Earthquakes and Structures, *Vol. 9, No. 3 (2015) 625-637*
DOI: http://dx.doi.org/10.12989/eas.2015.9.3.625

The effect of the vertical excitation on horizontal response of structures

Hosein Ghaffarzadeh* and Ali Nazeri

Department of Civil Engineering, University of Tabriz, Tabriz, Iran

(Received February 8, 2014, Revised April 21, 2015, Accepted April 27, 2015)

Abstract. It is usual in design and assessment of structures to isolate the effects of vertical and horizontal excitations by ignoring their coupling effects. In this situation, total structural response is obtained by employing the well-known combination rules whereby independent assumed response components of earthquakes are combined. In fact, the effects of the simultaneity of the ground motion components are ignored.

In this paper, the effect of vertical excitation on horizontal response of structures, the coupling of vertical and horizontal responses, has been evaluated. A computer program is prepared to perform nonlinear dynamic analysis based on the derived governing equations of coupled motions. In the case of simultaneous excitation the results show significant increases in spectral displacement in some periods of vibration in comparison to only horizontally excited systems. Moreover, whenever ratio of the vertical peak ground acceleration to horizontal one become larger, the significant increase in horizontal spectral displacements are observed.

Keywords: simultaneous excitation; coupled horizontal and vertical components; vertical earthquake

1. Introduction

Whereas earthquake waves are propagated in all directions of ground, the produced motions by these waves can be re-expressed in three-translational and three-rotational directions. Among six components of ground motions, only three translational motions consist of two horizontal and a vertical motion are measured through recording and evaluating process of earthquakes. Horizontal ground motions are major components of earthquakes. Although the vertical component of earthquakes has less importance than horizontal ones in seismic analysis and design of structures, current researches indicate that the vertical component has a significant role in some earthquakes especially in near fault regions earthquakes (Beresnev *et al*. 2002, Ambraseys and Douglas 2003, Yang and Lee 2007, Warn and Whittaker 2008, Kim *et al*. 2011). The vertical component significance is usually measured by maximum vertical acceleration to horizontal one ratio (V/H). The most codes propose a vertical acceleration equal to 2/3 of horizontal one to include in the seismic design, as postulated earlier by Newmark *et al*. (1973), and Newmark and Hall (1982).

*Corresponding author, Associate Professor, E-mail: ghaffar@tabrizu.ac.ir

http://www.techno-press.org/journals/eas&subpage=7 ISSN: 2092-7614 (Print), 2092-7622 (Online)

Therefore, the same frequency content for all earthquake components is considered in the mentioned codes. Nevertheless the suggested V/H ratio is not conservative in near fault zone.

The maximum vertical to horizontal (V/H) acceleration ratio of some earthquakes was reported more than 1 in near fault zone (Bozorgnia and Niazi 1993, Silva 1997, Bozorgnia *et al.* 1999). For instance, the maximum V/H ratio was recorded equal 1.79 in 1994 earthquake of Northridge and 1.63 in 1995 Kobe Earthquake. It has been specified that the maximum vertical acceleration depends on the soil conditions, site to epicenter distance and earthquake magnitude (Kusunoki 1995). Lixinle *et al*. (2007) stated the maximum V/H ratio for moderate earthquakes with magnitude between 6.5 and 7 is greater than that of for small earthquakes with magnitude between 4.5 and 6 and even more than that of for severe earthquake with magnitude of greater than 7.

The significant vertical acceleration of near fault earthquakes leads to a specific failures mode of structures located in the vicinity of causative faults. In the 1994 Northridge earthquake, a specific mode of failure was reported in structures. Most welded steel structures collapsed due to development of cracks at the location of connections. Although the major causes of failures were addressed to improper welding of connections, unqualified materials and poor detailing, recorded earthquakes showed that the vertical component was much larger than is usually considered normally in design. However, for similar structures located far from fault, the effect of the vertical component of ground motion was less and consequently they showed acceptable performance (Salazar and Haldar 2000). Two modes of shear and compression failures which caused by vertical component of ground motion has been reported in concrete structures. Based on the reports destruction in such structures due to effects of vertical component of earthquakes depend on the amount of increase or decrease of compression forces in RC columns and shear walls. It is evident that the variation of vertical forces inevitably gave arise to a reduction in shear (Papazoglou and Elnashai 1996). In order to include the vertical ground motion effects in design, recent efforts have consider the development of vertical ground motion seismic design spectra (Bozorgnia and Campell 2004, Kalkan and Gulkan 2004). These studies have developed vertical ground motion spectra and concentrated on its parallel use with horizontal ground motion spectra.

