DOI: http://dx.doi.org/10.12989/gae.2009.1.4.323

Fractal and laboratory analyses of the crushing and abrasion of granular materials

Luis E. Vallejo† and Zamri Chik‡

Department of Civil and Environmental Engineering, University of Pittsburgh, Pittsburgh, PA 15261, USA

(Received October 30, 2008, Accepted November 27, 2009)

Abstract. Gravels forming part of the base of flexible pavements experience abrasion and crushing as a result of static and dynamic loads. Abrasion takes place when the sharp corners of the particles of gravel are removed as a result of compressive and shear loads. As a result of abrasion, the particles change in shape. Crushing is caused by the fragmentation of the particles into a mixture of many small particles of varying sizes. In this study, the abrasion and crushing of gravels are evaluated experimentally and analytically. The laboratory component of this study involves gravels that were subjected to abrasion and dynamic compression tests. The evaluation of the abrasion and crushing experienced by the gravel was carried out using fractals. In this study, the fractal dimension concept from fractal theory is used to evaluate: (a) the changes in shape, and (b) the crushing (fragmentation) of the original particles of gravel. It was determined that the fractal dimension of the profile of the particles decreased as a result of abrasion. With respect to crushing, the fragmentation fractal dimension was found to increase with the degree of breakage of the gravel. To understand the influence of crushing on the permeability of the gravels, the hydraulic conductivity of the gravels was measured before and after crushing. The hydraulic conductivity of the gravels was found to decrease with an increase in their level of crushing. Also, changes in the angle of friction of the granular materials as a result of abrasion was calculated using the Krumbein's roundness chart. The angle of friction of the granular materials was found to decrease as a result of abrasion.

Keywords: crushing; abrasion; granular materials; laboratory analysis; fractals.

1. Introduction

Gravels form part of the base of flexible pavements. These gravels are subjected during their engineering lives to either static and dynamic loads (Brown and Pappin 1981). As a result of these loads, the gravels may experience abrasion and crushing. Because of sustained abrasion and crushing, the original engineering properties with which the base of a pavement structure was designed (i.e. hydraulic conductivity, shear strength, elastic moduli) will change during its engineering life. Changes in the original engineering properties could affect the stability of the structure and could make it unsafe. Thus, there is a need to understand the evolution of abrasion

† Professor, Corresponding author, E-mail: vallejo@pitt.edu
‡ Former Graduate Student

and crushing in granular materials. In this study, the evaluation of abrasion and crushing of gravels is conducted using fractal theory. Laboratory experiments in the form of abrasion and dynamic compression tests are used to induce abrasion and crushing in the gravels.

1.1 The abrasion and crushing of granular materials

Granular materials form part of engineering structures such the base of flexible pavements, highway embankments, and foundations. The granular materials forming part of these structures are subjected during their engineering lives to either static or dynamic loads. As a result of these loads, particle abrasion and particle breakage occur (Lee and Farhoomand 1967, Lade *et al*. 1996, Coop 1999, Bolton 1999, Raymond 2000). According to Lee and Farhoomand (1967) and Coop (1999), particle breakage or crushing seems to be a general feature for all granular materials. Grain crushing is influenced by grain angularity, grain size, uniformity of gradation, low particle strength, high porosity, and by the stress level and anisotropy (Ramamurthy 1969, Bohac *et al*. 2001).

When a granular mass is subjected to a compressive load, the particles resist the load through a series of contacts between the grains. The particles with highly loaded contacts are usually aligned in chains (Cundall and Strack 1979). Crushing starts when these highly loaded particles fail and break into smaller pieces that move into the voids of the original material. This migration causes the settlement of a granular assembly (Fig. 1). Also, on crushing, fines are produced and the grain size distribution curve becomes less steep. Consequently, with continuing crushing, the granular material becomes less permeable and more resistant to crushing. Grain size distribution is a suitable measure of the extent of crushing (Lade *et al*. 1996).

Lade *et al.* (1996) found that if a uniform granular material is crushed, the resulting grain size distribution approaches that of a well graded soil for very large compressive loads. Bolton (1999), and McDowell *et al.* (1996) established that the grain size distribution of a granular assembly that has been crushed under large compressive loads is a fractal distribution. A well graded particle distribution or a fractal distribution represents a granular structure that is made of grains of all sizes

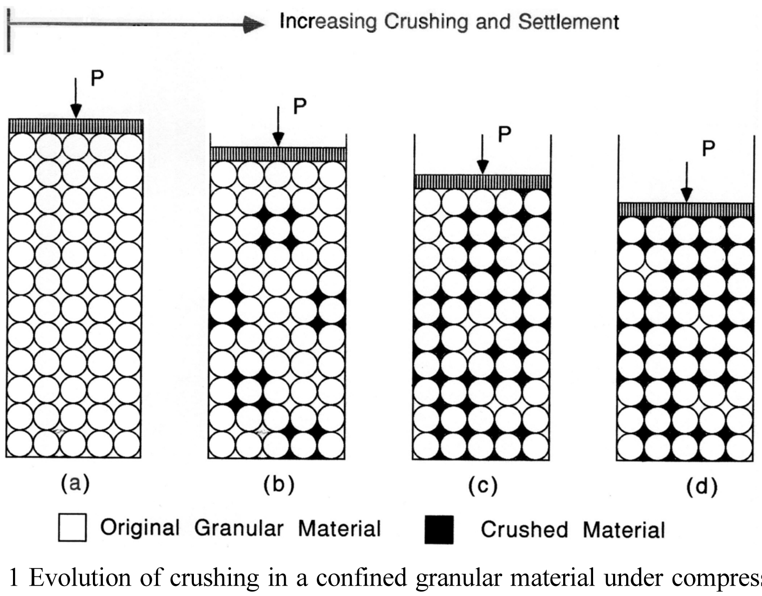


Fig. 1 Evolution of crushing in a confined granular material under compression

including the original unbroken grains.

