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Probabilistic multi-objective optimization of a corrugated-core sandwich structure

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Abstract. Corrugated-core sandwich panels are prevalent for many applications in industries. The researches performed with the aim of optimization of such structures in the literature have considered a deterministic approach. However, it is believed that deterministic optimum points may lead to high-risk designs instead of optimum ones. In this paper, an effort has been made to provide a reliable and robust design of corrugated-core sandwich structures through stochastic and probabilistic multi-objective optimization approach. The optimization is performed using a coupling between genetic algorithm (GA), Monte Carlo simulation (MCS) and finite element method (FEM). To this aim, Prob. Design module in ANSYS is employed and using a coupling between optimization codes in MATLAB and ANSYS, a connection has been made between numerical results and optimization process. Results in both cases of deterministic and probabilistic multi-objective optimizations are illustrated and compared together to gain a better understanding of the best sandwich panel design by taking into account reliability and robustness. Comparison of results with a similar deterministic optimization study demonstrated better reliability and robustness of optimum point of this study.

Keywords: sandwich structure; corrugated core; optimization; probabilistic; genetic algorithm; finite element method

1. Introduction

Corrugated-core sandwich panels are a type of low weight and high strength structures that are extensively used in a variety of applications in aerospace, marine and automotive industries. Due to desirable energy absorption, acoustic, thermal and cooling characteristics as well as durability, sandwich panels have been very desirable for industrial applications. Mechanical properties of sandwich panels highly depend on their core configuration (Wadley *et al*. 2003).

Due to the above-mentioned characteristics, these structures have drawn considerable attention in recent years. Lu *et al*. (2001) designed and optimized a flexural actuator consisting of a triangular corrugated core with shape memory alloy (SMA) faces. They concluded that cantilever actuators consisting of SMA face sheets and triangular corrugated cores can successfully operate against large restraining moments at almost low weight compared to pertaining concepts, such as bimorphs. Wadley *et al*. (2003) demonstrated the possibility of fabrication of robust metallic sandwich panels with periodic, open cell, cores by protocols based both on the sheet forming of

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trusses and textile assembly. They also showed that sandwich panels constructed with these cores sustain loads at weights greatly superior to stochastic foams. Valdevit *et al*. (2004) performed single objective optimization of sandwich panels with corrugated cores considering non-dimensional form of the structure weight as objective function and occurrence of yielding and buckling in face and core sheets as optimization constraints. Tan and Soh (2007) performed multi-objective optimization of a new type of sandwich panels with prismatic cores considering the lowest weight and maximum heat transfer performance of the sandwich panel. They used genetic algorithms and obtained a set of compromised solutions, known as the trade-off surface. Biagi and Bart-Smith (2012) used analytical, numerical, and experimental methods to characterize the failure response of sandwich panels with corrugated core under in-plane loading. Bartolozzi *et al*. (2015) proposed a simplified multilayered model instead of a fully detailed 3D one for aluminum sandwich panel with a sinusoidal corrugated core. Wei *et al*. (2014) fabricated ZrO_2 ceramic corrugated-core sandwich panels using gelcasting technique and pressureless sintering. They measured the three-point bending strength, the compressive strength and the specific bending strength of this type of sandwich panel with corrugated core. Chang *et al*. (2006) also developed the elasto-plastic modeling of corrugated-core sandwich panels based on incremental theory of plasticity. They employed a combination of Hill's criterion for naturally orthotropic material and Ilyushin's criterion for isotropic plates and shells as the yield criterion. Dynamic response of a sandwich beam with foam or functionally graded cores was also investigated for a single impact loading considering arbitrary impacted face sheet and locations (Malekzadeh *et al*. 2015).

Optimal design of sandwich panels deals with optimizing two objective functions including weight and stiffness of the structure. However, these objective functions are known to represent conflicting trend, meaning that improving one leads to deteriorating another. Such problems concerned with optimization of conflicting objective functions are recognized as multi-objective optimization problems (MOP). Multi-objective optimization problems are very prevalent in advanced design and engineering practices. Some of the recent works include utilizing multi-objective optimization algorithms for minimizing the power consumption of piezoelectric patches, while obtaining the highest vertical displacement of smart FML panels using method of modified multi-objective Elitist-Artificial Bee Colony (E-ABC) algorithm. (Ghashochi-Bargh and Sadr 2014), as well as multi-objective optimization of steel frames to obtain both the lowest damage and minimum cost (Kaveh *et al*. 2013). In another study, multi-objective optimization was performed on a parametric ship hull form represented by B-Spline curves via Genetic Algorithm (Guha and Falzarano 2015). In addition, a modified particle swarm approach was employed for multi-objective optimization of laminated composite structures (Sepehri *et al*. 2012). Method of Non-dominated Sorting Genetic Algorithm II (NSGA II) was also used to simultaneously optimize size and topology of a geometrically nonlinear dome structure by minimizing its weight & joint displacements and maximizing load-carrying capacity (Targul 2012).

Optimization of the sandwich panel for gaining the best structure by incorporating low weight and high strength is actually a multi-objective optimization problem encountered with conflicting objective functions. Although many researches in the literature have conducted numerical and experimental investigations on modeling and analysis of these panels, only a few research works can be found connected with performing optimization.

Different methods for solving MOPs have been proposed by previous researchers (Collette and Siarry 2013, Lee *et al*. 2011). Method of NSGA-II proposed by Deb (2001) and Srinivas and Deb (1994) which is a Pareto based approach, has been found an effective algorithm for solving MOPs. It generates a set of non-dominated solutions (Pareto solutions), where a non-dominated solution has better characteristics in at least one criterion than the other solutions. To improve NSGA-II, Nariman-Zadeh (Nariman-Zadeh *et al*. 2006) proposed modified NSGA-II which uses ε-elimination algorithm instead of crowding factor (Nariman-Zadeh *et al*. 2006). This method has been proved to work successfully in many recent studies (Khalkhali and Safikhani 2012, Khalkhali *et al*. 2014).

