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Influence of higher order modes and mass configuration on the quality of damage detection via DWT

Mohammadreza Vafaei[*1] and Sophia C Alih[2a]

[1]*Faculty of Civil Engineering, Center for Forensic Engineering, Universiti Teknologi Malaysia, Skudai, 81310, Johor, Malaysia*
[2]*Faculty of Civil Engineering, Institute of Noise and Vibration, Universiti Teknologi Malaysia, Skudai, 81310, Johor, Malaysia*

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Abstract. In recent decades, wavelet transforms as a strong signal processing tool have attracted attention of researchers for damage identification. Apart from the wide application of wavelet transforms for damage identification, influence of higher order modes on the quality of damage detection has been a challenging matter for researchers. In this study, influence of higher order modes and different mass configurations on the quality of damage detection through Discrete Wavelet Transform (DWT) was studied. Nine different damage scenarios were imposed to four cantilever structures having different mass configurations. The first four mode shapes of the cantilever structures were measured experimentally and analyzed by DWT. A damage index was defined in order to study the influence of higher order modes. Results of this study showed that change in the mass configuration had a great impact on the quality of damage detection even when the changes altered natural frequencies slightly. It was observed that for successful damage detection all available mode shapes should be taken into account and measured mode shapes had no significant priority for damage detection over each other.

Keywords: damage detection; discrete wavelet transform; cantilever structures; higher order modes; mass configuration

1. Introduction

In recent decades, damage identification has attracted attention of researchers for condition assessment of in-service structures. Efforts have been made to propose robust and precise methods while maintaining simplicity and practicality of these techniques. A powerful signal processing tool that is referred to as Wavelet Transform (WT) has opened a new window toward damage identification methods. When a structure is damaged, a sudden change occurs in the structural responses close to the damaged sites (Liew and Wang 1998, Wang and Deng 1999). By applying WT to the measured responses, obvious spikes appear in the analyzed signals reflecting the discontinuity of the signals due to the damage. This property of decomposed signals has been

∗Corresponding author, Ph.D., E-mail: m.reza.vafaei@gmail.com
[a]Ph.D., E-mail: sophiacalih@gmail.com

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employed by researchers to detect damage via WT. Rucka and Wilde (2006) worked on the damage localization of a cantilever beam through Continuous Wavelet Transform (CWT). The deflected shape of the beam was decomposed by CWT. Then, the location of the crack was determined by a peak in the spatial variation of the transformed static deflection. They found that the imposed crack can be localized with a good precision when measurements were performed with a high accuracy. Zhong and Oyadiji (2011) compared application of Discrete Wavelet Transform (DWT) and Stationary Wavelet Transforms (SWT) for damage localization in a simply supported beam. Modal displacements were employed to identify the location of crack. They observed that the SWT of modal data provided better indications of damage when compared with the DWT of modal data. In addition, they concluded that the proposed method was sensitive to the spatial sampling intervals. Peng *et al.* (2013) studied a free-spanning damage to subsea pipelines by means of wavelet packet transform. The change in the wavelet packet energy was defined as the damage index. It was shown that the proposed method could accurately localize the free-spanning damage of subsea pipelines.

A review of the literature shows that mode shapes have been widely employed by researchers for damage detection through wavelet transforms. Beside studies that show the efficiency of wavelet transforms for damage identification (Vafaei *et al.* 2014a, b), efforts have been also made to understand the influence of higher order modes on the quality of damage detection. Jiang *et al.* (2012) used the slope of the mode shapes to localize damage in beam type structures. Continuous wavelet transform and complex CWT were employed to analyze the mode shape and the slope of the lumped crack model of the beam. It was found that complex CWT was more efficient at locating cracks even when the signals were contaminated by noise. Results also showed that the second mode shape and its corresponding slope were more preferable than the first mode for damage identification. Castro *et al.* (2006) studied the influence of mode order on damage detection in rods via CWT. They stated that higher order mode shapes could better detect damage in rods. In another study, Okafor and Dutta (2000) worked on damage detection of aluminum cantilever beams using CWT of experimental and numerical mode shapes. Wavelet transform of experimental mode shapes indicated that, the second mode shape was not sensitive to the presence of damage because damage fell in the vicinity of zero-crossing point. However, they showed that the first and second mode shapes clearly localized damage. Masoumi and Ashory (2013) used Generalized Flexibility Matrix (GFM) and Uniform Load Surface (ULS) in conjunction with DWT to find damage location in plate-type structures. They concluded that the number of mode shapes did not have any effect on the obtained results from ULS-based method while better results were obtained when three mode shapes were employed in the GFM-based approach. Rucka (2011) extracted the first eight mode shapes of a cantilever beam through laser-based vibration measurement technique in order to study the influence of higher order modes on damage detection through CWT of mode shapes. It was observed that, higher mode shapes were more sensitive to the presence of damage. However, they included more zero-crossing points in which they lost sensitivity to damage detection. Therefore, it was concluded that for reliable damage detection at least two mode shapes are necessary. Gentile and Messina (2003) by working on a classical Eulero-Bernoulli beam stated that since damage may occur at locations that have a poor sensitivity for certain mode shapes, no priority should be given to fundamental or higher mode shapes for damage detection through CWT and all available mode shapes should be analyzed.

