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Analysis of stress, magnetic field and temperature on coupled gravity-Rayleigh waves in layered water-soil model

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Abstract. In this study, the coupled effects of magnetic field, stress and thermal field on gravity waves propagating in a liquid layer over a solid surface are discussed. Due to change in temperature, initial hydrostatic stress and magnetic field, the gravity-sound Rayleigh waves can propagate in the liquid-solid interface. Dispersion properties of waves are derived by using classical dynamical theory of thermoelasticity. The phase velocity of gravity waves influenced quite remarkably in the presence of initial stress parameter, magneto-thermoelastic coupling parameter in the half space. Numerical solutions are also discussed for gravity-Rayleigh waves. In the absence of temperature, stress and magnetic field, the obtained results are in agreement with classical results.

Keywords: gravity-Rayleigh waves; magnetic field; thermoelasticity; initial stress

1. Introduction

Gravity waves are vertical waves which are generated at the interface between two mediums which has buoyancy. The example of such interface is ocean-earth interface. The examples of gravity waves are tsunamis, wind-generated waves on the water surface and ocean tides. The gravity waves which occur on air-sea interface of sea are called surface waves and those gravity waves which develop within the body of the liquid are called internal waves. Due to change in temperature, the gravity-sound Rayleigh waves can propagate in the liquid-solid interface.

Many papers on the subject of surface waves such as Rayleigh, Love waves, torsional waves have been published in many journals, due to drastic capabilities during earthquake and practical applications in the field of geophysical prospecting; unfortunately little literature is available on gravity-Rayleigh waves. This paper has been proposed to study the effect of temperature, magnetic field and initial stress on gravity waves in a liquid layer lying on the solid half-space and Rayleigh waves in the system. Sridharan *et al*. (2008) studied the effect of gravity waves and tides on mesospheric temperature inversion layers. Kumar and Kansal (2008) discussed Rayleigh waves in an isotropic generalized thermoelastic diffusive half-space subjected to rotation. Rehman and

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Khan (2009) derived a formula for the speed of Rayleigh wave speed in transversely isotropic medium. Sharma and Walia (2007) investigated Rayleigh waves in piezothermoelastic half space subjected to rotation. Sharma *et al.* (2009) studied Rayleigh waves in thermoelastic solids under the effect of micropolarity, microstretch and relaxation times. Kumar and Partap (2011) discussed vibration analysis of wave motion in micropolar thermoviscoelastic plate. Sharma and Kaur (2010) investigated Rayleigh surface waves in rotating thermo-elastic solids with voids. Sethi and Gupta (2011) studied influence of gravity and couple-stress on Rayleigh waves. Abd-Alla *et al.* (2011) studied the effect of initial stress and gravity field on Rayleigh surface waves in magneto-thermoelastic orthotropic medium. Singh and Bala (2007) studied Rayleigh surface wave at a stress free thermally insulated surface. Gupta and Gupta (2013) analyzed wave motion in an anisotropic initially stressed fiber reinforced thermoelastic media. Kakar (2014) analyzed the effect of gravity and nonhomogeneity on Rayleigh waves in higher-order elastic-viscoelastic half-space. Kakar and Gupta (2014) investigated the existence of Love waves in an intermediate heterogeneous layer placed in between homogeneous and inhomogeneous half-spaces using Green’s function technique.

In this work, we have investigated the coupled gravity-Rayleigh waves in a liquid layer lying on the gravitating elastic, solid half-space. The effect of magnetic field, thermal field and initial hydrostatic stress on gravity waves in a compressible liquid layer over an incompressible solid is examined at a particular value of Rayleigh wave velocity at different coupling coefficients of temperature and magnetic field. Biot’s equations are modified in context of classical dynamical theory of thermoelasticity with uniform magnetic field. The frequency equation is approximated and analyzed numerically to study the phase velocity of gravity waves with the help of MATLAB software (Version 7.6.0.324 (R2008a), Trademark of Mathworks. Inc. U.S. Patent).

2. Governing equations

The governing equations of magneto-thermoelastic solid with hydrostatic initial stress are

a. The stress-strain-temperature relation

$$s_{ij} = -P(\delta_{ij} + \omega_{ij}) + \bar{\lambda}\, e_{PP}\, \delta_{ij} + 2\bar{\mu} e_{ij} - \frac{\alpha}{k_T}(T + \alpha\dot{T})\delta_{ij}, \tag{1}$$

where, s_{ij} are the components of stress tensor, P is initial pressure, δ_{ij} is the Kronecker delta, ω_{ij} are the components of small rotation tensor, $\bar{\lambda}$, $\bar{\mu}$ are the counterparts of Lame parameters, e_{ij} are the components of the strain tensor, α is the volume coefficient of thermal expansion, k_T is the isothermal compressibility, $T = \Theta - T_0$ is small temperature increment, Θ is the absolute temperature of the solid half space, T_0 is the reference uniform temperature of the body chosen such that $\left|\frac{T}{T_0}\right| \langle\langle 1$

b. The displacement-strain relation

$$e_{ij} = \frac{1}{2}(\mathrm{u}_{i.j} + \mathrm{u}_{j,\mathrm{i}}), \tag{2}$$

where, $u_{i.j}$ are the components of the displacement vector

c. The small rotation-displacement relation

$$\omega_{ij} = \frac{1}{2}(\mathrm{u}_{i.j} - \mathrm{u}_{j,\mathrm{i}}), \tag{3}$$

where, $u_{i.j}$ are the components of the displacement vector

d. The modified Fourier's law

$$h_i + a^* \dot{h}_i = K\frac{\partial T}{\partial x_i}, \tag{4}$$

where, K is the thermal conductivity, $a, a^* \geq 0$ are the thermal relaxation times

e. The heat conduction equation

$$K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \rho c_p\left(\frac{\partial T}{\partial t} + \tau\frac{\partial^2 T}{\partial t^2}\right) + \gamma T_0\left(\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 v}{\partial y \partial t} + \tau_0 \delta_{ij}\left[\frac{\partial^3 u}{\partial x \partial t^2} + \frac{\partial^3 v}{\partial y \partial t^2}\right]\right) \tag{5}$$

where, K is the thermal conductivity, c_p is specific heat per unit mass at constant strain, τ_0 is the first relaxation time, τ is second relaxation time, δ_{ij} is the Kronecker delta, ρ is density and T is the incremental change of temperature from the initial state of the solid half space. Moreover the use of the relaxation times τ, τ_0 and a parameter δ_{ij} marks the aforementioned fundamental equations possible for the three different theories:

(1) Classical Dynamical theory: $\tau = \tau_0 = 0, \delta_{ij} = 0.$

(2) Lord and Shulman's theory: $\tau = 0,\ \tau_0 > 0,\ \delta_{ij} = 1.$

(3) Green and Lindsay's theory: $\tau \geq \tau_0 > 0,\ \delta_{ij} = 0.$

f. Maxwell's equations

$$\vec{\nabla}\cdot\vec{\mathrm{E}} = 0, \vec{\nabla}\cdot\vec{\mathrm{B}} = 0, \vec{\nabla}\times\vec{\mathrm{E}} = -\frac{\partial\vec{\mathrm{B}}}{\partial t}, \vec{\nabla}\times\vec{\mathrm{B}} = \mu_e\varepsilon_e\frac{\partial\vec{\mathrm{E}}}{\partial t} \tag{6}$$

where, $\vec{\mathrm{E}}$, $\vec{B}$, μ_e and ε_e are electric field, magnetic field, permeability and permittivity of the solid half space.

g. The components of electric and magnetic field

$$\vec{\mathrm{H}'}(0,0,\mathrm{H}') = \vec{\mathrm{H}}_0 + \vec{h}' \tag{7}$$

where, $\vec{h}'$ is the perturbed magnetic field over $\vec{\mathrm{H}}_0$.

h. Maxwell stress components

$$T_{ij} = \mu_e\left[H_i e_i + H_j e_j - (H_k e_k)\delta_{ij}\right] \text{ (where i, j, k =1, 2,3)} \tag{8}$$

where, H_i, H_j, H_k are the components of primary magnetic field, e_i, e_j, e_k are the stress components acting along x-axis, y-axis, z-axis respectively and δ_{ij} is the Kronecker delta.

Using Eq. (8), we get

$$T_{yy} = \mu_e H_0^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \text{ and } T_{xy} = 0 \tag{9}$$

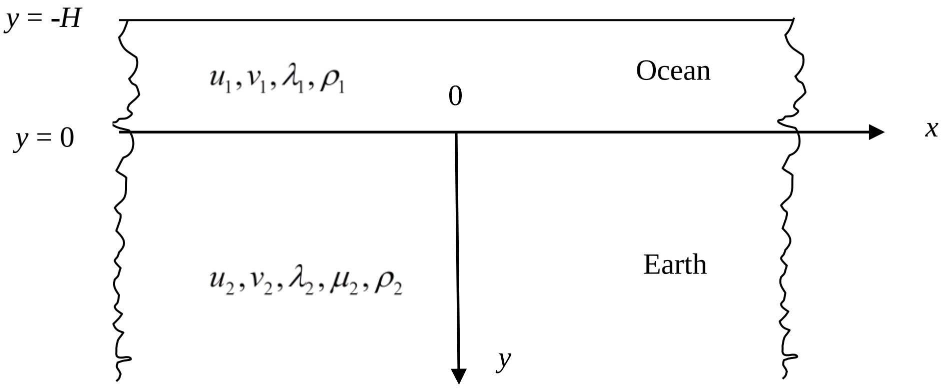

Fig. 1 Geometry of ocean-earth system

The dynamical equations of motion for the propagation of wave have been derived by Biot (1965) and in two dimensions these are given by

$$\frac{\partial s_{xx}}{\partial x} + \frac{\partial s_{xy}}{\partial y} - P \frac{\partial \omega}{\partial y} + B_x = \rho \frac{\partial^2 u}{\partial t^2} \tag{10}$$

$$\frac{\partial s_{xy}}{\partial x} + \frac{\partial s_{yy}}{\partial y} - P \frac{\partial \omega}{\partial x} + B_y = \rho \frac{\partial^2 v}{\partial t^2} \tag{11}$$

where, s_{xx}, s_{yy} and s_{xy} are incremental thermal stress components. The first two are principal stress components along x- and y-axes, respectively and last one is shear stress component in the x-y plane, ρ is the density of the medium and u, v are the displacement components along x and y directions respectively, B is body force and its components along x and y axis are B_x and B_y respectively. ω is the rotational component i.e., $\omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$ and $P = s_{yy} - s_{xx}$.

The body forces along x and y axis under constant primary magnetic field H_0 parallel to z-axis are given by

$$B_x = \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} \right) \tag{12}$$

$$B_y = \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} \right) \tag{13}$$

where, μ_e is permittivity of the medium.

Following Biot (1965), the stress-strain relations with incremental isotropy are

$$s_{xx} = (\lambda + 2\mu + P)e_{xx} + (\lambda + P)e_{yy} + 2\mu e_{xx} - \gamma\left(\mathrm{T} + \tau\frac{\partial \mathrm{T}}{\partial x}\right) \tag{14}$$

$$s_{yy} = \lambda e_{xx} + (\lambda + 2\mu)e_{yy} - \gamma\left(\mathrm{T} + \tau\frac{\partial \mathrm{T}}{\partial x}\right) \tag{15}$$

$$s_{xy} = 2\mu e_{xy} \tag{16}$$

where

$$e_{xx} = \frac{\partial u}{\partial x}, \qquad e_{yy} = \frac{\partial v}{\partial x}, \qquad e_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{17}$$

where, e_{xx} and e_{yy} are the principle strain components and e_{xy} is the shear strain component, γ=(3λ+2μ)α_t, α_t is the coefficient of linear expansion of the material, λ μ are Lame's constants, T is the incremental change of temperature from the initial state and τ is second relaxation time.