The main drawback of conventional optimization techniques lies in the fact that these approaches do not take into account design uncertainties, hence, a deterministic approach is taken toward the optimization process. Notably, it has been proved that such an optimization without considering uncertainties actually results in potentially high-risk solutions instead of optimal designs. Therefore, finding a reliable and robust design with low performance variation in the presence of uncertainties is highly crucial for realistic industrial applications. In this regard, researchers are going more and more towards utilization of probabilistic approaches incorporating reliability and robustness of design in structural optimization. (Sun and Betti 2015, Yang *et al*. 2015, Richardson *et al*. 2015, Li *et al*. 2015)

Generally, there are two stochastic approaches accommodating influence of uncertainties, namely robust design optimization (RDO) as well as reliability-based design optimization (RBDO) (Papadrakakis *et al*. 2004). Both approaches propose a probabilistic optimization process instead of deterministic formulations.

In RDO view, it is required to decrease the sensitivity of the robust performance to the random variation generated by uncertain design variables, so the performance degradation from ideal deterministic behavior becomes minimized. While, in RBDO approach, some pre-defined reliability metrics subjected to probabilistic constraints are met. In fact, a limiting index has been defined as the probability of failure of each design. Regardless the choice of any of corresponding approaches, the objective functions and the constraints of the optimal design should be assessed to reflect the effect of probabilistic nature of uncertain parameters in the system performance. By means of computational power, a great number of researches in the field of robust analysis and design leaded to the use of Monte Carlo simulation (MCS) (Lönn *et al*. 2009, Khakhali *et al*. 2010). In fact, MCS has also been an efficient tool for verification of the results of other methods in RDO or RBDO problems provided that the number of sampling is sufficient.

In this paper, firstly, regarding different uncertainties in design performance, geometrical design variables and material properties of the sandwich panel with corrugated core are considered as probabilistic parameters with Gaussian distribution. Dimensionless weight, the mean and standard deviation of the deflection of the sandwich panel structure are considered as three conflicting objective functions in the present MOP. Deflection of the panel based on geometrical design parameters has been calculated using an APDL code developed in ANSYS. In addition, by means of a coupling between MATLAB and ANSYS commercial software, the APDL code obtained in ANSYS is employed in MATLAB for the optimization process during the run time. Subsequently, modified NSGA-II algorithm is utilized for solving the MOP, where both RDO and RBDO approaches are taken into consideration. Concerning RDO approach in the optimization process, the standard deviation of the distribution of the panel deflection is to minimize, while, RBDO is achieved by considering some probabilistic reliability constrains, preventing the failure of the panels. Best possible combinations of the values for objective functions generate a set of optimal design points known as a Pareto front. Finally, to select some trade-off optimum design points among the obtained Pareto points, two methods including the Nearest to Ideal Point Method (NIP) and Technique for Order Performance by Similarity to Ideal Solution method (TOPSIS) are introduced and utilized in this study.

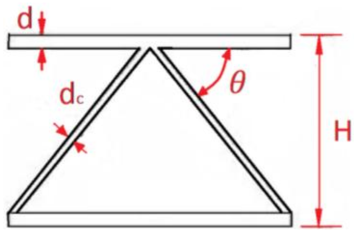


Fig. 1 Geometry of one unit of a sandwich panel with corrugated core and the corresponding design variables

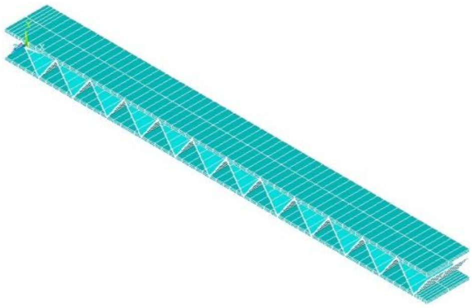


Fig. 2 Finite element model of the sandwich panel with corrugated core

2. Finite element analysis

Geometrical configuration as well as design variables for one unit of the considered sandwich panel with corrugated cores are depicted in Fig. 1.

Accordingly, the geometrical design variables of this model include d as the thickness of the panel faces, d_c as the thickness of the core member, and H as the value of the distance between the face sheets. Fig. 2 also indicates the associated FE model of the sandwich panel with corrugated core. FE simulation of the transverse loading and modeling of this panel are done using ANSYS commercial software, in which 3D beam elements (BEAM 189) are used for discretization.

The total length of the panel in all FE analyses is constantly equal to 1m. In FE simulations, one end of the sandwich panel is completely fixed and a downward transverse load with a magnitude of 6317.5 N is applied to the point at the other end. This value for force has been selected according to the results obtained by authors in (Valdevit *et al*. 2004). As mentioned earlier, unlike the methodology employed in this study, they carried out a deterministic single-objective optimization to exclusively optimize the weight of the sandwich panel structure. Based on the graphs represented by Valdevit *et al*. (2004) the amount of load applied to one of the optimum designs for a single-array sandwich panel was equal to 6317.5 N. After applying the load, maximum deflection of the panel corresponding to the design variables as inputs, is calculated by an APDL script developed in ANSYS. To check the accuracy of FE analysis, 5 different FE simulations were performed and deflection of the panel was compared to corresponding values reported by Valdevit *et al*. (2004). Obtained root mean square error was equal to 0.0039, indicating acceptable accuracy of FE modeling in this study.

To consider the effects of uncertainties in design variables, *Prob*. *Design* module in ANSYS is utilized. In fact, this module provides as a probabilistic finite element analysis, meaning, instead of deterministic values of geometrical and mechanical properties, a distribution of them are entered in this module to accommodate uncertainty from deterministic values. Based on the initial distribution of inputs in this module, outputs such as panel deflection will be calculated with a certain stochastic distribution instead of a specific value. Initial values for different samples as design variables are first produced in MATLAB. Subsequently using the APDL code in *Prob*. *Design*, a probabilistic finite element analysis is performed to obtained outputs including mean and standard deviation of panel deflection along with its dimensionless weight. Finally, these outputs are again imported to the optimization code to initiate the process of multi-objective optimization, producing a set of probabilistic outputs corresponding to a specific set of probabilistic inputs.

