation and presentation of a rheological model. This gap can be bridged using the Perzyna's viscoplasticity model. Furthermore, evolution of pore pressure was accurately modeled using this method. Here, a simple shape function is not enough and for a better approximation (even for FDM along the depth) a better performance is required. The main unreliability of this method is determination of rheological model parameters. Here using the laboratory rheimeters can be a solution to measure size of very small particles. Otherwise, the opposite analysis of the similar cases would be applied. The main advantage of depth-integrated model is its simplicity. Using this method, the complicated problems can be solved in PCs in less than 5 minutes. To select each of finite element, finite body, or SPH methods, we should see that which one is available for analysis. This method studies ground topography based on: (1) a depth-integrated mathematical model consisting of contact between soil structure and pore fluids; (2) rheological models suitable for describing the relation between stress and deformation tensors for fluidized soil; and (3) a mesh-free numerical SPH model with separate computational mesh (and or a set of computation nodes) using the meshes. This method is constitutive and then have high operation speed. After being validated, a depth-integrated SPH model was applied for landslide propagation in two landslide cases occurred in August 1995, in Hong Kong which their information is available. As the computed results are in good agreement with the observations, the proposed method can be applied for other cases.

Very common	Common	Rare	Standard	Availability in commercial structural mechanics software

Cascini *et al*. (2014) modelled SPH run-out of channelized landslides of the flow type. Correct modeling is necessary for the propagation step of risk analysis and management. Besides, the correct selection of mass rheology and variations of pore water pressure and bed entrainment along the landslide axis are two critical factors which are required to be considered in this method. This method can be entirely simulated using a quasi-3D (depth-integrated) coupled SPH model. In this process, such method was applied for a case history in Southern Italy for which a complete set of geotechnical data are available. The numerical results give a satisfactory analysis about the case history and can obviously predict bed entrainment rate, the extent of the erodible areas affected by propagation paths, and deposition heights and rate. Based on the obtained results, the overall perspective about these landslides was discussed considering various modelling assumptions for propagation modelling, and it was revealed that occurrence of probable risk must be investigated in coupled field evidence-mathematical modelling approach.

7. Other specific simulations by SPH method

Dong *et al*. (2006) mesh-free virtual particle SPH for assessing the shear performance of rocks in jointed and mixed area. Tunnel boring machine (TBM) has been very popular over the last decades and is applied for tunnel boring in rocky environments. Creation of an accurate and reliable index for disc cutter performance of TBM is a critical requirement particularly for ground with complicated conditions. SPH is able to simulate rock crushing, fragmentation, chipping, etc. and is free from constraints of traditional FDM and FEM meshing. The modified Drucker-Prager strength model was developed for rock. The nonlinear strength envelope of the model developed based on generally accepted Hoek-Brown rock failure criterion was generalized in this work. The rock-instrument interaction can be extracted using the force-displacement curve. As first stage, the force is almost linearly increasing as displacement increases. By development of Hertzian cone cracks and fragments the force is increased up to the residual value. The boring system is damaged once it is encountered with mixed ground conditions, particularly when boring is performed in rock-soil interface which is a very common event in tunneling industry. The strength difference in tunnel face leads to the difference in boring loads which typically result in shaking and damage to the system. In this study, the rock boring performance was evaluated in terms of rock strength index. The obtained results can be useful for estimation of accessible penetration rate, boring optimization, and overall design of TBM.

