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Structural reliability index versus behavior factor in RC frames with equal lateral resistance

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Abstract. The reliability or the safety index is a measure of how far a structure is from the state of collapse. Also it defined as the probability that a structure will not fail in its lifetime. Having any increase in the reliability index is typically interpreted as increasing in the safety of structures. On the other hand most of researchers acknowledged that one of the most effective means of increasing both the reliability and the safety of structures is to increase the structural redundancy. They also acknowledged that increasing the number of vertical seismic framing will make structural system more reliable and safer against stochastic events such as earthquakes. In this paper the reliability index and the behavior factor of a numbers of three dimensional RC moment resisting frames with the same story area, equal lateral resistant as well as different redundancy has been evaluated numerically using both deterministic and probabilistic approaches. Study on the reliability index and the behavior factor in the case study models of this research illustrated that the changes of these two factors do not have always the same manner due to the increasing of the structural redundancy. In some cases, structures with larger reliability index have smaller behavior factor. Also assuming the same ultimate lateral resistance of structures which causes an increase to a certain level of redundancy can enhance behavior factor of structures. However any further increase in the redundancy of that certain level might decrease the behavior factor. Furthermore, the results of this study illustrate that concerning any increase in the structural redundancy will make the reliability index of structure to be larger.

Keywords: redundancy; the deterministic overstrength index; the probabilistic overstrength index; the reliability index; the behavior factor

1. Introduction

Although the importance of structural redundancy and reliability has long attracted the attention of researchers, structural redundancy became the focus of research in the structural engineering community especially after the 1994 Northridge and the 1995 Kobe earthquakes. Since that time many definitions and interpretations of redundancy and the redundancy factor in both scopes of quantity and quality have been suggested. Among which the following definitions may be mentioned.

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1.1 Definitions of redundancy

Redundancy is the ability of a structural system to redistribute the loads among its members which can no longer be carried by some other damaged parts (Biondini *et al.* 2008). The second definition which described by Hendavi and Frangopol (1994) is the ratio of the probability of any first member yielding which may occurs in the intact structure minus the probability of the collapse of the intact structure to the probability of collapse of the intact structure. Yet, another definition indicates that in order to prevent immediate collapse of structures and to achieve progressive collapse, the redundancy can create alternative load path to transfer damaged member's load from minor members to major members of structure (Marhadi and Venkataraman 2009). Yet, other definition denotes that the number of plastic hinges of the structural system that fails when structure collapses, can be used to investigate the redundancy of frame structures based on both static and dynamic analyses (Bertero and Bertero 1999). According to this research, degree of redundancy is actually the number of plastic hinges in a structural system which continues to yield until the structure exceeds the allowable limit leading to emergency disasters like plastic displacement or complete collapse.

1.2 Redundancy factor

Some researchers defined an analytical parameter as redundancy factor according to the lines of vertical seismic framing at any direction. Moses (1974) studied the effect of redundant wind framing systems in a quantitative way. The degree of indeterminacy of the structure was suggested to be a parameter to evaluate the degree of redundancy of the structure, e.g., a weakest-link system (serial systems or statically determinant structure) versus a parallel system (statically indeterminate structure). In ATC-19 and ATC-34 (1995), it was denoted that the reliability of the framing system against seismic load depends on the number of lateral load resistant components. Also it was proposed that a behavior factor of R can be divided into three factors, a period dependent strength factor (R_s), a period dependent ductility factor (R_μ), and a redundancy factor (R_R). In all aforementioned references, for structures that consist of 4, 3 and 2 lines of vertical seismic framing in any principal direction, the amounts of the proposed redundancy factor are 1, 0.866 and 0.71 respectively. Husain and Tsopelas (2004) attempted to measure the structural redundancy of 2D RC frames. They considered two factors as the redundancy strength and the redundancy variation coefficients. They studied on the relation between these two indexes and the plastic rotation ductility factor. The effects of number of stories and bays, length of bays and story height were studied as well as the effect of redundancy on the behavior factor (R_R).

1.3 Reliability factor

It is noticeable that the reliability/redundancy factor, ρ, is added to NEHRP, UBC and IBC codes after 1997. This factor is applied to horizontal design earthquake load. The ρ factor is a function of the system configuration and number of seismic components and does not depend on the inherent structural parameters such as overstrength and ductility (Wen and song 2003, Liao and Wen 2004).The uncertainly in structural demands versus capacities of structural systems are evident in the most qualitative definitions that have been formulated for redundancy. Hence many researchers study the effects of redundancy on reliability index and behavior factor using both deterministic and probabilistic approaches. Wen *et al.* (2003) studied on the reliability/redundancy of structural behavior under SAC ground motions. They notified that when more elements are involved in resistance against lateral load, the probability of collapse of all elements at the same time is lower than the case when elements with equal resistance are involved.

In addition to the above definitions, other approaches have also been applied in the case of redundancy that the following can be noted. Okasha and Frangopol (2010) investigate the time-variant redundancy of structural systems. They studied structural reliability and redundancy affected by deterioration in structural resistance and increase in applied loads are conducted by using numerical examples. Kanno and Ben-Haim (2011) developed a quantitative and widely applicable concept of strong redundancy and showed its relation to the info-gap robustness of the structure.

The review of previous researches indicate that the general opinion in conjunction with the overstrength capacity of a structure which is explained by the process of first local yielding to total failure can be considered based on structural redundancy. Furthermore, results of some other researches indicate this part of overstrength capacity can be introduced as the redundancy factor. Yet, another notified criterion which has been obtained from the researches is a parameter according to the lines of vertical seismic framing and the theory of structural reliability which can be formulated as the redundancy factor. This new factor is independent of the overstrength reduction factor in the analytical definition of the behavior factor. Now, a general conceptual question is why structural redundancy has been regarded as a desirable property. Is it desirable that because of assigning an upper level of overstrength capacity to design process or just increasing the number of seismic resistant frames as well as increasing the reliability of the systems, would able to achieve to better level of structural performance. In this research upon to considering the strength factor as a random variable, both deterministic and probabilistic effects of redundancy on the reliability index and the behavior factor of structures with equal lateral resistance were evaluated.

2. Components of the behavior factor

The existence of some mechanical properties such as ductility, damping, overstrength and redundancy cause that structural systems have the ability of dissipation of earthquake input energy. It should be noted that the aforementioned characters would make structural systems able to dissipate all of the input design earthquake energy with an overall inelastic deformations and redistribution of the seismic forces. Having a review on early 1990s researchers show a comprehensive reformulation on accuracy and reliability of the behavior factor in which to analyze this factor into its constituent parameters. Most researchers and some seismic regulations agree on factors such as ductility capacity, overstrength, redundancy and structural damping. According to these studies a conceptual formula has been obtained to calculate the behavior factor (Yang 1991).

$$\mathrm{R_w} = \mathrm{R_s R_\mu R_R R_\xi} = \left(\frac{\mathrm{c_y}}{\mathrm{c_s}} \times \frac{\mathrm{c_s}}{\mathrm{c_w}}\right) \times \frac{\mathrm{c_{eu}}}{\mathrm{c_y}} \times 1 \times 1 \tag{1}$$

where R_w is the behavior factor using allowable stress design approach, R_s is total the overstrength reduction factor, R_μ is the ductility reduction factor, R_R is the redundancy factor and R_ξ is the damping factor. Assuming 5% damping ratio, the amount of R_ξ should be considered as unit. The symbol of R_R in some researches is defined as the overstrength capacity which is resulted from the process of first significant yielding until total failure.