In this paper, geometrical design variables, namely, H, d and d_c as well as module of elasticity E (for both cores and panel) are considered as probabilistic design variables with Gaussian distribution. Mean and variance of the distributions for each of the above-mentioned design variables will be further investigated in Section 6. Considering excessively high number of runs needed for this module in the optimization process, an APDL script has been developed particularly for the probabilistic analysis in *Prob*. *Design* module.

Moreover, Monte Carlo simulation method has been utilized in *Prob*. *Design* module and the number of samples has been regarded 1000 individuals. The mean deflection ($\bar{\delta}$) and its standard deviation (σ) have been considered as outputs of this FE analysis. To evaluate the effect of the number of samples in accuracy of probabilistic FE analysis, firstly, several analyses regarding different values for the number of samples including 100, 1000, 10000 and 100000 individuals were done, wherein the difference in the values of the outputs was investigated for each analysis. It was proved that 1000 individuals as the number of samples showed a good accuracy in the outputs, while for the numbers below 1000, output values had an unacceptable accuracy.

3. Reliability-based and robust stochastic methods

A comprehensive explanation of the probabilistic stochastic method employed in this study can be found by Khakhali *et al*. (2010). Briefly, to achieve a robust design, it is required to minimize the variability of a random process due to the presence of uncertainties in deterministic design variables. This robust approach toward the MOP can be given by

$$\text{Minimize} \quad \begin{gathered} \{\mu[f(x,d,p)],\, \upsilon[f(x,d,p)]\} \\ x^{(L)} \le x \le x^{(U)} \\ d^{(L)} \le d \le d^{(U)} \end{gathered} \tag{1}$$

Where f (x, d, p) is the performance or the cost function, μ is the mean value and υ is one stochastic dispersion index, namely variance (σ^2), standard deviation (σ) or coefficient of variation ($C_v = \sigma/\mu$), $\boldsymbol{x}$ is the vector of uncertain design variables, $\boldsymbol{d}$ is the vector of deterministic design variables and $\boldsymbol{p}$ is the vector of uncertain parameters which are not design variables.

To consider a reliability-based design view, some reliability metrics by means of some inequality constraints based on the probability of failure of the design are defined. A deterministic constraint is regarded in the form of $g_i(\boldsymbol{x},\boldsymbol{d},\boldsymbol{p}) \le \hat{g}_i$ where $\hat{g}_i$ is the limiting value of ith constraint. Using the definition of a random process, such a deterministic constraint can be converted into a probabilistic one as follows

$$G(x,d,p) \equiv \hat{g}_i - g_i(x,d,p), \tag{2}$$

The typical probability constraint is then given as

$$P_f^i = P\left[G(x,d,p) \geq 0\right] \leq \varepsilon_i \quad (i = 1, 2, 3, \ldots, m) \tag{3}$$

where P_f^i is the probability of failure of the ith reliability index and m is the number of inequality constraints and ε_i is the highest value of the desired admissible probability of failure. Clearly, the ideal value of each P_f^i is zero.

Consequently, a set of N different solutions is generated by the propagation of the known probabilistic distribution of the variation of $\boldsymbol{x}$ and $\boldsymbol{p}$ in the model. Afterwards, each constraint g_i can be investigated for each sample to check its possible violation. If r cases (of N) do not satisfy g_i constraints, the probability of failure of P_f^i of Eq. (3) can be simply computed by the value of r/N.

In addition, corresponding Eqs. for the mean value (μ) and standard deviation (σ) in Eq. (1) can be calculated as

$$\mu(X) = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{4}$$

and

$$\sigma(X) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \mu(X)\right)^2} \tag{5}$$

In the present reliability-based robust multi-objective optimization problem, there are several conflicting reliability-based metrics that should be minimized simultaneously. This methodology can be formulated as

$$\begin{aligned} &\text{Minimize} && \left\{\mu[f_i(x,d,p)],\ \upsilon[f_i(x,d,p)]\right\} \quad (i = 1, 2, 3, \ldots, m) \\ &\text{Subject to} && \left\{P\left[G(x,d,p) \geq 0\right] \leq \varepsilon_j\right\} \quad (j = 1, 2, 3, \ldots, m) \\ &&& x^{(L)} \leq x \leq x^{(U)} \\ &&& d^{(L)} \leq d \leq d^{(U)} \end{aligned} \tag{6}$$

In this paper, Hammersley Sequence Sampling (HSS) method has been used to generate samples for probability estimation of failures in the reliability-based robust multi-objective process. HSS is a low-discrepancy method capable of producing uniform distribution of n points in a k-dimensional hypercube (Kalagnanam and Diewekar 1997). This method has been proved to be an efficient and precise algorithm, in particular, for robust design and optimization under presence of uncertainties (Kalagnanam and Diewaker 1997, Diewekar and Urmila 2003). Furthermore, HSS technique has been widely used for generation of uniform samples in probabilistic optimization studies incorporating effect of design uncertainty (Fu *et al*. 2000, Subramanyan *et al*. 2004, Khalkhali *et al*. 2010, Lingshuang *et al*. 2013).

4. Reliability-based and robust multi-objective optimization of sandwich panels with corrugated core

In conventional MOPs of sandwich panels, it is usually desirable to minimize the dimensionless weight (ψ) and the deflection of the sandwich panel (δ) under transverse loading. However, for achieving a RDO, objective functions should be as insensitive as possible to the presence of uncertainties. This necessitates minimizing the standard deviation of deflection (σ) as another objective function. In addition, for acquiring a RBDO, several probabilistic failure constraints should also be met in the process of optimization. In a corrugated core sandwich panel, for the transverse loading case, the non-dimensional loading index which is based on the maximum value of the shear force can be calculated as follows

$$\Pi = \frac{V}{\sqrt{EM}} \tag{7}$$

where, V and M are maximum values of the shear force and bending moment, both per unit width, and E is the Young modulus of the panel constitutive material. Assuming that the shear force is undergone almost entirely by the core and the bending moment almost completely by the face sheets, the failure constraints are as follows (Valdevit *et al*. 2004)