It is customary in design and assessment of structures to isolate the effects of vertical and horizontal excitations by ignoring their coupling effects. Multi component excitation of structures was studied by Kalkan and Graizer (2007, 2008). They expanded the equation of a SDOF system to consider the effects of vertical and tilting motion on horizontal response. In this manner a SDOF motion equation was modified by some additional forces which reflect the effects of vertical and tilting ground motions. They presented the SDOF oscillator by a rigid bar and the flexural of system is lumped in a rotational spring at the base. Regarding to employing the rigid bar, the axial stiffness is ignored and the effect of vertical displacement of mass has been considered as second order term. Since the vertical component of earthquake continently change throughout the earthquake, using a constant term can be lead to misleading results.

The main target of current study is to investigate the effect of vertical excitation on horizontal response of structures. In this study, considering both the flexural and axial stiffness, a system with two degree of freedom is defined in which the stiffness of system is modeled with an elastic bar. In fact, the effect of vertical displacement is explicitly studied by employment of an elastic bar, rather than a rigid bar. The governing equations of vertical and horizontal vibration of the system with two degrees of freedom are derived. Based on the governing equations, a simple computer program is prepared to perform nonlinear time history analysis. In this regard, by changing the mass and stiffness various systems with different natural periods are developed. These systems are subjected 8 earthquake ground motion records having great vertical acceleration and the horizontal displacements are obtained.

2. Problem statement

A single degree of freedom (SDOF) is a spring-mass-damper system in which the mass is allowed to move in only one direction. The horizontal vibration of a single story building can be conveniently modeled as a single degree of freedom system. In structural dynamics, SDOF is used to study the dynamic response of complicated structures subjected to seismic ground motions. Response spectrum is a practical aspect of usage the single degree of freedom system. A response spectrum represents the response of SDOF system for a specified earthquake ground motion.

The dynamic equilibrium of a SDOF system with mass, viscous damper and stiffness, respectively named by *m*, *c* and *k* can be written as Eq. (1).

$$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=-m\ddot{u}_g(t) \tag{1}$$

In which u, $\dot{u}$ and $\ddot{u}$ are relative displacement, velocity and acceleration of oscillator and $\ddot{u}_g$ is ground motion acceleration. In general, m and c are constant coefficients while the stiffness matrix *k*, is varied with time for a nonlinear system. SDOF configuration and its horizontal component of motion have been shown in Fig. 1. As shown in the Fig. 1, the nonlinearity of system may be considered by bilinear modeling of system stiffness.

For considering both the flexural and axial stiffness, a system with two degree of freedom is defined in which the stiffness of system is modeled with an elastic bar. In Fig. 2 the effect of vertical displacement is depicted.

If the oscillator to be subjected to the vertical vibration along with horizontal vibration, the dynamic forces would be imposed to intensive mass in both direction asynchronously. The oscillator motions equations in this status could be written as following based on Fig. 2.

$$m\ddot{u}(t)+c_h\dot{u}(t)+k_hu(t)-\frac{m}{l}[\ddot{v}(t)+\ddot{v}_g(t)]u(t)=-m\ddot{u}_g(t) \tag{2}$$

