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Pseudo 3D FEM analysis for wave passage effect on the response spectrum of a building built on soft soil layer

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Abstract. Spatially variable ground motions can be significant on the seismic response of a structure due to the incoherency of the incident wave. Incoherence of the incident wave is resulted from wave passage and wave scattering. In this study, wave passage effect on the response spectrum of a building structure built on a soft soil layer was investigated utilizing a finite element program of P3DASS (Pseudo 3-dimensional Dynamic Analysis of a Structure-soil System). P3DASS was developed for the axisymmetric problem in the cylindrical coordinate, but it is modified to apply anti-symmetric input earthquake motions. Study results were compared with the experimental results to verify the reliability of P3DASS program for the shear wave velocity of 250 m/s and the apparent shear wave velocities of 2000-3500 m/s. Studied transfer functions of input motions between surface mat foundation and free ground surface were well-agreed to the experimental ones with a small difference in all frequency ranges, showing some reductions of the transfer function in the high frequency range. Also wave passage effect on the elastic response spectrum reduced the elastic seismic response of a SDOF system somewhat in the short period range.

Keywords: wave passage effect; finite element program of P3DASS; soft soil layer; anti-symmetric input earthquake motions; transfer function

1. Introduction

Spatial variation of seismic motions is investigated in the structural engineering since the 1970s. (Clough and Penzien 1975, Luco 1976) Effects of spatial variation of seismic motions are caused mainly by wave passage effect and wave scattering effects. Wave passage is a time delay of arriving seismic waves, causing a shift in the Fourier phases of earthquake motions. Wave scattering is a complex waveform scattering due to the heterogeneities along the travel path of seismic waves causing random variations of Fourier amplitude and phase in the earthquake motions. Some incoherence of the incident waves resulted from wave passage is deterministic (predictable), but some incoherence resulted from wave scattering is stochastic.

Spatially variable ground motions can be a significant component of the seismic demand on a variety of structures including long-span bridges, buried pipelines, tunnels (Lupoi *et al*. 2005,

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Ates *et al*. 2009, O’Rourke and Liu 1997, Hashash *et al*. 2005), and relatively brittle foundation systems such as poor reinforced concrete slabs (Ancheta 2010). The forced deformation between independent foundations can increase the deformation of a structure such as bridges.

Effects of non-vertically incident seismic waves on the response of structures were investigated obtaining significant different responses in translation and rocking from those with vertically incident SH-wave (Luco 1976, Luco and Sotiropoulos 1980). Also effects of spatially random ground motions on the response of a rigid surface foundation show those similar to wave passage reducing the translational response at high frequencies and creating rocking and torsional responses (Luco and Wong 1986). Another study on the spatially varying ground motions was performed to optimize long-span frames for specified seismic responses by Ohsaki (2001) founding that the spatial variation of ground motions leads to the increase of the optimal structural volume under constraints on member strains, and Tsai and Hashash (2010) reviewed the procedures for generating spatially variable input motions.

On the other hand, Abrahamson (1992) prepared the basis for most modern practice on spatially variable ground motion and improved the procedure to generate spatially variable accelerograms. Ancheta (2010) modified his procedure to generate realistic waveforms that are variable in space with respect to both phase and amplitude using Frequency Dependent Windowing method. Also Ancheta *et al.* (2011) examined empirical relations for the three sources of spatially variable ground motions of apparent horizontal wave velocity, random phase variation of ground motions (lagged coherency) and Fourier amplitude variability (standard deviation) of ground motions, using the data from the Borrego Valley Differential Array in California and the LSST array in Taiwan. They showed that lagged coherency and Fourier amplitude variability model residuals are uncorrelated and frequency-to-frequency Fourier amplitude and coherency difference between array station pairs are weakly correlated.

In this study, a finite element program of Pseudo 3-dimensional Dynamic Analysis of a Structure-soil System (P3DASS) was modified to analyze the wave passage effect applying phase shifted (time delayed) earthquake input motions to the loading points distributed along the bedrock surface. The modified program was verified to justify the applicability of a program comparing the analysis results with the experimental study results.

2. Modification and verification of P3DASS program

The finite element program of P3DASS was originally developed to perform a pseudo 3-dimensional dynamic analysis of a structure-soil system considering vertically propagating waves from the bedrock. P3DASS was coded using a cylindrical coordinate system for the axisymmetric system in the frequency domain as shown Fig. 1 (Roesset and Kim 1987, Kim and Roesset 2004, Kim 2012) and utilizing the consistent lateral boundary to reproduce the far field developed by Kausel (1974) and also used as a consistent transmitting boundary by Lee *et al.* (2012). This P3DASS program was modified to simulate the wave passage along the bedrock due to the horizontally propagating bedrock earthquake. Wave passage is a time delay in wave arrivals between different locations and makes a shift in the Fourier phases of input earthquake motions. (Zerva 2009)

The dynamic equations of motion of the structure-soil system can be written in the frequency domain as follows.