The above mentioned studies confirm the importance of appropriate selection of mode shapes for damage detection through continuous wavelet transforms. While some studies recommend usage of higher order mode shapes for better detection of damage (Castro *et al.* 2006), for reliable damage identification, other studies, necessitate consideration of a minimum of two mode shapes (Rucka 2011). Meanwhile, some research outcomes ask for analyzing of all available mode shapes (Gentile and Messina 2002). Such discrepancy in the obtained results implies that appropriate selection of order of mode shapes for damage identification needs more research. Herein, the influence of the order of modes on the quality of damage detection through discrete wavelet transform (Vafaei *et al.* 2014c, Ovanesova *et al.* 2004) is presented. While the focuses of previous studies have been on CWT this study has concentrated on DWT. In addition, the effect of different mass configurations on the quality of damage detection, which has not been addressed in previous works, is studied.

2. Discrete Wavelet Transform (DWT)

The decomposition of a signal in DWT starts by high-pass and low-pass filtering of the signal. The original signal $u(t)$ can be represented by its approximation and detail coefficients as follows (Staszewski 1998)

$$u(t) = X_j + \sum_{j \subseteq j} Y_j \tag{1}$$

X_j is the approximation at level j and is obtained by

$$X_j = \sum_{j \supset J} Y_j \tag{2}$$

Y_j is the detail at level j and is defined as

$$Y_j = \sum_{k \in Z} a_{j,k} \psi_{j,k}(t) \tag{3}$$

$\psi_{j,k}(t)$ is the basis function and is represented as

$$\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k) \tag{4}$$

Eqs. (1) to (4) display that the signal is divided into two separated components including decompositions (Y_j) and approximations (X_j). The approximations are low-frequency components and the details are high-frequency components. For many signals, the low frequency content is the most important. It has been demonstrated that detail coefficients contain the information necessary to detect damage to structures (Ovanesova and Suarez 2004, Zhong and Oyadiji 2011). Appropriate selection of wavelet functions is of great importance for damage identification. Symmetry, regularity and capability for exact reconstruction of the analysed signal are important characteristics that can be considered for the selection of wavelet functions (Ovanesova and Suarez 2004). Zhong and Oyadiji (2011) suggested the number of vanishing moments and the effective support of a wavelet as two other main criteria for the selection of wavelet functions. A wavelet function that created the maximum number of close to zero wavelet coefficients was selected by Rucka and Wilde (2006, 2010) for damage detection in beams and plates. As the literature shows, there is no systematic approach for appropriate selection of wavelet functions and often trial and error is used (Reda Taha *et al.* 2006). In this study, after performing several rounds of trial and error, a bi-orthogonal wavelet (bior 3.1) was selected as the wavelet function. The bi-orthogonal