Using both FEA and SPH methods, El Gindy *et al*. (2011) conducted a set of tests on simple soil considering it as elastoplastic solid. The pressure sinkage tests were carried out to detect the differences between these two modeling approaches. The results of this test showed that FEA model supports the surface elements by creation of compressive forces through the particle compaction. The SPH soil is typically analyzed using a shear box simulation, as the test cannot be simply simulated using the FEA soil modeling. These real shear tests have shown that the behavior of SPH soil is majorly similar to that of clay against the initial shear and sand in the case of increase shear level induced by vertical loading. Furthermore, both shear-displacement and pressure shrinkage tests revealed that the higher particle density is not a necessary parameter. The SPH soil model applied in this research presents a “proof-of-concept” for the model. It has been shown that the SPH method can be applied for a material resistance model but the SPH soil is also required to be modified. The SPH soil model parameters are majorly defined through its comparison with FEA model, which suffers from many limitations. On the other hand, SPH soil model is required to be parameterized using the simple virtual tests such as pressure-shrinkage and shear-displacement for finding SPH parameters leading to favorable results. Once detecting the favorable material and material parameters, simulation of rolling resistance is required to be conducted along the other real in-plane and out-plane tests such as steering and slip tests. These results can be compared with FEA soil as a reference; however, this analysis is not proper for validation purposes.

Zhong *et al*. (2012) studied the SPH-based Lagrangian methods for forward and reverse-rotational rotary blades using the LS-DYNA software. As previously mentioned, SPH is a mesh-free method, which enables studying high distortions in soil shear problems. For most terrains subject to movement of machineries such as bulldozers or tillage, the work is done by a blade. Therefore, for the agricultural systems, the precise prediction of the forces inserted on the blade is a critical variable which can enhance productivity. The structural parameters of the reverse-rotational rotary blades are studied and optimized using the orthogonal and numerical simulation processes. The results show that the forward and reverse-rotational rotary blades can be independently applied for shear of dense soil with high resistance to various factors. Application of composite rotary tiller is completely feasible for compacted soil deep-tilling with low engine power. Simulation was carried out using the nonlinear analysis code of transitional dynamic FEM in LS-DYNA. The simulation showed that power consumption can be considerable lowered using the optimized structural parameters and phase angle of these two blades. Using specific parallel computations, it is possible to reduce the computational costs. The method offers a theoretical ground for instrument design.

Gao and Jin (2011) developed a virtual prototype of oblique rotary tilling using the SPH method. The digital simulations and in situ tests of the oblique rotary tilling were carried out in this work. The digital simulation results show that the soil is mainly torn to failure in the toe of oblique rotary tilling, so that the tilling can store the energy. A comparison between energy consumption in the digital simulations and indoors test, the validity of the vertical prototype is confirmed. The optimum oblique angle and phase angle were simulated using a rotational speed of 200 rpm and velocity of 0.4 m/s. Using the soil throw simulation in the tilling toe, it was found that the velocity of the thrown soil is not the same, but is done in lean to tangent blade’s direction. This simulation result must be considered, specifically in the cases that the toe has an oblique design. The furrow bottom shape was also obtained using the simulation results, where the height of convex part of furrow is computed as 9.6% of the toe depth. All results indicate that the oblique rotary tilling can be a potential tilling which help energy economization.

Mabssout and Herreros (2012) compared two different time integration designs using a refined SPH model for shock wave propagation in elasto viscoplastic media. They show that it is possible to minimize the intrinsic numerical problems of standard SPH using the stress-velocity formula and a refinement SPH for spatial discretization. This refined formula involves a Lagrangian kernel to prevent numerical instability and refinement of estimation function and its components to prevent boundary shortage problems. Since it is required to consider only the basic boundary conditions, applying the stress-velocity formula can prevent treatment of the natural boundary conditions. To deal with shock propagation problem, the authors of this paper proposed an alternative method: Taylor-SPH (TSPH). This method is composed of a two-step time discretization algorithm using the Taylor expansion and refined SPH for spatial discretization. Both Lagrangian kernel and its gradient are refined to satisfy stability conditions. The main advantages of TSPH, in comparison with Runge–Kutta time integration method, are as follows:

(1) TSPH has higher performance in comparison to Runge–Kutta for numerical damping and dispersion properties when dealing with discrete functions. For the shock wave, trailing oscillations and damping is minimized using the TSPH method.
(2) For localization problems in soft viscoplastic soils, TSPH indicate higher performance for taking the shear band. However, once applying the Runge–Kutta time integration method, the plastic shear strain is not local, unless a very tight configuration is applied for adjustment of the validated particles where TSPH is more competent that Runge–Kutta method.