3. Formulation of the problem

Let us consider gravity and Rayleigh waves in a compressible liquid layer of uniform thickness H over a solid half-space (Fig. 1). We assume the following assumptions;

a. Both media (compressible liquid layer and solid soil layer) under consideration are homogeneous in nature and gravity acts on them.

b. In liquid layer pressure is proportional to the degree of compression and in the solid half space stress and deformation are related through Hooke's law.

c. Displacements in the compressible liquid are small as compared to the compressible liquid layer thickness and characteristic wavelengths.

d. Deformations are small in the compressible liquid.

The wave is propagating along the direction of x-axis, y-axis is taken vertically downward and y=0 is the surface of the half space. The half space is under an initial stress P, magnetic field H_0 and initial temperature T_0.

4. Solution of the problem

4.1 For upper liquid surface

The wave equation for liquid surface satisfying velocity potential $\overline{\phi}_1$ is given by (Ewing *et al.* 1957)

$$\frac{\partial^2 \overline{\phi}_1}{\partial t^2} = \alpha_1^2 \nabla^2 \overline{\phi}_1 + g\frac{\partial \overline{\phi}_1}{\partial t} \tag{18}$$

where, $\alpha_1^2 = \dfrac{\lambda_1}{\rho_1}$; $\lambda_1$is Lame's constant and ρ_1 is the density of the liquid. g is acceleration due to gravity acting on the liquid.

Eq. (1) can be solved by taking plane harmonic waves travelling along x-axis as

$$\bar{\phi}(x,y,t) = A_1(y)\mathrm{e}^{i(\omega \mathrm{t} - \mathrm{kx})} \tag{19}$$

From Eq. (18) and Eq. (19)

$$\alpha_1^2 \frac{d^2 A_1}{dy^2} + g\frac{dA_1}{dy} - (k^2\alpha_1^2 - \omega^2)A_1 = 0 \tag{20}$$

The solution of Eq. (20) is

$$A_1(y) = e^{-\left(\frac{gy}{2\alpha_1^2}\right)}\left(Ae^{-i\xi y} + Be^{i\xi y}\right) \tag{21}$$

where, $\xi = \left(k_{\alpha_1}^2 - k^2 - \dfrac{g^2}{4\alpha_1^4}\right)^{\frac{1}{2}}$ and $k_{\alpha_1} = \dfrac{\omega}{\alpha_1}$.

From Eq. (19) and Eq. (21)

$$\bar{\phi}_1 = e^{\left(-\left(\frac{gy}{2\alpha_1^2}\right) + i(\omega t - kx)\right)}\left(Ae^{-i\xi y} + Be^{i\xi y}\right) \tag{22}$$

The velocity components in the liquid along x-axis and y-axis are given by

$$\frac{\partial\bar{\phi}_1}{\partial x} = ike^{\left(-\left(\frac{gy}{2\alpha_1^2}\right) + i(\omega t - kx)\right)}\left(Ae^{-i\xi y} + Be^{i\xi y}\right) \tag{23}$$

$$\frac{\partial\bar{\phi}_1}{\partial y} = ike^{\left(-\left(\frac{gy}{2\alpha_1^2}\right) + i(\omega t - kx)\right)}\left[\left(i\xi + \frac{gy}{2\alpha_1^2}\right)Ae^{-i\xi y} - \left(i\xi - \frac{gy}{2\alpha_1^2}\right)Be^{i\xi y}\right] \tag{24}$$

4.2 For lower half space

From Eq. (12), Eq. (13), Eq. (14), Eq. (15), Eq. (16) and Eq. (17), we get

$$(\lambda_2 + 2\mu_2)\frac{\partial^2 u_2}{\partial x^2} + (\lambda_2 + \mu_2)\frac{\partial^2 v_2}{\partial x \partial y} + \mu_2\frac{\partial^2 u_2}{\partial^2 y} + \mu_e H_0^2\left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial x \partial y}\right) = \rho_2\frac{\partial^2 u_2}{\partial t^2} + \gamma\left(\frac{\partial T}{\partial x} + \tau\frac{\partial^2 T}{\partial t \partial x}\right) \tag{25}$$

$$(\lambda_2 + 2\mu_2)\frac{\partial^2 v_2}{\partial y^2} + (\lambda + \mu_2)\frac{\partial^2 u_2}{\partial x \partial y} + \mu_2\frac{\partial^2 v_2}{\partial^2 x} + \mu_e H_0^2\left(\frac{\partial^2 u_2}{\partial x \partial y} + \frac{\partial^2 v_2}{\partial y^2}\right) = \rho_2\frac{\partial^2 v_2}{\partial t^2} + \gamma\left(\frac{\partial T}{\partial y} + \tau\frac{\partial^2 T}{\partial t \partial y}\right) \tag{26}$$

where, λ_2, μ_2 are Lame's constants for the lower solid half space and ρ_2 is its density.

