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Earthquake response spectra estimation of bilinear hysteretic systems using random-vibration theory method

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Abstract. A theoretical procedure to estimate spectral displacement of a hysteretic oscillator with bilinear stiffness excited by band-limited excitation is presented. The stochastic method of ground-motion simulation is combined with the random vibration theory to compute linear and nonlinear structural response. The response is obtained by computing the root-mean-square oscillator response using dissipation energy balancing by integrating over all energy levels of system weighting with the stationary probability density of the energy. The results are presented in a convenient form, and the accuracy of the procedure is assessed by comparison with results obtained with the time-domain method using the recorded data. The model shows little or no bias at the structural period of engineering interest.

Keywords: bilinear hysteretic; random-vibration; displacement; stochastic; point-source

1. Introduction

The application of elastic response spectra is extensive in earthquake engineering as such spectra reflect both the response of the structures and the frequency content of the ground motion (Hudson 1962). In severe earthquake events, many types of structures exhibit inelastic behavior, represented by hysteretic characteristics. Therefore, an elastic response spectrum, although a very important concept with widespread applications, is limited in its ability to predict structural damage and some fundamental features of inelastic dynamic behavior (Bozorgnia *et al*. 2010).

One of the challenging issues in civil engineering is stochastic structural analysis involving uncertainty or stochastic variables. The seismic demand of a structure due to uncertainties in ground motion and in structural properties needs to be properly characterized in earthquake engineering (Yazdani and Eftekhari 2012). In this case the structural response will also be a stochastic process and must be described in probabilistic terms. In the stochastic methods, the earthquake response spectra can be obtained using time-domain (TD) analysis by averaging the response of a suite of acceleration time series or by using random-vibration (RV) method, working directly with the spectra (Boore 2003). RV simulations are usually thousands of times faster than TD simulation (Boore and Thompson 2012) and shorter computational times are important in

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stochastic structural analysis.

Strong-motion seismology is concerned with the measurement and estimation of strong motion generated by potentially damaging earthquakes. The stochastic methods that assume the ground motion as band-limited finite-duration Gaussian white noise using seismological information are widely used for the simulation of Fourier spectra of ground motions for regions of the world in which recordings of motion from potentially damaging earthquakes are not available (Hanks and McGuire 1981, Boore 1983, 2003). In RV methods, the ground motion is characterized by the Fourier amplitude spectra (FAS) and a description of ground motion duration (Kuwamura *et al*. 1994, Takewaki 2001, Yamamoto *et al*. 2011). Boore and Joyner (1984), Liu and Pezeshk (1999), Boore and Thompson (2012) modified the measure of duration of excitation in RV calculations based on comparisons of TD and RV simulations on the basis that the ground shaking includes not only a stationary part but also a nonstationary part. In dynamic reliability analysis, stochastic analysis based on the nonlinear random vibration theory is required. The bilinear hysteresis model is one of the most widely used inelastic models in structural engineering. The purpose of this study is to estimate elastic and inelastic displacement spectrum on the basis of FAS of ground motion using information on the seismic source, seismic wave propagation through the earth, and geological site conditions that affect ground motion. Estimation of response displacement spectrum is a great importance in displacement-based seismic design of structures. The presented procedure based on RV theory can be applied to regions where strong ground motion data are limited in availability regarding the magnitude and distance range of engineering interest. Also, in regions where there are sufficient strong ground motion data, by the fact that they are based on combined recorded data sets from different earthquakes recorded in different regions, stochastic-based methods provide a conceptual framework for understanding some basis underlying physical parameters that control observed ground-motion amplitudes (Toro *et al*. 1997, Atkinson and Silva 2000). The procedure is validated by comparison with ground motion data from the M 6.7 1994 Northridge, California earthquake.

2. Randomly excited hysteretic system

In structural safety analysis, the stochastic seismic response of a nonlinear system based on the random vibration theory is rarely known, except for simple systems under idealized excitations (Koliopulos *et al*. 1994). With certain restrictive conditions, the state space vector is a Markov process, and the state space variable can be obtained as the solution to the corresponding Fokker-Planck equation (Rudinger and Krenk 2003). Approximate solution techniques are typically needed because such restrictive conditions can rarely be met in practical cases. The most frequently used approximation scheme is the equivalent linearization procedure (Roberts and Spanos 1990). When the system is highly nonlinear, the equivalent linearization procedure is considered unsuitable because the probability distribution of the system response is usually far from being Gaussian. Lutes (1970) proposed the method of equivalent nonlinear systems to improve the accuracy of approximate solutions for highly nonlinear hysteretic systems. Cai and Lin (1988) developed another approximation procedure, dissipation energy balancing, as an application of the general scheme of weighted residuals that focuses directly on the unknown probability density rather than calculation of the approximate statistical moment.