These original large grains do not break based on the fact that with more small size particles surrounding them, the average contact stress acting on these large grains tends to decrease (Lade *et al*. 1996). However, before the granular structure reaches a well graded or a fractal particle size distribution, the granular structure will experience gradual changes in particle sizes depending on the magnitude of the compressive load applied to it (Cecconi and Viggiani 2001). Triaxial compression tests conducted by Cecconi *et al.* (2002) on crushable granular material (*Pozzolona Nera)* revealed that the material experienced a reduction in their frictional properties as a result of crushing. DeSimone and Tamagnini (2005) developed constitutive models that included crushing for the case of granular materials subjected to varying static triaxial compressive stresses.

Pavements are structures that are difficult to design (Cedergreen 1994). Water enters through their tops, bottoms, and sides, but because pavements are relatively flat, the water flows out again very slowly unless they are well drained under their full width (Cedergreen 1994). The most serious problems occur in asphalt pavements when their granular bases are unable to remove the water that enters the pavement. Fig. 1(a) represents a well drained granular base assuming drainage goes vertically or laterally. In Fig. 1(b) the loose zones that drain the water are interconnected (the dense zones filled with crushed material are not connected). Thus, drainage in the vertical or horizontal direction is still possible. In Figs. 1(c) and 1(d), the loose zones that drain the granular base in either the vertical or horizontal direction are no longer connected. These loose zones must be interconnected in order for water to drain from underneath the pavement. In Figs. 1(c) and 1(d), the dense zones made of crushed material are the ones that are interconnected. The dense zones made of crushed granular material surround and isolate the loose zones that promoted drainage.

Thus, when the granular base reaches the conditions of Figs. 1(c) and 1(d) as a result of crushing, serious problems will develop in pavements. Due to traffic loads, the material in the loose isolated zones will act as closed hydraulic systems that will develop excess pore water pressures, ultimately producing the failure of the granular base as well as the pavement (Cedergreen 1994). Next, a theoretical method based on fractal theory for evaluating abrasion and complete fragmentation in gravels is presented.

2. Fractals and the concept of the fractal dimension

The shape of forms in nature is usually analyzed using Euclidean geometry. According to this kind of geometry, straight lines are perfectly straight lines and curves are arcs of perfect circles. However such perfection is seldom found in natural forms. Most of the time, the shapes of natural forms are irregular. Fractals are a relatively new mathematical concept to describe the geometry of irregularly shaped objects in terms of a fractional number (the fractal dimension) rather than an integer. In this study the fractal dimension concept from fractal theory is used to measure the degree of irregularity of particle profiles. Fractals are also used to evaluate the size distribution in a granular material subjected to varying crushing levels.

2.1 The fractal dimension of closed (particle) profiles: abrasion measurement

Many methods have been developed to measure the fractal dimension of open and closed form profiles such as those constituting part of rock joints, geomembranes, pavements, sands, gravels,

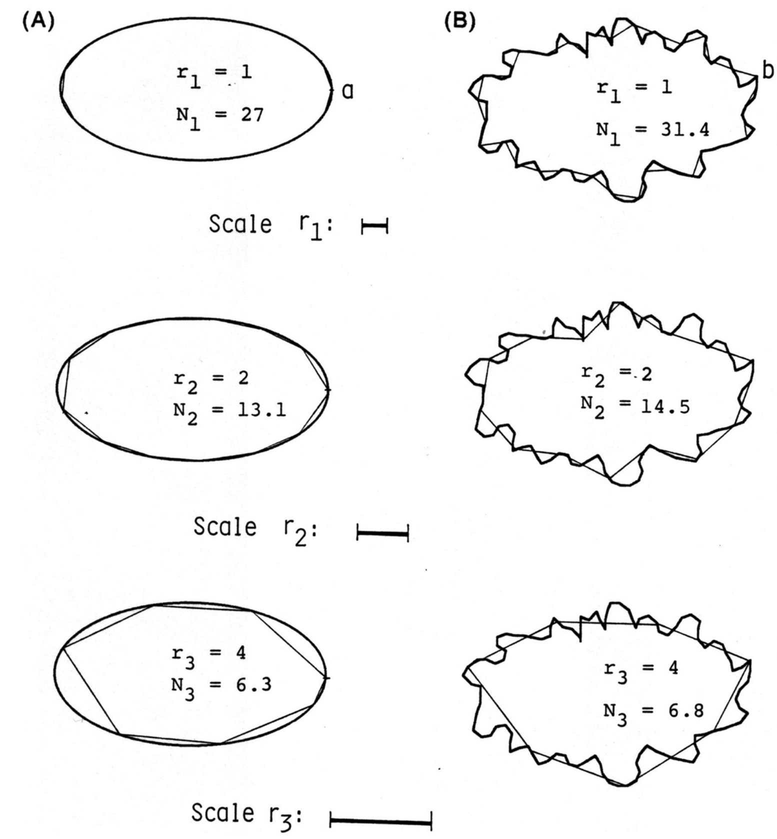


Fig. 2 Smooth and rough particle for fractal analysis. The rough profile represent a particle before abrasion. The smooth profile represents a particle after abrasion.

and voids in soils (Yeggoni *et al.* 1996, Vallejo 2001). The most commonly used methods are: (a) the divider method, (b) the box method, (c) the area-perimeter method, and (d) the spectral method (Vallejo 1995, 1996, Hyslip and Vallejo 1997). Next, the divider method is presented as a way of measuring the fractal dimension of a closed profile (Fig. 2).

