

Searching for critical failure surface in slope stability analysis by using hybrid genetic algorithm

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(Received January 9, 2009, Accepted March 5, 2009)

Abstract. The radius and coordinate of sliding circle are taken as searching variables in slope stability analysis. Genetic algorithm is applied for searching for critical factor of safety. In order to search for critical factor of safety in slope stability analysis efficiently and in a robust manner, some improvements for simple genetic algorithm are proposed. Taking the advantages of efficiency of neighbor-search of the simulated annealing and the robustness of genetic algorithm, a hybrid optimization method is presented. The numerical computation shows that the procedure can determine the minimal factor of safety and be applied to slopes with any geometry, layering, pore pressure and external load distribution. The comparisons demonstrate that the genetic algorithm provides a same solution when compared with elasto-plastic finite element program.

Keywords: hybrid genetic algorithm; slope stability; critical factor of safety; hybrid optimization.

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1. Introduction

The stability of slopes has received wide attention due to its practical importance in the design of excavations, embankments, earth and rock-fill dams and tailing dams. There are numerous methods available for the stability analysis of slopes. The majority of slope stability analyses performed in practice still use traditional limit equilibrium approaches. In a survey of equilibrium methods of slope stability analysis reported by Duncan, the characteristics of a large number of methods were summarized (Griffiths 1999), including the ordinary method of slices (Fellenius 1936), Bishop's Modified Method (Bishop 1955), force equilibrium methods (Lowe and Karafiath 1960), Janbu's generalized procedure of slices (Janbu 1968), Morgenstern and Price's method (Morgenstern and Price 1965) and Spencer's method (Spencer 1967). Some of these methods satisfy only overall moment, like the ordinary and simplified Bishop methods and are applicable to a circular slip surface, while Janbu's method satisfies only force equilibrium and is applicable to any shape slope. Spencer's method, however, satisfies both moment and force equilibrium and it is applicable to failure surface of any shape. It is considered as one of the rigorous and accurate methods for solving stability problems. A difficulty with all the equilibrium methods is that they are based on the assumption that the failing soil mass can be divided into slices (Al-Karni 2000). The general procedure in all these methods may be summarized as follows: 1) the postulation of a slip surface; 2) the static analysis of the shear stress applied to the slip surface; 3) the calculation of the factor of safety k , which is defined as the ratio of the shearing strength available to the applied shear stress; and 4) the determination of the critical slip surface giving the minimum $k_{\min}$ by performing multiple searches. Considerable work has been done on searching for critical factor of safety in slope stability analysis by using numerical methods (Chugh 1981). Uncertainty and reliability analysis applied to slope stability problems have been respectively developed by Juang (1998) and Malkawi (2000). And model uncertainty is addressed by evaluating the relative performance of the slope stability methods, which include Ordinary, Bishop, Janbu and Spencer's method. Chen (2001) proposed a three-dimensional slope stability analysis method by using the upper bound theorem and used the simulated annealing algorithm to search for the critical failure mode. The dynamic programming method was developed by Pham (2003) for analyzing the slope stability.

Existence of more than one numerical solution to the slope stability problems derived on the basis of traditional nonlinear optimization techniques is indicated. The main shortcoming of these techniques lies in the uncertainty as to robustness of the algorithms to locate the global minimum factor of safety rather than the local minimum factor of safety for complicated and non-homogenous geological subsoil conditions. Attempts at using traditional optimization method were unsuccessful and failed to converge to global minimum for factor of safety (Anthony 1999). Genetic algorithm is an approach to optimization based upon the concepts of genetics, in which an optimum solution evolves through a series of generations. Genetic algorithm has the super ability of global convergence and parallel searching so that the problem of local optimum can be avoided (Caosta 2001). The objective of this paper is to describe that there exist a few of solutions for the complicated geotechnical slopes, but only one solution is the global minimum for factor of safety and the corresponding slip surface is called the critical slip surface. The other objective of this paper is to study the use of hybrid genetic algorithm in solving practical slope stability problems. In order to demonstrate the effectiveness of proposed searching method, numerous example problems have been solved by using genetic algorithm and elasto-plastic finite element program.