In dissipation energy balancing, a nonlinear system is replaced by another nonlinear system so that the average dissipated energy in the two systems remains the same (Lin and Cai 2004). An

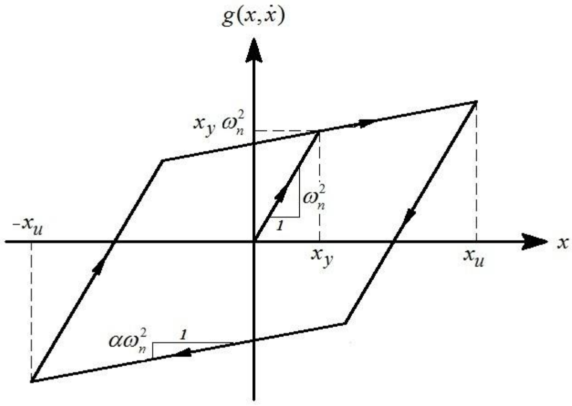


Fig. 1 Bilinear hysteretic restoring force as a function of displacement

important advantage of this procedure is that the form of the replacement system need not be preselected. The equation of motion for a bilinear hysteretic oscillator is expressed as follows

$$\ddot{x} + 2\zeta\omega_n\dot{x} + g(x,\dot{x}) = W(t) \tag{1}$$

where x is the displacement and a dot indicates the derivative with respect to time. $W(t)$ is an excitation and $g(x,\dot{x})$ is the stiffness function. In this equation, ω_n, ζ, and α are the natural circular frequency, the damping ratio within the linear range and the post- to pre-yielding stiffness ratio, respectively. The bilinear restoring force is illustrated in Fig. 1. The yield and maximum displacements are represented by x_y and x_u, respectively. In this oscillator, the total energy for a hysteretic system is determined as the sum of the kinetic and potential energies as follows

$$\lambda = \frac{1}{2}\dot{x}^2 + U(x) \tag{2}$$

The potential energy of the system, $U(x)$ is the recoverable part of the total energy, obtained by integration of the stiffness function. For the hysteretic characteristics in which the unloading rigidity is equal to the initial rigidity (Suzuki and Minai 1988), the potential energy of the system can be expressed as follows

$$U(x) = \begin{cases} \frac{1}{2}\omega_n^2 x^2 & ;\mathrm{x} \le \mathrm{x_y} \\ \frac{1}{2}\alpha\omega_n^2 x^2 + \frac{1}{2}\omega_n^2(1-\alpha)(x + x_u - x_y)^2 & ;\mathrm{x} \ge \mathrm{x_y}\ ,\ \text{-}\,\mathrm{x_u} \le x \le 2x_y - x_u \\ \frac{1}{2}\alpha\omega_n^2 x^2 + \frac{1}{2}\omega_n^2 x_y^2(1-\alpha) & ;\mathrm{x} \ge \mathrm{x_y}\ ,\ \ 2x_y - x_u \le x \le x_u \end{cases} \tag{3}$$

The approximate stationary probability density of total energy is given by the following equation (Caughey 1971)

$$p(\lambda) = C\exp(-\int \phi'(\lambda)d\lambda) \tag{4}$$

where C is a normalization constant and $\phi(\lambda)$ is the stationary potential, which is assumed to be a function of the total energy. For a bilinear hysteretic system, the probability potential function at level λ, $\phi'(\lambda)$, is derived as follows (Cai and Lin 1990)

$$\phi'(\lambda) = \frac{2\zeta\omega_n}{\pi S_f} + \frac{A_r}{2\pi S_f \int_{-x_u}^{x_u} \sqrt{2(\lambda - U(x))}dx} \tag{5}$$

where A_r is the area enclosed by a hysteresis loop and S_f is the power spectral density (PSD) of the excitation. By substituting Eq. (5) into Eq. (4), the probability density of energy at each energy level can be obtained. The mean square value of the displacement may be computed as follows

$$E(X^2) = \int_0^\infty p(\lambda)d\lambda \int_{-x_u}^{x_u} \frac{2x^2}{\sqrt{2(\lambda - U(x))}} dx \tag{6}$$

The PSD of the excitation, which is a more fundamental description of the frequency content of ground motion, is calculated in most practical methods of simulation of earthquake ground motions.