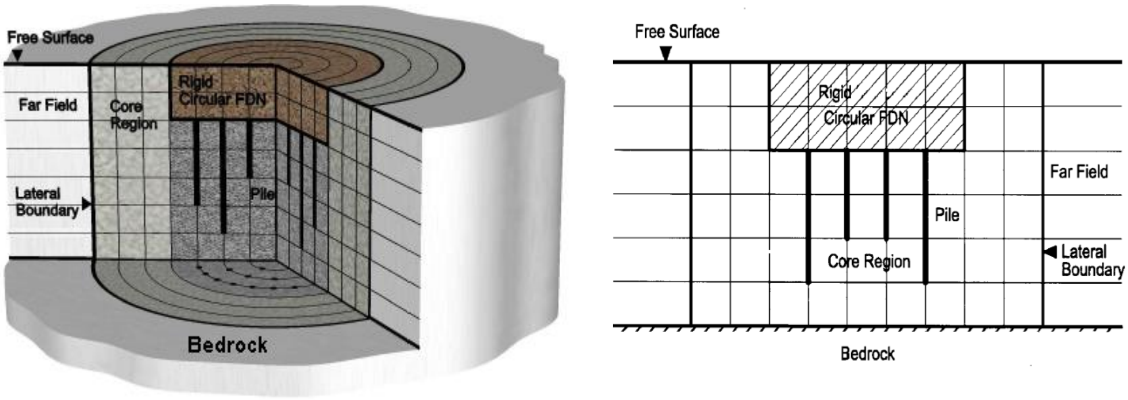


Fig. 1 Soil-foundation model for P3DASS

$$\begin{bmatrix} \bar{\mathbf{S}}_{\mathbf{SS}} & \bar{\mathbf{S}}_{\mathbf{SB}} \\ \bar{\mathbf{S}}_{\mathbf{BS}} & \bar{\mathbf{S}}_{\mathbf{BB}} \end{bmatrix} \begin{Bmatrix} \tilde{\mathbf{U}}_{\mathbf{S}} \\ \tilde{\mathbf{U}}_{\mathbf{B}} \end{Bmatrix} = -\begin{bmatrix} \bar{\mathbf{M}}_{\mathbf{SS}} & \bar{\mathbf{M}}_{\mathbf{SB}} \\ \bar{\mathbf{M}}_{\mathbf{BS}} & \bar{\mathbf{M}}_{\mathbf{BB}} \end{bmatrix} \begin{Bmatrix} \tilde{\mathbf{0}} \\ \tilde{\ddot{\mathbf{U}}}_{\mathbf{g}} \end{Bmatrix} \tag{1}$$

In this Eq. (1), subscripts of S and B denote a structure-soil system and a bedrock boundary, and a subscript of g denotes a bedrock input motion. As the displacement of a bedrock is negligible, $\{\tilde{\mathbf{U}}_{\mathbf{B}}\} = \tilde{\mathbf{0}}$. Then the responses of a structure-soil system can be found applying time delayed bedrock input acceleration motions along the bedrock interface.

The vertically propagating wave mechanism in the soft soil layer can be explained by Snell's Law as shown in Fig. 2. A horizontally propagating shear wave in the crustal rock with the shear wave velocity of V_{app} will refract into the bedrock which has a shear wave velocity of V_{s1} and a refracting angle of α, and will change the propagating direction upward.

According to the Snell's Law

$$\frac{sin\alpha}{sin\beta} = \frac{V_{S1}}{V_{app}} \tag{2}$$

As a shear wave in the crustal rock travels horizontally, β is $\frac{\pi}{2}$ and then

$$sin\alpha = \frac{V_{S1}}{V_{app}} \tag{3}$$

And V_{app} is assumed as 2500 m/s and the shear wave velocity of the bedrock (V_{s1}) can be taken as 1000 m/s for the soil site of S_B in IBC (International Building Code), then $sin\alpha$ is 0.4. Also the refraction at the surface between soft soil layer and bedrock can be expressed as follows

$$\frac{sin\gamma}{sin\alpha} = \frac{V_S}{V_{S1}} \tag{4}$$

If V_s is taken as 250 m/s for the soft soil layer, $sin\gamma = 0.1$. Then γ is 5.74° which is small, and it means that the wave propagates almost vertically.