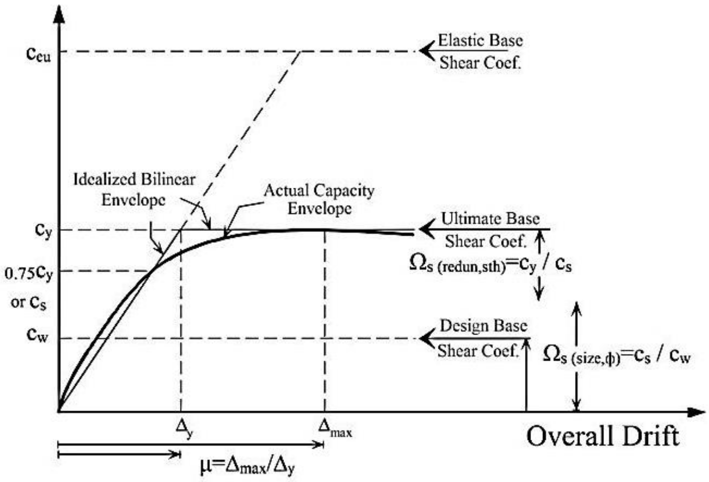


Fig. 1 The capacity curve and components of the overstrength

2.1 Total overstrength reduction factor

The results of researches show that the overstrength capacity which is displayed by a framed structural system is composed of two parts according to Fig. 1. The first part of the overstrength capacity is obtained from the design requirement base shear coefficient until that base shear coefficient which first local yielding is formed. This part of the overstrength is arising from restricted choices for member sizes, rounding up of sizes and dimensions and differences between the nominal and the factored resistances. This part of the overstrength can be calculated according to Eq. (2) (Yang 1991, Massumi *et al.* 2004).

$$\Omega_{s(\text{size},\varphi)} = \frac{c_s}{c_w} \tag{2}$$

The second part of the overstrength capacity of a structure should be explained by the process of first local yielding to total failure which is due to structural redundancy and steel strain hardening. When structural members yielding started, then the redistribution of internal force will happen. This process is related to structural redundancy and has obvious influences on failure mechanism. Hence the overstrength capacity that is created after first local yielding formed until total failure (i.e., state of instability) can be calculated according to Eq. (3) (Yang 1991, Massumi *et al.* 2004).

$$\Omega_{s(\text{redu},\text{sth})} = \frac{c_y}{c_s} \tag{3}$$

After determination of the two components of overstrength, the total value of the overstrength reduction factor is obtained according to Eq. (4).

$$R_s = \Omega_{s(\text{size},\varphi)} \cdot \Omega_{s(\text{redu},\text{sth})} = \frac{c_s}{c_w} \times \frac{c_y}{c_s} = \frac{c_y}{c_w} \tag{4}$$

It is evident that the main reason to cause overstrength in Eq. (3) is the structural redundancy. Hence in some researches this part of overstrength introduced as the redundancy factor (Husain and Tsopelas 2004, Fallah *et al*. 2009). Additionally, in the recent both aforementioned researches this part of overstrength introduced as the redundancy factor that included both deterministic and probabilistic effects of the redundancy in 2D resisting moment frames. In the present paper both deterministic and probabilistic effects of the redundancy in 3D resisting moment frames with equal lateral resistance has been evaluated as the overstrength due to the redundancy.

2.2 Ductility reduction factor

Based on idealizing the capacity curve to an ideal elastic - perfectly plastic curve as shown in Fig. 1, the overall ductility capacity can be expressed according to Eq. (5) (Yang 1991, Massumi and Tasnimi 2006).

$$\mu = \frac{\Delta_{max}}{\Delta_y} \tag{5}$$

The structural system is able to dissipate input seismic energy because of the structural ductility. Furthermore, based on the energy dissipation capacity of the structure the elastic design forces can be reduced to level of total failure (i.e., state of yielding).

$$R_\mu = \frac{c_{eu}}{c_y} \tag{6}$$

Several researches in devising and establishing an analytical relation between μ and R_μ have been done. In the present paper, the results of research done by Miranda and Bertero (1994) are used. The equation for the ductility reduction factor introduced by Miranda and Bertero was obtained from a study of 124 ground motions recorded on a wide range of soil conditions. The soil conditions were classified as rock, alluvium and very soft sites which characterized by low shear wave velocity. A 5% of critical damping was assumed. The expressions for the period - dependent force reduction factors R_μ are given by

$$R_\mu(T, \mu) = \frac{\mu - 1}{\varphi} + 1 \tag{7}$$

where Φ is a function of total ductility, period of system and soil conditions which obtained from Eq. (8). The factor of Φ in Eq. (8) is for the alluvium soil condition.

$$\Phi = 1 + \frac{1}{T(12 - \mu)} - \frac{2}{5}\exp\left[-2(\ln T - 0.2)^2\right] \tag{8}$$

3. Indices for evaluating the effects of redundancy

For the sake of assessing both deterministic and probabilistic effects of the redundancy on the reliability index and the behavior factor, a couple of indices which named the deterministic overstrength index and the probabilistic overstrength index are introduced. The deterministic overstrength index Ω_{det} reflects the ability of a structural system to redistribute loads from failed or yielded components towards more resistant components. This factor is a function of static indeterminacy, ductility of components, strain hardening, and average strength of elements of the structural system. The second index is a probabilistic overstrength index Ω_{pro}. This index quantifies the effects of probabilistic variables of element strength on the structural system strength. This index is also a function of static indeterminacy of the structural system, and the correlation coefficient between elements strength. The overall effects of redundancy on the base shear coefficient of a structural system could be quantified through the subsequent ratio (Husain and Tsopelas 2004, Fallah *et al.* 2009).

$$\frac{c_{redu}}{c_{non,redu}} \tag{9}$$

where c_{redu} is the ultimate base shear coefficient of the redundant structure; and $c_{non\text{-}redu}$ is the base shear coefficient of the same structural system as if it was non-redundant (statically determinate). A structural system composing of ideal elastic-brittle elements could be dependably used to model a non-redundant structural system. In such a determinate structure, initial yielding would cause the state of collapse if the strength reserves of the undamaged elements have been exhausted. Accordingly, assuming elastic brittle behavior of structural elements, the point of first yielding in a structural system can be considered as a reasonable approximation of the base shear coefficient for the determinate structure. Therefore $c_{non\text{-}redu}$ in Eq. (9) can be substituted by c_s, which is the base shear coefficient of the structural system at the point of the first yielding. The deterministic overstrength index Ω_{det} is defined as the ratio of average ultimate to average yield base shear coefficient, as follows (Husain and Tsopelas 2004, Fallah *et al.* 2009).

$$\Omega_{det} = \frac{\bar{c}_{redu}}{\bar{c}_{non,redu}} = \frac{\bar{c}_y}{\bar{c}_s} \tag{10}$$

In Eq. (10) assuming the base shear coefficient as a random variable, to evaluate the deterministic overstrength index, the average ultimate base shear coefficient $\bar{c}_y$ and the average base shear coefficient of structural system at the point of first yielding $\bar{c}_s$ were used. In this paper $\bar{c}_y$ and $\bar{c}_s$ are calculated based on the base shear coefficient versus overall drift curve resulting from the nonlinear static pushover analysis (Fig. 2). Therefore Ω_{det} considered as a deterministic value of the overstrength due to both the structural configurations and the redundancy. Ω_{det}=1 indicates a determinate (non-redundant) structure. Also values higher than 1 indicate redundant structures (Husain and Tsopelas 2004, Fallah *et al.* 2009).

3.1 The probabilistic overstrength Index

To obtain a formulation for the probabilistic overstrength index, a two dimensional plane frame which has particular failure mode is considered as a general structural system. The selection of the failure mode would be significant, since it could lead to non-accurate estimates of Ω_{pro} but for simplicity in the current derivation, a sway type failure mode is assumed as shown in Fig. 3. This failure mode is based on the strong column and weak beam assumption which can achieve under some lateral loadings such as monotonic loadings.

The relation between the structural system base shear coefficient and the local component strengths could be formulated using principles of plastic analysis. The frame ultimate base shear coefficient for selected failure mode could be represented by the following expression