Fig. 2 represents the profile of two particles having the same cross sectional area but different profiles. Fig. 2(A) shows the two dimensional profile of a smooth, ellipsoidal particle repeated twice. Fig. 2(B) shows the profile of a rough, ellipsoidal particle, also repeated twice. Suppose we wish to measure the length L of the simple and complex closed profiles shown in Fig. 2 using a ruler or yardstick of fixed length, r. We may begin by setting two arms of a divider to a known distance (step or segment length r) and step off the outline of the profiles as shown in Fig. 2. The length of the profiles, L, is obtained from the product of the number of segments, N, and the chosen segment length, r. Three different segment lengths, r, were used to measure both the simple and complex closed profiles. The scales for the length of these segments are shown in Fig. 2. The number of segments, N, of each length, r, to cover the profile of the particles is also shown in Fig. 2. According to Mandelbrot (1977), if a linear relationship develops between the values N and r when plotted on log-log paper, the profiles analyzed are fractal profiles. The absolute value of the slope of the linear relationship between N and r values represents the fractal dimension, D, of the profiles. The number of segments, N, and the corresponding length of the segments, r, are plotted

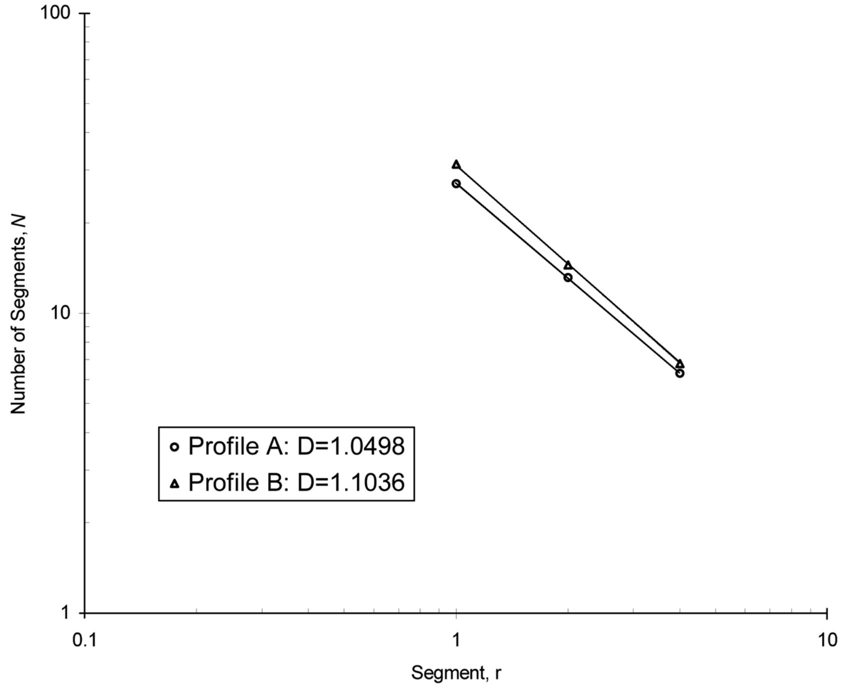


Fig. 3 Fractal dimension, D, for particles shown in Fig. 2

on log-log paper (Fig. 3).

The slope of the best fit line passing through the points relating N and r represents the fractal dimension D of the profiles. As expected, the fractal dimension, D, of the rough profile (Fig. 2(B)) is greater than the fractal dimension, D, for the smooth profile (Fig. 2(A)). The fractal dimension of the rough profile is equal to 1.1036, and the fractal dimension of the smooth profile is equal to 1.0498 (Fig. 3).

Fig. 2(B) can represent the profile of *one* particle *before abrasion* occurs. Fig. 2(A) can represent the profile of *one* particle *after abrasion* occurs. Thus, the fractal dimension concept can be used to measure abrasion in the particles forming part of granular bases under flexible pavements.

Fig. 3 indicates the way to evaluate the fractal dimension, D, for the case of *one* particle. To evaluate the average fractal dimension of a *group of particles*, the area-perimeter method is recommended (Hyslip and Vallejo 1997). The area perimeter-method involves the measurement of the areas and the respective perimeters of the *multiple particles forming a group*. One then plots on log-log paper the areas and the perimeters of the individual particles. The slope, m, of the best fit line passing through the plotted points is used to calculate the average fractal dimension, D, of the group of particles analyzed. The average fractal dimension, D, is equal to the ratio (2/m) (Hyslip and Vallejo 1997). The area-perimeter method is used in this study to measure abrasion levels in gravels.