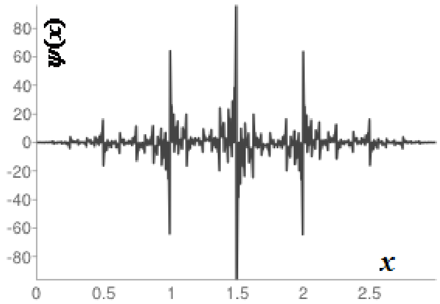

Fig. 1 Basis Function of bior 3.1

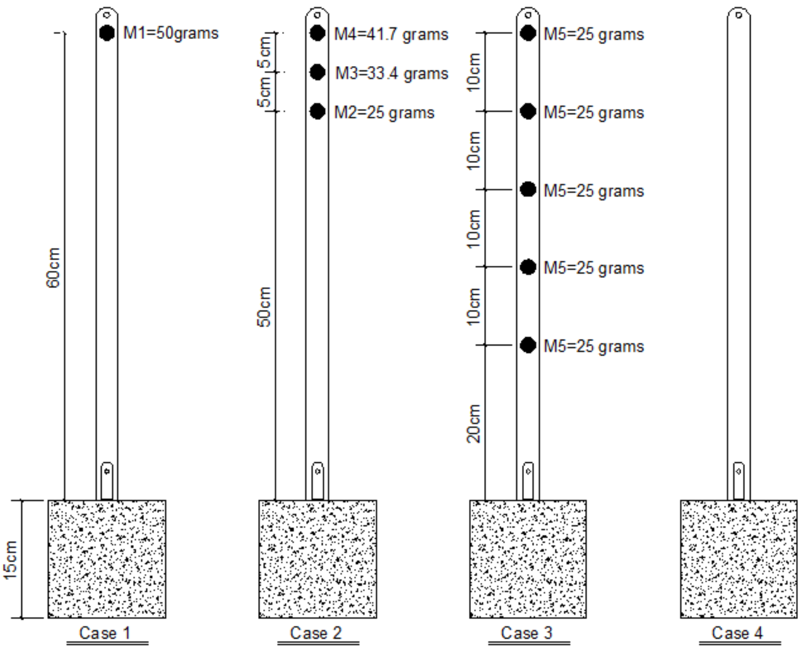

Fig. 2 Mass configurations of cantilever structures

wavelet family exhibits the property of linear phase that is needed for the reconstruction of signals (Mallat 1999). Fig. 1 displays the basis function of bior 3.1.

3. Experimental set up and testing of laboratory structures

3.1 The test structures

In this study, four cantilever steel structures referred to as Case 1 to Case 4 were tested experimentally for damage detection via DWT. Structures composed of an identical steel plate with the dimensions of 625 mm by 28 mm by 1.5 mm but with different mass configurations. Fig. 2 displays the mass quantities and locations where they were installed on the steel plates. It should be mentioned that the mass of the steel plates, for all structures, was 122 grams. The support system of all the structures included two steel angles that had their horizontal legs bolted to a concrete cube with the dimensions of 150 mm by 150 mm by 150 mm and their vertical legs clamped to the steel plate. The clamps were installed 40mm above the surface of concrete cube.

3.2 Damage scenarios

Three different damage severities located at three different levels were considered as damage scenarios. Damage was introduced to the structures with a saw cut at both edges of the steel plates. The width of all notch type damage was 1 mm while the lengths were 2 mm, 4 mm and 6 mm. The damage severities correspond to a cross section loss of the second moment of area, I, of 14.29%, 27.27%, and 42.86% that were termed as Light (L), Medium (M) and Severe (S), respectively. Damage locations were found at the 12 cm, 25 cm, and 45 cm heights of the steel plates and they were referred to as B, M and T, respectively. The nine different damage scenarios were introduced to all cantilever structures similarly.

3.3 Experimental modal test set up

Roving Hammer (RH) test was used to extract modal properties of structures before and after damage occurrence. Fig. 3 shows the experimental modal test design for Case 3. The impact hammer that was used to excite the cantilever structures was a KISTLER model 9722A2000. Moreover, the accelerometer used in this study was a DAYTRAN, model 3110A with the sensitivity of 95.2 mV/g. Data acquisition system was an imc Mebsysteme model CS-3008 that