From Eq. (25) and (26) by using classical dynamical theory we get

$$(\lambda_2+2\mu_2)\frac{\partial^2 u_2}{\partial x^2}+(\lambda_2+\mu_2)\frac{\partial^2 v_2}{\partial x\partial y}+\mu_2\frac{\partial^2 u_2}{\partial^2 y}+\mu_e H_0^2\left(\frac{\partial^2 u_2}{\partial x^2}+\frac{\partial^2 v_2}{\partial x\partial y}\right)=\rho_2\frac{\partial^2 u_2}{\partial t^2}+\frac{\partial}{\partial x}(\gamma T) \quad (27)$$

$$(\lambda_2+2\mu_2)\frac{\partial^2 v_2}{\partial y^2}+(\lambda+\mu_2)\frac{\partial^2 u_2}{\partial x\partial y}+\mu_2\frac{\partial^2 v_2}{\partial^2 x}+\mu_e H_0^2\left(\frac{\partial^2 u_2}{\partial x\partial y}+\frac{\partial^2 v_2}{\partial y^2}\right)=\rho_2\frac{\partial^2 v_2}{\partial t^2}+\frac{\partial}{\partial y}(\gamma T) \quad (28)$$

Eq. (27) and Eq. (28) can be solved by choosing potential functions ϕ_2 and ψ_2 as

$$u_2=\frac{\partial\phi_2}{\partial x}-\frac{\partial\psi_2}{\partial y}\ \text{and}\ v_2=\frac{\partial\phi_2}{\partial x}+\frac{\partial\psi_2}{\partial y} \quad (29)$$

From Eq. (27), (28) and (29), we get

$$\nabla^2\phi_2=\frac{\rho_2}{(\lambda_2+2\mu_2+\mu_e H_0^2)}\frac{\partial^2\phi_2}{\partial t^2}+\frac{\gamma T}{(\lambda_2+2\mu_2+\mu_e H_0^2)} \quad (30)$$

$$\nabla^2\psi_2=\frac{\rho_2}{\mu_2}\frac{\partial^2\psi_2}{\partial t^2} \quad (31)$$

where, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

By using classical dynamical theory: $\tau=\tau_0=0$, $\delta_{ij}=0$ Eq. (5) reduces to

$$K\nabla^2 T=\rho_2 c_p\frac{\partial T}{\partial t}\ +\ \gamma T_0\frac{\partial}{\partial t}\left(\frac{\partial u_2}{\partial x}+\frac{\partial v_2}{\partial y}\right) \quad (32)$$

Introduce Eq. (29) in Eq. (32), we get

$$\nabla^2 T-\frac{c_p\rho_2}{K}\frac{\partial T}{\partial t}-\frac{\lambda T_0}{c_p}\nabla^2\frac{\partial\phi}{\partial t}=0 \quad (33)$$

From Eq. (30) and Eq. (32), eliminating T, we get

$$\left(\nabla^2-\frac{1}{C_1^2}\frac{\partial^2}{\partial t^2}\right)\left(\nabla^2-\frac{c_p\rho_2}{K}\frac{\partial}{\partial t}\right)\phi_2-\chi\eta\nabla^2\left(\frac{\partial\phi_2}{\partial t}\right)=0 \quad (34)$$

where, $C_1^2=\frac{(\lambda_2+2\mu_2+\mu_e H_0^2)}{\rho_2},\chi=\frac{\gamma}{(\lambda_2+2\mu_2+\mu_e H_0^2)}$ and $\eta=\frac{\gamma T_0}{K}$

From Eq. (31) and Eq. (32), eliminating T, we get

$$\left(\nabla^2-\frac{1}{C_2^2}\frac{\partial^2}{\partial t^2}\right)\psi_2=0 \quad (35)$$

where, $C_2^2 = \dfrac{\mu_2}{\rho_2}$

Eq. (34) and Eq. (35) can further be solved by plane harmonic waves travelling along *x*-axis as

$$\phi_2(x,y,t) = A_2(y)\,\mathrm{e}^{i(\omega \mathrm{t}-\mathrm{kx})} \tag{36}$$

$$\psi_2(x,y,t) = B_2(y)\,\mathrm{e}^{i(\omega \mathrm{t}-\mathrm{kx})} \tag{37}$$

where, k is wave number and ω is frequency of oscillation of the harmonic wave.

From Eq. (34) and Eq. (36), we get

$$\left(\frac{\partial^2}{\partial y^2} - \lambda_1^2\right)\left(\frac{\partial^2}{\partial y^2} - \lambda_2^2\right)A_2(y) = 0 \tag{38}$$

$$\left(\frac{\partial^2}{\partial y^2} - \nu^2\right)B_2(y) = 0 \tag{39}$$

where, $\lambda_1^2 = k^2 - \alpha^2, \lambda_2^2 = k^2 - \beta^2$ and $\nu^2 = k^2 - \delta^2$.

Here $\delta^2 = \dfrac{\omega^2}{C_2^2}$ and α^2, β^2 are the roots of following biquadratic equation.

$$\Lambda^4 - \Lambda^2[\sigma^2 + \mathrm{q}(1+\varepsilon)] + \sigma^2\,\mathrm{q} = 0 \tag{40}$$

where, $\Lambda^2 = -\nabla^2$ and the roots α^2, β^2 are

$$\alpha^2 = \mathrm{q}\left[1 + \frac{qm}{\sigma^2 - q}\right] \text{ and } \beta^2 = \sigma^2\left[1 - \frac{qm}{\sigma^2 - q}\right] \tag{41}$$

Here, $\sigma^2 = \dfrac{\omega}{C_1^2}, q = \dfrac{-i\omega c_p \rho_2}{K}$ and $m = \dfrac{\gamma^2 T_0}{K\rho_2(\lambda_2 + 2\mu_2 + \mu_e H_0^2)}$ are magneto-thermoelastic coupling parameters.