2.2 Fractal dimension of the grain size distribution: fragmentation measurement

Grain size distribution of naturally occurring soils has been found by Tyler and Wheatcraft (1992) and Hyslip and Vallejo (1997) to be fractal. Tyler and Wheatcraft (1992) have developed a relationship that uses the results of a standard sieve analysis to calculate the fractal dimension, D_F, of the size distribution of natural soils. This relationship is:

$$\frac{M(R<r)}{M_T}=\left(\frac{r}{r_L}\right)^{3-D_F} \tag{1}$$

where $M(R<r)$ is the cumulative mass (weight) of particles with size R smaller (finer) than a given comparative size r; M_T is the total mass (weight) of particles; r is the sieve size opening; r_L is the maximum particle size as defined by the largest sieve size opening used in the sieve analysis; and D_F is the fragmentation fractal dimension. The results of a sieve analysis tests using Eq. (1) can be plotted on log-log paper. The slope, m, of the best fitting line through data obtained using Eq. (2) and the fractal dimension, D_F, are related as follows:

$$D_F = 3 - m \tag{2}$$

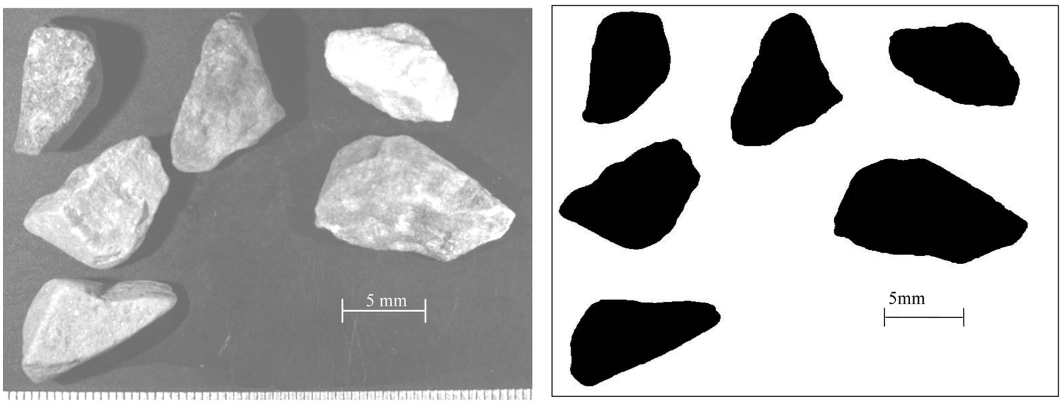
Eqs. (1) and (2) are used to obtain the fractal dimension of the size distribution in a gravel subjected to crushing. The crushing is the result of dynamic compressive loads. Changes in the degree of crushing of the gravel are reflected in the value of the fractal dimension, D_F, of the size distributions.

3. Laboratory tests and related fractal dimensions

3.1 Abrasion tests

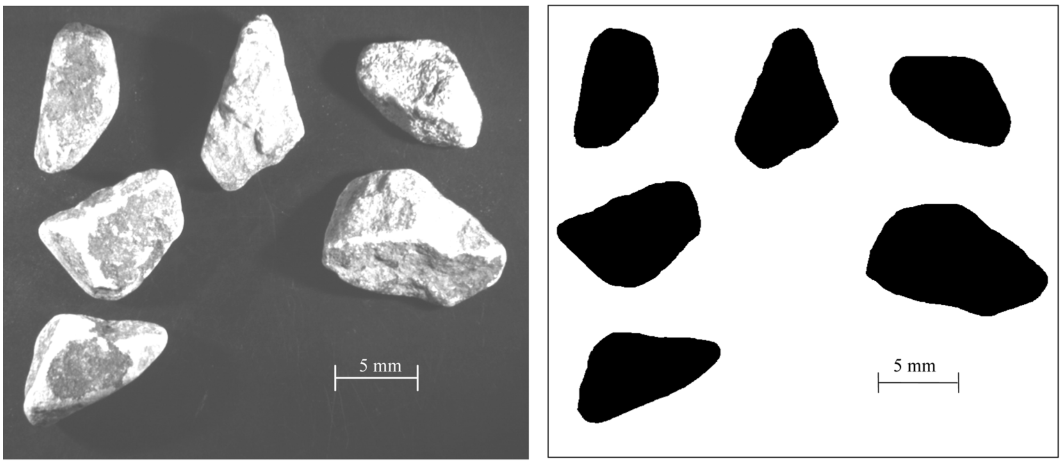
A fine gravel ($d_{50} = 7$ mm) and a specific gravity, Gs, equal to 2.67 was subjected to abrasion tests. The abrasion tests were performed in a cylindrical jar mill made of ceramic material. The jar mill's diameter was equal to 15.24 cm, and its length was also equal to 15.24 cm. Inside the jar mill, there were 65 ceramic balls (charges), each measuring 2 cm in diameter (Fig. 4). Twenty pieces of the gravel under dry conditions were placed inside the cylinder together with the ceramic balls. After this was done, the cylinder was rotated for a period of 1 hour. The frequency of rotation

Fig. 4 Jar milling equipment used in the abrasion tests



(a) (b)

Fig. 5 Rough gravel before the abrasion test: (a) real and (b) binary image



(a) (b)

Fig. 6 Rough gravel after the abrasion test: (a) real and (b) binary image

was 50 rpm. During rotation, the interaction between the ceramic balls and the gravel caused the abrasion of the surface of the particles of gravel. This abrasion produced small changes in the profile of the rock samples. Fig. 5 shows a photograph of six rock pieces before the abrasion test (real and binary images). Fig. 6 shows the same rock pieces after the abrasion tests (Real and binary images).