2. Computation for the stability analysis of slopes

Most problems in slope stability are statically indeterminate, and as a result, some simplifying assumptions are made in order to determine a unique factor of safety. Due to the differences in assumption, various methods have been developed. For the ordinary method of slices, which is considered the simplest method of slices, the factor of safety is directly obtained. The method assumes that the inter-slice forces are parallel to the base of each slice (Makawi 2000). Slope stability analysis using the methods of slices involves passing a slip surface through the earth mass and dividing the inscribed portion into vertical slices, shown as Fig. 1. According to the Bishop's simplified method of slice, the safety factor is determined by the equation (Al-Karni 2000)

$$k = \sum \frac{[(W_n - u_n b_n) \tan \varphi_n + c_n b_n]}{(\sin \theta_n + \cos \theta_n \tan \varphi_n / k)} / \sum (W_n \cos \theta_n) \quad (1)$$

Where n is the slice number, w_n represents the total weight of slice, b_n denotes the width of slice, φ_n is angle of friction, θ_n is the angle between the horizontal and the line connecting the midpoint of the base of the slice and the center of rotation of failure circle, c_n is the cohesion of the soil for slice, u_n is the pore pressure and k is the factor of safety.

In Bishop's method the factor of safety is determined by trail and error, using an iterative process, since the factor of safety appears in both sides of equation. The inter-slice shear forces are neglected, and only normal forces are used to determine the inter-slice forces. When taking into account of earthquake action, the earthquake inertial force is represented using horizontal seismic coefficient k_h , and the safety factor is expressed as follows

$$k = \frac{\sum \frac{1}{m_{\theta_n}} \left\{ c_n b_n + \left[W_n \left(1 \pm \frac{1}{3} k_H C_z \right) - u_n b_n \right] \tan \varphi_n \right\}}{\sum \left\{ \left(W_n \left(1 \pm \frac{1}{3} k_H C_z \right) \sin \theta_n \right) + \sum W_n k_H C_z \frac{e_n}{R} \right\}} \quad (2)$$

$$m_{\theta_n} = \cos \theta_n + \frac{\tan \varphi_n \sin \theta_n}{k} \quad (3)$$

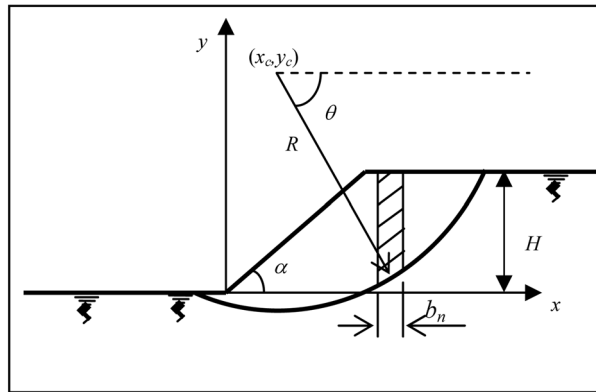


Fig. 1 General slope stability problem description

Where R is the radius of slip circle; e_n is the vertical distance from the slice center to slip center. k_H is horizontal seismic coefficient; C_z is the comprehensive affection coefficient of earthquake action. Spencer developed a slope stability analysis technique based on the method of slices, which satisfies all equilibrium equations. Spencer's method is suitable for a failure surface of arbitrary shape (Bolton 2003).

When the coordinates of the circle center and radius of slip surface are determined, the factor of safety for slope stability can be calculated according to Eq. (1) or Eq. (2). The location of the critical failure surface can be viewed as a form of nonlinear and non-smooth global optimization problem and the objective function to be minimized is the factor of safety function. The critical slip surface is defined as the slip surface that yields the minimum value of safety factor in slope stability analysis. Minimization techniques using optimization methods have been proposed to search for the critical slip surface in slope stability computations. The objective function of the safety factor k is non-smooth and can be non-convex in nature. The constraints which include kinematically acceptable shapes of failure surfaces, rock and soil profile may also be non-smooth, non-convex functions. It appears that the existence of multiple local minima is the fundamental feature of slope stability problem (Cheng 2003).