(3) TSPH indicate appropriate performance even when dealing with viscoplastic shear strain component and requires no special treatment of the source.
(4) TSPH needs few particles for yielding precise results.

Therefore, superiority of TSPH over the Runge-Kutta time integration pattern for SPH is confirmed, as it is shown that TSPH is more stable, stronger, and efficient and needs only few particles to obtain accurate results.

Bojanowski (2014) applied such numerical method to model (soil structure interaction) SSI problems in presence of large soil deformations. The performance of present design for SSI problems is based on assuming linear elastic properties of soil and ignoring the nonlinear geometrical part, and treating the displacements as small. However, there are a large number of problems which cannot be treated with this approach, as the need a more sophisticated method. Simulation using the Lagrangian finite element, element-free Galerkin, SPH, and multi-material arbitrary Lagrangian Eulerian for two previously conducted tests: (1) large-scale steel pad

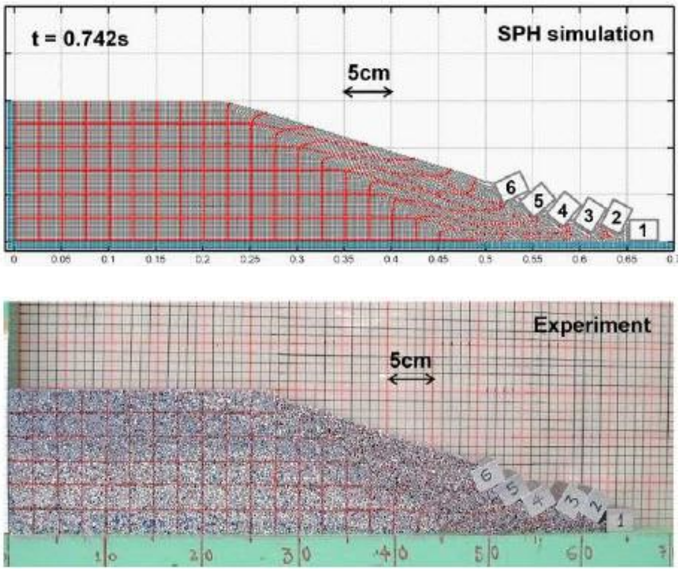


Fig. 32 Comparison between the SPH simulation and the experiment for the final configuration of the MRW system

Table 1 Advantages and disadvantages of Lagrangian EFG, SPH, and ALE methods for SSI simulation

ALE	SPH	EFG	Lagrangian	
High	High	High	Low	Suitability for simulating large deformations
Slow	Very slow	Fast	Very fast	Speed of calculations
Balanced	Low	Moderate	High	Sensitivity to mesh density
No	Yes	Yes	-	Able to merge with Lagrange elements
Moderat (coupling)	High (nodes to surface)	High (nodes to surface)	High (surface to surface)	Ability to model contact with Lagrangian structure (used contact type)

Table 2 A comparisons between papers conducted on SPH in geotechnical engineering