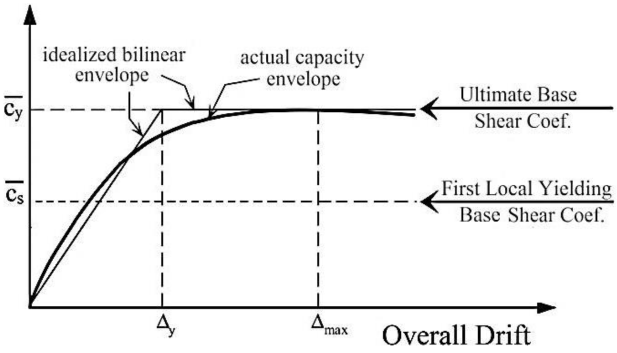


Fig. 2 How to get $\overline{c}_y$ and $\overline{c}_s$

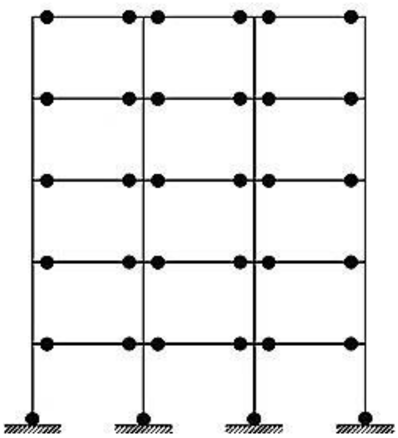


Fig. 3 Sway type failure mode of a general 2D frame

$$c = \Sigma_{i=1}^{n} \Psi_i \cdot M_i \tag{11}$$

where c is frame ultimate base shear coefficient; n is the number of plastic hinges in the frame resulting from the sway failure mode; M_i is yield moment of the structural element where plastic hinge "i" is formed; and Ψ_i is coefficient with unit of radians/(length. Mass) that is a function of the plastic rotation and geometry of the structure. Eq. (11) is the form of the strength (i.e., base shear coefficient) equation of a parallel system that its total base shear coefficient is obtained from summations of elements strength. The average value of the frame ultimate base shear coefficient can be derived from the following formula (Husain and Tsopelas 2004, Fallah *et al.* 2009)

$$\bar{c} = \sum_{i=1}^{n} \Psi_i \cdot \overline{M}_i \tag{12}$$

where $\overline{M}_i$ is average value of the strength of the structural element where plastic hinge "i" is formed. Accordingly the standard deviation of the frame base shear σ_f can be calculated from following Equation

$$\sigma_f = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} \Psi_i \Psi_j \rho_{ij} \sigma_{Mi} \sigma_{Mj}} \tag{13}$$

where ρ_{ij} is the correlation coefficient between the strengths M_i and M_j. The parameter of σ_{Mi} is the standard deviation of the yield moment M_i. Also ρ_{ij}=1 for $i=j$. To simplify the above derivation, a regular multistory multi-bay frame with the following properties is considered.

1. The frame is made up of components with exactly the same normally distributed strengths.

$$\overline{M}_i = \overline{M}_j = \overline{M}_e$$

2. The correlation coefficient between the strength of any two pairs of components is the same.

$$\rho_{ij} = \rho_e$$

3. The bays of the frame have equal spans and the stories have same height which result in

$$\Psi_i = \Psi_j = \Psi$$

Hence, both Eqs. (12) and (13) become

$$\bar{c} = n.\Psi.\overline{M}_e \tag{14}$$

$$\sigma_f = \Psi.\sigma_e\sqrt{n + n(n-1)\rho_e} \tag{15}$$

The following relationship between the coefficient of variation (COV) of the frame strength v_f and the COV of the element strength v_e is calculated through dividing Eq. (15) by Eq. (14) (Husain and Tsopelas 2004)

$$v_f = \frac{\sigma_f}{\bar{c}} = \frac{\sigma_e}{\overline{M}_e} \cdot \sqrt{\frac{1+(n-1)\rho_e}{n}} = v_e\sqrt{\frac{1+(n-1)\rho_e}{n}} \tag{16}$$

The probabilistic overstrength index Ω_{pro} is defined as the ratio between v_f and v_e

$$\Omega_{pro} = \frac{v_f}{v_e} = \sqrt{\frac{1+(n-1)\rho_e}{n}} \tag{17}$$

For a parallel system with unequally correlated elements, ρ_e could be substituted with the average correlation coefficient $\bar{\rho}$ which defined as

$$\bar{\rho} = \frac{1}{n(n-1)}\sum_{\substack{i,j \\ i \neq j}}^{n} \rho_{ij} \tag{18}$$

Therefore, Eq. (17) can be modified using the average correlation coefficient of strengths of the structural elements, as follows

$$\Omega_{pro} = \sqrt{\frac{1+(n-1)\bar{\rho}}{n}} \tag{19}$$

Hence the probabilistic overstrength index, Ω_{pro}, is a function of the number of plastic hinges, n, and the average correlation coefficient between their strengths, $\bar{\rho}$, and presents a way for measuring probabilistic effects of the redundancy on the system strength. Its values range are between 0 and 1; for a framed structure where a single plastic hinge causes total failure (n=1), Ω_{pro}=1 and the structure under consideration is non-redundant. The other extreme value Ω_{pro}=0 reveals an infinitely redundant structural system and is reached either when an infinite number of plastic hinges are required to cause total failure or when element strengths in a structure are uncorrelated. Ω_{pro} can be estimated from a deterministic pushover analysis or an incremental dynamic analysis for a particular value of the average correlation coefficient of the structural member's strength (Husain and Tsopelas 2004, Fallah *et al.* 2009).

Eq. (19) represents the probabilistic overstrength index for 2D frames or in other words, for a single line of resistance within three-dimensional lateral load resisting systems. In real structures, which are composed of a number of 2D frames, the probabilistic effects of the redundancy due to plurality of the lateral lines of resistance on the structural strength have to be accounted in the probabilistic overstrength index Ω_{pro}. Hence, it is considered a structural system with m parallel identical rigid plane frames. Assuming that 3D framed structures collapse after the demolition of $m-1$ plane frames, then the overall system completely loses its both lateral and torsional resistances and becomes instable. The 3D frame now can be modeled as a parallel or a Daniel's system (Gollwitzer and Rackwitz 1990). The mathematical representation of the coefficient of variation of its strength is given from the following equation (Husain and Tsopelas 2004).

$$V_s = v_f \sqrt{\frac{1+(m-2)\bar{\rho}_f}{m-1}} \tag{20}$$

where v_s is COV of the strength of the 3D structural system; v_f is COV of the strength of the plane frames; and $\bar{\rho}_f$ is average correlation between the strengths of the plane frames. The probabilistic overstrength index Ω_{pro} of a 3D structural system is defined as the ratio between v_s and v_e, which can be written as follows

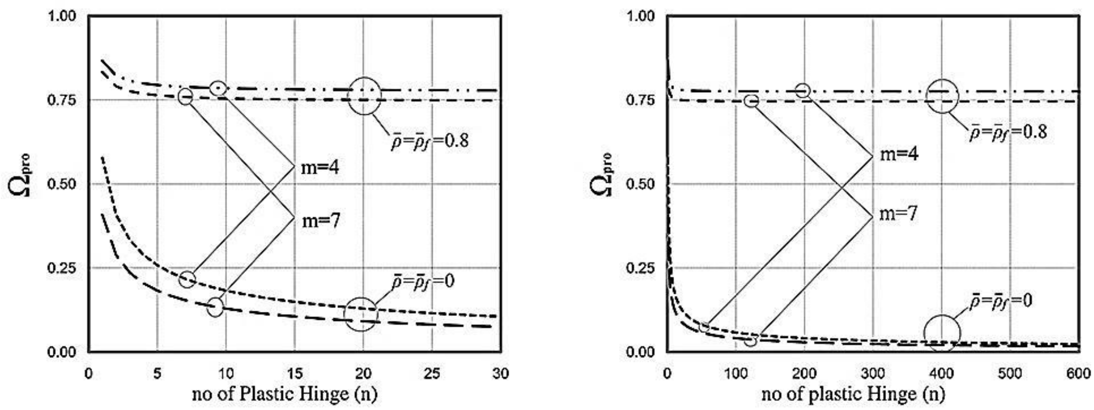
$$\Omega_{pro} = \frac{v_s}{v_e} = \frac{v_s}{v_f} \times \frac{v_f}{v_e} \tag{21}$$

where v_s and v_f are defined above and v_e is COV of the strength of the elements comprising the 3D structure (beams, columns, etc.). By virtue of Eqs. (20) and (21) the probabilistic overstrength index of a 3D structural system can be expressed as

$$\Omega_{pro} = \frac{v_s}{v_e} = \sqrt{\frac{1+(n-1)\bar{\rho}}{n} \times \frac{1+(m-2)\bar{\rho}_f}{m-1}} \tag{22}$$

where n is number of plastic hinges formed in one frame (structure consists of identical 2D frames) of the structure at its ultimate state; and m is number of plane frames that contribute to the lateral resistance of the 3D structure (Husain and Tsopelas 2004).

Fig. 4(a), (b) plots Eq. (22) for two values of the average correlation coefficient for the frame and element strengths, 0 (uncorrelated) and 0.8 (strongly correlated). In addition, it is considered that 3D structures with four and seven parallel plane frames. Fig. 4(a) plots Eq. (22) for system with low degree of indeterminacy. Also Fig. 4(b) plots Eq. (22) for system with high degree of indeterminacy.