The shear strength reduction technique is a commonly used method of conducting slope stability analysis using the finite element method (FEM) or the finite difference method. This technique was used as early as 1975 by Zienkiewicz *et al.*, and has since been applied by Duncan, Matsui and San, Ugai and Leschinsky, Griffiths and Lane, Dawson *et al.* and many others. Many experiences have been accumulated and detailed comparisons between the strength reduction technique and the limit equilibrium methods have been made. The progress made in this study lies in two aspects. First, it proves that any elasto-plastic material must obey Mohr-Coulomb's yield criterion. Second, an initial value problem of a system of ordinary differential equations for critical slide lines is formulated and a robust numerical procedure for the initial value problem is proposed (Zheng 2005).

For slopes, the factor of safety k is traditionally defined as the ratio of the actual soil shear strength to the minimum shear strength required to prevent failure. As Duncan bring forward that k is the factor by which the soil shear strength must be divided to bring the slope to the verge of failure. Since it is defined as a shear strength reduction factor, the way of computing k with the finite element is simply to reduce the soil shear strength until collapse occurs. So the result of the k is the ratio of the soil's actual shear strength to reduced shear strength at failure. In the mid 1970s, techniques for applying the FEM to slope stability analysis started appearing in geotechnical literature. They were mostly based on an approach that flows naturally from the definition of slope factor of safety, and is now commonly referred to as the Shear Strength Reduction (SSR) technique. By definition, the factor of safety of a slope is the "ratio of actual soil shear strength to the minimum shear strength required to prevent failure," or the factor by which soil shear strength must be reduced to bring a slope to the verge of failure. In the SSR finite element technique elasto-plastic strength is assumed for slope materials. The material shear strengths are progressively reduced until collapse occurs. The parameter, c_f and φ_f , are given by

$$c_f = c/k \quad (4)$$

$$\varphi_f = \arctan\left(\frac{\tan \varphi}{k}\right) \quad (5)$$

For Mohr-Coulomb materials, the steps for systematically searching for the critical factor of safety value, k , which brings a previously stable slope to the verge of failure, are as follow: Step 1: Develop an FE model of a slope, using the deformation and strength properties established for the slope materials. Compute the model and record the maximum total deformation in the slope. Step 2: Increase the value of F and calculate factored Mohr-Coulomb material parameters as described above. Enter the new strength properties into the slope model and re-compute. Record the maximum total deformation. Step 3: Repeat Step 2, using systematic increments of k , until the FE model does not converge to a solution, i.e., continue to reduce material strength until the slope fails. The critical k value just beyond which failure occurs will be the slope factor of safety.

3. Application of genetic algorithm to the stability analysis of slopes

Many approaches have been developed to automate the search for the critical slip surface. This is not too complicated for circular slip surface because it only involves the optimization of three variables, which consist of the horizontal and vertical coordinates of the slip surface and the radius of the circle. Traditional mathematical optimization methods that have been used include dynamic programming, conjugate-gradient, random search, and simplex optimization. The gradient searching algorithms can not guarantee that the critical failure surface as obtained is the global minimum because the multiple minima exist in a feasible solution domain. The simple genetic algorithm has been applied to slope stability analysis (McCombie 2002, Zolfaghari 2005).

Let m represent the optimization variable vector, $m = \{x_c, y_c, R_c\}^T$, and x_c , y_c and R_c denote x and y coordinate and radius of slip surface, respectively. The solution of the searching problem of slope stability analysis consists in obtaining a minimum of an objective function, which is the factor of safety for slope stability analysis

$$\min \{k(m), \quad m \in \mathbb{R}^P; g^j < 0\} \quad (6)$$

Where m represents the variable vector, which belongs to the space of admissible parameters $\mathbb{R}^P$, g^j are inequality constraints, which define the feasible domain D :

$$D = \{m \in \mathbb{R}^P, g^j < 0\} \quad (7)$$

Genetic algorithm (GA) is a search method based on Darwin's theory of evolution and survival of the fittest. Based on the concept of genetics, GA simulates the evolutionary process numerically. Genetic algorithms strongly differ in conception from other search methods, including traditional optimization methods and other stochastic search methods. The basic difference is that while other methods always process single points in the search space, genetic algorithms maintain a population of potential solutions (Firswell 1998). Genetic algorithms constitute a class of search methods especially suited for solving complex optimization problems. Search algorithms in general consist of systematically walking through the search space of possible solutions until an acceptable solution is found. Genetic algorithms transpose the notions of natural evolution to the world of computers, and imitate natural evolution. They were initially introduced by John Holland for explaining the adaptive processes of natural systems and for creating new artificial systems that work on similar bases. In Nature new organisms adapted to their environment develop through evolution. The genetic algorithm has been widely used in the identification, short-term load forecasting, the design optimization, dynamic channel assignment, the parameter identification of inelastic constitutive

models (Costa 2001, Friswell 1998, Garcia 1998). Genetic algorithms evolve solutions to the given problem in a similar way. The main evolutionary processes of genetic algorithm include the evaluation of fitting function, selection operation, crossover operation, mutation and elitist strategy.