The binary images of the photographs were then used to obtain the fractal dimension of the group. These fractal dimension values were obtained using ImageJ, and open source software. The fractal dimension of each group of gravel shown in Figs. 5 and 6 was obtained using the area-perimeter method, as previously explained (Figs. 7 and 8). An analysis of Figs. 7 and 8 indicates that as a result of abrasion, the fractal dimension of the gravel profiles decreased from 1.0588 to 1.0106.

3.2 Crushing tests

For the crushing tests, the rough gravel shown in Fig. 5 was used. The gravel was placed in a

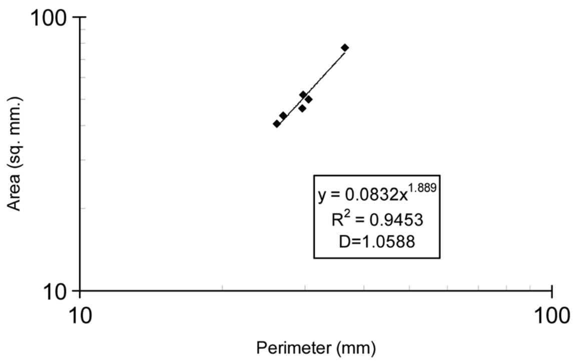


Fig. 7 The area-perimeter method to obtain the fractal dimension, *D*, for gravel before abrasion tests (Fig. 5)

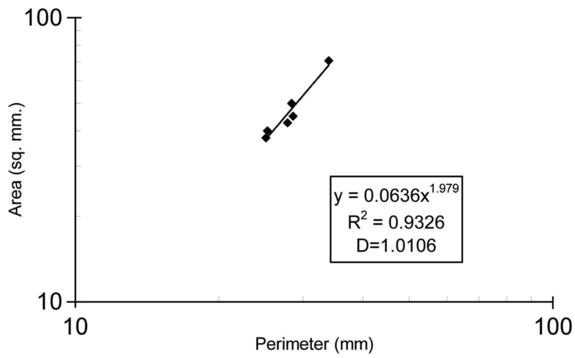


Fig. 8 The area-perimeter method to obtain the fractal dimension, *D*, for gravel after abrasion test (Fig. 6)

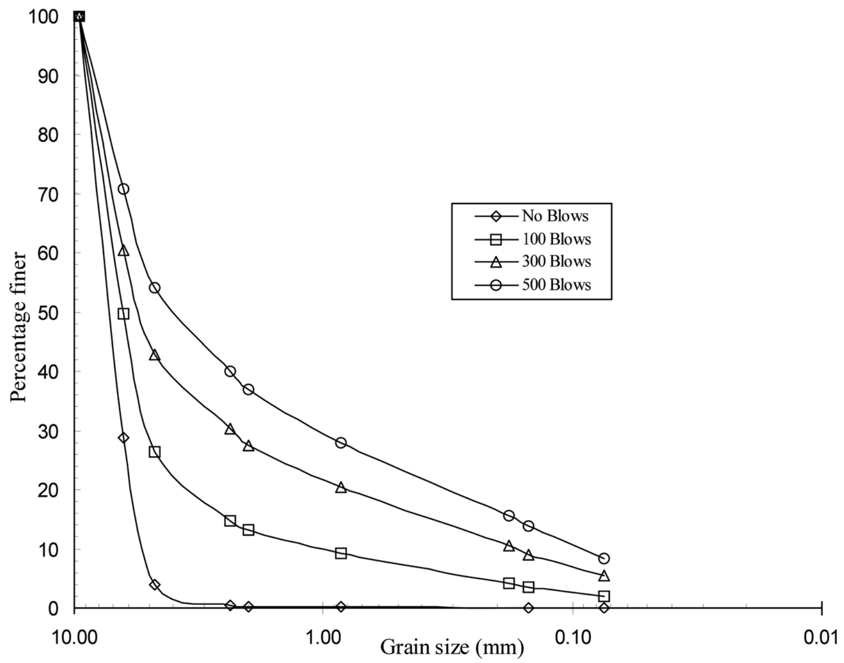


Fig. 9 Grain size distribution before and after dynamic compressive crushing

metallic cylinder having a diameter equal to 15 cm. The height of the sample in the cylinder measured 20 cm. Using a Standard Proctor hammer, the gravel was subjected to 100, 300, and 500 compressive blows. As a result of the dynamic compressive stresses induced by the falling of the hammer on the samples (simulating traffic loads), the gravel experienced fragmentation. A sieve analysis was conducted on the samples before and after the dynamic compression. The result of the sieve analysis is shown in Fig. 9.