The requirement that the stresses and hence the functions ϕ_2 and ψ_2 vanish as $(x^2+y^2)\to\infty$ leads to the following solutions of Eq. (38) and Eq. (40)

$$A_2(y) = \frac{D}{i\omega}\mathrm{e}^{-\lambda_1 y} + \frac{E}{i\omega}\mathrm{e}^{-\lambda_2 y} \tag{42}$$

$$B_2(y) = \frac{F}{i\omega}\mathrm{e}^{-\nu y} \tag{43}$$

Introducing Eq. (42) and Eq. (43) in Eq. (36) and Eq. (37), we get

$$\phi_2(x,y,t) = \left(\frac{D}{i\omega}\mathrm{e}^{-\lambda_1 y} + \frac{E}{i\omega}\mathrm{e}^{-\lambda_2 y}\right)\mathrm{e}^{i(\omega t - kx)} \tag{44}$$

$$\psi_2(x,y,t)=\left(\frac{F}{i\omega}\mathrm{e}^{-\nu y}\right)\mathrm{e}^{i(\omega t-kx)} \tag{45}$$

From Eq. (29), Eq. (44) and Eq. (15) we get

$$u_2=\frac{1}{i\omega}\left(ik(D\,\mathrm{e}^{-\lambda_1 y}+E\,\mathrm{e}^{-\lambda_2 y})+\nu F\,\mathrm{e}^{-\nu y}\right)\mathrm{e}^{i(\omega t-kx)} \tag{46}$$

$$v_2=-\frac{1}{i\omega}\left(ikF\,\mathrm{e}^{-\nu y}+\lambda_1 D\,\mathrm{e}^{-\lambda_1 y}+\lambda_2 E\,\mathrm{e}^{-\lambda_2 y}\right)\mathrm{e}^{i(\omega t-kx)} \tag{47}$$

From Eq. (40) and Eq. (47), we get the velocity components in lower half space, given by

$$\frac{\partial u_2}{\partial t}=-\left(ik(D\,\mathrm{e}^{-\lambda_1 y}+E\,\mathrm{e}^{-\lambda_2 y})-\nu F\,\mathrm{e}^{-\nu y}\right)\mathrm{e}^{i(\omega t-kx)} \tag{48}$$

$$\frac{\partial v_2}{\partial t}=-\left(\lambda_1 D\,\mathrm{e}^{-\lambda_1 y}+\lambda_2 E\,\mathrm{e}^{-\lambda_2 y}+ikF\,\mathrm{e}^{-\nu y}\right)\mathrm{e}^{i(\omega t-kx)} \tag{49}$$

From Eq. (30)

$$T=\frac{(\lambda_2+2\mu_2+\mu_e H_0^2)}{\gamma}\left[\nabla^2\phi_2-\frac{1}{C_1^2}\frac{\partial^2\phi_2}{\partial t}\right] \tag{50}$$

Eq. (44) and Eq. (50), we get

$$T=\frac{(\lambda_2+2\mu_2+\mu_e H_0^2)}{\gamma}\frac{1}{k^2}\left[(\sigma^2-\alpha^2)D\,\mathrm{e}^{-\lambda_1 y}+(\sigma^2-\beta^2)E\,\mathrm{e}^{-\lambda_2 y}\right]\mathrm{e}^{i(\omega t-kx)} \tag{51}$$

5. Boundary conditions and dispersion equation

The initial conditions are supplemented by the following boundary conditions. Since the vertical component together with the normal and tangential stresses is continuous at the surface y=0 also pressure is zero at the free surface, therefore conditions are

$$\begin{aligned}
&\text{i. } P=0 \text{ at } y=-H,\\
&\text{ii. } \frac{\partial u_2}{\partial t}=\frac{\partial v_2}{\partial t} \text{ at } y=0,\\
&\text{iii. } \nabla f_x=s_{12}-P\frac{\partial v_2}{\partial x}=0 \text{ at } y=0,\\
&\text{iv. } \nabla f_y=-P \text{ , where, } \nabla f_y=s_{22}-P\frac{\partial u_2}{\partial x}-g\rho_2(v_2)_{y=0}\\
&\text{v. } \frac{\partial T}{\partial y}+hT=0 \text{ at } y=0.
\end{aligned} \tag{52}$$

where ∇f_x and ∇f_y are incremental boundary forces per unit initial area and *h* is the ratio of heat transfer coefficient and thermal conductivity.

By considering the deformation of the free surface and representing the vertical displacement by v_1, the first boundary condition of Eq. (52) gives

$$\frac{\partial \bar{\phi}_1}{\partial t} + g v_1 = 0 \tag{53}$$

Using Eq. (23), (24) and $\dfrac{\partial \bar{\phi}_1}{\partial y} = i\omega v_1$ at $y = -H$, Eq. (53) becomes

$$A\left(-\omega^2 + i\xi g + \frac{gy}{2\alpha_1^2}\right) + B\left(-\omega^2 - i\xi g + \frac{gy}{2\alpha_1^2}\right) = 0 \tag{54}$$

Using Eq. (24) and Eq. (40), the second boundary condition of Eq. (52) becomes

$$A\left(-i\xi g - \frac{gy}{2\alpha_1^2}\right) + B\left(i\xi g - \frac{gy}{2\alpha_1^2}\right) + D\lambda_1 + E\lambda_2 + F\left(ik\right) = 0 \tag{55}$$

Using Eqs. (14), (15), (16), (17), (29) and Eq. (47), the third boundary condition of Eq. (52) becomes

$$D(1+\mathrm{S})ik\lambda_1 + E(1+\mathrm{S})ik\lambda_2 + F\left[\frac{\delta^2}{2} - (1+\mathrm{S})k^2\right] = 0 \tag{56}$$

where, $S = \dfrac{P}{2\mu_2}$ is dimensionless initial stress parameter.