3. Stochastic modeling of power spectral density of ground motions

The main goals of engineering seismology in interpretation of ground motions are to improve the understanding of the physical processes that control ground motions and to develop reliable estimates of ground motions for use in engineering analyses. The stochastic methods that model ground motions as a random process and band-limited white noise are suitable for engineering applications for intermediate- to high-frequency structures. The well-known simple Kanai-Tajimi filter (Kanai 1957, Tajimi 1960) and a more recently developed seismological model (Brune 1970, 1971, Hanks and McGuire 1981, Boore and Atkinson 1987, Boore 2003) apply a band-pass filter in the frequency domain to mold the FAS and the corresponding PSD of the earthquake ground motion.

Seismological methods use stochastic models of the seismic source and wave propagation to simulate ground motions. The widely used point-source methods, which do not require information about the fault geometry, can predict high-frequency ground motions with acceptable accuracy (Boore 2003). To overcome the limitations of point-source models, stochastic finite-fault models should be used to simulate ground motions in the frequency range of engineering interest (Beresnev and Atkinson 1998, Motazedian and Atkinson 2005). In finite-fault simulations, the fault is subdivided into a number of subfaults, each of which is modeled using a point-source model. The point-source approach offers the advantages of simplicity and stability; the finite-fault model involves more parameters and requires average simulations over many azimuths and slip distributions. Atkinson and Silva (2000) postulated that the use of a point-source model with a two-corner source spectrum is equivalent to the use of a finite-fault model comprised of point-source subfaults. They indicated that two-corner point-source and finite-fault stochastic models will generate similar median ground motions, when averaged over all azimuths. In a seismological model, the FAS can be expressed as the product of a number of functions (Boore 2003)

$$Y(f) = E(M_0, f)G(f)A(f)\exp(-\gamma(f)R)\exp(-\pi f \kappa) \tag{7}$$

where f and M_0 are the frequency and the seismic moment, respectively.

R is equal to $R = \sqrt{d^2 + h^2}$ with d being closest distance to the fault plane and h is equivalent point-source depth is a function of fault size, and hence earthquake magnitude (Atkinson and Silva 2000). The terms $E(M_0,f)$, $G(f)$, and $A(f)$ are the earthquake source spectrum, the geometric spreading function, and the upper crust amplification factor, respectively. The anelastic attenuation, $\gamma(f)$, is determined from the regional wave transmission quality factor, namely, the Q factor. The high-frequency amplitudes are reduced by near-surface attenuation, which is assumed to be independent of distance, through the *kappa* factor. The two-corner source spectrum can be described using the following functional form (Atkinson 1993)

$$E(M_0, f) = C(2\pi f)^2 M_0((1-\varepsilon)/[1+(f/f_a)^2] + \varepsilon/[1+(f/f_b)^2]) \tag{8}$$

The constant C indicates the effect of the radiation pattern, the partition of total shear wave energy into horizontal components, the effect of the free surface, and the density and shear-wave velocity in the vicinity of the earthquake source. In this equation, the lower corner frequency, f_a, is related to the size of the finite fault and is determined by the source duration, and the higher corner frequency, f_b, is related to the subfault size and is the frequency at which the spectrum attains 1/2 of the high-frequency amplitude level (Atkinson 1993). The parameter ε is a relative weighting parameter whose value lies between 0 and 1. These two corner frequencies and ε can be derived by regression analysis using recorded data, after correcting for path and site effects in different regions. The PSD function can be calculated from the above-simulated FAS as follows

$$S(f) = |Y(f)|^2 / T_w \tag{9}$$

where T_w is the earthquake ground motion duration. The time duration is related to the earthquake size and the propagation distance. A simplified form of the distance-dependent term ($0.05R$) is adopted in this study, and the rupture duration part is assumed to be predicted by $1/(2f_a)$ (Boatwright and Choy 1992).