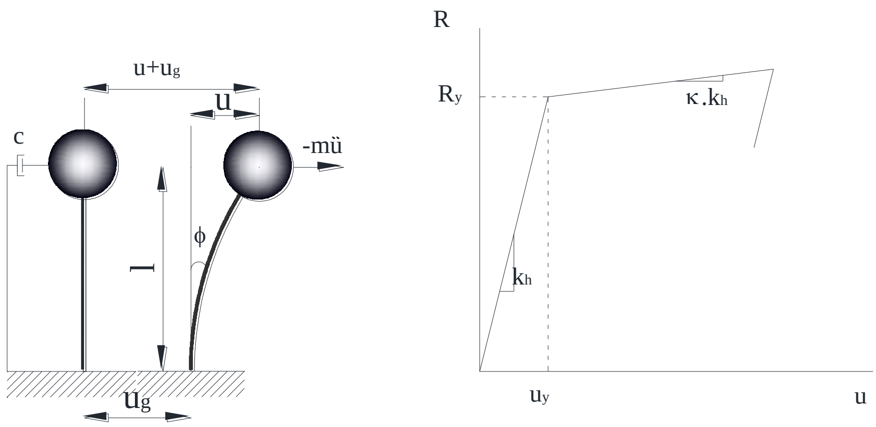


Fig. 1 Horizontal motion of SDOF and its nonlinear stiffness model

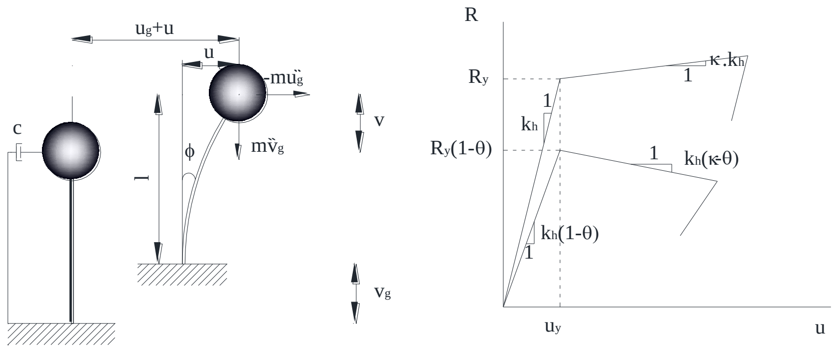


Fig. 2 Accompanying of horizontal and vertical motions and its material and geometric nonlinearity in stiffness model

$$m\ddot{v}(t)+c_v\dot{v}(t)+k_v v(t)=-m\ddot{v}_g(t) \tag{3}$$

Where subscripts h and v refer to horizontal and vertical motion respectively, v is relative vertical displacement of oscillator and $\ddot{v}_g$ is ground motion acceleration in vertical direction.

The Eqs. (2) and (3) show a set of two coupled simultaneous equations, in which the horizontal vibration of oscillator would be affected by vertical response of oscillator and the vertical ground acceleration. This can be seen by aberrance of coupled term in Eq. (2).

Eq. (3) can be re-written as follows

$$m\ddot{u}(t)+c_h\dot{u}(t)+\{k_h-\frac{m}{l}[\ddot{v}(t)+\ddot{v}_g(t)]\}u(t)=-m\ddot{u}_g(t) \tag{4}$$

The later equation represents the real horizontal vibration of oscillator. Comparing Eq. (4) with Eq. (1) indicates to a change in the coefficient of $u(t)$ in asynchronous condition. As shown in the Eq. (5), this coefficient can be interpreted as the system effective stiffness

$$k_h'=k_h-\frac{m}{l}[\ddot{v}(t)+\ddot{v}_g(t)] \tag{5}$$

In accordance with Eq. (5), it is obvious the system effective stiffness may be theoretically zero in specific conditions. This can be occur when intensive mass quantity increased, oscillator vertical acceleration response amount increased or by increasing ground gravity acceleration in vertical direction. In this status, there is a dynamical instability in system, hence the stability coefficient can be considered to system as θ