$$\text{Face sheet yielding,} \quad g_1 : \frac{V^2}{EM}\frac{E}{\sigma_y}\frac{l}{d}(\frac{H}{l}-\frac{d}{l})^{-1}-1\leq 0 \tag{8}$$

$$\text{Face sheet buckling,}\ g_2 : \frac{1}{nSin\theta}\frac{V^2}{EM}\frac{E}{\sigma_y}\frac{l}{d_c}-1\leq 0 \tag{9}$$

$$\text{Core member yielding,} \quad g_3 : \frac{48}{K_f\pi^2 n^2 \tan^2\theta}\frac{V^2}{EM}\frac{E}{\sigma_y}(\frac{H}{l}-\frac{d}{l})(\frac{l}{d})^3-1\leq 0 \tag{10}$$

$$\text{Core member buckling,} \quad g_4 : \frac{12}{K_c\pi^2 n^3 Sin^3\theta}\frac{V^2}{EM}\frac{E}{\sigma_y}(\frac{H}{l}-\frac{d}{l})^2(\frac{l}{d_c})^3-1\leq 0 \tag{11}$$

Parameter l is defined according to the length of loading and the boundary conditions and can be calculated as

$$l = \frac{M}{V} \tag{12}$$

In this paper, the sandwich panel has been considered as a cantilever, so the effective value of l is equal to the total length of the beam. Furthermore, K_f and K_c which are the Euler's Eq. correction factors, are defined as follows (Valdevit *et al*. 2004).

$$K_f = \left(\frac{2.4Cos\theta(d_c/d)^3+1}{1.2Cos\theta(d_c/d)^3+1}\right)^2 \tag{13}$$

$$K_c = 1.375\left(\frac{2.2+1.2(d/d_c)^3/Cos\theta}{1.6+0.6(d/d_c)^3/Cos\theta}\right)^2 \tag{14}$$

In the present MOP, dimensionless weight (ψ), mean deflection ($\bar{\delta}$) and standard deviation of deflection (σ) of the sandwich panel have been considered as three objective functions. The dimensionless weight has been calculated using a simple mathematical equation as

$$\psi = \frac{W}{\rho l^2} = 2\frac{\bar{d}}{l} + \frac{1}{Cos\theta}\frac{\bar{d}c}{l} \tag{15}$$

Where ρ and θ denote the density of the constitutive material and the core sheet angle, respectively. In the present study, θ is considered equal to 54.7° in which the structure shear strength will be maximized (Wadley *et al*. 2003). To calculate the mean and standard deviation of the deflection of the sandwich panels, Prob. Design module in ANSYS has been used. To do this, the optimization script, which is written in MATLAB program, is connected to the APDL script, written in ANSYS program. As a result, outputs of finite element analysis, which are objective function of this study, are imported to the optimization algorithm. In each generation, design variables are produced by the optimization script and afterwards the APDL script is employed for computation of the three objective functions. To achieve a reliable design for sandwich panels with corrugated core, the probability of failure of constraints g_1 to g_4 has been regarded less than 10%. In this MOP, there are three geometrical design variables with probabilistic normal distributions including: the face sheet thickness (d), the core sheet thickness (d_c) and the face sheet distance (H).

Moreover, two different uncertain parameters which are not design variables are also considered with probabilistic normal distributions including: the elasticity module (E) and yield stress (σ_y). It is assumed that the standard deviations of d, d_c, H, E and σ_y are equal to 0.0001 m, 0.0001 m, 0.01 m, 10 MPa and 2 GPa, respectively. These values have been considered according to 0.3 mm, 0.3 mm, 30 mm, 30 MPa and 6 GPa tolerance in the face sheet thickness, core sheet thickness, sheet distance, yield stress and elasticity module, respectively, based on 3σ as the value of tolerance. The mean of elasticity module and yield stress of the material are considered equal to 70 GPa and 490 MPa, respectively. These values are equal to those deterministic values that were considered by Valdevit *et al*. (2004) for aluminum alloy as the constitutive material of the sandwich panel. The present probabilistic optimization problem can be defined in accordance with equations (Bartolozzi *et al*. 2015) as follows

$$\begin{cases} \text{Minimize} \quad \psi = 2\bar{d} + \frac{\bar{d}c}{\cos\theta} \\ \text{Minimize} \quad \bar{\delta} = \mu[f_1(H, d, d_c, E)] \\ \qquad \text{(Calculated using ANSYS APDL script, Prob. Design module)} \\ \text{Minimize} \quad \sigma = v[f_1(H, d, d_c, E)] \\ \qquad \text{(Calculated using ANSYS APDL script, Prob. Design module)} \\ \text{Inequality constraints} \\ \qquad P[\frac{V^2}{EM}\frac{E}{\sigma_y}\frac{l}{d}(\frac{H}{l} - \frac{d}{l})^{-1} - 1 \geq 0] \leq 0.1 \\ \qquad P[\frac{1}{nSin\theta}\frac{V^2}{EM}\frac{E}{\sigma_y}\frac{l}{d_c} - 1 \geq 0] \leq 0.1 \end{cases} \tag{16}$$

$$\begin{cases} P[\dfrac{48}{K_f \pi^2 n^2 \tan^2\theta}\dfrac{V^2}{EM}\dfrac{E}{\sigma_y}(\dfrac{H}{l}-\dfrac{d}{l})(\dfrac{l}{d})^3 - 1 \geq 0] \leq 0.1 \\ P[\dfrac{12}{K_c \pi^2 n^3 Sin^3\theta}\dfrac{V^2}{EM}\dfrac{E}{\sigma_y}(\dfrac{H}{l}-\dfrac{d}{l})^2(\dfrac{l}{d_c})^3 - 1 \geq 0] \leq 0.1 \\ \text{Boundary of each design variable} \\ 0.001 \prec \dfrac{d}{l} \prec 0.01,\ \ 0.0001 \prec \dfrac{d_c}{l} \prec 0.003,\ \ 0.01 \prec \dfrac{H}{l} \prec 0.103 \end{cases} \tag{16}$$