Using the results presented in Fig. 9 together with Eqs. 1 and 2, the fragmentation fractal dimensions, D_f, were obtained. The fragmentation fractal dimensions before and after the crushing tests are shown in Fig. 10. An analysis of Fig. 10 indicates that the fragmentation fractal dimension

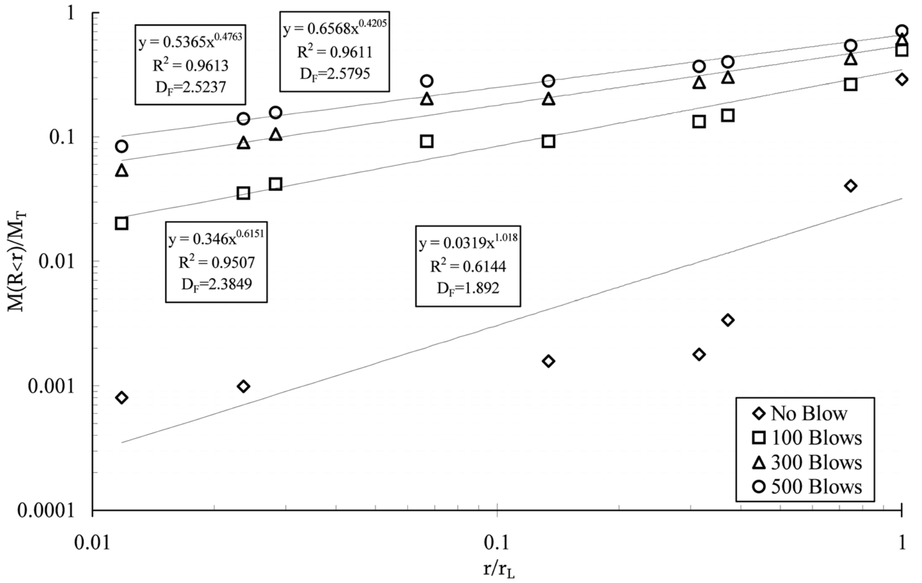


Fig. 10 Fractal fragmentation calculation for the gravels before and after their dynamic crushing

values, D_f, increased in value with the number of blows delivered to the samples. The fragmentation fractal dimension, D_f, for the original sample was equal to 1.892. It increased to 2.3849 after 100 blows, to 2.5237 after 300 blows, and to 2.5795 after 500 blows. In other words, the size distribution became increasingly more fractal with the level of dynamic compression exerted on the gravel.

3.3 Hydraulic conductivity tests

Before and after each of the crushing tests, constant head hydraulic conductivity tests were performed on the samples of gravel. The testing procedure follows the recommendation of the

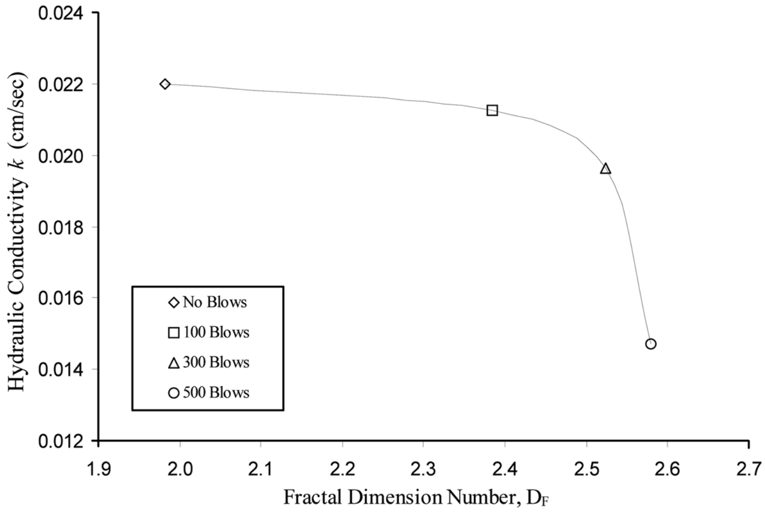


Fig. 11 Relationship between the hydraulic conductivity of the samples and their fragmentation fractal dimension