Authors	Year	The applied numerical method	Project goal	Application, validation, and conclusion
Wang *et al.*	2004	SPH–FEM coupled method	A full coupled numerical model for simulation of underground structure response under the explosion loading	The presented model is proved for empirical predictions and can be applied for parametric studies and verification of practical models and special design conditions in which a much details of reactions are required.
Lu *et al*.	2005	SPH and FEM	Presentation of a numerical full coupled model for simulati concrete structure experiencing the underground explosion, by an emphasis on comparative performance of 2D and 3D model	This research can be useful in design of facilities installed for reducing the structure shock hazard to them.
Xu and Liu	2008	SPH and FEM	Application of coupled SPH-FEM method for analysis of structure response to explosive charges	Analyses were carried out for two numerical cases: one for unrestrained ground explosion; and another for response of structure under the explosive charges.
Bui *et al*.	2007	SPH	Development of SPH modelling for simulation of mutual behavior of soil-water	Soil excavation was simulated using a water jet under dry and saturated conditions. Due to the test difficulty, numerical results are not quantitatively compared with empirical data; however, the computations are stable and the results are acceptable.
Bui *et al*.	2008	SPH	Simulation of progressive slope failure induced by the quick rising of water table due to the heavy rainfalls and determination of safety factor of slope and maximum post-failure displacement of soil	Slope stability was studied using the analysis of measured in-situ data, empirical models, and results of numerical simulations.
Pastor *et al*.	2008	SPH and FEM	Simulation of catastrophic avalanching slides and their produced waves in fjords, lakes, and reservoirs	(a) Landslide triggering; (b) landslide propagation; (c) water and slide interaction; and (d) far-field propagation were analyzed.

Table 2 Continued

Authors	Year	The applied numerical method	Project goal	Application, validation, and conclusion
Lenaerts and Dutre	2009	SPH	Simulation of fluid-sand model in SPH framework unit	Different impacts such as alteration of dry soil to mud induced by rainfall and erosion occurred in rigid sand structures by waves are studied in this work.
Sakai *et al*.	2009	SPH	Effect of air bubbles development in pore water was studied on seepage-based failure	A new numerical simulation is developed for computing flowage deformation and interaction of soil-water-air bubbles using the SPH method. The simulation results are verified by comparing them with test results.
Ulrich *et al*.	2011	SPH	Investigating SPH for interaction of multiphase soil/water by dynamic coarsening of the particles.	This method is applied for harbor and ocean engineering by an emphasis on sediment scouring. Verification is performed using different water/soil interaction tests.
Dakssa and Harahap	2012	SPH	Prediction of landslide velocity in runoff release along a saturated slope such as debris flow and avalanche	SPH codes were developed in FORTRAN for simulation run.
Stefanova *et al*.	2012	SPH	Simulation of mutual effect of water-soil system using the SPH	Effect of fluid flow induced by lowered gravity based foundation was studied for off-shore wind turbines in soil surface and jet injection.
Bui and Fukagawa	2013	SPH and LEM	Applications of SPH for saturated soil problems including water pore pressure and hydrostatic pressure	Effect of dilation angel was studied in failure mechanism of an embankment with two-sided slope and surface failure was compared with the one predicted using the limit equilibrium (Bishop's modified) method.
Dakssa and Harahap	2013	SPH	Simulation of surface runoff triggering along a soil slope in terms of predicting fast-moving slides such as debris flows and avalanche	Source SPH code of the flow written in FORTRA was presented for simulation run. Then, the modeling and its comparison with standard empirical formulas were conducted.

Table 2 Continued

Authors	Year	The applied numerical method	Project goal	Application, validation, and conclusion
Huang *et al*.	2013	SPH	Evaluation of coupled soil-water SPH for flow analysis in the fluidized soils	Simulation of simple shear test and simulation of the mutual forces between water and soil through the falling-head permeability test and physical flow modelling of the fluidized were carried out. An embankment failure was applied as a real fluidization example to extend SPH method.
Zhang and Maeda	2014	SPH	Design of a test model and simulation using the SPH for detecting the infiltration mechanism and slope and levee failure trend under the heavy rainfall considering the soil-water-air coupling	Four rainfall intensities and two base layers (under drained and undrained) conditions are considered in the model test.
Bui *et al*.	2008	SPH	Analysis of large deformations and simulation of geomaterial failure in SPH framework for interaction between soil and structure	The bending mechanism of the reinforced pile and stress distribution in its solid structure was simulated using the SPH method. The obtained results and qualitatively valid for the entire process.
Das and Cleary	2013	SPH	Modelling of gravity dam failure under fluctuating dynamic earthquake loads	SPH prediction from crack initiation point and propagation pattern was compared with available FEM predictions and the laboratory results of physical models.
Huang and Dai	2013	SPH and FEM	Simulation of large deformations and failure for geo-disasters	Presentation of a general view of SPH applications for solving a wide range of large deformation and failure problems
Fredj and Dinovitzer	2010	MM-ALE and SPH	Development and verification of 3D continuous modelling techniques for evaluation of pipeline systems prone to large soil displacements	In this paper, a 3D continuous model was proposed using the MMALE and SPH methods, and then developed and applied based on LS-DYNA technique. The obtained results are compared with laboratory data.