$$\theta=\frac{\frac{m}{l}[\ddot{v}(t)+\ddot{v}_g(t)]}{k_h} \tag{6}$$

By presenting the stability coefficient, θ, Eq. (5) can be written

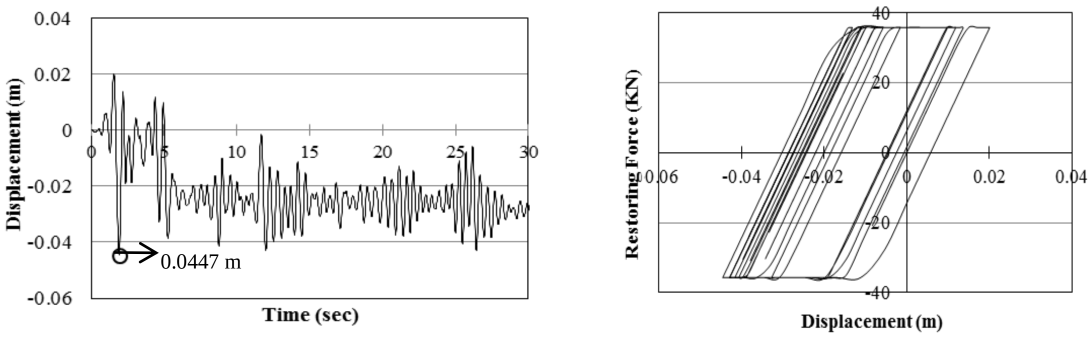
$$k_h'=k_h(1-\theta) \tag{7}$$

Eq. (5) shows the effect of stiffness reduction in the case of accompanying vertical vibration with horizontal vibration. Fig. 2 demonstrates stiffness reduction in the bilinear stiffness model of system, in which stiffness tend to diminish and following it, strength value, R_y dropped to the amount of $R_y(1-\theta)$. Stiffness and strength reduction indicate to geometric nonlinearity of system. It is noticeable that yield displacement, u_y remains unchanged, since u_y is directly related to the moment-rotation curvature behavior at any section level of member and is independent from geometric nonlinearity of system caused by the vertical component acceleration effect.

3. Numerical solution

The direct numeric integration methods are effective approaches for the solution of coupled simultaneous nonlinear Eqs. (2) and (3). The Newmark method is the family of single-step integration methods widely used for the solution of dynamic motion problems. It attempts to satisfy dynamic equilibrium at discrete points in time. Based on the Newmark's method, the dynamical equilibrium conditions at the beginning of each time step is imposed and then the response to be calculated. The performance of Newmark's method in the solution of nonlinear systems has been widely assessed (Newmark *et al*. 1973). The nonlinearity of stiffness coefficient at the dynamic equilibrium equation can be considered by its re-calculating at beginning of each time step and comparing with the calculated response at the end of time step.

The vertical dynamic equilibrium equation, Eq. (3), can be straightforwardly solved. Based on noticeable stiffness amount in vertical direction, it can be assumed to behave in the linear range. Geometric and material nonlinearity should be imposed to the Eq. (2). This can be taken into account by checking stiffness values at each time step with the exact known stiffness-displacement relationship of the system. To implement the procedure to find the dynamic response of the system subjected to simultaneous vertical and horizontal motion, a computer program, VH-Synch, was written using MATLAB computing language environment. Computing of the nonlinear time histories of structural responses subjected to the given time histories of an earthquake was the main object of the program. Second order effects due to interaction of vertical component of ground motion to be investigated by the program. Although, the program was mainly implemented



(a) (b)

Fig. 3 VH-Synch results obtained for the SDOF example [19] with $\mu = 3.0, T = 0.5\,Sec$: (a) Time history of displacement response (b) Inelastic force displacement response for two degrees of freedom systems, it can be generalized to calculate dynamic response of systems with multi degrees of freedoms. To evaluate the accuracy of the program, the example of chapter 7 in reference (Chopra 2002) was considered to be examined by VH-Synch. In the reference, the nonlinear response of a SDOF system with elastic-perfectly plastic stiffness behavior subjected to the El Centro earthquake was plotted for different ductility factors. The program was executed with the dynamic characteristics of the reference system with ductility factor of 3 and natural oscillation period equals 0.5 second. Fig. 3 demonstrates the time history of displacement response along with force displacement hysteresis loops. In comparison with the results of the reference, the calculated results indicate to acceptable accuracy of the program.

4. Results from simultaneous vertical and horizontal excitation

To investigate the effects of vertical excitation on the response of horizontally vibrated systems, several two degrees of freedom systems to be considered for dynamic analysis by the VH-Synch program. The Inelastic responses of systems were computed under synchronous vertical and horizontal components of selected earthquakes as listed in Table 1. The main criterion in the selection of the records of earthquakes was emphasis on the variety of the vertical to horizontal PGA ratio of earthquakes. The V/H ratio of selected earthquakes in Table 1 varied from 0.5 to 1.62.

4.1 Nonlinear time history analysis of a bridge model

In the previous sections, the governing equations of motions for a system of two degrees freedom were developed considering the combinations of horizontal and vertical input motion components, simultaneously. Based on the equations, an inelastic system response subjected to uncoupled and coupled combinations of the horizontal and vertical components can be determined. Results from the inelastic transient analyses will be presented in a comparative way to distinguish the relative impacts produced by each component.