The probability of survival of any individual is determined by its fitness: through evolution the fitter individuals overtake the less fit ones. In order to evolve good solutions, the fitness assigned to a solution must directly reflect its 'goodness', i.e. the fitness function must indicate how well a solution fulfills the requirements of the given problem. The evaluation of the fitness can be conducted with a linear scaling, where the fitness of each individual is calculated as the worst individual of the population subtracted from its objective function value. Fitness assignment can be performed in several different ways: We define a fitness function and incorporate it in the genetic algorithm. When evaluating any individual, this fitness function is computed for the individual.

$$f_j = \max\{k_j | j = 1, 2, \dots, S\} - k_j \quad (8)$$

Where f_j is the fitting function; S is the population size. k_j is the factor of safety of j -th individual.

Selection, also called reproduction, is simply the copying of quality solution in proportion to their effectiveness. Here, since the goal is to minimize the objective function, several copies of candidate solutions with small objective functions are made; solutions with large objective functions tend not to be replicated. The intrinsic principle of the genetic algorithm is Darwin's natural selection principle. Selection is the impetus of the genetic algorithm, by which, the superior individual are selected into the next generation while the inferior ones are washed out. A part of the new population can be created by simply copying without change selected individuals from the present population. This gives the possibility of survival for already developed fit solutions. The selection pressure is the intrinsic criteria, if the pressure is too excessive, the searching process will end in premature, while, if the pressure is too less, the convergent speed will be too low. The selection process can produce a new population, extracting with repetition individuals from the old population.

In tournament selection, a set of n individuals are chosen from the population at random. Then the best of the pool is selected. The higher is the value of fitness value, the more directed the selection is towards better individuals. The extraction can be carried out in several ways. Another selection is ranked selection. The problem of fitness-proportional selection is that it is directly based on fitness. In most cases, we cannot define an accurate measure of goodness of a solution, so the assigned fitness value does not express exactly the quality of a solution. Still, an individual with better fitness value is a better individual. In rank based selection, the individuals are ordered according to their fitness. The individuals are then selected with a probability based on some linear function of their rank. Fitness-proportional selection is commonly used in the population reproduction. When using this selection method, a solution has a probability of selection directly proportional to its fitness. The mechanism that allows fitness proportional selection is similar to a roulette wheel that is partitioned into slices. Each individual has a share directly proportional to its fitness. When the roulette wheel is rotated, an individual has a chance of being selected corresponding to its share. One of the most commonly used is the roulette wheel selection, where individuals are extracted in probability following a Monte Carlo procedure. The extraction probability of each individual is proportional to its fitness as a ratio to the average fitness of all the individuals. In the selection process, the reproduction probabilities of individuals are given by their relative fitness

$$p_i = f_i / \sum f_i \quad (9)$$

Where p_i is the reproduction probability of the i th individual.

Recombination, also called crossover, is a process by which information contained in two candidate solutions is combined. In the recombination, each individual is first paired with an individual at random. New individuals are generally created as offspring of two parents (as such, crossover being a binary operator). One or more so-called crossover points are selected (usually at random) within the chromosome of each parent, at the same place in each. The parts delimited by the crossover points are then interchanged between the parents. The individuals resulting in this way are the offspring. Beyond one point and multiple point crossover there exist more sophisticated crossover types. Let a pair of present individuals be given by $[m'_\alpha, m'_\beta]$. a new pair $[m^{t+1}_\alpha, m^{t+1}_\beta]$ is then created in terms of a phenomenological recombination formula (Costa 2001)

$$m^{t+1}_\alpha = (1 - \mu)m^t_\alpha + \mu \cdot m^t_\beta \quad (10)$$

$$m^{t+1}_\beta = (1 - \mu)m^t_\beta + \mu \cdot m^t_\alpha \quad (11)$$

Where μ is a random number changing from 0 to 1.