The tangential stress on the side of the liquid is given by (Ewing et al. 1957)

$$P = \rho_1\left(\frac{\partial \bar{\varphi}_1}{\partial t}\right) + \rho_1 (v_1)_{y=0} \tag{57}$$

i.e.

$$P = \left[A\left(-i\omega - \frac{g\xi}{\omega} - \frac{g^2}{i\omega 2\alpha_1^2}\right) - B\left(i\omega - \frac{g\xi}{\omega} + \frac{g^2}{i\omega 2\alpha_1^2}\right)\right]\rho_1 e^{i(\omega t - kx)} = 0 \tag{58}$$

Using Eqs. (14), (15), (16) and Eq. (47), the fourth boundary condition of Eq. (52) becomes

$$\begin{aligned} &A\left(-i\omega - \frac{g\xi}{\omega} - \frac{g^2}{i\omega 2\alpha_1^2}\right)\rho_1 + B\left(i\omega - \frac{g\xi}{\omega} + \frac{g^2}{i\omega 2\alpha_1^2}\right)\rho_1 - D\left(i\omega\left[1 - \frac{2k^2}{\delta^2}\right] - \frac{ig\lambda_1}{\omega} - \frac{iP^2k^2}{\omega\rho_2}\right)\rho_2 \\ &-E\left(i\omega\left[1 - \frac{2k^2}{\delta^2}\right] - \frac{ig\lambda_2}{\omega} - \frac{iP^2k^2}{\omega\rho_2}\right)\rho_2 - F\left(\frac{2k\omega\upsilon}{\delta^2} - \frac{gk}{\omega} - \frac{k\upsilon P}{\omega\rho_2}\right)\rho_2 = 0 \end{aligned} \tag{59}$$

Using Eq. (51), the fifth boundary condition of Eq. (52) becomes

$$(h-\lambda_1)(\sigma^2-\alpha^2)D+(h-\lambda_2)(\sigma^2-\beta^2)E=0 \tag{60}$$

Now eliminating *A*, *B*, *D*, *E* and *F* from Eq. (54), Eq. (55), Eq. (56), Eq. (59) and Eq. (60), we get fifth order determinant

$$\left|a_{ij}\right|=0 \quad \text{(i,j =1,2,3,4,5)} \tag{61}$$

where

$$\begin{aligned}
&a_{11}=\left(-\omega^2+i\xi g+\frac{gy}{2\alpha_1^2}\right), a_{12}=\left(-\omega^2-i\xi g+\frac{gy}{2\alpha_1^2}\right), a_{13}=0, a_{14}=0, a_{15}=0,\\
&a_{21}=\left(-i\xi g-\frac{gy}{2\alpha_1^2}\right), a_{22}=\left(i\xi g-\frac{gy}{2\alpha_1^2}\right), a_{23}=\lambda_1, a_{24}=\lambda_2, a_{25}=\left(ik\right), a_{31}=0,\\
&a_{32}=0, a_{33}=(1+\mathrm{S})ik\lambda_1, a_{34}=(1+\mathrm{S})ik\lambda_2, a_{35}=\left[\frac{\delta^2}{2}-(1+\mathrm{S})k^2\right],\\
&a_{41}=\left(-i\omega-\frac{g\xi}{\omega}-\frac{g^2}{i\omega 2\alpha_1^2}\right)\rho_1, a_{42}=\left(i\omega-\frac{g\xi}{\omega}+\frac{g^2}{i\omega 2\alpha_1^2}\right)\rho_1,\\
&a_{43}=-\left(i\omega\left[1-\frac{2k^2}{\delta^2}\right]-\frac{ig\lambda_1}{\omega}-\frac{iP^2k^2}{\omega\rho_2}\right)\rho_2, a_{44}=-\left(i\omega\left[1-\frac{2k^2}{\delta^2}\right]-\frac{ig\lambda_2}{\omega}-\frac{iP^2k^2}{\omega\rho_2}\right)\rho_2,\\
&a_{45}=-F\left(\frac{2k\omega\upsilon}{\delta^2}-\frac{gk}{\omega}-\frac{k\upsilon P}{\omega\rho_2}\right)\rho_2, a_{51}=0, a_{52}=0, a_{53}=(h-\lambda_1)(\sigma^2-\alpha^2),\\
&a_{54}=(h-\lambda_2)(\sigma^2-\beta^2), a_{55}=0.
\end{aligned} \tag{62}$$

Expanding Eq. (62), we get

$$\begin{aligned}
&\left[1-\frac{gk^2\tan\xi H}{\omega^2\xi}+\frac{gy\tan\xi H}{2\alpha_1^2\xi}\right]\left[\left(\frac{2k^2}{\delta^2}-1\right)^2+\frac{4k^2}{\delta^2}\left(\frac{2k^2}{\delta^2}-1\right)S+\frac{4k^4}{\delta^4}S^2\right]\\
&\times\left[h\left((\sigma^2-\beta^2)-(\sigma^2-\alpha^2)\right)+\left(\lambda_1(\sigma^2-\alpha^2)-\lambda_2(\sigma^2-\beta^2)\right)\right]\\
&-\left[\frac{g}{\omega^2}+\frac{4k^2}{\delta^4}(k^2-\delta^2)^{\frac{1}{2}}(1+S)^2\right]\times\left[\begin{aligned}&h\left(\lambda_1(\sigma^2-\beta^2)-\lambda_2(\sigma^2-\alpha^2)\right)\\&+\lambda_1\lambda_2\left((\sigma^2-\alpha^2)-(\sigma^2-\beta^2)\right)\end{aligned}\right]\\
&+\frac{\rho_1\tan\xi H}{\rho_2\xi}\left[1-\left(\frac{gK}{\omega^2}\right)^2\right]\left[h\left(\lambda_1(\sigma^2-\beta^2)-\lambda_2(\sigma^2-\alpha^2)\right)+\lambda_1\lambda_2\left((\sigma^2-\alpha^2)-(\sigma^2-\beta^2)\right)\right]=0
\end{aligned} \tag{63}$$

where, $(k^2-\delta^2)^{\frac{1}{2}}=\nu$ and $\xi=\left(k_{\alpha_1}^2-k^2-\dfrac{g^2}{4\alpha_1^4}\right)^{\frac{1}{2}}$