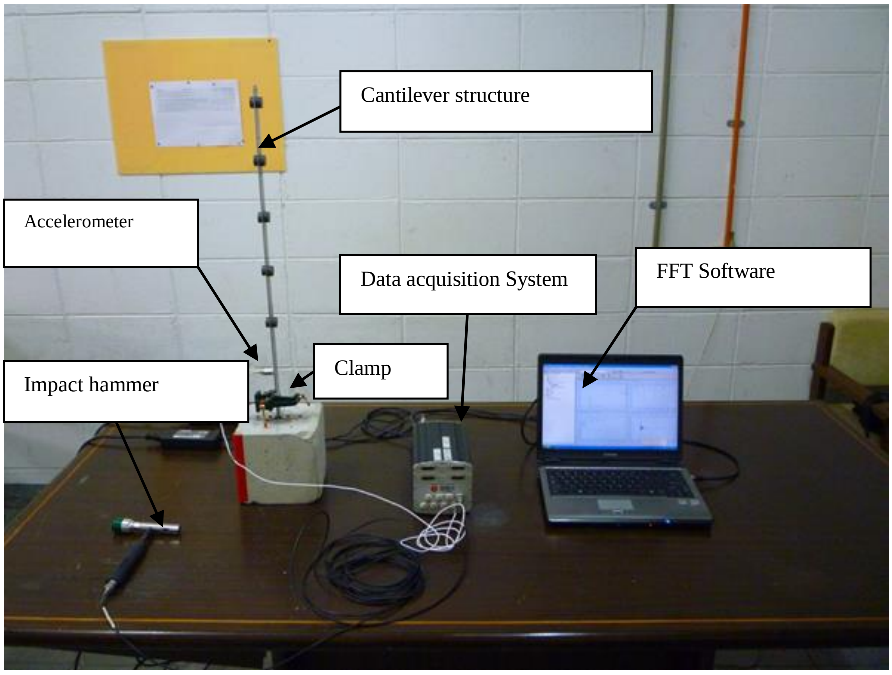

Fig. 3 Experimental modal test set up of Case 3

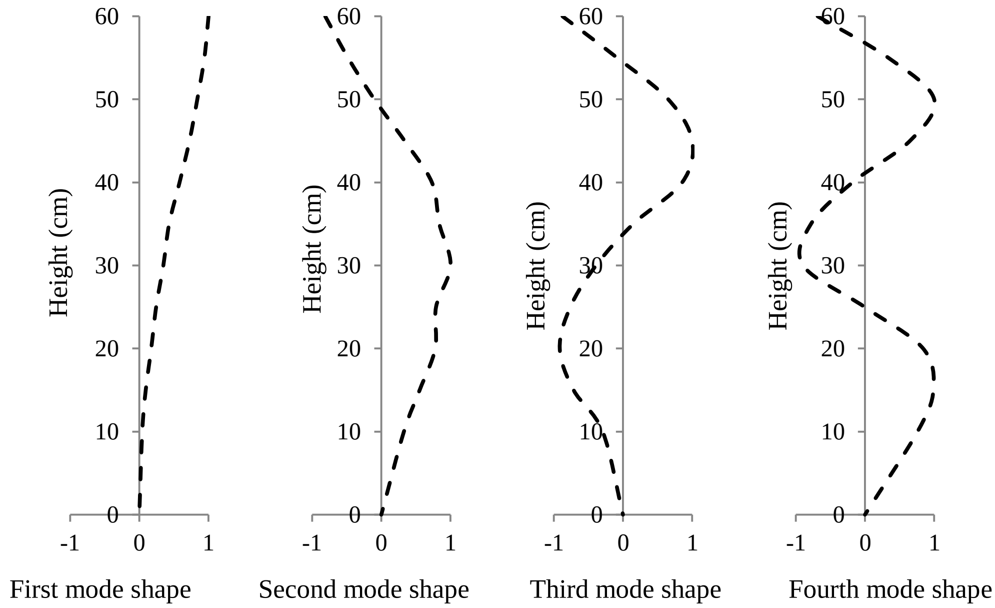

Fig. 4 The first four flexural mode shapes of Case 3 for undamaged state

Table 1 Modal parameters for undamaged state of the cantilever structures

	Natural Frequency (Hz)				Damping ratio (%)			
Structure	Mode1	Mode2	Mode3	Mode4	Mode1	Mode2	Mode3	Mode4
Case 1	2.28	16.80	49.40	81.20	5.20	3.34	4.24	3.07
Case 2	2.19	16.90	45.20	73.90	5.77	3.11	4.14	2.81
Case 3	2.46	15.05	39.30	68.80	5.22	4.76	4.43	2.87
Case 4	3.42	19.40	51.70	83.70	4.37	4.77	3.50	3.01

was equipped with FFT analyzer software called “imc Wave”. Each excitation was repeated three times and the average of three ensembles per test was employed for the extraction of modal parameters. Mode shapes were measured every 20 mm along the length of the structures and then a pricewise cubic spline data interpolation was used to decrease the sampling distance of the measured mode shapes from 20 mm to 2 mm. Fig. 4 displays the first four mode shapes of the Case 3 for undamaged state. Table 1 shows natural frequencies and damping ratios for undamaged state of cantilever structures.