A new individual is created by making modifications to one selected individual. The modifications can consist of changing one or more values in the representation or in adding/deleting parts of the representation. In genetic algorithms mutation is a source of variability, and is applied in addition to crossover and reproduction. Mutation is a process by which vectors resulting from selection and recombination are perturbed. The mutation is conducted with only a small probability by definition. An individual, after this mutation, m^{t+1}_i , is described as

$$m^{t+1}_i = rand\{m_{down}, m_{up}\} \quad (12)$$

Where m_{down} and m_{up} represent lower and upper bounds of parameters; $rand\{.\}$ represents the random selection from the reasonable solution domains. At different stages of evolution, one may use different mutation operators. At the beginning mutation operators resulting in bigger jumps in the search space might be preferred. Later on, when the solution is close by, a mutation operator leading to slighter shifts in the search space could be favored. However, the above mutation operation is a random one with no clear aim.

Simulated annealing is another important algorithm which is powerful in optimization and high-order problems (Alkhanmis 1999). It uses random processes to help guide the form of its search for minimal energy states. Simulated annealing is a generalization of a Monte Carlo method for examining the equations of state and frozen states of n-body systems. The concept is based on the manner in which liquids freeze or metals re-crystallize in the process of annealing. In an annealing process a melt, initially at high temperature and disordered, is slowly cooled so that the system at any time is approximately in thermodynamic equilibrium (Sahab 2005). As cooling proceeds, the system becomes more ordered and approaches a “frozen” ground state at $T=0$. The paper provides a mutation method based on the simulating annealing algorithm, which makes the average fitness of the population tend to be optimized. Firstly, we define a neighborhood structure, then select a new solution in the neighborhood structure of the intermediate solution, that is to say, getting a new solution by cause a disturb on the old one.

$$m_{new} = m_{old} + \Delta m \quad (13)$$

Where, Δm is a random disturb. Then, reject or accept the new solution according the Metropolis rule, the probability of accepting the new generated solution is expressed as the follows (Jeong 1996)

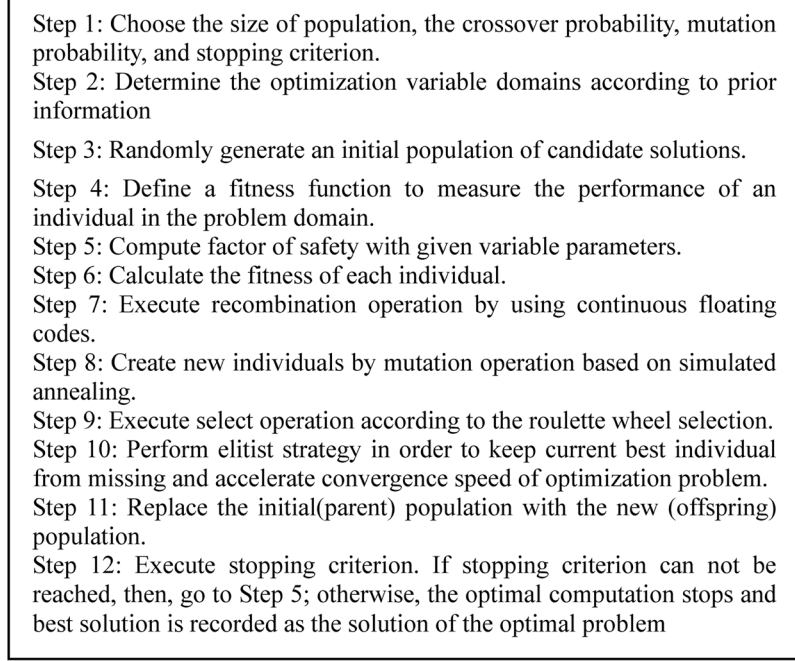


Fig. 2 Fundamental structure of hybrid genetic algorithm with simulated annealing

$$p_{new} = \begin{cases} 1 & k_{old} \geq k_{new} \\ \exp[-\delta k / T_k] & k_{old} < k_{new} \end{cases} \quad (14)$$

where, δk is the increasement of the objective function, $\delta J = k_{new} - k_{old}$, T_k is the annealing temperature, which tends to be dropped during the evolutionary process. The probability of rejecting the new solution is