For naturally occurring waves $\frac{g}{k_{\alpha_1}^2}\langle\langle 1$, therefore Eq. (63) reduces to

$$\left[1-\frac{gk^2\tan\xi' H}{\omega^2\xi'}+\frac{gy\tan\xi H}{2\alpha_1^2\xi}\right]\left[\left(\frac{2k^2}{\delta^2}-1\right)^2+\frac{4k^2}{\delta^2}\left(\frac{2k^2}{\delta^2}-1\right)S+\frac{4k^4}{\delta^4}S^2\right]$$

$$\times\left[h\left((\sigma^2-\beta^2)-(\sigma^2-\alpha^2)\right)+\left(\lambda_1(\sigma^2-\alpha^2)-\lambda_2(\sigma^2-\beta^2)\right)\right]$$

$$-\left[\frac{g}{\omega^2}+\frac{4k^2}{\delta^4}(k^2-\delta^2)^{\frac{1}{2}}(1+S)^2\right]\times\begin{bmatrix}h\left(\lambda_1(\sigma^2-\beta^2)-\lambda_2(\sigma^2-\alpha^2)\right)\\+\lambda_1\lambda_2\left((\sigma^2-\alpha^2)-(\sigma^2-\beta^2)\right)\end{bmatrix}+\frac{\rho_1\tan\xi' H}{\rho_2\xi'} \tag{64}$$

$$\times\left[1-\left(\frac{gK}{\omega^2}\right)^2\right]\left[h\left(\lambda_1(\sigma^2-\beta^2)-\lambda_2(\sigma^2-\alpha^2)\right)+\lambda_1\lambda_2\left((\sigma^2-\alpha^2)-(\sigma^2-\beta^2)\right)\right]=0$$

where, $\xi'=\left(k_{\alpha_1}^2-k^2\right)^{\frac{1}{2}}$

$$\text{Let } \beta_1^2=1-\frac{\alpha^2}{k^2}\ ,\ \beta_2^2=1-\frac{\beta^2}{k^2} \text{ and } \beta_3^2=1-\frac{\delta^2}{k^2} \tag{65}$$

$$\text{If } H\to\infty \text{ then } \frac{\tan\xi' H}{\xi'}\to 1 \tag{66}$$

With the help of Eq. (65) and Eq. (66), Eq. (64) reduces to

$$\begin{aligned}&\left[2(1+S)-\frac{C^2}{C_2^2}\right]^2\left[\frac{C^2}{C_1^2}+\beta_1^2+\beta_2^2+\beta_1^2\beta_2^2-1\right]-4(1+S)^2\beta_1^2\beta_2^2\beta_3^2(\beta_1^2+\beta_2^2)\\&\left(\frac{\delta^2}{k^2}+\frac{c^2}{\beta_2^2}\right)++\frac{\rho_1}{\rho_2}\frac{C^4}{C_2^4}\beta_1^2\beta_2^2(\beta_1^2+\beta_2^2)-\left(1-\frac{\rho_1}{\rho_2}\right)\left(\frac{gK}{\omega^2}\right)\frac{C^4}{C_2^4}\beta_1^2\beta_2^2(\beta_1^2+\beta_2^2)\\&=\frac{h}{k}\left[2(1+S)-\frac{C^2}{C_2^2}\right]^2(\beta_1+\beta_2)-4(1+S)^2\beta_3\left(1+\beta_1\beta_2-\frac{C^2}{C_1^2}\right)\\&+\frac{\rho_1}{\rho_2}\frac{C^4}{C_2^4}\left(1+\beta_1\beta_2-\frac{C^2}{C_1^2}\right)-\left(1-\frac{\rho_1}{\rho_2}\right)\left(\frac{gK}{\omega^2}\right)\frac{C^4}{C_2^4}\left(1+\beta_1\beta_2-\frac{C^2}{C_1^2}\right)\end{aligned} \tag{67}$$

From Eq. (40), we get

$$\alpha^2+\beta^2=\sigma^2+\mathrm{q}(1+m) \text{ and } \alpha^2\beta^2=\sigma^2\,\mathrm{q} \tag{68}$$

From Eq. (65) and Eq. (68), we get

$$\alpha_1^2+\alpha_2^2=2-\frac{C^2}{C_1^2}-\frac{ic^2}{\varpi C_1^2}\left(1+m-\frac{C^2}{C_1^2}\right) \text{ and } \alpha_1^2\alpha_2^2=1-\frac{C^2}{C_1^2}-\frac{iC^2}{\varpi C_1^2}\left(1+m-\frac{C^2}{C_1^2}\right) \tag{69}$$

where, $\varpi=\dfrac{K\omega}{c_p\rho_2 C_1^2}$, is reduced frequency.