4. Damage detection

Discrete wavelet transform was applied on the measured mode shapes and the first and second level detail coefficients of decomposed signals were calculated. Spikes that appeared in the obtained detail coefficients displayed the location of imposed damage. Fig. 5 displays the obtained results for Case 1 when severe damage occurs at 25 cm height. As can be seen, in addition to the

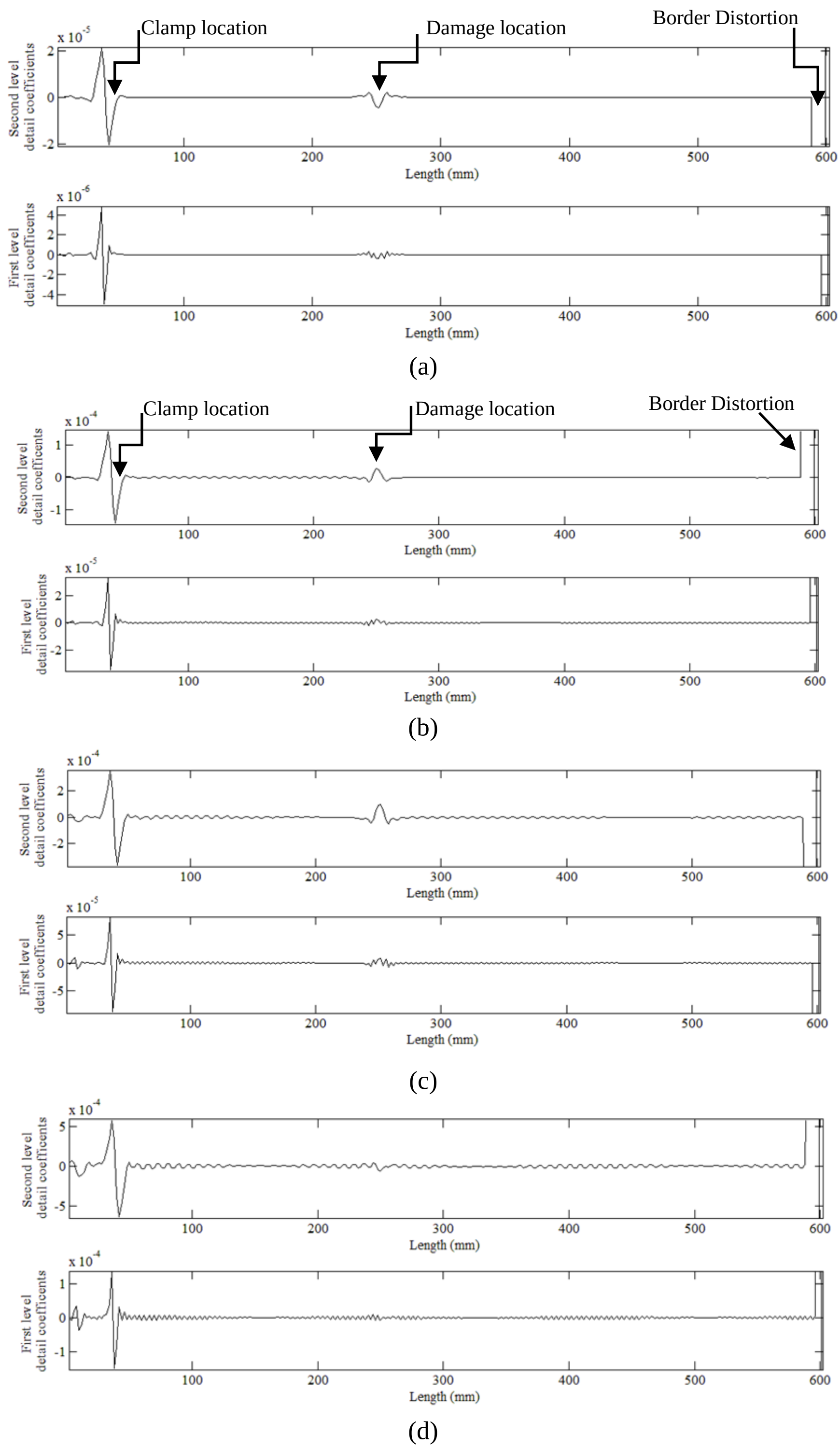

Fig. 5 Discrete wavelet transform of experimental mode shapes of Case 1 for severe damage at 25 cm height. (a) First mode shape (b) Second mode shape (c) Third mode shape (d) Fourth mode shape