Table 2 Continued

Authors	Year	The applied numerical method	Project goal	Application, validation, and conclusion
Fredj and Dinovitzer	2010	MM-ALE and SPH	Development and verification of 3D continuous modelling techniques for evaluation of pipeline systems prone to large soil displacements	Effect of soil restraint on responses of pipe/soil systems (e.g., pipeline wrinkles and bending and stress demand)
Lescoe *et al*.	2010	SPH and FEM	Development of new models for soil-tire interaction	Resistance to rolling test was carried out to study differences between FEA, SPH, and FEA/SPH methods.
Fredj and Dinovitzer	2012	SPH	Studying the mechanical behavior of pipeline buried in the soil in relation with active landslide	A comparison between modeling results and the results of this work was performed in full laboratory scale.
Pang *et al*.	2012	SPH	Study of soil parameters for showing interaction mechanism of pile/soil	The pile entrainment process was simulated using a dynamic software. By tracing SPH particles, the interaction and analysis of soil particles was obtained for different times and depth induced by extrusion and shear effect of pile.
Nguyen *et al*.	2013	SPH	Presentation of a new numerical method for simulation of large and post-failure deformations of modular block retaining wall system	The numerical modelling was carried out and compared with the corresponding tests for verifying the validity of numerical framework.
Bui *et al*.	2013 2014	SPH	Simulation of large deformations and post-failure behavior of the soil and blocks of retaining wall in SRW system	2D simulation of SRW collapse was carried out and the obtained results were compared with empirical data obtained under similar conditions.
Wang *et al.*	2013	SPH	Presentation of a simulation model for the frictional contact between soil and rigid structure or deformability in the SPH framework	Four numerical tests were proposed to study the proposed methods.
Zhang *et al*.	2013	SPH	Analysis of boundary deficiency mechanisms for SPH simulation of frictional contact	Four numerical tests are presented for studying the proposed method.

Table 2 Continued

Authors	Year	The applied numerical method	Project goal	Application, validation, and conclusion
Pastor *et al*.	2008	SPH	Presentation of a model for coupling between pore pressure and solid skeleton inside the avalanching mass	Model accuracy was tested using a set of criteria and simulation of flow-like catastrophic Campania landslide was performed.
Huang *et al*.	2012	SPH	Analyzing the run-out of flow-like landslides	Dam break and granular flows are simulated and validated to assess accuracy of SPH model. Then, analysis of earthquakes with recorded information was performed.
Bui and Fukagawa	2013	SPH and LEM	Applications of SPH for saturated soil problems including water pore pressure and hydrostatic pressure	Effect of dilation angel was studied in failure mechanism of an embankment with two-sided slope and surface failure was compared with the one predicted using the limit equilibrium (Bishop's modified) method.
Nguyen *et al*.	2013	SPH	Presentation of a new numerical method for simulation of large and post-failure deformations of modular block retaining wall system	The numerical modelling was carried out and compared with the corresponding tests for verifying the validity of numerical framework.
Pastor *et al*.	2013	SPH	The numerical models for triggering and propagation mechanism of fast catastrophic avalanches and other related problems such as waves caused by landslides	Two alternative numerical modelling are presented to describe landslide triggering and propagation.
Das and Cleary	2013	SPH	Modelling of gravity dam failure under fluctuating dynamic earthquake loads	SPH prediction from crack initiation point and propagation pattern was compared with available FEM predictions and the laboratory results of physical models.
Dhillion	2013	FEM and SPH	Rubber-soil simulation using the nonlinear FEM analysis code and their verification to use in static and dynamic tests such as response to steering input	The new mesh-free SPH model was proposed and accuracy of SPH and FEA models were compared. Finally, the results were compared with empirical data.