Introducing Eq. (69) into Eq. (67), expanding the quantities β_1 and β_2 in the series of ϖ and neglecting the terms of the order $\varpi^{\frac{1}{2}}$, we get expression for complex frequency equation of gravity-Rayleigh waves, the real part are

$$\left[2(1+S)-\frac{C^2}{C_2^2}\right]^2-4(1+S)^2\left\{\left[1-\frac{C^2}{C_2^2}\right]\left[1-\frac{C^2}{(1+m)C_1^2}\right]\right\}^{\frac{1}{2}}$$
$$=\left[\left(1-\frac{\rho_1}{\rho_2}\right)\left(\frac{gK}{\omega^2}\right)-\frac{\rho_1}{\rho_2}\right]\frac{C^4}{C_2^4}\left[1-\frac{C^2}{(1+m)C_1^2}\right]^{\frac{1}{2}} \tag{70}$$

Let $\dfrac{C_2^2}{C_1^2(1+m)}=N$, $G=\left(\dfrac{gK}{\omega^2}\right)$ and $v_p=\dfrac{C^2}{C_2^2}$, then Eq. (70) becomes

$$\left[2(1+S)-v_p\right]^2-4(1+S)^2\left\{\left[1-v_p\right]\left[1-v_pN\right]\right\}^{\frac{1}{2}}=\left[\left(1-\frac{\rho_1}{\rho_2}\right)G-\frac{\rho_1}{\rho_2}\right]v_p^2\left[1-v_pN\right]^{\frac{1}{2}} \tag{71}$$

Also, $N=\dfrac{C_2^2}{C_1^2(1+m)}=\dfrac{C_2^2}{C_0^2(1+R)(1+m)}$

Here, $R=\dfrac{c_a^2}{c_0^2}$ is dimensionless magneto pressure number, $c_a^2=\dfrac{\mu_e H_0^2}{\rho_2}$ is dimensionless magneto wave velocity, $v_p=\dfrac{C^2}{C_2^2}$ is dimensionless phase velocity of Rayleigh waves, $c_0^2=\dfrac{\lambda_2+2\mu_2}{\rho_2}$ is dimensionless isothermal dilatational, $G=\left(\dfrac{gK}{\omega^2}\right)$ is the phase velocity of gravity waves, $S=\dfrac{P}{2\mu_2}$ is dimensionless initial stress parameter and m is dimensionless thermoelastic coupling parameter.

6. Numerical analysis

We have taken magnetic field of earth equal to 50 μT and $\dfrac{\rho_1}{\rho_2}=\dfrac{1327}{5337}$ i.e., ρ_1 density of sea water and ρ_2 density of earth, $v_p=\dfrac{C^2}{C_2^2}$ =0.906 at 0.5 *kH*, various curves are plotted to study the variation of gravity waves in the system. The curves indicate that as the temperature increases, the phase velocity of gravity wave also increases in non-linear form. On the increase of magnetic field the phase velocity increases provided the initial stress is not changed.

Fig. 2: Variation of G (the phase velocity of gravity waves) with S (initial stress parameter) for different values of N and v_p=.906 (phase velocity) of Rayleigh waves.

Fig. 3: Variation of $G=\left(\frac{gK}{\omega^2}\right)$ (the phase velocity of gravity waves) with $S=\frac{P}{2\mu_2}$ (initial stress parameter) for different values of $R=\frac{c_a^2}{c_0^2}$ (magneto pressure number) constant and $v_p=\frac{C^2}{C_2^2}=.906$ (phase velocity) of Rayleigh waves.

Fig. 4: Variation of $G=\left(\frac{gK}{\omega^2}\right)$ (the phase velocity of gravity waves) with $S=\frac{P}{2\mu_2}$ (initial stress parameter) for different values of m (thermoelastic coupling parameter) constant and $v_p=\frac{C^2}{C_2^2}=.906$ (phase velocity) of Rayleigh waves.

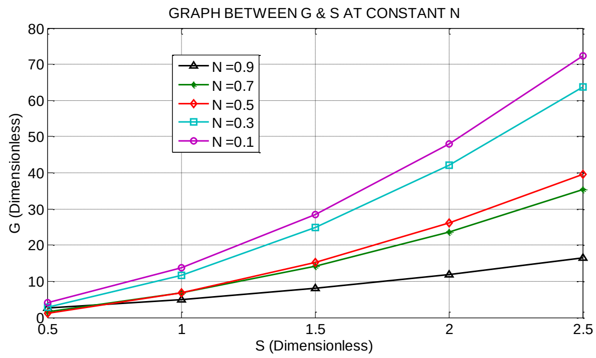

Fig. 2

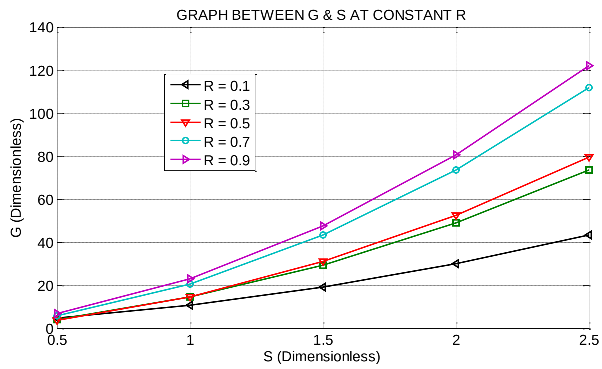

Fig. 3

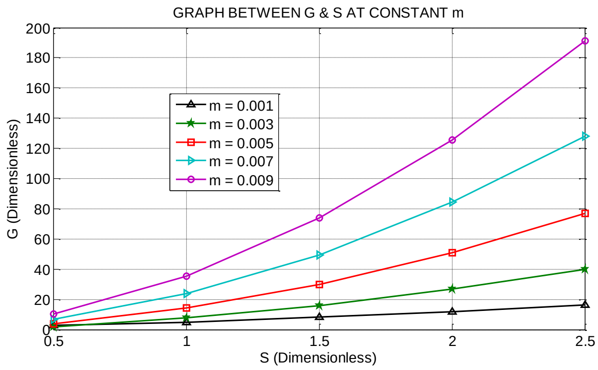

Fig. 4

7. Conclusions

It can be concluded that the magnetic field, temperature as well as initial compressive hydrostatic stress have significant influence on the phase velocity of gravity waves as well as Rayleigh waves in the system. This study also shows that the magnitude of phase velocity increases as the temperature increases. The gravity wave phase velocity is higher for higher magnetic stress parameter.

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