4. Numerical results

The stochastic procedure has been applied to the estimation of the root mean square displacement and displacement spectrum of bilinear systems with nonlinear restoring forces, as shown in Fig. 1. The accuracy of the procedure used to calculate the linear and nonlinear response is assessed by comparison with the results of conventional TD response analysis for recorded earthquakes.

The multiple free-field stations that recorded strong motions from the M_w 6.7 1994 Northridge, California mainshock are considered (Chang *et al*. 1996, Beresnev and Atkinson 1998). The locations of the 16 rock stations used in the validation are shown in Fig. 2, surrounding the fault at various azimuths. The distribution of these stations provides reasonable coverage of the directivity effects from the mainshock rupture (Table 1).

Raoof *et al*. (1999) determined that the California attenuation can be modeled by bilinear geometrical spreading of R^{-1} to a distance of 40 km, with $R^{-0.5}$ spreading for distances greater than 40 km. The anelastic attenuation associated with this spreading model is represented by a frequency-dependent regional quality factor given by Q=180$f^{\,0.45}$.

Atkinson and Silva (2000) adopted the value of κ=0.03 to reduce the high-frequency

Table 1 The stations used in the validation

Station name	Lat.	Long.	Location	Distance[1]	Agency
BCY	34.204	-118.302	USC #59	16.88	USC[2]
GPK	34.120	-118.300	Los Angeles~3riffith Observatory	23.77	USGS[3]
L09	34.610	-118.560	Lake Hughes #9	25.36	CDMG[4]
LF5	34.127	-118.405	USC #14	18.36	USC
LPK	34.109	-119.065	Point Mugu-Laguna Park	41.93	CDMG
LUH	34,062	-118.198	Los Angeles-University Hospital Grounds	34.2	CDMG
LV3	34.596	-118.243	Leona Valley #3	37.33	CDMG
LWE	34.114	-118.380	USC #17	20.3	USC
LWS	34.089	-118.435	USC #16	20.81	USC
MCN	34.087	-118.693	Malibu Canyon-Monte Nido Fire Station	27.4	USGS
MSM	34.086	-118.481	USC #15	20.45	USC
MTW	34.224	-118.057	Mt. Wilson-Caltech Seismic Station	35.88	CDMG
ORR	34.560	-118.640	Castaic-Old Ridge Route	20.72	CDMG
RHE	33.787	-118.356	Rolling Hills Estates-Rancho Vista School	49.32	CSMIP
SSA	34.231	-118.713	Santa Susana	16.74	DOE[5]
SCT	34.106	-118.454	Stone Canyon Reservoir Dam	19.5	SCEC[6]

[1]Closest distance to fault in kilometers, [2]University of Southern California, [3]United States Geological Survey, [4]California Division of Mines and Geology, [5]DOE: Department of Energy, [6]Southern California Earthquake Center

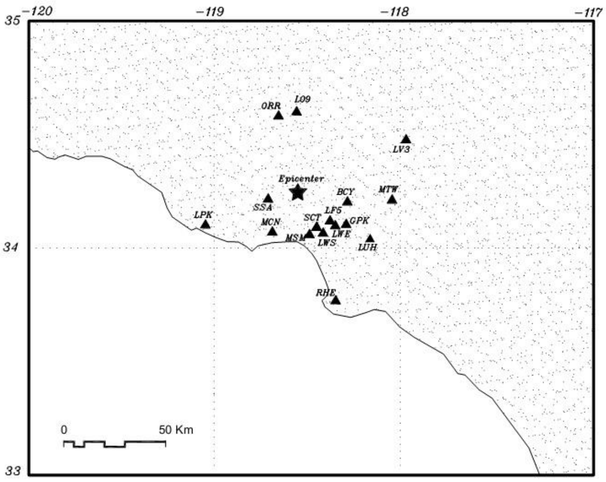


Fig. 2 The locations of the stations used in the validation, surrounding the fault at various azimuths. The trace-by-trace simulations are presented in Figs. 3 and 4. The dark triangles mark the rock stations, surrounding the fault plane at different azimuths. The epicenter is marked with a star