And the distance of x to apply a time delayed seismic motion considering a time step (Δt) of an exciting earthquake motion can be estimated as follows.

$$x = V_{app} \cdot \Delta t \tag{5}$$

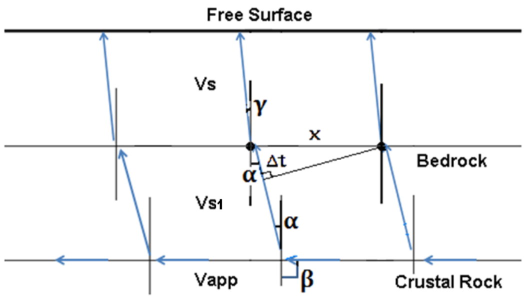


Fig. 2 Distance for time shift

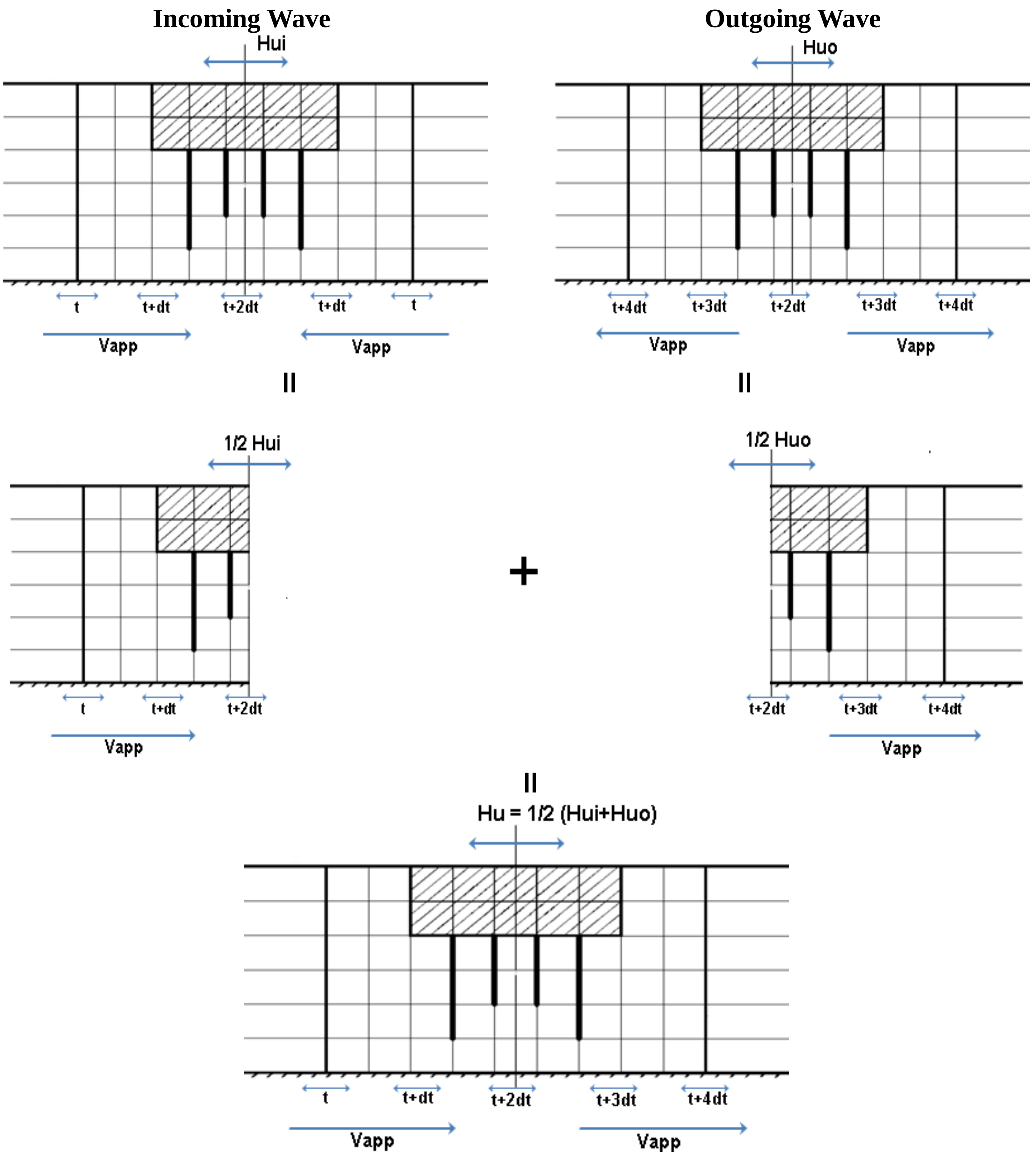
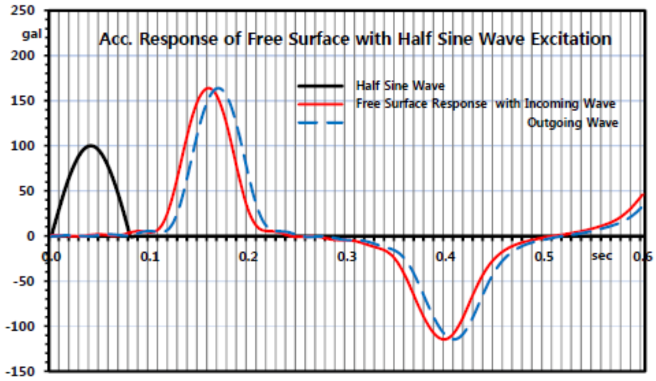
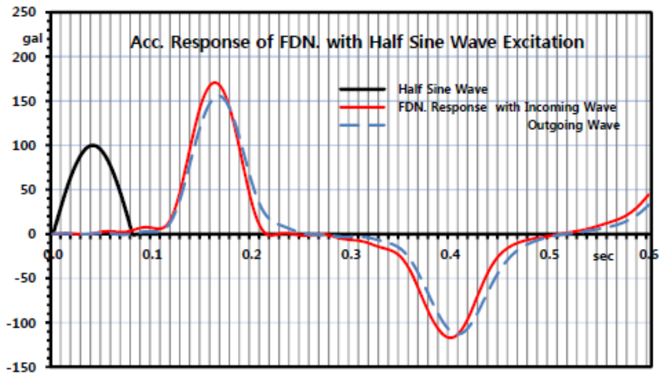