In order to investigate the coupled response, a pier of a simply supported bridge was used as the

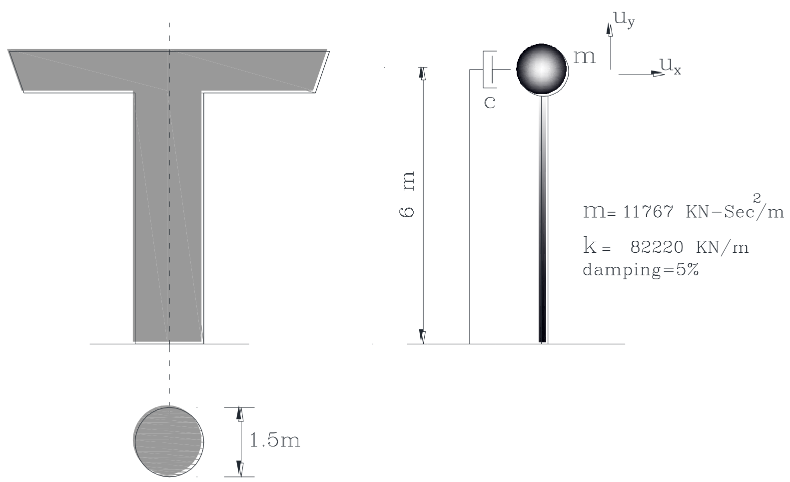
Table 1 Specifications of selected records

No.	Earthquake	Station	Mw	Epic. Dist. (km)	Ver. PGA (g)	Hor. PGA (g)	V/H
1	Bam, 2003	Bam	6.5	8.0	0.99	0.77	1.30
2	Cape Mendocino, 1992	Cape Mendocino	7.1	8.5	0.75	1.50	0.50
3	Kobe, 1995	Takatori	6.9	0.3	0.27	0.61	0.44
4	Manjil-Rudbar, 1990	Abbar	7.4	12.6	0.54	0.51	1.05
5	Northridge, 1994	Sylmar	6.7	6.4	0.53	0.84	0.63
6	Northridge, 1994	Arleta	6.7	9.2	0.55	0.34	1.62
7	Tabas, 1978	Tabas	7.4	-	0.69	0.85	0.81
8	San Fernando, 1971	Pacoima Dam	6.6	2.8	0.70	1.20	0.58

first case of our consideration. The configuration of the pier has been shown in Fig. 4(a). For the purpose of response evaluation in the horizontal and vertical directions, the pier can be idealized as a two degrees of freedom system as shown in Fig. 4(b). Dynamic characteristics of the bridge in terms of mass, stiffness, damping and height were shown in the Fig. 4(b). Inelastic material behavior of the model was characterized by the elastic perfectly plastic model.

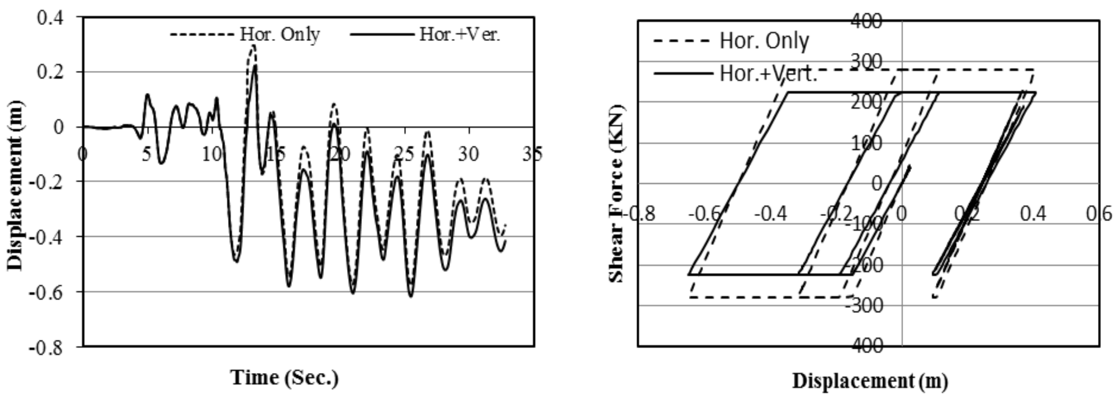
Nonlinear time history analyses of the pier subjected to horizontal and vertical components of the 1978 Tabas earthquake was accomplished by VH-Synch program for ductility factor of 2. Displacement time history plots for the ductility factors were demonstrated in Fig. 5(a). Base shear force variation against displacement correspond to the factor of ductility has been shown in Fig. 5(b).