Those design vectors which violate the constraints are eliminated from the optimization process by means of a penalty function. This function multiplies the real values of the objective functions by a very large coefficient. A mathematical formulation for such a function can be presented as

$$F_i(x) = f_i + 10^{10} G(x) \tag{17}$$

where

$$G(x) = \sum_{1}^{4} \lambda_i \qquad \begin{cases} \lambda_i = 0 & \text{If the } i\text{th constraint is satisfied for design vector } x \\ \lambda_i = 1 & \text{If the } i\text{th constraint is not satisfied for design vector } x \end{cases}$$

5. Results and discussion

The evolutionary-based probabilistic multi-objective optimization was performed using a population of 30 individuals with a crossover probability (P_c) of 0.7 and mutation probability (P_m) of 0.07 has been used in 400 generations that no further improvement has been achieved for such population size. Solving the optimization problem described by Eq. (30) using modified NSGA-II algorithm gives a set of non-dominated optimal design vectors, known as Pareto points. It should be emphasized that while these obtained Pareto points are non-dominated to each other, they provide better performance, reliability as well as robustness in design in comparison to other vectors of design variables, which are not Pareto points, within the search space. Corresponding Pareto fronts of $\psi - \bar{\delta}$ and $\sigma - \bar{\delta}$ are shown in Figs. 3 and 4, respectively. As evident from Fig. 3, an increase in the weight of the sandwich panel structure results in a decrease in its mean deflection. This finding indicates that unlike some previous studies by Valdevit *et al*. (2004) the weight of the sandwich panel structure cannot be considered as the only objective function in case of single-objective optimization of the structure because a sandwich panel with the lowest weight has the maximum deflection. In addition, Fig. 4 indicates a direct relationship between the mean value and standard deviation of the panel deflection. It is clear that in case of a probabilistic optimization, when uncertainties are accounted, the lower the weight of the sandwich panel structure, the more its mean as well as standard deviation of the deflection. This result again shows that due to an increase in the standard deviation of the sandwich panel deflection and resultantly a decrease in the design robustness, exclusively regarding the weight of sandwich panels as the only objective function is not accurate.

It is now desirable to select some optimal trade-off points among all presented Pareto fronts using TOPSIS and NIP methods. TOPSIS is a popular multi-criteria decision making method

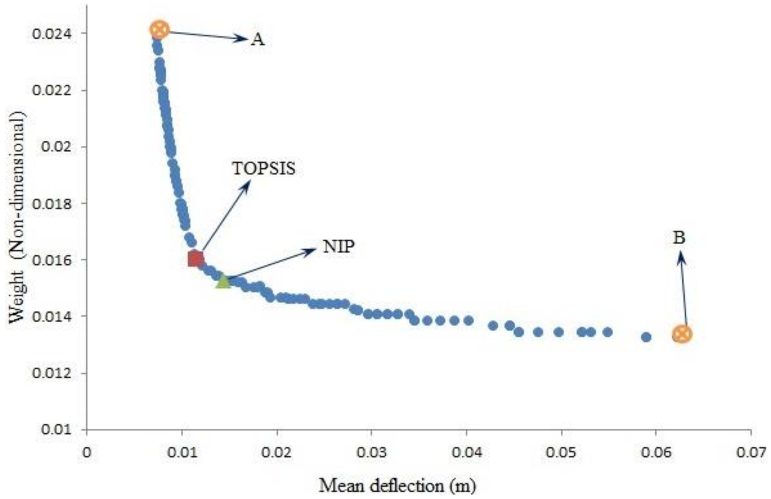


Fig. 3 Pareto front of non-dimensional weight vs. mean deflection of the sandwich panel

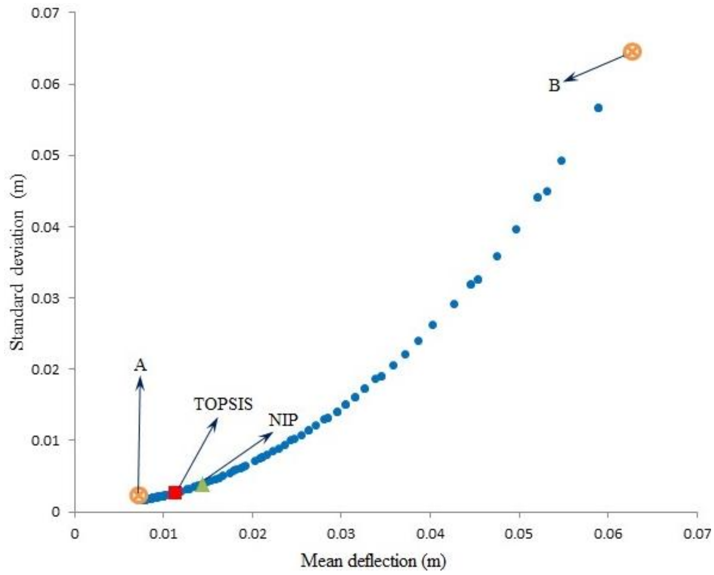


Fig. 4 Pareto front of standard deviation vs. mean deflection of the sandwich panel

widely used for engineering optimization in multi-objective problems (Shidpour *et al*. 2013, Athawale and Chakraborty 2010, Gadakh 2012). A comprehensive review about its areas of application as well as explanation of underlying concept behind TOPSIS can be found in (Behzadian *et al*. 2012).

Moreover, NIP method works based on minimizing distance of a set of non-dominant points relative to an ideal point with best characteristic for each objective function. This method has been widely used for finding the trade-off point in multi-objective problems (Khalkhali 2015, Khalkhali *et al*. 2014, 2016).