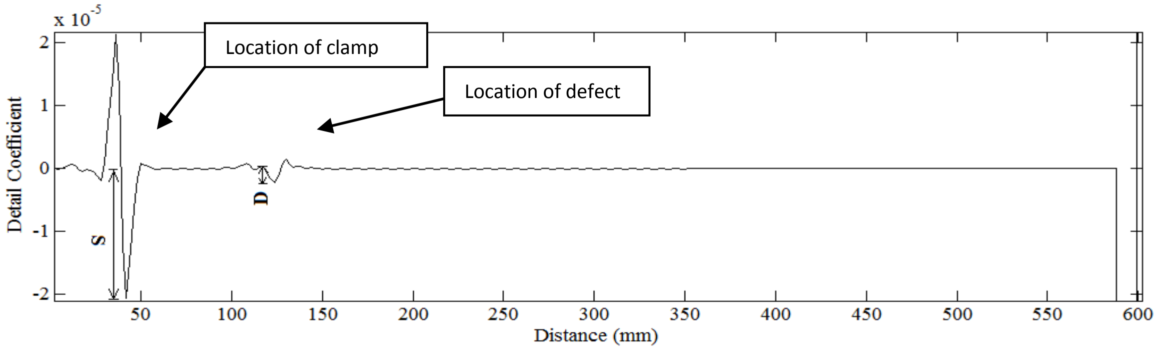

Fig. 6 Second level detail coefficients for the first mode of Case 1

damage location a more obvious spike appears at almost 4 cm height. This spike indicates the location of clamp where support angles connect to the steel column (see Fig. 3). At the clamp location because of abrupt change in the lateral stiffness, detail coefficients of decomposed mode shapes display another spike. A distortion in the obtained detail coefficients can be also seen at the free end of the cantilever structure. The border distortion problem has been observed by other researchers (Rucka and Wilde 2006) and it is due to the finite length of mode shapes. DWT is a product of wavelet and a signal of infinite length, therefore, when mode shapes are decomposed by DWT, at the free end of signals detail coefficients turn to very large values. This problem can be solved by extending signals beyond the boundary of structures. However, since in this study border distortion did not affect the obtained results it was ignored. Fig. 5 shows that the second level detail coefficients can signify the presence and location of damage better than the first level detail coefficients. Therefore, they are preferable for damage detection more than the first level detail coefficients.

In order to investigate the influence of higher order modes and different mass configurations on the quality of damage detection through DWT, a damage index was established. It should be mentioned that, the introduced damage index only highlights the presence of damage and is not meant to be a new method for damage identification. As it is shown in Eq. (5) the damage index takes advantage of second level detail coefficients at the locations of clamp and the imposed damage. This way the damage index normalizes the size of observed spike at the damage location with respect to the size of spike that appears at the clamp location. This means that, the proposed damage index shows how visible the spike at the damage location can be when compared with the spike at the clamp location. Assuming the spike at the clamp location as a permanent severe damage to the cantilever structures, such normalization considers the effect of multiple damage when studying the influence of different mass configurations and higher order modes. It is worth mentioning that, the size of spike at the clamp location changes as the order of modes or mass configurations are altered. It is evident from Eq. (5) that, the higher the value of damage index, the better the quality of damage detection. Fig. 6 displays how the size of spike at the clamp location “S” and at the damage location “D” are obtained from the second level detail coefficients of the first mode of the Case 1 when damage was located at 12 cm height

$$\text{Damage Index} = \left|\frac{D}{S}\right| \times 100 \tag{5}$$

5. Discussion on the obtained results

The first four mode shapes of damaged structures were analyzed by DWT and for all damage scenarios the value of the damage indices at the damage locations were calculated. Figs. 7 to 10 depict the obtained results for Case1 to Case 4, respectively. In these figures TS, TM and TL stand for severe, medium, and light damage at the top of structure, respectively. Moreover, MS, MM, and ML show severe, medium, and light damage in the middle of structure, respectively. Meanwhile, BS, BM and BL represent severe, medium, and light damage at the bottom of structure, respectively. It should be mentioned that, when the value of damage index fell below 5% it was difficult to distinguish the presence of damage because of other spikes that appeared in the detail coefficients. However, since the aim of this study was to compare the influence of mode orders, even when the value of damage index was less than 5% it was calculated at the imposed damage locations in order to be compared with other damage indices.

It is evident from Figs. 7 to 10 that, regardless of damage locations and mass configurations, light damages have the smallest damage indices and severe damages have the biggest. The highest value of the damage index obtained for low, medium and severe damages are 6.8%, 15.1% and 33.4%, respectively. Meanwhile, the lowest value of damage index obtained for low, medium and severe damages are 0.73%, 2% and 3.7%, respectively. The significant differences between the obtained damage indices indicate that regardless of damage intensity the quality of damage detection is highly influenced by mode orders.