$$P_{old} = 1 - p_{new} \quad (15)$$

One feature that is currently missing in this selection procedure is that it does not guarantee the best individual always survives into the next generation, particularly when many individuals have fitness close to that of the best individual. The elitist strategy, where the best individual is always survived into the next generation on behalf of the worst individual, can compensate for some disadvantages of missing the best individual in selection operation or mutation operation. With the elitist strategy, the best individual always moves in a descent direction, thereby a stable convergence is obtained. The gradient search algorithm adopted in genetic algorithm is the most popular quasi-Newton method with the BFGS algorithm. The individual after the recombination is formulated as follows (Furukawa 2002)

$$m_{new} = \begin{cases} -A^{-1} \nabla f(m_{old}) & \text{if } f(m_{new}) > f(m_{old}) \\ m_{old} & \text{otherwise} \end{cases} \quad (16)$$

Where A is a well-known positive-definite matrix used on behalf Hessian matrix. The main steps for searching problem of slope stability by using hybrid genetic algorithm are shown in Fig. 2.

4. Case studies

To illustrate the effectiveness of the proposed searching method based on genetic algorithm, three test problems are conducted, and the searching results of factor of safety by using GA are compared with by using FEM. The first example is a simple slope consisting of homogeneous soil, as shown in Fig. 3. The total height of slope is 10.0 m. The slope angle is 26.565° . The soil mechanical parameters are shown in Table 1. The genetic algorithm parameters are shown as follows: number of generations 200, population size 80, crossover rate 0.85, and mutation rate 0.03. Minimal factor of safety results by using different method are shown in Table 2. Fig. 4 depicts the comparison of critical failure surfaces founded by using GA and FEM for the homogeneous slope example. The solid line is critical slide searched by genetic algorithm. The color contours are maximum shear strain lines computed by FEM. The stability of a slope can also be expressed in terms of development

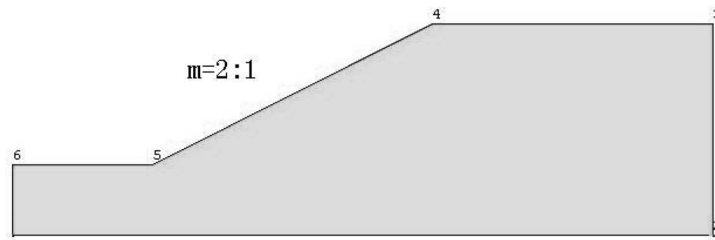


Fig. 3 Geometry of homogeneous slope

Table 1 Mechanical parameters of soil layer for the homogeneous slope example

Unit weight/($\text{kN}\cdot\text{m}^{-3}$)	Cohesion/kPa	Angle of shearing resistance/ $^\circ$
20.0	3.0	19.6

Table 2 Factor of safety results for the homogeneous slope example

method	Bishop	Spencer	Janbu	FEM
factor of safety	0.989	0.988	0.991	0.990

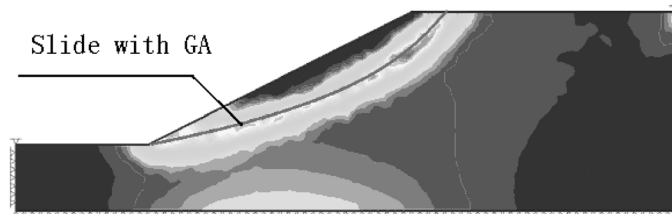


Fig. 4 Comparison of critical failure surfaces founded by using GA and FEM for the homogeneous slope example

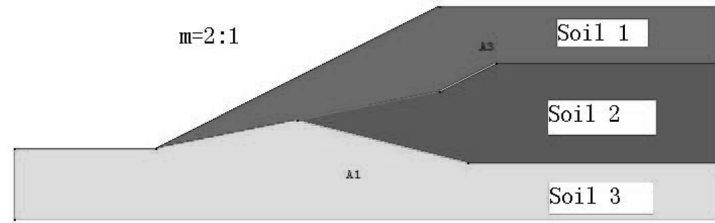


Fig. 5 Geometry of three-material slope

Table 3 Mechanical parameters of soil layers for the three-material slope example

Soil layer number	Unit weight/($\text{kN}\cdot\text{m}^{-3}$)	Cohesion/kPa	Angle of shearing resistance/ $^{\circ}$
1	19.5	0.0	38.0
2	19.5	5.3	23.0
3	19.5	7.2	20.0

Table 4 Factor of safety results for the three-material slope example

method	Bishop	Spencer	Janbu	FEM
factor of safety	1.407	1.377	1.358	1.360

of strains. Hence, the failure of the slopes can also be seen as shear strain rate contours as shown in the Fig. 4. It is clearly evident that the slopes developed a circular failure with high shear strain rate along the failure plane. The critical slide line searched by genetic algorithm agrees well with maximum shear strain computed by FEM.