The best trade-off points are obtained and represented in Figs. 3 and 4. Additionally, it may be

Table 1 Trade off design points found by TOPSIS and NIP methods along with single-objective optimization under uncertainties

Method	$\bar{h}$	$\bar{d}$	$\overline{d_c}$	ψ	$\bar{\delta}$	σ
TOPSIS	0.099	0.0062	0.0021	0.016034	0.0114	0.0026
NIP	0.090	0.0059	0.0020	0.015261	0.0143	0.0039
A	0.100	0.0097	0.0026	0.023899	0.0073	0.0016
B	0.050	0.0050	0.0019	0.013289	0.0621	0.0642
Refraction point recommended by the deterministic study	0.094	0.0057	0.002	0.014861	0.0136	0.0034
NIP point recommended by the deterministic study	0.082	0.0055	0.0018	0.014115	0.0185	0.0059

noteworthy to find the optimum points in regard to a single-objective optimization problem. Subsequently, points A and B are the obtained optimal design points through the single-objective optimization of the weight and the mean deflection of the sandwich panel structure, respectively. These points are highlighted in Pareto fronts represented in the above-mentioned figures. Associated values of design variables as well as objective functions for corresponding points are tabulated in Table 1.

A similar two-objective, however, deterministic, optimization problem has been conducted by Khalkhali *et al*. (2014). The optimization algorithm, geometrical configurations and boundary conditions of the study are very close to that of the present study. We have also considered the FE model analogous to that of the mentioned study, for a meaningful comparison. However, in that study, authors have ignored the presence of uncertainties by taking a deterministic approach, furthermore, they have used refraction method instead of TOPSIS for finding the best trade-off point.

It can be very conducive to compare the results acquired from probabilistic optimization to those of deterministic optimization. To this aim, the weight and deflection of the optimum points obtained from probabilistic optimization can be calculated in the deterministic environment. Corresponding values have been compared to the results of the deterministic study by Khalkhali *et al*. (2014) and designated as *Prob. to Deter*. in Fig. 5.

These points are directly entered into the deterministic optimization code by knowing their design variables, as inputs. To import probabilistic optimum points into the deterministic space, mean of the design variables for these points have been entered into a deterministic space. This method provides meaningful comparison of both probabilistic and deterministic optimum designs in the deterministic space.

Moreover, trade-off design points found by TOPSIS and NIP methods in the present study and those found using refraction point method and NIP by Khalkhali *et al*. (2014) are addressed in this figure. As obvious, for points with deflection of more than 0.01 m, the weight of the sandwich panel, in case of probabilistic design, is higher than that of deterministic one represented by the deterministic study for the same value of the deflection. This increase in the sandwich panel weight in comparison to deterministic optimization is due to the presence of constraints considered in probabilistic optimization problem. In fact, in the presence of design uncertainties and failure constraints, sandwich panel structures are needed to have some added weight to guarantee a safer design. Also this figure indicates that there is no optimal and reliable design vector with the mean

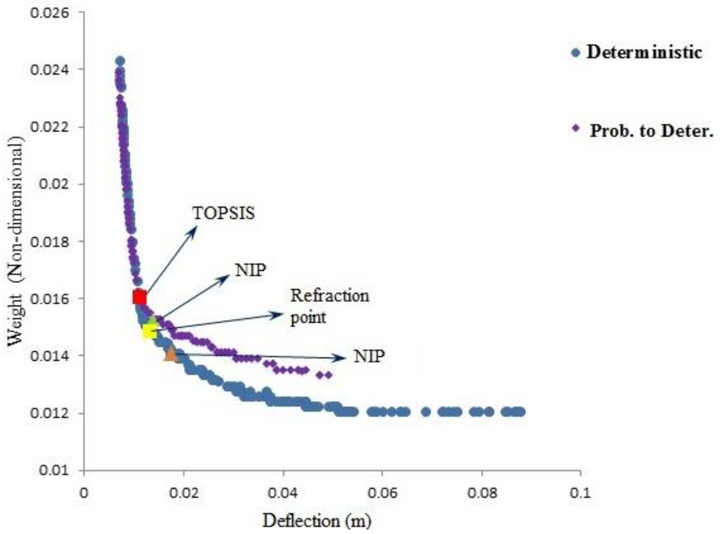


Fig. 5 Non-dimensional weight vs. deflection of the panel. Green triangle refers to NIP point of this study, while the orange one shows that of the deterministic study

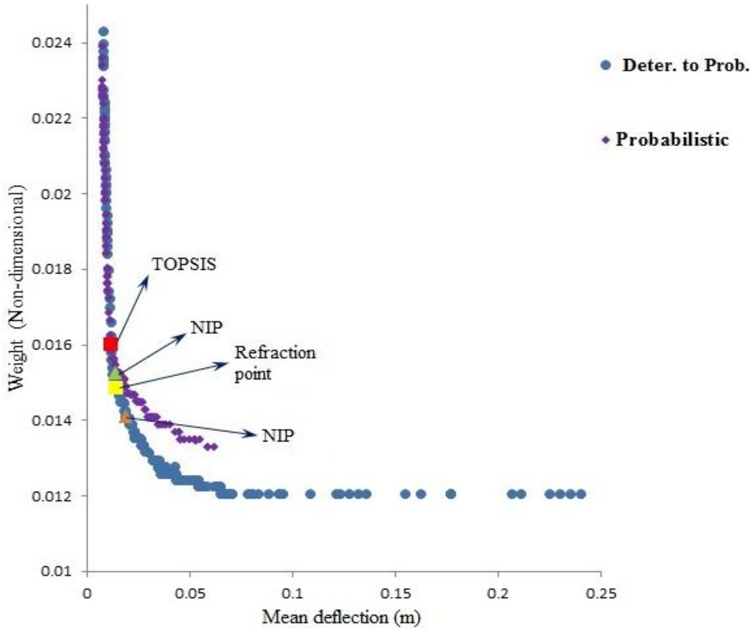
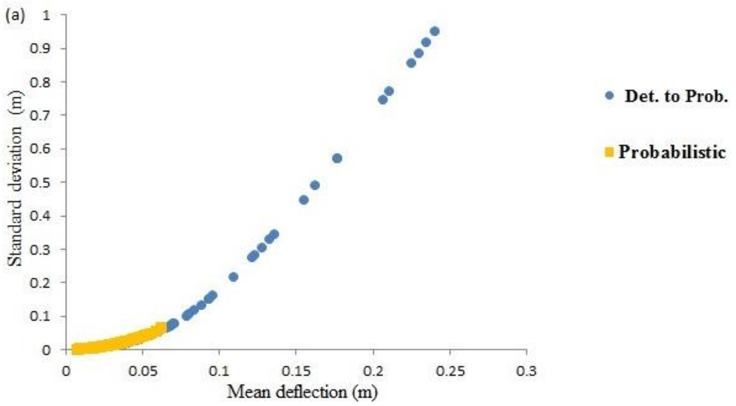