(a) Low degree of indeterminacy (b) High degree of indeterminacy

Fig. 4 Variation of the probabilistic overstrength index versus the number of plastic hinge and the average correlation coefficient

Any changes of Ω_{pro} in systems that fails with more plastic hinges (true-scale and more realistic structures) in comparison with those that fails with less plastic hinges (ideal systems with low degree of indeterminacy) are more dependent on the average correlation coefficient of resistance. Also it is evident from the figs that structures with correlated element strengths cannot claim the advantage of increased redundancy whereas structures with uncorrelated elements experience increased redundancy due to its probabilistic effect.

3.2 Redundancy based overstrength using deterministic and probabilistic approaches

The overall effects of redundancy on the structural strength can be completely described by the ratio of the ultimate base shear coefficient of a structural system over the base shear coefficient of the same, but determinate (non-redundant) structure. Thus in this study the formulation of the overstrength due to the redundancy, i.e., Ω_{redu} using deterministic and probabilistic approach is based on the following expression

$$\Omega_{redu} = \frac{c_y}{c_{non,redu}} \tag{23}$$

where c_y is the ultimate structural system base shear coefficient which includes all the effects of the redundancy and $c_{non\text{-}redu}$ is base shear coefficient of the same but non-redundant structural system. Assuming that the strength of a structure is normally distributed, the characteristic or design strength of the structural system can be expressed as a function of its mean value, its standard deviation, and its standard form k. Therefore both c_y and $c_{non,redu}$ can be written as follows in Eqs. (24) and (25)

$$c_y = \bar{c}_y - k\sigma_f \tag{24}$$

$$c_{non,redu} = \bar{c}_{non,redu} - k\sigma_{non,redu} \tag{25}$$

where σ_f is the standard deviation of the frame strength; $\sigma_{non,redu}$ is the standard deviation of the non-redundant frame strength; $\bar{c}_y$ is average of the ultimate frames base shear coefficient; and $\bar{c}_{non,redu}$ is average of the non-redundant base shear coefficient. An expression for σ_f could be obtained as follows

$$\sigma_f = \Omega_{pro}\Omega_{det}v_e\bar{c}_{non,redu} \tag{26}$$

Substituting recent relation with the c_y formula, the following equation is obtained

$$c_y = \Omega_{det}\bar{c}_{non,redu} - k\Omega_{pro}\Omega_{det}v_e\bar{c}_{non,redu} = \Omega_{det}(1 - \Omega_{pro}v_e)\bar{c}_{non,redu} \tag{27}$$

and finally with placing the above relations in Eq. (23), then a conceptual relation for Ω_{redu} is obtained.

$$\Omega_{redu} = \Omega_{det}\left(\frac{1 - k\Omega_{pro}v_e}{1 - kv_{non,redu}}\right) \tag{28}$$

where $v_{non,redu} = \frac{\sigma_{non,redu}}{\bar{c}_{non,redu}}$ defined as the COV of the strength of the non-redundant frame. A non-redundant structure could be modeled as a parallel system consisting of ideal elastic brittle elements where failure of one element results in the system collapse and the safety index of the system is equal to the safety index of the element. For a non-redundant system (i.e., n=1) $v_e=v_{non,redu}$. Therefore the overstrength due to the redundancy using deterministic and probabilistic approaches, i.e., Ω_{redu} can be expressed as follows (Husain and Tsopelas 2004, Fallah *et al.* 2009)

$$\Omega_{\mathrm{redu}} = \Omega_{\mathrm{det}}\left(\frac{1-\mathrm{k}\Omega_{\mathrm{pro}}\mathrm{v}_{\mathrm{e}}}{1-\mathrm{kv}_{\mathrm{e}}}\right) \tag{29}$$

For normally distributed strength with the probability between 90% and 95%, k varies between 1.5 and 2.5. Without any loss of generality, the following values of the COV of the element strength could be used, v_e=0.08–0.14, which were derived for gravity load effects and not used for extreme load effects. Whence, an average value of 0.2, for the product of kv_e, could be used with reasonable accuracy in evaluating the effect of Ω_{det} and Ω_{pro} on Ω_{redu}. According to recent equation and amount intended to kv_e, Ω_{redu} has been rewritten as follows (Fallah *et al.* 2009).

$$\Omega_{\mathrm{redu}} = 1.25\Omega_{\mathrm{det}}\left(1-0.2\Omega_{\mathrm{pro}}\right) \tag{30}$$

Fig. 5 plots Ω_{redu} versus Ω_{pro} for different values of Ω_{det}. The probabilistic effect of redundancy on a structural system could be assessed from Fig. 5. The curve for Ω_{det}=1.0 corresponds to a "non-redundant" system or a system consisting of ideal elastic-brittle elements. For kv_e=0.2 the probabilistic effect of redundancy on the strength of a system could not exceed 25%, that is for a system with Ω_{pro}=0, and accordingly the highest value for Ω_{redu} is 1.25. On the other hand, for a given structure with minimal probabilistic redundancy effect, Ω_{redu} is proportional to Ω_{det}.

Fig. 6 describes qualitatively overall effect of the redundancy on the structural strength in terms of the previously introduced redundancy indices. The deterministic effect captured by the deterministic overstrength index Ω_{det} (mainly effects due to structural indeterminacy) shifts the probability density function of the non-redundant system strength towards higher values to the right, without any shape changes. On the other hand, the probabilistic effects represented by Ω_{pro} result in reduction of the strength uncertainty and accordingly change the shape of the probability density curve without changing its average value (Husain and Tsopelas 2004, Fallah *et al.* 2009).

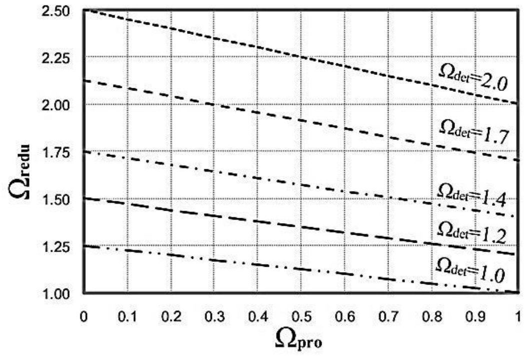


Fig. 5 Variation of Ω_{redu} with respect to Ω_{det} and Ω_{pro} for structural systems with kv_e=0.2

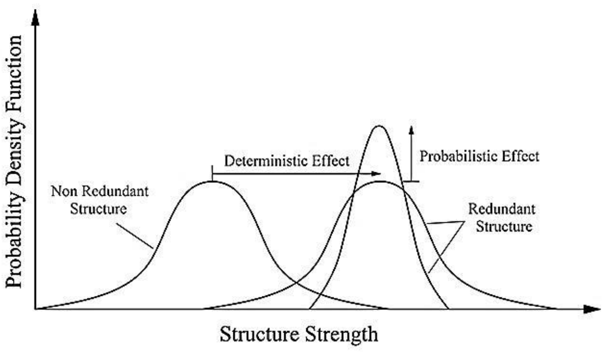


Fig. 6 Effects of redundancy or the overstrength indices Ω_{det} and Ω_{pro} on structural system strength

4. The reliability index

An expression for the structural reliability index β as a function of the overstrength indices Ω_{det} and Ω_{pro} is derived as bellows. The reliability or the safety index β is a measure of how far a structure is from the state of collapse. The reliability index is a function of both the structural strength and the loads which is given by the formula Eq. (31).

$$\beta = \frac{\bar{c}_y - \bar{L}}{\sqrt{\sigma_f^2 + \sigma_L^2}} \tag{31}$$

where $\bar{c}_y$ and σ_f are the average and the standard deviation of the system ultimate base shear coefficient. $\bar{L}$ and σ_L are the average and the standard deviation of the applied loads on weight of structure (load/weight). Substituting $\bar{c}_y = \Omega_{det}\bar{c}_{non,redu}$ and $\sigma_f = \Omega_{pro}\Omega_{det}v_e\bar{c}_{non,redu}$ in Eq. (31) and using coefficient of variations for the strength and the loads, then the following expression is obtained (Husain and Tsopelas 2004, Fallah *et al.* 2009)

$$\beta = \frac{\Omega_{det}.\bar{c}_{coc,redu} - \bar{L}}{\sqrt{v_f^2 \cdot \Omega_{det}^2 \cdot \bar{c}_{non,redu}^{\;2} + \bar{L}^2 v_L^2}} \tag{32}$$

where v_f and v_L are COV of the base shear coefficient and the load per weight of structure of the frame. By virtue of v_f=$\Omega_{pro}\cdot v_e$, Eq. (32) becomes

$$\beta = \frac{\Omega_{det} \cdot \bar{c}_{non,redu} - \bar{L}}{\sqrt{\Omega_{pro}^2\Omega_{det}^2 \cdot v_e^2\bar{c}_{non,redu}^{\;2} + \bar{L}^2 v_L^2}} \tag{33}$$

Assuming $v = \frac{v_L}{v_e}$ and $l = \frac{\bar{L}}{c_{non,redu}}$ then reliability index is obtained as

$$\beta = \frac{\Omega_{det} - 1}{v_e\sqrt{\Omega_{pro}^2 \cdot \Omega_{det}^2 + l^2 v^2}} \tag{34}$$

Thus the reliability index β is a function of (a) The system redundancy, by virtue of its two measures Ω_{det} and Ω_{pro}, (b) The COV of the elements of the structure, (c) The deterministic ratio of the average load to the average non redundant strength (l), and (d) The ratio of the COV of the load to the COV of the element strength (Husain and Tsopelas 2004, Fallah *et al.* 2009).