The second example is a complex slope comprising of 3 layers of soil, as shown in Fig. 5. The height of the slope is 10.0 m. The slope angle is 26.565° . The mechanical parameters of soil layers are listed in Table 3. The genetic algorithm parameters are chosen as same as example 1. Table 4 lists factor of safety results by using different method for the three-material slope example. Fig. 6 shows the comparison of critical failure surfaces founded by using GA and FEM for the three-material slope example. The solid line is critical slide searched by genetic algorithm. The color contours are maximum shear strain lines computed by FEM. The critical slide line searched by genetic algorithm agrees well with maximum shear strain computed by FEM.

The third example models a non-homogeneous, three layer slope with material properties given in Table 3 and geometry as shown in Fig. 5. A horizontal seismically induced acceleration of 0.15 g is included in the analysis. Table 5 lists the minimal factor of safety by using different method for the

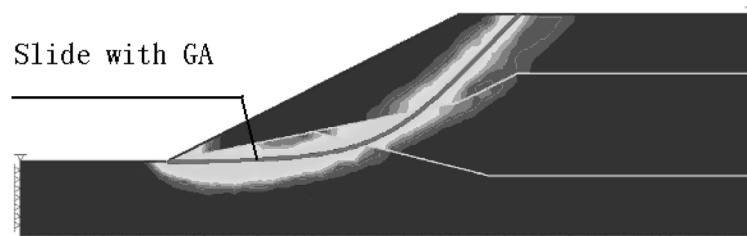


Fig. 6 Comparison of critical failure surfaces founded by using GA and FEM for the three-material slope example

Table 5 Minimal factor of safety for the case with a seismic action

method	Bishop	Spencer	Janbu	FEM
factor of safety	1.016	0.997	0.987	0.970

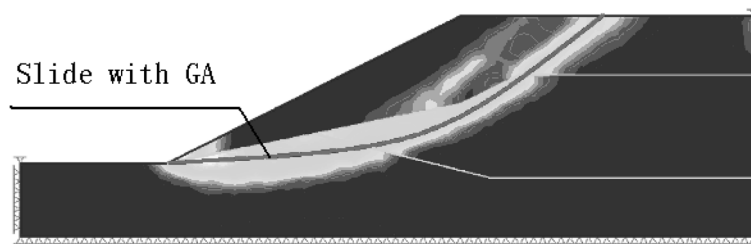


Fig. 7 Comparison of critical failure surfaces founded by using GA and FEM for the case with a seismic action

case with a seismic action. Fig. 7 shows the comparison of critical failure surfaces founded by using GA and FEM for the case with a seismic action. The solid line is critical slide searched by genetic algorithm. The color contours are maximum shear strain lines computed by FEM. The critical slide line searched by genetic algorithm agrees well with maximum shear strain computed by FEM.

5. Conclusions

The objective of the work has been the development of a global search procedure for slope stability problem. Genetic algorithm performs a multidirectional search of a population of many potential solutions, not just a single solution. The search for critical factor of safety in slope stability analysis will normally not become trapped in local optima. Some examples have been presented to demonstrate the effectiveness and robustness of the genetic algorithm. The proposed hybrid genetic algorithm can be applied to find the slip surface with lowest or near-lowest factor of safety when compared with simple genetic algorithm and gradient-based optimization. From the example, it has been concluded that the search procedure is able to locate a minimal factor of safety and the corresponding slip surface for slope stability problem. The computational results show that the proposed searching method can be used in slope stability analysis of earth dams, natural slopes and any other geotechnical problems with multi-layers, external loads, and earthquake action.

Acknowledgments

This research is funded by the National Natural Science Foundation of China (No. 90815023) and by The National Basic Research Program (973 Program) (No. 2007CB714006).

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