amplitudes. In building codes, the lack of information concerning actual amplification factors can be overcome by the use of mean amplification factors based on the average 30-m shear wave velocity to characterize local site conditions. As Lee and Trifunac (2010) have mentioned, the average 30-m shear wave velocity may be an inadequate and hugely uncertain parameter by which to characterize local site conditions, but its effect is considered in this study in light of its widespread usage in building codes. Thus, the crustal amplification is modeled by multiplying the spectrum by the frequency-dependent crustal amplification factors of Boore and Joyner (1997) for California sites. The values of the regional physical constants of crustal density and shear wave velocity in the source region are taken to be 2.8 gm/cm^3 and 3.5 km/sec, respectively (Boore and Joyner 1997). The linear functions to predict the magnitudes of the dependent corner frequencies f_a and f_b and the relative weighting factor ε, derived from California data, are listed as follows (Atkinson and Silva 1997)

$$\begin{aligned} \log f_a &= 2.181 - 0.496 M_w \\ \log f_b &= 1.778 - 0.302 M_w \\ \log \varepsilon &= 2.764 - 0.623 M_w \end{aligned} \tag{10}$$

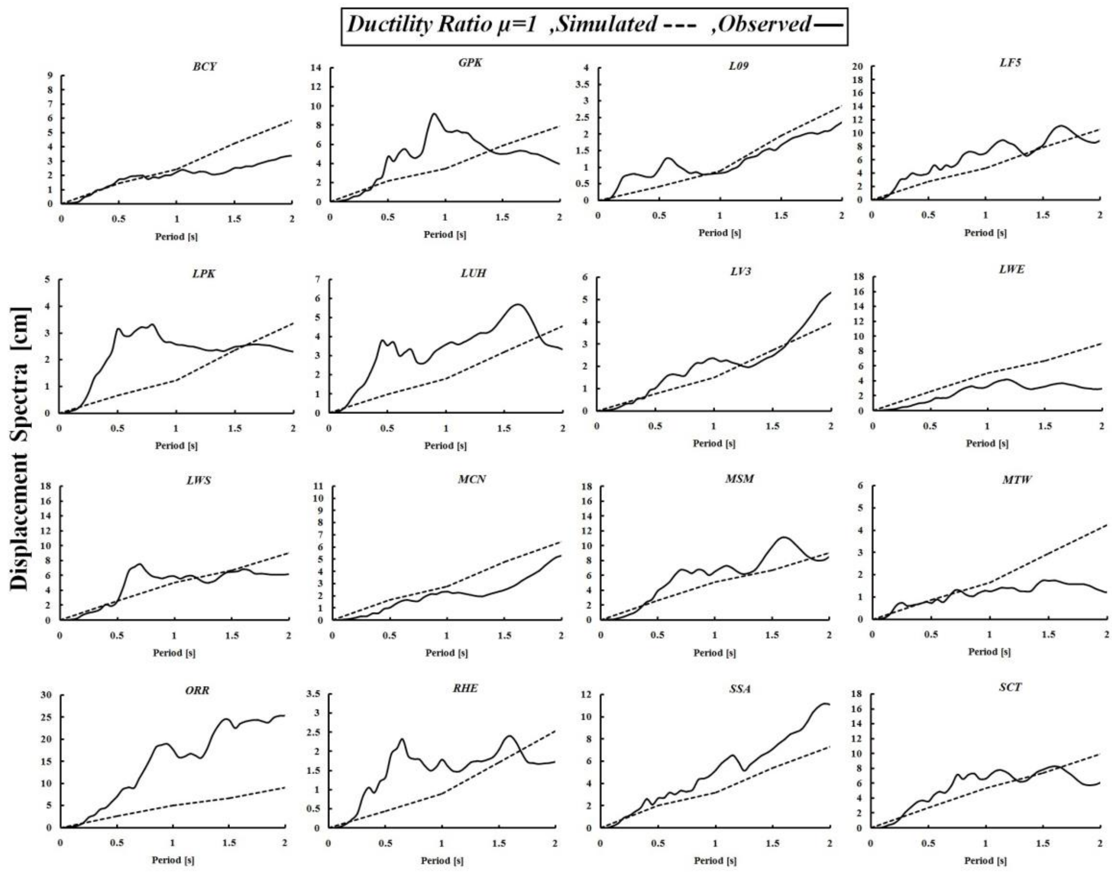


Fig. 3 Recorded and simulated 5%-damped linear displacement spectra (ductility ratio equal to one) at the stations surrounding the fault plane. The station locations are shown in Fig. 2. The observed spectra computed by the TD method and simulated spectra computed by RV procedure are shown by the solid and dashed lines, respectively

where M_w is the moment magnitude. The seismic moment M_0 is often expressed in terms of the moment magnitude M_w, according to the following equation (Kanamori 1977)

$$\log M_0 = 1.5M_w + 16.05 \tag{11}$$

The values of the damping ratio and the post- to pre-yielding stiffness ratio are assumed to be 0.05 and 0.03, respectively.