The maximum difference between displacements of only horizontal excitation case and coupled horizontal and vertical excitation was 0.047 meter.



(a) (b)

Fig. 4 (a) A pier of a simply supported bridge and (b) its modeling with two degrees of freedom



(a) (b)

Fig. 5 Results from bridge analysis for ductility factor of 2: (a) Time history of displacement response (b) Inelastic shear force displacement response

4.2 Two-component displacement response spectrum

In structural Dynamics, the response spectrum is the maximum response of structural systems against certain earthquake for various vibration periods. In seismic design of structures response spectrum is an important and essential key. Without spectral acceleration response spectrum, calculating the code based shear forces, which induced in structures through seismic events, is impossible. Meanwhile, spectral displacement response spectrum has also important key in recently developed seismic design method; performance based seismic design method (PBSD). The performance based seismic design method is based on converting a multi degrees of freedom structure to a single degree of freedom system. Spectral displacement response of SDOF is implemented to estimate target displacement and also performance point of structures by PBSD. It is customary to construct horizontal response spectrum by neglecting interaction effects of vertical vibration on horizontal response. In some cases, constant values of stability coefficient θ to be used to characterize the P-Δ effects in generating horizontal spectrum (MacRae 1994, Bernal 1998), although in many cases such effects are completely disregarded.

In order to identify the effect of vertical excitation on horizontal vibration, regular response spectrum (based on the horizontal motion only) and two-component based displacement spectrum were computed. For nonlinear time history analysis the 5 percent of critical damping ratio was used for each case, and two components of selected records as listed in Table 1, were used as the input. Three levels of ductility factors including 1, 2 and 4 to generate nonlinear response spectra. The VH-Synch program was run for several values of mass and stiffness. Maximum displacement response of each case was calculated from time history of systems response for the selected earthquake records. Figs. 6 and 7 demonstrate maximum displacement response spectrum calculated for the components of Northridge earthquake at station Sylmar and Manjil-Rudbar (Abbar) earthquake, respectively. The response spectrum based on the horizontal motion only was also plotted by dash line together with coupled horizontal motion spectra for different ductility factors. As seen in the Figs, there were some differences in the curves for two cases. Differences in some periods were significant. Maximum increases in the response spectra for the Northridge earthquake at Symar station were 0.0354, 0.0187 and 0.005 meter for ductility factors 1, 2 and 4 respectively. Maximum spectral displacements were in period about 2.6 second. Displacement response spectra for the other earthquakes of Table 1 were also calculated. The summary of displacement maximum increase for response spectra has been shown in Table 2.

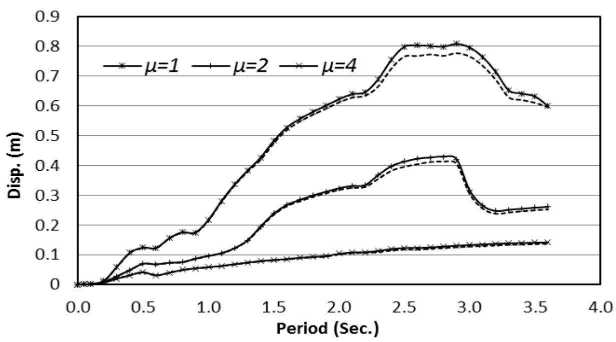


Fig. 6 The 5% damping displacement response spectrum of Northridge earthquake at Sylmar station for three ductility factors