Fig. 6 Non-dimensional weight vs. mean deflection of the panel; Comparison of the present study with the deterministic study. Green triangle refers to NIP point of this study, while the orange one shows that of deterministic study

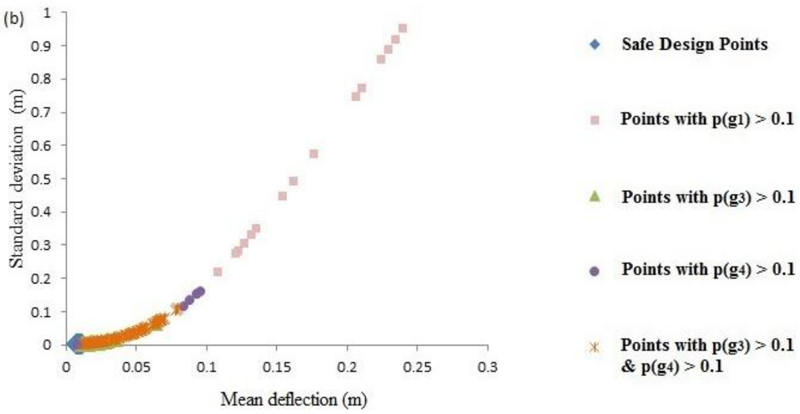
deflection of more than 0.05 m. This finding is because of the fact that points with the mean deflection of more than 0.05 m violate some of probabilistic failure constraints. Similarly, Figs. 6 and 7 show a comparison between the results obtained from both probabilistic and deterministic optimizations imported to the probabilistic environment.

It must be pointed out that similar to points designated by *Prob*. *to Det*., points identified by *Det*. *to Prob*. are the optimum deterministic points that have been directly entered into the probabilistic optimization code based on their design variables. In fact, to import deterministic results to probabilistic space, design variables of these points have been entered, as new inputs, into the probabilistic optimization code. Consequently for each set these design variables, probabilistic objective functions such as mean and standard deviation of deflection are calculated. This method serves as an advantageous asset for comparing results of deterministic optimization with probabilistic one, as both solutions are evaluated considering the same uncertain environment.

In Fig. 7(a), the probability of failure for optimal points achieved by deterministic optimization have been investigated. As it is depicted in this figure, for all of the optimum points including the trade-off ones proposed by the deterministic study with the mean deflection of more than 0.03 m, at least one of the failure constraints is violated. Notably, according to Fig. 6, from optimal NIP point obtained in this study to that of the deterministic study by (Khalkhali *et al*. 2014), the weight of the sandwich panel has increased by almost 7%, while the percentage of the change in the



(a) Optimum deterministic points in probabilistic space are designated by *Det*. *To Prob*. points



(b) Investigation of reliability of results for each constraint between deterministic and probabilistic studies

Fig. 7 Standard deviation vs. mean deflection of panel. Comparison of the present results with the deterministic study (Khalkhali *et al*. 2014)

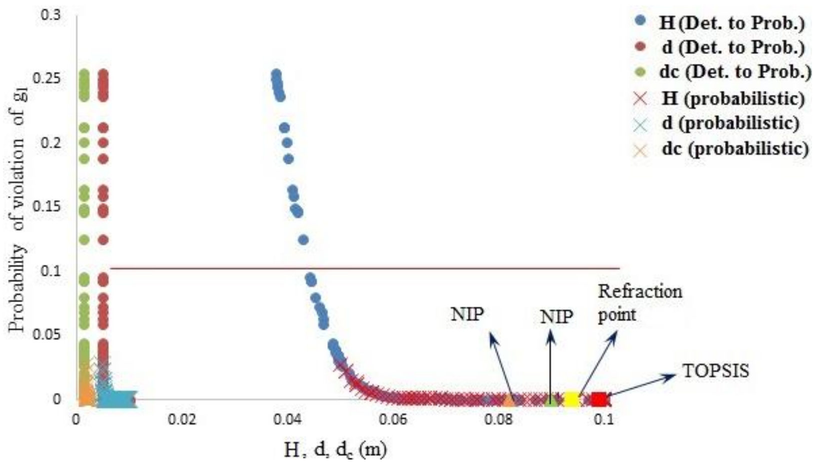


Fig. 8 Relation between design variables and Probability of violation of g_1. Comparison of the present results with the deterministic study. Green triangle refers to NIP point of this study, while the orange one shows that of deterministic study

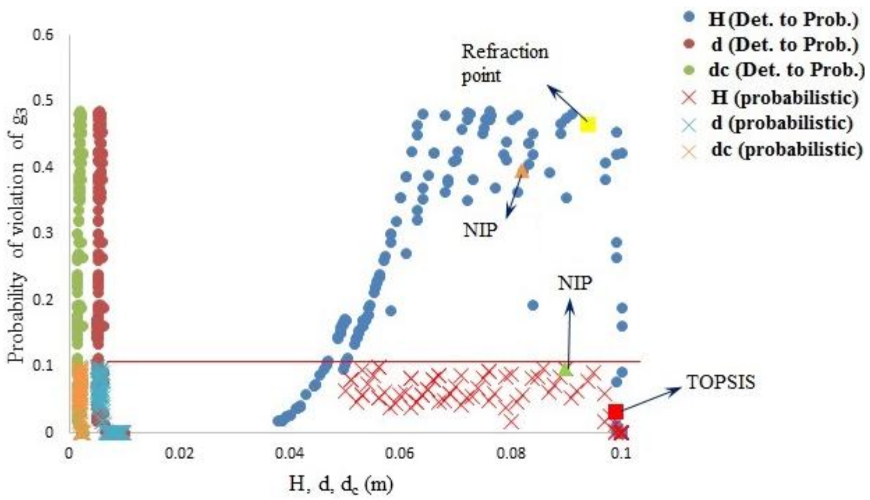


Fig. 9 Relation between design variables and Probability of violation of g_3. Comparison of the present results with the deterministic study. Green triangle refers to NIP point of this study, while the orange one shows that of deterministic study