Figs. 7 to 10 display that for all studied structures, regardless of damage intensity, when damage occurs at the top of structures the differences between damage indices obtained from different mode shapes are more than the time when damages occur in the middle or bottom of structures. This shows that some mode shapes are more sensitive to the damages that occur at the top of structures while other mode shapes are not sensitive enough to detect the presence of damages. For instance, when the light damage occurs anywhere along the height of structures the fourth mode shape almost always results in the highest quality for damage detection; however, when the intensity of damage increases to the severe level, for the bottom part of structures the first mode shape provides better results compared to the fourth mode shape. Furthermore, when severe damage occurs in the middle and top of the structures, the third mode shape provides the best quality for the presence of damage.

Figs. 7 to 10 also show that, Case 4, Case 3 and Case 2 have the highest damage indices for low, medium and severe damages, respectively. On the other hand, Case 4 has the smallest damage index for light damage and Case 3 has the smallest damage index for medium and severe damages. Considering this fact that all studied structures have had similar damage scenarios this can be concluded that change in the mass configuration has also significant impact on the quality of damage detection through DWT. Table 1 shows that the difference between the measured natural frequencies for Case 1 and Case 2 are negligible (less than 4% for the first mode and less than 9% for the fourth mode). However, as can be seen from Figs. 7 and 8, for all damage scenarios, the damage indices obtained from the fourth mode shape of Case 2 are lower than those obtained from the fourth mode shape of Case 1. Moreover, while for the MM damage scenario the obtained damage index for the first mode of Case 2 is significantly lower than that of Case 1, the differences between other damage indices are insignificant. These results imply that even when the change in the mass configuration results in a small change in the natural frequency, the quality of damage detection through decomposition of mode shapes can be influenced significantly.

In short, from Figs. 7 to 10 it can be concluded that when DWT is employed for the purpose of