To assess the accuracy of the proposed procedure, the displacement response spectra for constant ductility ratios equal to 1 and 4 at the different stations were computed by the RV method, using the simulated FAS based on given seismological parameters, and by the TD method, using the recorded ground motions shown in Figs. 3 and 4. For comparison, the geometric average of the spectra of the two horizontal components of recorded data is used. On average, the mach is satisfactory but not necessarily accurate. Taking into account the simplicity of the model and the effect of variability of ground motion variables, the fit can be considered satisfactory at the

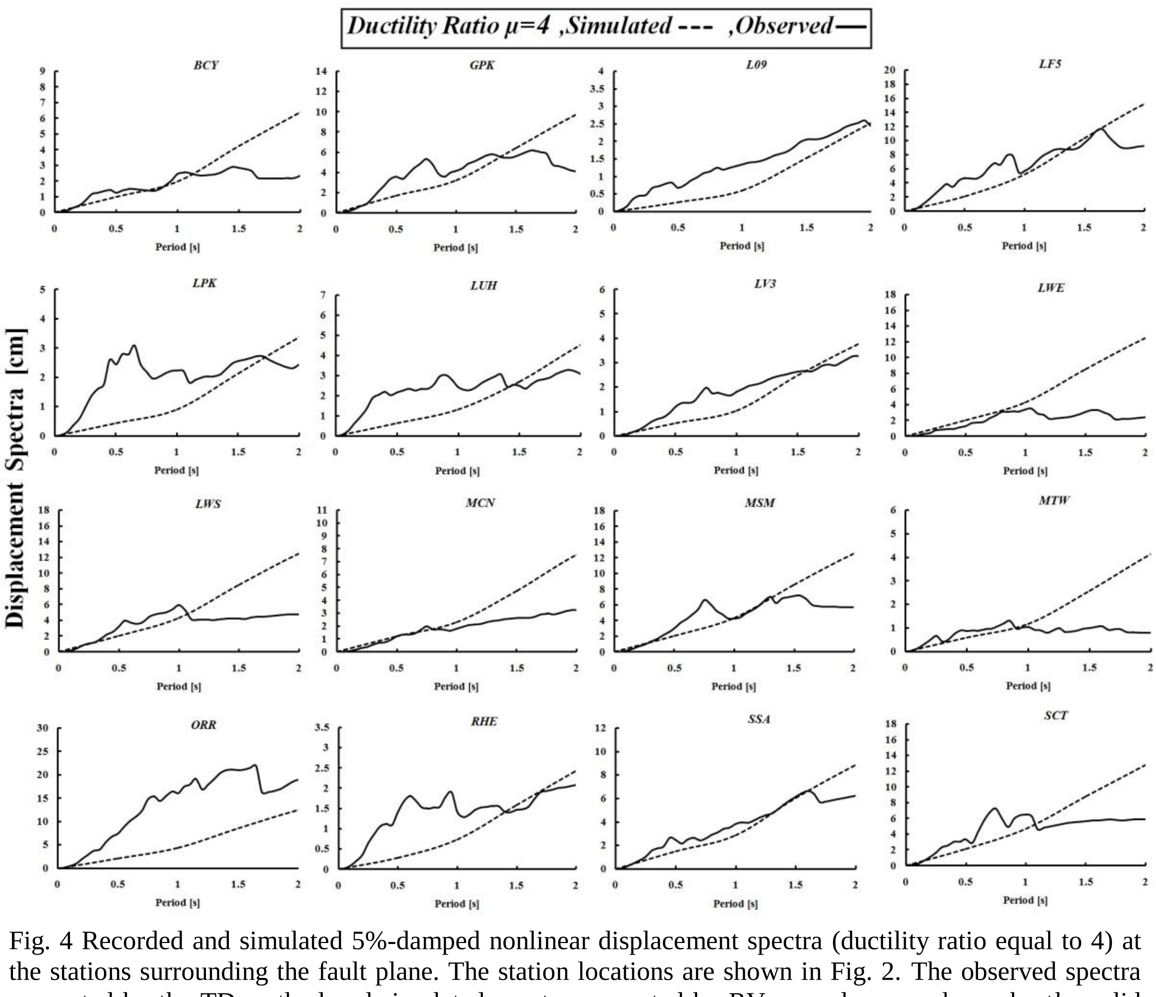


Fig. 4 Recorded and simulated 5%-damped nonlinear displacement spectra (ductility ratio equal to 4) at the stations surrounding the fault plane. The station locations are shown in Fig. 2. The observed spectra computed by the TD method and simulated spectra computed by RV procedure are shown by the solid and dashed lines, respectively