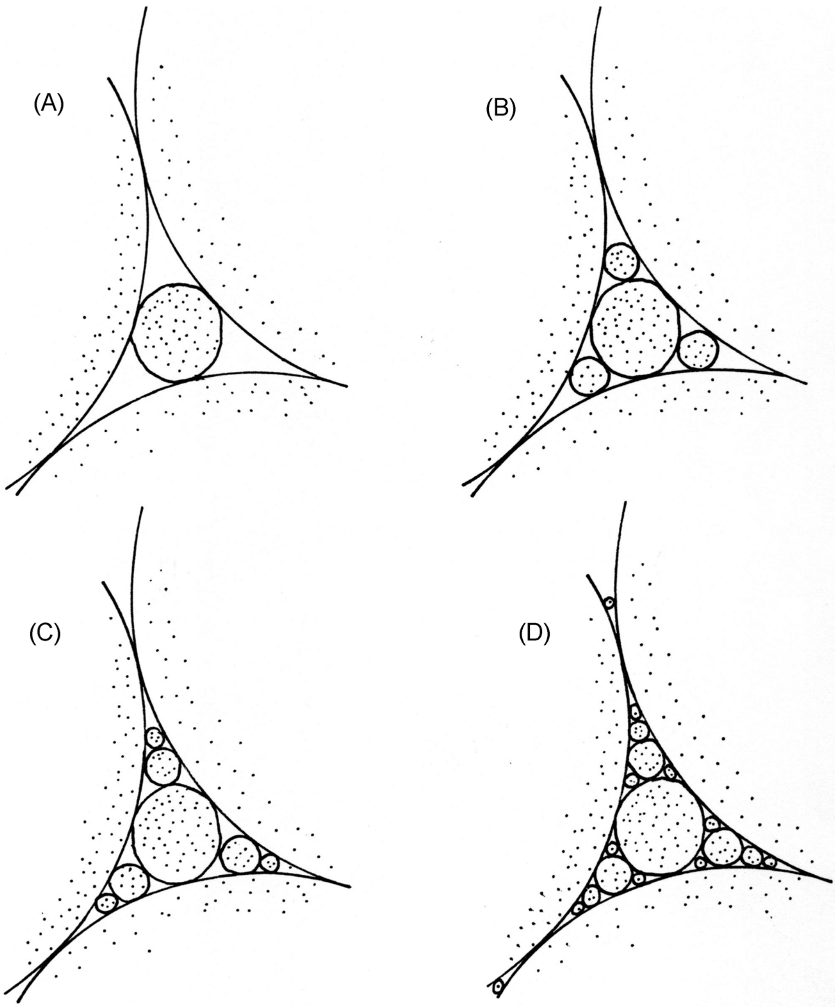


Fig. 12 A pore being filled by a fractal granular material

ASTM D 2434. Fig. 11 shows a plot of the hydraulic conductivity tests and the fragmentation fractal dimension values. Fig. 11 indicates that the hydraulic conductivity values decreased with the degree of fragmentation of the gravels in the crushing tests. The degree of fragmentation of the gravels was measured by their fragmentation fractal dimension, D_f.

The changes in hydraulic conductivity experienced by the gravel under the compressive stresses in the compaction cylindrical apparatus and the influence that these changes have on the hydraulic conductivity of the crushed gravel can be best explained using Fig. 12. This figure shows what happens to a pore located within three large gravel particles when it is gradually filled by smaller and smaller grains resulting from the gradual crushing of the larger particles. The particles shown in Fig. 12 are self-similar with respect to their sizes and represent a gravel with a fractal size distribution. Fig. 12(a) shows the pore between the three grains when it is filled by one small grain. The same pore continues to be filled by smaller and smaller grains as one goes from Fig. 12(b) to Fig. 12(d). The pore space decreases gradually until it becomes completely blocked (Fig. 12(d)). Thus, when the pore reaches the condition shown in Fig. 12(d), water can not move through the pore. Thus, the filling of the pore space by a smaller gravel particles with a fractal size distribution will influence the hydraulic conductivity of the pore and the sand that contains it.

4. Friction angle of the gravel before and after abrasion tests

Next, the influence of abrasion on the fundamental angle of frictional resistance, ϕ, of the gravels is analyzed. The shape of granular materials can be described by either their fractal dimension, D, (Figs. 7 and 8) or by the use of the Krumbein's angularity or roundness number, R. Krumbein (1941) has produced a chart that gives values of the roundness number, R. for nine group of particles. This chart is shown in Fig. 13. Depending of the degree of roughness of the particle profiles, their angularity or roundness number R changes from, a value of 0.1 for very rough particles, to a value of 0.9 for particles with round profiles (Fig. 13).

Santamarina and Cho (2004) has produced a relationship between the friction angle between the particles shown in Fig. 13 and the roundness number R. This relationship is as follows:

$$\phi = 42 - 17R \tag{1}$$

In order to obtain the angle ϕ for the rough gravels before and after the abrasion test (Figs. 5 and 6), one needs to obtain their roundness numbers, R. To obtain the roundness numbers, the binary images of the gravel shown in Figs. 5(b) and 6(b) were used. For the rough gravels before the abrasion test a value of R = 0.3 is obtained when one compares Fig. 5(b) with Fig. 13. Using this R

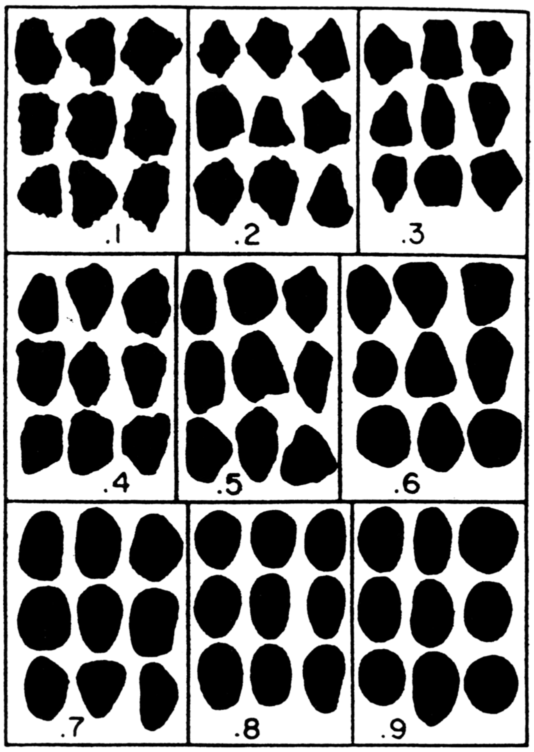


Fig. 13 Krumbein's chart with roundness numbers, R (Krumbein 1941)

value and Eq. (1), the ϕ for the gravel before the abrasion test is equal to 36.9 degrees. For the case of the rough gravel after the abrasion test a value of $R = 0.6$ is obtained after comparing Figs. 6(b) and 13. Using this R value and Eq. (1), the ϕ for the gravel after the abrasion test is equal to 31.8 degrees. Therefore, abrasion caused a decrease in the fundamental angle of friction between the particles. This fundamental angle of friction is important in the calculation of the angle of repose of granular materials (Santamarina and Cho 2004).

With respect to the crushing tests on the gravels, the changes in their angle of internal friction as a function of the intensity of crushing were not measured. However, Discrete Element simulations of the crushing of granular materials in the direct shear test conducted by Lobo-Guerrero and Vallejo (2005) as well as triaxial compression tests on crushable granular materials conducted by Cecconi *et al*. (2002) indicates that the angle of friction of granular materials decreases with an increase in their degree of crushing.