5. Case study structures

The performance criteria must be defined for structures or structural components to monitor response parameters during analyses as well as to estimate the reliability index and the behavior factor, too. In this research the interstory limitation were used to stop nonlinear static pushover analysis. The interstory drift ratio is limited to 2.5% in nonlinear analysis. This criterion varies between 2% and 3% in building codes. The Iranian standard 2800 edition 2 limited interstory drift to 3% and the Iranian standard 2800 edition 3 limited interstory drift to 2% for buildings that their period are bigger than 0.7s. Previous researches illustrated that RC resisting moment frames have capacity to make more overstrength after reaching interstory drift to 2%. However RC frames cannot make more overstrength when interstory drift exceeds from 2.5% (Massumi and Tasnimi 2006). It is worth mentioning that design methodology of structures based on the standard 2800 is according to force based criterion. Nevertheless, design methodology according to this code is complying with the life safety performance level approximately. On the other hand, some of codes such as IBC2000 cited that reaching interstory drift to 2.5% is comply with the collapse prevention performance level approximately. Therefore, the calculated reliability index and the behavior factor in this research are evaluated based on reaching interstory drift to 2.5%.

To achieve the aim of this study, a number of three-dimensional reinforced concrete framed structures with the same story area and the same equal lateral resistance were designed. A parametric study was devised involving pushover analyses for RC 3D frames with different structural characteristics to reflect different levels of system redundancy. All of the RC frames with 6 and 9 stories and 3, 4, 5, and 6 bays in any direction were analyzed. In order to compute the overstrength indices, 8 frame samples from 3 bays to 6 bays at each direction and with six and nine stories were designated. The SAP2000 software was used for nonlinear static analyses. Furthermore, the defined FEMA356 hinges are also assigned to beams and columns of all models.The interacting P-M_2-M_3 and the moment M_3 and shear V_2 hinges were used for nonlinear behavior of columns and beams, respectively. For nonlinear static analysis, a set of eight designed frame structures with high ductility demand were selected. The reason of selecting these models is to consider the influence of the number of bays and stories in overall response of 3D bending frames with the same of both story area and equal lateral resistance. The bays lengths are 4, 4.8, 6 and 8 meters respectively and the story height is 3 meters. The case study models were designed subjected to an incrementally increasing lateral loads with the pattern of inverted triangular distribution as well as constant set of gravity loads. It is noticeable that the ultimate base shear coefficient and ultimate lateral resistance of all models should be considered as equal when the level of overall drift could reach to 2.5%. For this purpose, an iterative process of trial and error was used for selecting sections, analysis, design and amount of sections reinforcement. The aforementioned parameters were selected so that the ultimate base shear coefficient and the ultimate lateral resistance of all case study models become identical. Considering these constraints of the design process, it can be seen that the behavior factors of all 3D frames with allowable stress design approach (without considering the probabilistic effects of redundancy), and the total overstrength factor (R_s) between the models are constant. Therefore, it is concluded that any changes in amount of the calculated behavior factors, would not be related to R_s, and could be due to different ductility reduction factors that influenced by the number of bays (redundancy). Equal lateral resistance and equal base shear coefficient in case study models clarify the role of redundancy and separate the role of redundancy from that of the total overstrength reduction factor. Lateral loading in the nonlinear static pushover analysis applied with an inverted triangular distribution according to code 2800. This process was done based on imposing a full part of 100% of the forces and displacements in *x* direction as well as a part of 30% of the forces along with the *y* direction, simultaneously. After obtaining the structural capacity curve, the process of bilinear idealization of the capacity curve was performed according to the following recommendations which denoted by Park (1989) for reinforced concrete members. Accordingly, the effective elastic stiffness is obtained as the slope of the line which connects the orient to either the point of first yielding or 75% of the ultimate load, whichever is the less. The capacity curve, the bilinear idealization, the geometry and reference code of case study models are shown in Fig. 7.

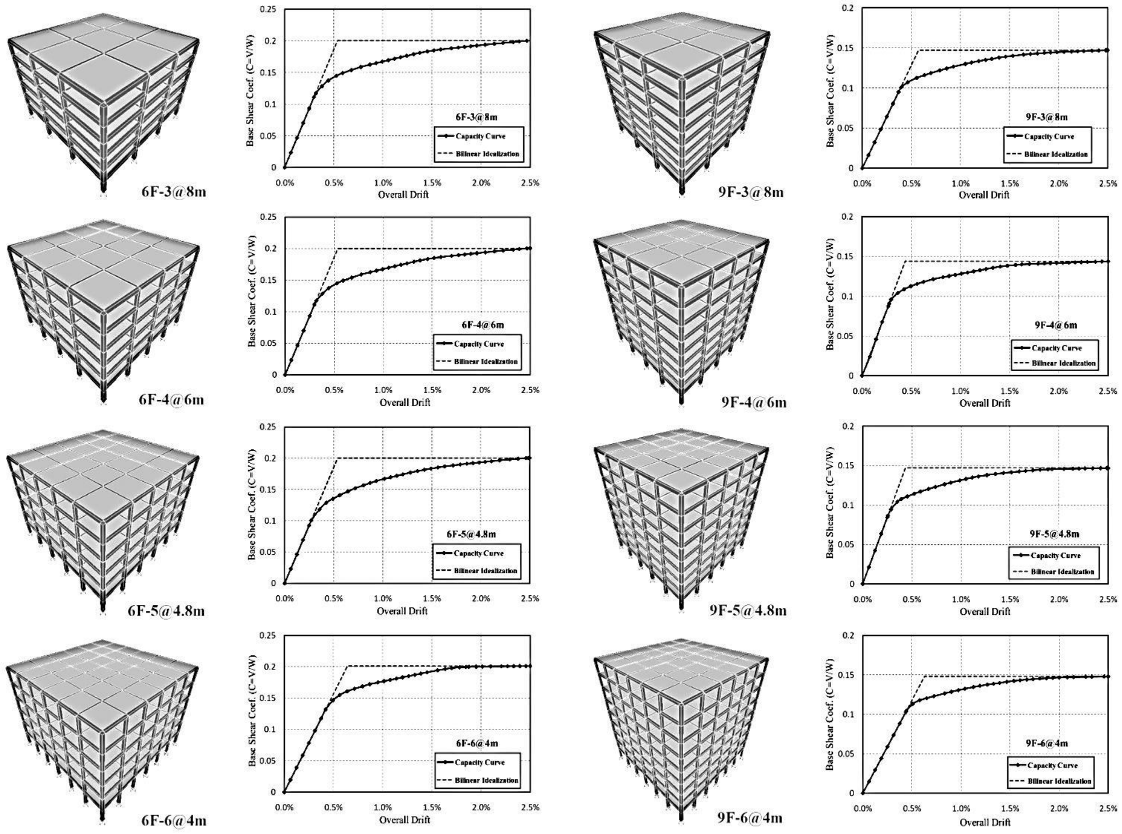


Fig. 7 The capacity curve, bilinear idealization, geometry and reference code of case study models

6. Results of nonlinear static analysis

The designed structures were analyzed under incrementally increasing lateral loads with inverted triangular distribution and constant gravity loads, to estimate essential parameters of the behavior factors under static inelastic load conditions. The parameters such as total overstrength reduction factors, overall ductility, ductility reduction factor, behavior factor with allowable design approach, behavior factor with ultimate strength design approach and ultimately the reliability index of structures were estimated numerically. The results are shown in Table 1. As it is shown, the total overstrength factors without considering the probabilistic effects of redundancy are approximately equal in both of the six floor and the nine floor structures. Although, assuming the same ultimate lateral resistance for all of the studied structures, which causes an increase in redundancy for up to 5 spans at each direction, can enhance both the overall ductility and the inelastic capacity, any further increase in the redundancy (6 spans at each direction) might have a negative effect on the overall ductility and behavior factor. It should be implicated that for the structures with equal lateral resistance, whatever the force level at which the first plastic hinge is formed to be less, then the overall ductility coefficient and the overstrength due to the redundancy will be larger.