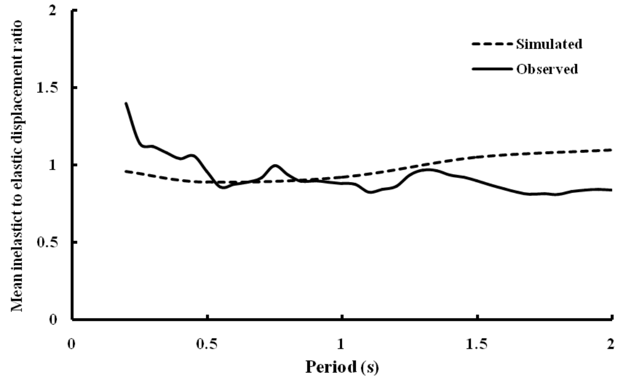
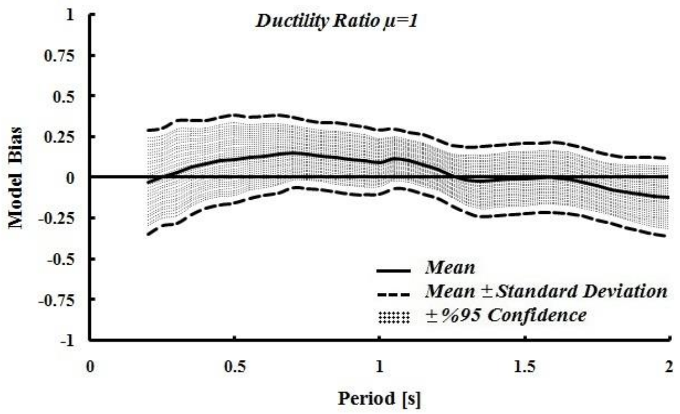
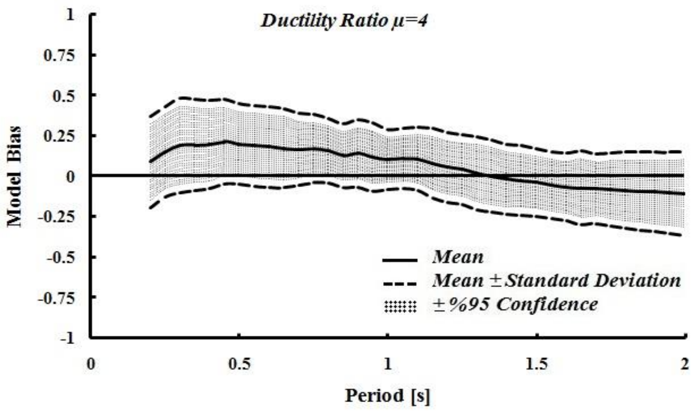


Fig. 5 The mean ratio between inelastic and elastic displacement spectra averaged over all stations. The ratios of inelastic-to-elastic observed spectra computed by the TD method and simulated spectra computed by RV procedure are shown by the solid and dashed lines, respectively



(a) μ=1



(b) μ=4

Fig. 6 Model bias showing the ratio of the observed to simulated response spectrum, averaged over all stations shown in Fig. 2. The observed spectrum at each site is calculated as the geometric average of the spectra of two horizontal components. The dashed lines indicate a band ±1 standard deviation wide. The hatched bands indicate the 95% confidence limits of the mean structural period of engineering interest. The scatter in these figures does not necessarily indicate the weakness of the used RV procedure, since the input parameters to the seismological model only define the average conditions and do not provide information on their variability.