mean deflection has remained negligible. In addition, optimal TOPSIS point has the highest weight among all optimal points, highlighting the fact that in case of deterministic optimization of the panel weight and deflection, because of the lower weight of the achieved trade-off design points, they are actually high risk designs instead of optimal ones. This clearly indicates the necessity of taking RBDO and RDO approaches in optimization procedure. Furthermore, Fig. 7(b) compares the standard deviation of the panel deflection and the range of the feasibility and violation of each constraint in two studies. This figure shows that the standard deviation of deflection for all of the optimal values found in the present study are less than 0.07 m; whereas the corresponding parameters in case of deterministic optimization reaches to 1 m. This low amount of variation in case of the probabilistic optimization shows the robustness of the design.

Figs. 8-10 give a comparison between the probability of failure of constraints $g_1 - g_4$ introduced in Eqs. (22)-(25) for various design variables in both probabilistic and deterministic approaches. It should be noted that g_2 is never violated within the range of design variables in neither of approaches, so that no such a figure is illustrated for this constraint. The horizontal red line in Figs. 8-10 exhibits the purported limiting value of the probability of violation for each constraint. Accordingly, none of the proposed optimal trade-off points using NIP and TOPSIS methods and other Pareto optimal points in probabilistic optimization exceed the limitation. Whereas, a portion

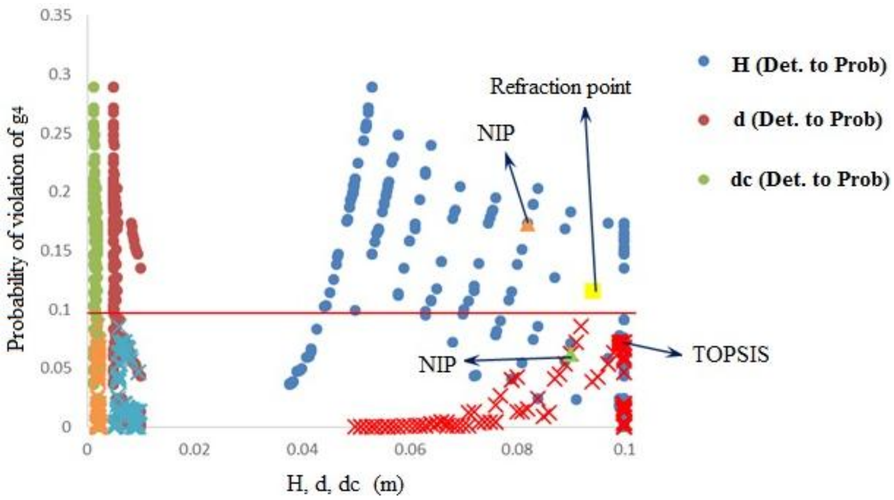


Fig. 10 Relation between design variables and Probability of violation of g_4. Comparison of the present results with the deterministic study. Green triangle refers to NIP point of this study, while the orange one shows that of deterministic study

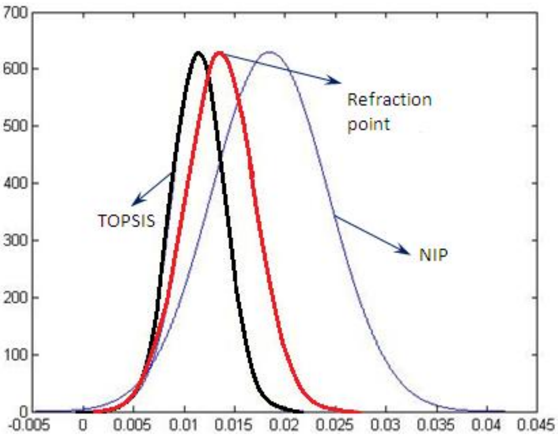


Fig. 11 A comparison between normal (Gaussian) distributions graphs for the mean deflection of the panel for probabilistic optimal trade-off points. NIP and refraction point correspond to the deterministic study

of the optimal design points including the trade-off optimal points found by Khalkhali *et al*. (2014) exceed this limiting line. Fig. 8 indicates that for sandwich panel designs with the face sheet distance higher than 0.0445 m and face sheet thickness higher than 0.005 m, the reliability is guaranteed in terms of the face sheet yielding failure.

The normal (Gaussian) distribution graphs for the mean deflection of the panel for probabilistic optimal trade-off points are shown in Fig. 11. As shown, the optimum point found by TOPSIS in the present study has more robustness and better performance in comparison to that of the deterministic study and can successfully be used by designers. In addition, based on findings in the present study, it is vitally important to consider the effect of uncertainties to achieve a robust and reliable optimum design.

6. Conclusions

Results achieved in this paper can be briefly summarized as:

- Modified NSGA-II algorithm and its coupling with Prob. Design module in ANSYS were successfully employed, and the good efficacy of the corresponding methodology was proved. It was also highlighted that such couplings between FE-based and computational programs can serve as an extremely conducive asset.
- Several non-dominated design points (Pareto points) considering optimum trends in all of the three conflicting objective functions were obtained. These points can be used for design purposes with respect to prevalence of either of objective functions in different applications.
- Using TOPSIS along with NIP methods, several compromising trade-off optimum points were selected and analyzed among all of the optimum points.
- The importance of taking a probabilistic approach instead of a deterministic one in engineering optimization was proved to be significant. This was accomplished through a detailed comparison between results of this study and that of a deterministic-based study with almost similar modeling and problem domain.

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