Table 1 Results of static pushover analysis

Ref. Code	T	C_w	C_s	C_y	Δ_s	Δ_y	$\Omega_{s\,(redu,sth)}$	$\Omega_{s\,(size,\phi)}$	R_s	μ	R_μ	R_w
6F-3@8m	1.13	0.0756	0.1261	0.2013	0.0035	0.0057	1.597	1.648	2.632	4.4	5.42	14.27
6F-4@6m	1.09	0.0756	0.1111	0.2003	0.003	0.0054	1.803	1.456	2.625	4.6	5.73	15.04
6F-5@4.8m	1.07	0.0756	0.1006	0.2004	0.0027	0.0054	1.992	1.315	2.620	4.6	5.73	15.01
6F-6@4m	1.20	0.0756	0.1317	0.2008	0.0042	0.0065	1.525	1.722	2.625	3.8	4.70	12.34
9F-3@8m	1.61	0.0624	0.0951	0.1472	0.0037	0.0058	1.548	1.524	2.358	4.3	4.82	11.37
9F-4@6m	1.38	0.0624	0.0906	0.1468	0.0028	0.0044	1.587	1.486	2.358	5.7	6.62	15.61
9F-5@4.8m	1.39	0.0624	0.0864	0.1470	0.0026	0.0044	1.701	1.385	2.356	5.7	6.60	15.55
9F-6@4m	1.68	0.0624	0.1030	0.1479	0.0044	0.0063	1.437	1.641	2.359	4.0	4.39	10.36

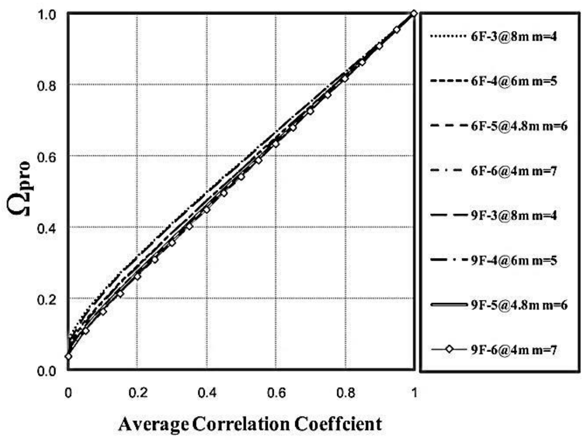
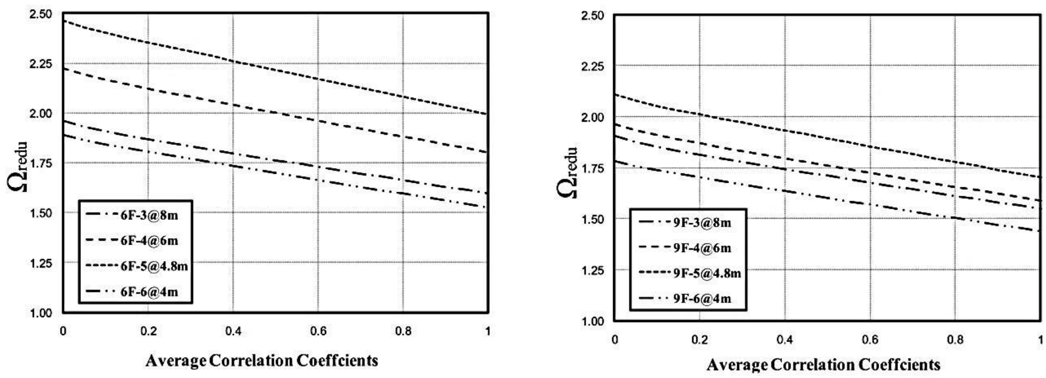


Fig. 8 Variation of Ω_{pro} versus the average correlation coefficient and the number of plastic hinge

7. Effects of redundancy on the overstrength and the behavior factor

The Overstrength due to the redundancy which stated in Table 1 is equal to the deterministic overstrength index in section 3.1. If it is assumed that the average correlation coefficient of members of the 2D frames $(\bar{\rho})$ and the average correlation between the strength parameters of the plane frames $(\bar{\rho}_f)$are equal to unit, then it can be concluded that Ω_{redu} values with values of $\Omega_{s(redu,sth)}$ (Table 1) will be equal. Therefore, probabilistic effects of the redundancy on the overstrength of case study models are calculated. The parameter Ω_{pro} in 3D frames is a function of four variables which are introduced as n that is the number of plastic hinges formed in one 2D frame of the structure at its ultimate state; and m that is the number of plane frames which contribute to the lateral resistance of the 3D structure; $\bar{\rho}$ that is the average correlation coefficient of all members of the 2D frame, and $\bar{\rho}_f$ which is the average correlation due to the strengths of plane frames. In this study, the extent of variation of the probabilistic overstrength index by changing the average correlation coefficients is much broader than that by changing the number of plastic hinges in the real scaled case study models. Assuming the same correlation coefficient of resistance for structural members of 2D frames increases the number of plastic hinges in which causes very slight decrease in the Ω_{pro} index. Fig. 8 illustrates all changes in the probabilistic



(a) 6 floor models (b) 9 floor models

Fig. 9 variation of Ω_{redu} versus the average correlation coefficients in 6 and 9 floors models

overstrength index for three dimensional models of this study based on the assumption of $\bar{\rho}$ is equal to $\bar{\rho}_f$.

As can be seen, varying the amount of Ω_{pro} in eight different models with the assumption of equality correlation coefficients ($\bar{\rho}$ and $\bar{\rho}_f$). For a fixed value of them, it is very small and negligible. It is reasonable to expect that the values of correlation coefficient of moment frames structural systems with more redundancy that designed properly (Strict observance of the weak beam and strong column criterion) are lower than the values of correlation coefficient of structures with less redundancy. $\bar{\rho}_{more,redundant} \leq \bar{\rho}_{less,redundant}$

Thus, the reasonable conclusion is that any increased redundancy causes a reduction in the amount of Ω_{pro} index and ultimately causes that the index of Ω_{redu} to be increased .Fig. 9(a), (b) illustrate the changes in the overstrength due to the redundancy in all three-dimensional 6 and 9 story models of this study ($\bar{\rho}$=$\bar{\rho}_f$).

Based on assuming that all values of the correlation coefficient are equal to unit; the values of Fig. 9(a), (b) should be equal to amounts of $\Omega_{s\,(redu,sth)}$as indicated in Table 1. As can be seen the increased redundancy is associated with lower correlation coefficients and causes an increase in Ω_{redu} index. It should be noted that, increased redundancy until 5 spans in each direction has been able to increase the amount of Ω_{redu}, although if this redundancy increases until 6 spans in each direction can reduce the amount of Ω_{redu}. This event can be caused by not observing weak beam strong column criteria alike for case study models. Also it can reduce local ductility of beams in the models with 6 spans in each direction. Fig. 10 illustrates the changes in total overstrength reduction factor in 6 and 9 floor, three-dimensional models of this study ($\bar{\rho}$=$\bar{\rho}_f$).

Assuming the values of the correlation coefficient is equal to unit; then the values of Fig. 10 should be equal to the amounts of R_s in Table 1. It is evident that the total overstrength has some values in lieu of a certain coefficient in all 6 and 9 floors models individually. Hence, it would be notified that the values of correlation coefficients of moment frames with more redundancy which have designed properly (satisfying the strong column-weak beam criterion) are lower than corresponding values of correlation coefficient due to the structures with less redundancy. Thus it is concluded that an increased redundancy according to probabilistic effects would cause the more overstrength capacity in structural models which have more redundancy. Fig. 11(a), (b) illustrates the changes in the behavior factor based on the allowable stress design approach.