The stochastic seismic response based on seismological information of the excitation is particularly useful for prediction of response for regions of the world in which recordings of motion from potentially damaging earthquakes are not available. The stochastic method is useful for estimating mean response of systems expected for a suite of earthquakes having a specified magnitude and fault station distance.

Fig. 5 indicate the mean ratio between inelastic-to-elastic displacements tend to be equal to 1 in the period range from 0.3 s up to 2 s, as already observed in previous studies (Miranda 2000, Borzi *et al*. 2001, Ruiz-Garcia and Miranda 2003). The results confirm the established observation that elastic and inelastic systems in structural period of engineering interest reach similar maximum displacements.

Fig. 6 presents the mean modeling bias for the all stations shown in Fig. 2. The bias is defined as the ratio of the observed to the simulated (Abrahamson *et al*. 1990, Atkinson and Boore 1998) response spectra, averaged over all stations. The traces of this figure present the model bias calculated for different ductility ratios. The standard deviation provides a measure of the prediction uncertainty for individual stations, while the confidence band of the mean provides a measure of the uncertainty of the mean bias of the model. The dashed lines indicate a band ±1 standard deviation wide. The hatched bands indicate the 95% confidence limits of the mean, calculated on the basis of the *t* distribution. The modeling bias for the event is not significantly different from zero, considering the width of the confidence interval. From Fig. 6, it is concluded that the (logarithm base 10 of) the mean ratio of the simulated to the observed spectra is close to zero at the structural period of engineering interest. This means that the stochastic procedure employed adequately predicts the linear and nonlinear response on average, although the uncertainties in predictions at individual sites may be relatively large.

5. Conclusions

The earthquake response spectra can be simulated using either TD method or RV theory simulations. In the estimation of dynamic responses, a suite of ground motion time histories needs to be utilized in linear and nonlinear TD analysis. These ground motions are limited by the amount of available strong motion data and by the fact that they are based on combined recorded data sets from different earthquakes recorded in different regions. In some locations for which there is a lack of sufficient recorded data, well-known stochastic models are customarily used to simulate strong motions for the purpose of structural analysis. Stochastic simulations of ground motion can be used for generic or region-specific applications rather than path-specific applications, for which the average motions from a suite of earthquakes rather than a scenario earthquake are desired (Boore 2009). On the other hand, many researchers introduced the critical ground motion inputs to determine the worst case scenario in structural analysis (Takewaki 2002a, 2002b, 2005, 2006, Moustafa *et al*. 2010). The stochastic point-source model with a two-corner source spectrum, which was used in this study, despite possessing some theoretical deficiencies, yields results similar to those obtained using finite-fault methods for the ground motion frequencies at moderate and large distances from the fault that are of most interest to engineers (Beresnev and Atkinson 1999, Atkinson and Sillva 2000, Boore 2009).

The uncertainty in basic variables depends on the circumstances and can be categorized as aleatory or epistemic. The uncertainty due to inherent variability in structural variables or the earthquake source and path and site condition variables is aleatory in nature and cannot be reduced by acquiring additional data or information. Model uncertainty may arise from missing variables in the mathematic model, perhaps due to our lack of knowledge about these missing variables or our desire to exclude them from the model for the sake of simplicity. Model uncertainty, often called statistical uncertainty, is epistemic in nature. Insufficient knowledge of the mean values of structural and seismological variables, which can be classified as epistemic uncertainty, plays an important role in the bias of the model. Yazdani and Takada (2011) showed analytically that earthquake source and soil condition variables are the main sources of uncertainty in predicting spectral response. Thus, the aleatory uncertainty of the earthquake variables and the epistemic uncertainty of the used procedure are responsible for the uncertainty in the prediction of nonlinear spectral response.

This study presents a procedure based on RV theory for estimation of the stochastic seismic response of hysteretic systems based on seismological information for the site of interest. Despite the uncertainties mentioned, formulating a response spectrum using RV simulation on the basis of information on the frequency excitation opens the door for wider use of seismological theory in understanding the relationship between the linear and nonlinear response spectra and the seismological variables of interest. The approximate solution for the bilinear hysteretic oscillator considered in this study provides a basis for obtaining exact solutions for realistic and non-stationary excitations, such as earthquake ground motions. Due to the simplicity and computational efficiency of the method, it provides an accurate prediction of the observed nonlinear response spectra on average for structural periods of engineering interest.

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