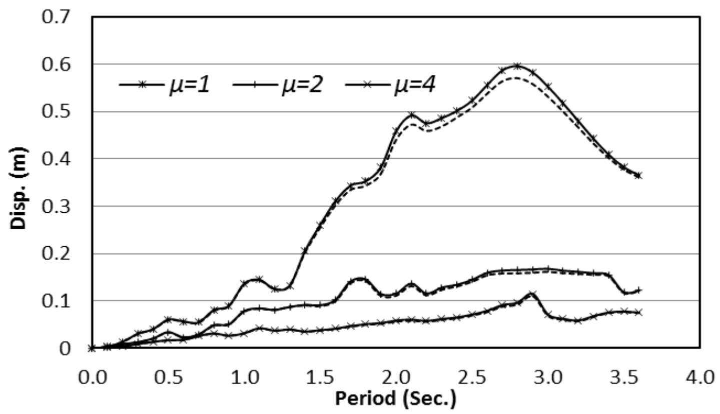


Fig. 7 The 5% damping displacement response spectrum of Manjil earthquake for three ductility factors

Table 2 Maximum increase of spectral displacement for response spectra

Earthquake Records	Maximum displacement increase (m)		
	μ=1	μ=2	μ=4
Cape Mendocino	0.0173	0.0055	0.0034
Kobe	0.0160	0.0090	0.0040
Manjil	0.0254	0.0071	0.0050
Tabas	0.0284	0.0175	0.0117
San Fernando	0.0114	0.0050	0.0022
Bam	0.0672	0.0530	0.0236
Northridge Arleta	0.0103	0.0045	0.0024
Northridge Sylmar	0.0354	0.0187	0.0050

The results indicate that spectral displacement increase in the case of coupled horizontal and vertical excitation. Although, increases were observed in all earthquakes records, but the rate of increase is not similar. Comparison of the rate of increase with maximum V/H PGAs ratio showed that there is not any correlation between the coupled response and V/H ratio. Whereas, the V/H ratio of the Northridge earthquake at Arleta station is greater than Sylmar station, the spectral displacement increase due to earthquake components at Arleta is lesser than Sylmar. Sep by step tracking of the time histories of responses revealed that the dependency of increase measure to the simultaneity of maximum PGAs in vertical and horizontal components was more than the V/H ratio of an earthquake.

4.3 Dependency of coupled responses to mass and height of system

The governing equation of the coupled vertical and horizontal vibration of system in Eq. (4) shows some degree of dependency to mass and the height of mass situation from the base. Although the period of system is also related to these parameters and any changes in mass and height values can be expressed by period, third term in Eq. (4) indicates to additional response dependency to the mass and height of system. The system response was assessed for several values of mass in the ranges between 30 to 400 $KN - Sec^2/m$ and the heights from 3 to 8 meters. Fig. 8 shows the maximum displacement increase against mass variation calculated by time history analysis of a system with different values of mass subjected to the simultaneous vertical and horizontal components of the Northridge earthquake. The displacement increase was drawn for three ductility factors. The analysis of system was also performed against the simultaneous vertical and horizontal components of earthquakes listed in Table 2. As shown in Fig. 9, increases in displacement response trend to grow with increasing mass. It can be addressed to the decreasing of the effective stiffness of system (Eq. (5)) when mass value increases.

The dependency of response to the height was demonstrated in Fig. 10 and Fig. 11. Simultaneous excitation of the system, with assumed constant natural period of vibration, by the vertical and horizontal components of Northridge earthquake for different values of height was accomplished. Fig. 10 demonstrates the maximum displacement increase versus the system height for three ductility factors. The results indicate that displacement difference in simultaneous case in comparison to the case with only horizontal excitation trend to diminish with increasing height at all considered ductility factors for. Similar results were computed form analysis of the system for other records of earthquakes as shown in Fig. 11.

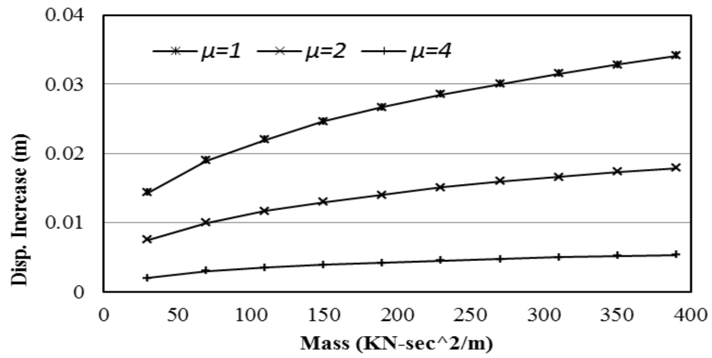


Fig. 8 Variation of Northridge earthquake maximum displacement responses for different ductility factors versus mass