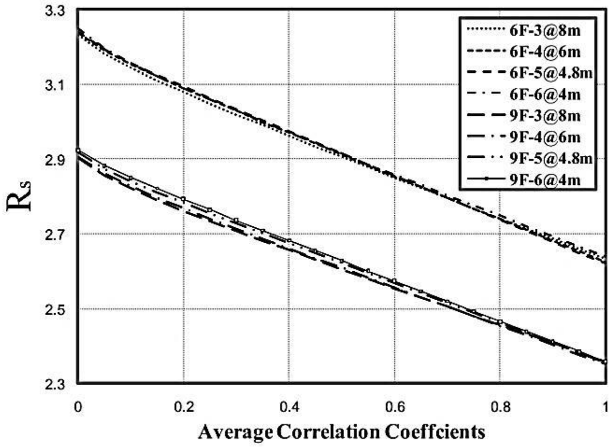
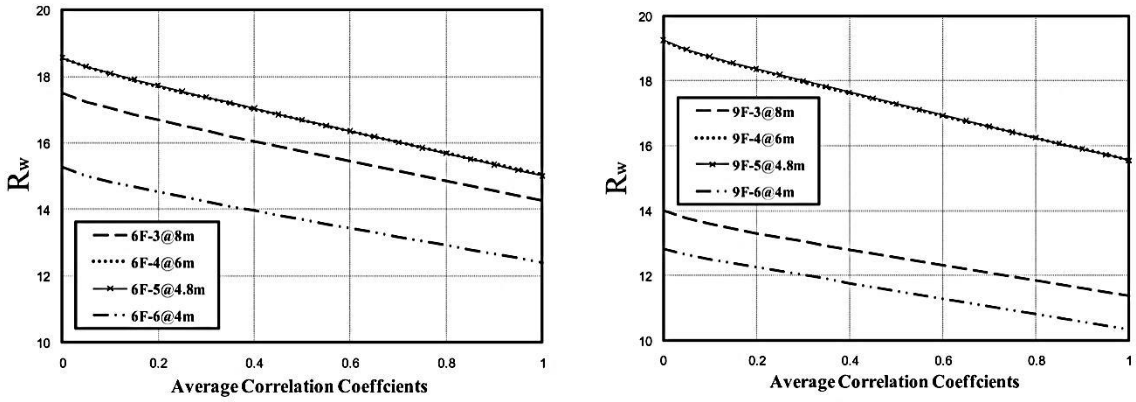


Fig. 10 Variation of total overstrength coefficient versus the average correlation coefficients



(a) 6 floor models (b) 9 floor models

Fig. 11 variation of the Behavior Factor versus the average correlation coefficients

Based on assuming the values of the correlation coefficient to be equal to unit, then the values of Fig. 11 will be equal to the amounts of R_w which are given in Table 1. It is noted that an increased redundancy (i.e., increase in the number of earthquake resistant frames) is associated with reducing the mean correlation coefficient of resistance of frame members. Thus the behavior factor will be increased. But this increase in the behavior factor is not enough to cover the negative effect of excessive redundancy in local ductility of members. Therefore it should be noted that how the structural ductility changes with increasing of redundancy in structures. This is because of excessive redundancy which has influences upon local ductility of members.

8. Evaluation of the reliability index

The reliability index of structures was calculated in a comparative study and separately for 6 and 9 floors structures. As previously mentioned, assuming an elastic brittle behavior for structural elements, the point of first yielding in a structural system can be considered as a reasonable approximation of the base shear coefficient for a determinate structure. Therefore $c_{non\text{-}redu}$ in Eq. (34) can be substituted by c_s, which is the base shear coefficient of the structural system at the point of the first yielding. Therefore, the parameter of l in Eq. (34) is considered as follows for 6 floors structures.

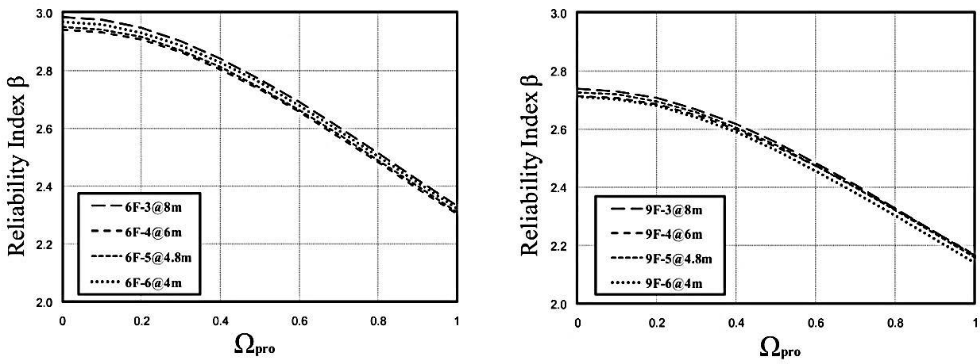
$$\text{If}\ \left(l=\frac{\bar{L}}{\bar{c}_{non-redu}}\right)_{6F-3@8m}=\left(l=\frac{\bar{L}}{\bar{c}_{s}}\right)_{6F-3@8m}=1\ \text{then}\begin{cases}\left(l=\frac{\bar{L}}{\bar{c}_{s}}\right)_{6F-4@6m}=1.135\\\left(l=\frac{\bar{L}}{\bar{c}_{s}}\right)_{6F-5@4.8m}=1.253\\\left(l=\frac{\bar{L}}{\bar{c}_{s}}\right)_{6F-6@4m}=0.957\end{cases}$$

and similarly for the 9 floors structures, the parameter of l in Eq. (34) is considered as bellow.

$$\text{If}\ \left(l=\frac{\bar{L}}{\bar{c}_{non-redu}}\right)_{9F-3@8m}=\left(l=\frac{\bar{L}}{\bar{c}_{s}}\right)_{9F-3@8m}=1\ \text{then}\begin{cases}\left(l=\frac{\bar{L}}{\bar{c}_{s}}\right)_{9F-4@6m}=1.050\\\left(l=\frac{\bar{L}}{\bar{c}_{s}}\right)_{9F-5@4.8m}=1.101\\\left(l=\frac{\bar{L}}{\bar{c}_{s}}\right)_{9F-6@4m}=0.924\end{cases}$$

Therefore, based on assuming v=2, v_e=0.1 and the values of l that obtained as above, the reliability index (β) can be calculated separately for 6 and 9 floors structures. Fig. 12(a), (b) present the reliability index β of case study models, which varies with $\mathbf{\Omega}_{pro}$ for various values of $\mathbf{\Omega}_{det}$. The reliability index is calculated for a different ratio of average load to average strength level. Therefore the values of l for each structure are different.

Both parts of Fig. 12 illustrate the changes of the reliability index due to 6 and 9 story modeled structures based on application of the same values of probabilistic overstrength index. This means



(a) 6 floor models (b) 9 floor models

Fig. 12 Variation of the reliability index (β) of structure with respect to $\mathbf{\Omega}_{det}$ and $\mathbf{\Omega}_{pro}$ for v_e=0.1, v=2 that considered changes in the reliability index of structures which have the same ultimate lateral resistance, is only due to the probabilistic effects of redundancy. As previously mentioned the index of Ω_{pro} in the more redundant structures takes smaller amounts in comparison with less redundant structures. Then the general conclusion is that more redundant structures due to the probabilistic effects of redundancy would be corresponded to a more reliability index.

9. Conclusions

In this study, both deterministic and probabilistic effects of the redundancy on the behavior factor and the reliability index of a number of studied structures with the same ultimate resistance were assessed. The general results are as follows:

• Considering an increased redundancy in moment resisting frame (regular in plan and height) with a specific plan (same story area) can reduce the amount of Ω_{pro} and increase the amount of Ω_{redu}.

• Based on assuming the same ultimate lateral resistance for structures which would cause an increase to a certain level of redundancy (i.e., an increase in the number of resistant frames up to 5 spans in each direction) is associated with enhancing both the ductility capacity and the inelastic capacity indeed. However any further increase in the redundancy of that certain level might have a negative effect on the ductility capacity.

• Increased redundancy (i.e., an increase in the number of earthquake resistant frames) is associated with reducing the average correlation coefficient related to the resistance of frame members. Thus the behavior factor can be increased but this increase is not enough to cover negative effect of excessive redundancy on local ductility of members. Therefore it is important to notify that how the structural ductility changes in regards to increasing the redundancy in structures. It is because of probable excessive redundancy which is able to change the local ductility demands. Furthermore, it can reduce the positive effects of increased redundancy.