Table 2 Continued

Authors	Year	The applied numerical method	Project goal	Application, validation, and conclusion
Cascini *et al*.	2014	SPH	A quasi-3D (depth-integrated) coupled SPH model for simulation of channelized landslides of the flow type	The theories on the channelized landslides of the flow type are discussed considering different assumptions for propagation modelling.
Dong *et al*.	2006	SPH	Evaluation of shear performance of rock in jointed and mix media using the SPH	Shear performance of rock was evaluated in terms of rock strength criterion
El-Gindy *et al*.	2011	SPH and FEM	Soil simulation using the FEA and SPH methods for tire-soil interaction	Compression-sinkage and shear-displacement tests were carried out for studying accuracy of the obtained results.
Zhong *et al*.	2012	SPH and FEM	SPH modelling of forward and reverse-rotational rotary blades using the LS-DYNA software	This method provides a theoretical basis for design of agricultural instruments.
Gao and Jin	2012	SPH	Development of a virtual prototype of oblique rotary tilling using SPH method	Digital simulation and field test of oblique rotary tilling was performed.
Mabssout and Herreros	2012	Runge-Kutta and Taylor-SPH	A comparison between two different time-integration designs was carried out using the refined SPH for shock waver propagation in elasto viscoplastic media	Superiority of Taylor-SPH over the time integration Runge-Kutta design was verified. The integration Taylor-SPH is more stable, stronger, and efficient and needs only few particles for obtaining accurate results.
Bojanowski	2014	MM-ALE, EFG, and FE	Application of this method for modelling SSI problems in presence of large soil deformations	Simulations were carried out for large-scale steel pad penetration in to the silt-clay with sand and standard cone penetration test on poorly graded sand.

8. Conclusions

Based on the points mentioned in introduction, the following items can be considered as the advantages and shortcomings of SPH method. SPH is applicable for a wide range of large deformation and failure problems known as geo-disasters; e.g., dam break, slope failure, soil liquefaction, seepage damage, dynamic erosion, response of underground structure and soil under the explosion loadings, propagation leading the avalanching disastrous landslide, rock failure, and large deformations of soil. This method has many advantageous over the traditional mesh-based numerical modellings when dealing with geo-disasters with large deformations, free surface, displacement interface, and deformable boundaries. Effect of pore water pressure can be properly presented using the SPH simulation. SPH can predict the maximum post-failure displacement of soil in a given slope, where the rise of multiphase (layer-by-layer) failure of the slope triggered by heavy rainfalls is probable. Also, Prediction of path, rate, and depth of flow-like catastrophic landslides is possible using the SPH method. Seepage failure induced by development of air bubbles in the pore water pressure can be modelled using the SPH method. Application of this technique was shown in 3-phase problems for prediction of flood disaster and development of failure prevention methods. Evaluation of pipeline system performance undergoing large soil displacement, and interaction between tire-soil interaction, tire-instrument, and soil-structure systems can be modelled using this method. Coupled with other numerical methods, SPH can be applied for soil modelling. The SPH method is able to deal with behavior and interaction of various (liquid and solid) materials and is capable for modelling multiphase soils. Also, the saturated and submerged soils can be simulated using SPH method. In this regard, SPH can be used in a wide range of geotechnical applications, particularly for the case of submerged complicated ground surface. SPH is able to analyze water-induced deformation, which is not an easy task using FDM and FEM methods due to the mesh distortion and other computational problems. Table 2 presents the evolving trend of SPH studies performed on problems dealing with soil interaction with other materials (water, structure, instruments, etc.) and demonstrates their potential. Obviously, the method requires to be evolved to expand its applications in different geotechnical fields.

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