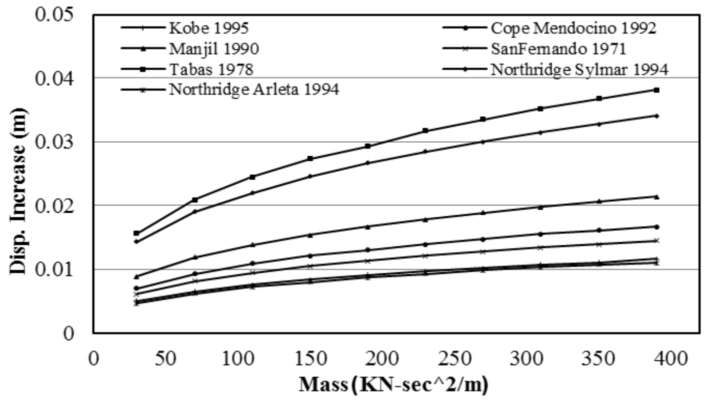


Fig. 9 Variation of maximum displacement responses for $\mu = 2$ versus mass calculated for all earthquake records

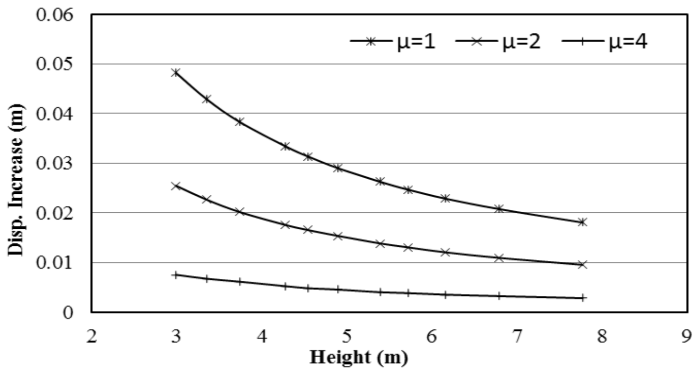


Fig. 10 Variation of Northridge earthquake maximum displacement responses for different ductility factors versus height

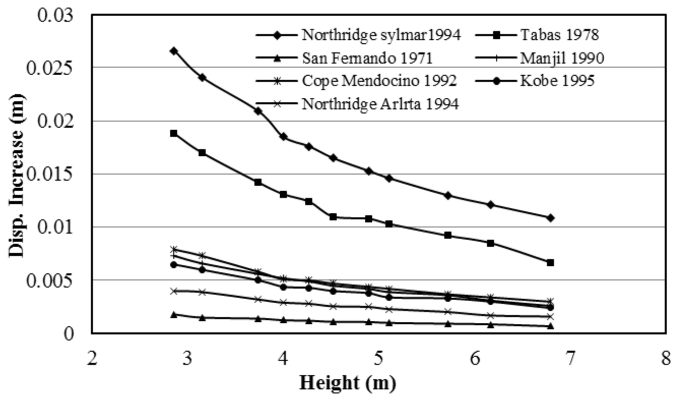


Fig. 11 Variation of maximum displacement responses for $\mu = 2$ versus height calculated for all earthquake records

5. Conclusions

In this paper, the governing equations of the coupled vertical and horizontal vibration of a system with two degrees of freedom were derived. Based on the governing equations, nonlinear dynamic analysis of some structural system was accomplished to study of the effect of vertical excitation on horizontal response. The plots of two component response spectra showed an increase in structural response in comparison with conventional response spectra. The results of this study can be summarized as follows.

• The system effective stiffness is a parameter that depends on vertical vibration response, vertical ground motion acceleration, mass and height of structure.

• System horizontal displacement increase by considering the vertical and horizontal components of ground motion simultaneity.

• There are increases in response spectrum values in the case of two component excitation. The increase amount was not concordant with the vertical to horizontal ratio of an earthquake.

• Displacement response in some earthquake records with lower V/H ratio was greater than the records with larger ratio. Sep by step tracking of computation process of the time history responses revealed that the dependency of increase measure to the simultaneity of maximum values in vertical and horizontal components was more than the V/H ratio of an earthquake.

• Structural response depends on the mass and height as well as period.

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