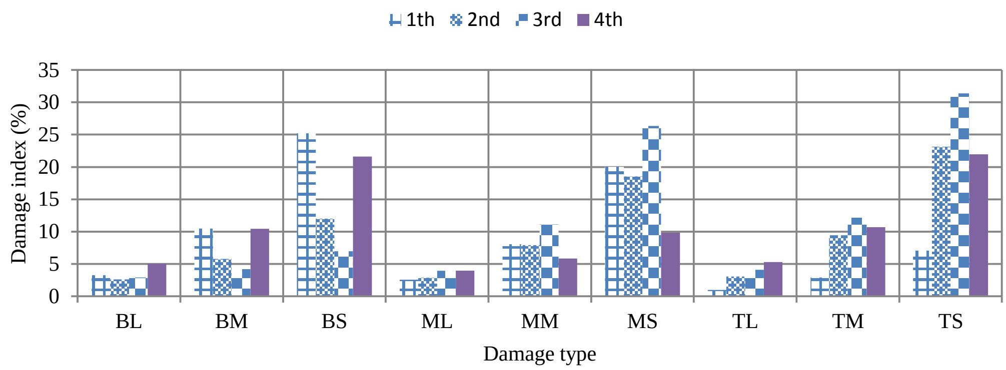

Fig. 7 Damage indices calculated for the first four mode shapes of Case 1

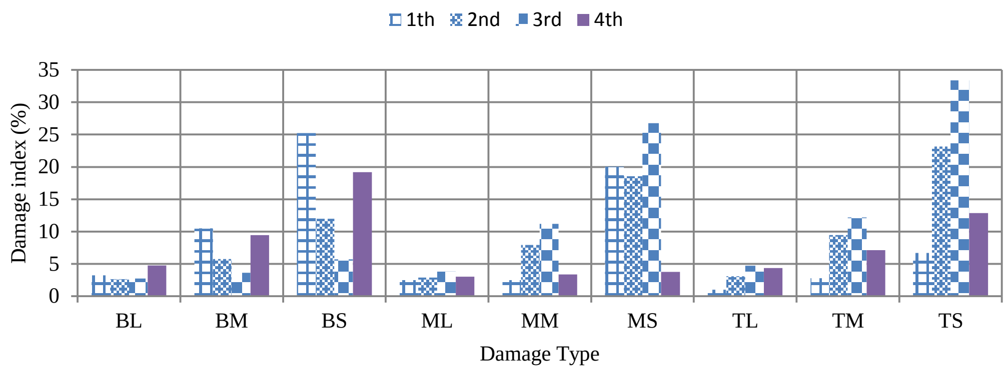

Fig. 8 Damage indices calculated for the first four mode shapes of Case 2

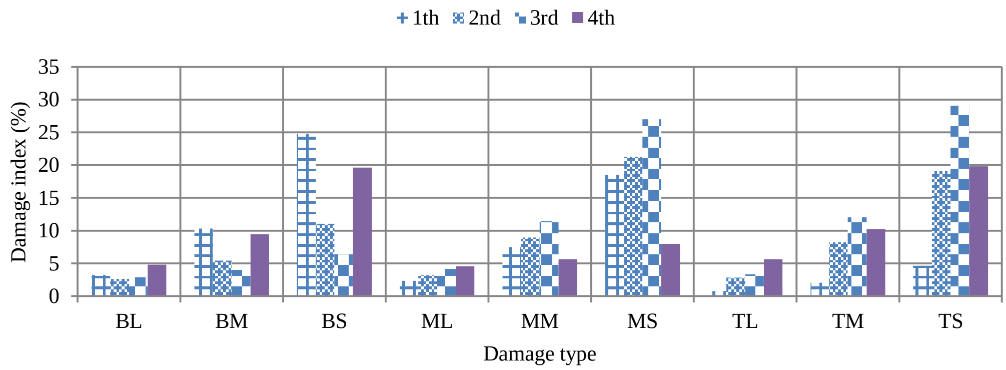

Fig. 9 Damage indices calculated for the first four mode shapes of Case 3

damage detection all available mode shapes should be measured and analyzed. Considering only the fundamental mode shape or higher order modes cannot lead to the best quality for damage

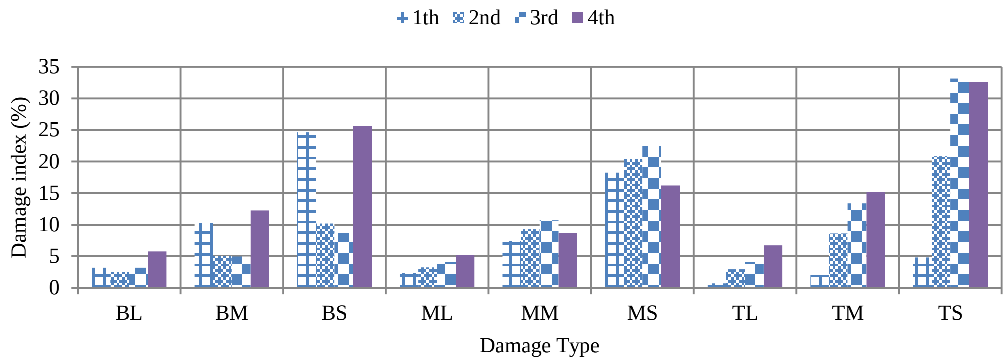

Fig. 10 Damage indices calculated for the first four mode shapes of Case 4

detection. This observation relies on this fact that not only sensitivity of mode shapes to the presence of damage varies with the change in the intensity of imposed damage but also it varies with the change in the location of imposed damage. This finding is in line with the findings of other researchers for damage detection through CWT of mode shapes (Gentile and Messina 2003).

6. Conclusions

In recent decades, wavelet transforms, as a power signal processing tool, have attracted attention of researchers for damage identification. Frequency and time domain data can be analyzed by wavelet transforms to identify imposed damages. Results of previous studies indicate that wavelet transforms can successfully identify damages when applied to mode shapes. However, appropriate selection of the order of mode shape for damage detection needs further studies. Herein, the influence of higher order modes and different mass configurations on the quality of damage detection via DWT is studied. Four cantilever structures with different mass configurations were selected and their first four mode shapes were extracted experimentally. Nine different damage scenarios were imposed to the selected structures and changes in the natural frequencies and mode shapes were recorded. In order to identify damages, the measured mode shapes were analyzed by discrete wavelet transform. To discuss on the quality of identified damages a damage index was defined based on the second level detail coefficients of decomposed mode shapes. It was observed that, the sensitivity of mode shapes to the presence of damage changed when the intensity or the location of damage was altered. Results of the analyzed mode shapes indicated that for damage detection all available mode shapes should be analyzed and priority shall not be given to the fundamental mode shape or higher order modes. It was also observed that, a small change in the mass configuration could significantly alter the sensitivity of mode shapes to the imposed damages.

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