5. Conclusions

The fractal dimension concept from fractal theory has been presented to evaluate abrasion and fragmentation of granular materials. Abrasion tests changed the profile of the gravel from a rough profile to a smoother one. These changes in profile were reflected by their fractal dimension values. Fragmentation was produced by conducting dynamic compression tests on fine gravel. As a result of the compressive loads, the size distribution of the gravel changed from that of a low fractal material to that of a high fractal one. The changes in the particle size distribution in the sand had a large influence on the hydraulic conductivity. The hydraulic conductivity decreased as the particle size distribution changed from a low fractal distribution to a high fractal one. Also, the effect of abrasion influenced the angle of friction of the gravel. An increase in abrasion caused a decrease in their angle of internal friction.

Acknowledgements

The work described in this study was sponsored by Grants CMS-0124714 and CMS-0301815 to the University of Pittsburgh from the National Science Foundation, Washington, D.C. This support is gratefully acknowledged.

References

Bohac, J., Feda, J. and Kuthan, B. (2001), "Modelling of grain crushing and debonding", *Proceedings of the 15th International Conference on Soil Mechanics and Geotechnical Engineering*, Istanbul, August.

Bolton, M.R. (1999), "The role of micro-mechanics in soil mechanics", *Proceedings of the International Workshop on Soil Crushability*, Yamaguchi, July.

Brown, S.F. and Pappin, J.W. (1981), "Analysis of pavements with granular bases", *Transport. Res. Rec.*, **810**, 17-23.

Cedergren, H.R. (1994), "America's pavements: world's longest bathtubs", *Civil Eng*., **64**(9), 56-58.

Cecconi, M. and Viggiani, G.M.B. (2001), "Structural features and mechanical behaviour of a pyroclastic weak rock", *Int. J. Numer. Anal. Meth. Geomech.*, **25**, 1525-1557.

Cecconi, M., DeSimone, A., Tamagnini, C. and Viggiani, G.M.B. (2002), "A constitutive model for granular material with grain crushing and its application to pyroclastic soil", *Int. J. Numer. Anal. Meth. Geomech.*, **26**, 1531-1560.

Coop, M.R. (1999), "The influence of particle breakage and state on the behavior of sands", *Proceedings of the International Workshop on Soil Crushability*, Yamaguchi, July.

Cundall, P.A. and Strack, O.D.L. (1979), "A discrete numerical model for granular Assemblies", *Geotechnique*, **29**(1), 47-65.

DeSimone, A. and Tamagnini, C. (2005), "Stress-dilatancy based modeling of granular materials and extensions to soils with crushable grains", *Int. J. Numer. Anal. Meth. Geomech.*, **29**, 73-101.

Hyslip, J.P. and Vallejo, L.E. (1997), "Fractal analysis of the roughness and size distribution of granular materials", *Eng. Geol.*, **48**(3-4), 231-244.

Krumbein, W.C. (1941), "Measurement and geological significance of shape and roundness of sedimentary particles", *J. Sediment, Pet.*, **11**, 64-72.

Lade, P.V., Yamamuro, J.A. and Bopp, P.A. (1996), "Significance of particle crushing in granular materials", *J. Geotech. Eng.*, **122**(4), 309-316.

Lee, K.L. and Farhoomand, J. (1967), "Compressibility and crushing of granular soils in anisotropic triaxial compression", *Can. Geotech. J.*, **4**(1), 68-86.

Lobo-Guerrero, S. and Vallejo, L.E. (2005), "DEM simulation of crushing of granular materials under direct shear stress conditions", *J. Geotech. Geoenviron*, **131**(10), 1295-1300.

Mandelbrot, B.B. (1977), *Fractals: forms, chance and dimension.* San Francisco, Freeman.

McDowell, G.R., Bolton, M.D. and Robertson, D. (1996), "The fractal crushing of granular materials", *Int. J. Mech. Phys. Soild.*, **44**(12), 2079-2102.

Ramamurthy, T. (1969), "Crushing phenomena in granular soils", *J. Indian Soc. Soil Mech. Found. Eng.*, **8**(1), 67-86.

Raymond, G.P. (2000), "Track and support for a mine company railroad", *Can. Geotech. J.*, **37**, 318-332.

Santamarina, J.C. and Cho, G.C. (2004), "Soil behaviour: the role of particle shape", *Proceedings of the Skempton Conference*, London, March.

Tyler, S.W. and Wheatcraft, S.W. (1992), "Fractal scaling of soil particle-size distibution analysis and limitations", *Soil. Sci. Soc. Am. J.*, **56**(2), 47-67.

Vallejo, L.E. (1995), "Fractal analysis of granular materials", *Geotechnique*, **45**(1), 159-164.

Vallejo, L.E. (1996), "Fractal analysis of the fabric changes in a consolidating clay", *Eng. Geol.*, **43**, 281-290.

Vallejo, L.E. (2001), "Fractal assessment of the surface texture of pavements", *Int. J. Pavement. Eng.*, **2**(2), 149-156.

Yeggoni, M., Button, J.W. and Zollinger, D.G. (1996), "Fractals of aggregates correlated with creep in asphalt concrete", *J. Transp. Eng-ASCE*, **122**(1), 22-27.

CC