• If having more redundancy for a group of structures with the same story area, would reduce the level of formation of first plastic hinge (i.e., c_s), then the increased redundancy would be an ideal property which will increase the behavior factor.

• Based on the assumption of considering same ultimate lateral resistance, hence those structures which are more redundant, have a greater reliability index. However, this outcome is not observed for the behavior factor. In general, any criteria in conjunction with the reliability index which does not cover the structural inherent parameters such as the ductility demand, cannot able to explain the real mode of behavior of structure and would be misleading the real situation of the structures.

References

ATC (1995), *A critical review of current approaches to earthquake-resistant design*, ATC-34 Report, Applied Technology Council, Redwood City, California.

ATC (1995), *Structural response modification factors*, ATC-19 Report, Applied Technology Council, Redwood City, California.

Bertero, R.D. and Bertero, V.V. (1998), “Redundancy in earthquake-resistant design: How to define it and quantify its effects.”, *Proceedings of 6th U.S. National Conference on Earthquake Engineering, CD*.

Bertero, R.D. and Bertero, V.V. (1999), “Redundancy in earthquake-resistant design”, *J. Struct. Eng.*, ASCE,

125(1), 81-88.
BHRC (2000), *Iranian Code of Practice for Seismic Resistant Design of Buildings*, Standard No.2800-05, 2nd edition, Building and Housing Research Center, Tehran.
BHRC (2005), *Iranian Code of Practice for Seismic Resistant Design of Buildings*, Standard No.2800-05, 3rd edition, Building and Housing Research Center, Tehran.
Biondini, F., Frangopol. D.M. and Restelli, S. (2008), "On structural robustness, redundancy and static indeterminacy", *Proceedings of the 2008 ASCE-SEI Structures Congress*, Vancouver.
Fallah, A.A., Sarvghad-Moghadam, A. and Mohammadzadeh, S. (2009), "Reliability index for reinforced concrete frames using nonlinear pushover and dynamic analysis", *Int. J. Adv. Struct. Eng.*, **1**(2), 135-151.
Fallah, A.A., Sarvghad-Moghadam, A. and Massumi, A. (2009), "A nonlinear dynamic based redundancy index for reinforced concrete frames", *J. Appl. Sci.*, **9**(6), 1065-1073.
FEMA356 (2000), *Prestandard and commentary for Seismic Rehabilitation of Buildings*, Federal Emergency Management Agency.
Gollwitzer, S. and Rackwitz, R. (1990), "On the reliability of Daniels systems", *Struct. Saf.*, **7**(2-4), 229-243.
Hendavi, S. and Frangopol, D.M. (1994), "System reliability and redundancy in structural design and evaluation", *J. Struct. Saf.*, **16**(1), 47-71.
Husain, M. and Tsopelas, P. (2004), "Measures of structural redundancy in reinforced concrete buildings. I: Redundancy Indices", *J. Struct. Eng.*, ASCE, **130**(11), 1651-1658.
Husain, M. and Tsopelas, P. (2004), "Measures of structural redundancy in reinforced concrete buildings. II: redundancy response modification factor R_R", *J. Struct. Eng.*, ASCE, **130**(11), 1659-1666.
IBC 2000 (1998), *International Building Code, Inter.Code Council*, Falls Church, Final Draft, VA.
Liao, K.W. and Wen, Y.K. (2004), "Redundancy in steel moment frame systems under seismic excitation", Department of civil and environmental engineering, University of Illinois at Urbana-Champaign Illinois.
Marhadi, K. and Venkataraman, S. (2009), "Surrogate measures to optimize structures for robust and predictable progressive failure", *Struct. Multidiscip. Optimiz.*, **39**(3), 245-261.
Massumi, A. and Tasnimi, A.A. (2006), "Estimation of response modification factors for RC moment resisting frames", *Building and Housing Research Center (BHRC)*, Publication No.: R-436, Tehran, Iran.
Massumi, A., Tasnimi, A.A. and Saatcioglu, M. (2004), "Prediction of seismic over-strength of concrete moment resisting frames using incremental static and dynamic analyses", *Proceedings of 13th World Conference on Earthquake Engineering*, Vancouver, BC, Canada.
Mirenda, E. and Bertero, V.V. (1994), "Evaluation of strength reduction factors for earthquake-resistant design", *Earthq. Spectra*, **10**(2), 357-379.
Moses, F. (1974), "Reliability of structural systems", *J. Struct. Div.*, ASCE, **100**(ST9), 1813-1820.
Okasha, N.M. and Frangopol, D.M. (2010), "Time-variant redundancy of structural systems", *J. Struct. Infrastruct. Eng.*, **6**(1-2), 279-301.
Park, R. (1989), "Evaluation of ductility of structures and structural assemblages from laboratory testing", *Bull. NZ. Natl. Soc. Earthq. Eng.*, **22**(3), 155-166.
SAP2000 Version 14.2.0 (2010), *A computer program for integrated finite element analysis and design of structures*, University of California, Berkeley.
Song, S.H. and Wen, Y.K. (2000), "Structural redundancy of dual and steel moment frame systems under Seismic Excitation", A Report on a Research Project Sponsored by the National Science Foundation (Under Grant NSF EEC-97-01785), University of Illinois at Urbana, Illinois.
Uang, C.M. (1991), "Establishing R and Cd factors for building seismic provisions", *J. Struct. Eng.*, **117**(1), 19-28.
Wen, Y.K. and Song, S.H. (2003), "Structural reliability/redundancy under earthquakes", *J. Struct. Eng.*, ASCE, **12**(1), 56-66.
Yoshihiro, K. and Yakov, B.H. (2011), "Redundancy and robustness, or when is redundancy redundant?" *J. Struct. Eng.*, ASCE, **137**(9), 935-945.

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Notation

c	: Ultimate base shear coefficient
c_{eu}	: Elastic base shear coefficient
$c_{non,redu}$	: Ultimate base shear coefficient of non-redundant structure
c_{redu}	: Ultimate base shear coefficient of redundant structure
c_s	: First local yielding base shear coefficient
c_w	: Design base shear coefficient
c_y	: Ultimate base shear coefficient
$\bar{c}_{non,redu}$	: The average of ultimate base shear coefficient of non-redundant structure
$\bar{c}_{redu}$	: The average of ultimate base shear coefficient of redundant structure
$\bar{c}_s$	: The average of first local yielding base shear coefficient
$\bar{c}_y$	: The average of ultimate base shear coefficient
$\bar{c}$	: The average of frame ultimate base shear coefficient
k	: The standard form of normal distribution
$\bar{L}$	: The average ratio of applied loads to weight of structure
m	: The number of plane frames that contribute to the lateral resistance of 3D structure
M_i	: Yield moment of structural element when plastic hinge "i" is formed
$\bar{M}_i$	: The average value of yield moment of the structural element when plastic hinge "i" is formed
n	: The number of plastic hinges in the frame resulting from the sway failure mode
R_R	: Redundancy factor
R_s	: Total overstrength reduction factor
R_w	: Allowable stress design behavior factor
R_μ	: Ductility reduction factor
R_ξ	: Damping factor
T	: Fundamental period of structure
v_e	: The coefficient of variation of the element strength
v_f	: The coefficient of variation of the strength of the plane frames
$v_{non,redu}$	: The coefficient of variation of lateral strength of the non-redundant frame
v_s	: The coefficient of variation of the strength of 3D structural system
β	: Structural reliability index
$\Delta_{\max}$	: Maximum overall drift
Δ_y	: Overall drift at yield
μ	: Overall ductility
$\bar{\rho}$	: The average correlation coefficient
$\bar{\rho}_f$	: The average correlation between lateral strength of the plane frames
ρ_{ij}	: The correlation coefficient between M_i and M_j
σ_f	: The standard deviation of the frame base shear
σ_L	: The standard deviation of ratios of applied loads to weight of structure
σ_{Mi}	: The standard deviation of M_i
σ_{Mj}	: The standard deviation of M_j
Φ	: A function of total ductility, period of system and soil conditions
Ψ_i	: A coefficient with unit of radians/(length$\cdot$ mass)

Ω_{det} : Deterministic overstrength index

Ω_{pro} : Probabilistic overstrength index

$\Omega_{s(redu,sth)}$: Overstrength arising from redundancy (until a collapse mechanism is formed) and steel strain hardening

$\Omega_{s(size,\phi)}$: Overstrength arising from restricted choices for member sizes, rounding up of sizes and dimensions, and differences between nominal and factored resistances