Fig. 3 Concept of P3DASS analysis for wave passage



(a) Free surface motion



(b) Foundation motion

Fig. 4 Test of seismic responses by half sine wave

For the wave passage analysis, P3DASS program coded with axisymmetric elements in the cylindrical coordinate was modified for the anti-symmetric input earthquake motions performing two seismic analyses with incoming and outgoing input motions. Incoming wave propagates from far field to core center and outgoing wave propagates from core center to far field with time steps of Δt. The schematic concept to solve the wave passage effect is illustrated in Fig. 3. The foundation response (Hu) including the wave passage effect can be obtained averaging two seismic responses (Hui and Huo) of a foundation which are calculated with incoming and outgoing waves having an apparent wave propagation velocity of V_{app} in the underlying bedrock. V_{app} is assumed to be 2.5 km/s. V_{app} is approximately 2.0-3.5 km/s in the typical bedrock beneath the soil site.

To check the applicability of P3DASS program for the wave passage analysis, free surface and foundation motions were investigated applying time delayed half sine wave of $100 sin 12.5\pi t$ cm/s^2 (gal) to a bedrock. The radius of a foundation was assumed to be 12.5 m and the lateral boundary was also placed at a distance of 12.5 m from the edge of a foundation to simulate a delayed time of 0.005 seconds.

The acceleration responses of free surface and foundation motions are plotted in Fig. 4 for incoming and outgoing waves with a time delay of 0.005 seconds. The free surface responses of an outgoing wave are shifted two time steps of 0.01 second from that of an incoming wave due to the time delay as shown in Fig. 4(a).

And foundation responses with an incoming wave are a little bit amplified due to the energy accumulation in the core region, but foundation responses with an outgoing wave are de-amplified a little bit due to the energy dissipation to the far field, showing a time delay of 0.005 seconds as shown in Fig. 4(b). Acceleration responses of free surface and foundation with time delayed bedrock excitations seem to be reasonable and P3DASS program can be applicable for the analysis of the wave passage effect.

3. Transfer function with time delayed excitations

Transfer function of seismic responses of a surface mat foundation with respect to those of a free ground surface was investigated to verify the modified P3DASS program.

Mylonakis *et al*. (2006) synthesized the transfer function of a surface mat foundation using the theoretical models as follows.

$$H_u = \frac{sin[a_0^k\left(\frac{V_s}{V_{app}}\right)]}{a_0^k(\frac{V_s}{V_{app}})} \quad \text{if} \quad a_0^k \leq \frac{\pi}{2}\frac{V_{app}}{V_s} \quad \text{or} \quad H_u = \frac{2}{\pi} \quad \text{if} \quad a_0^k \geq \frac{\pi}{2}\frac{V_{app}}{V_s} \tag{6}$$

In here, $a_0^k = \frac{\omega B_e^A}{V_s}$ and B_e^A is a half of the length of a square foundation.

Later, the above transfer functions were modified by Ancheta *et al*. (2011) as follows, assuming V_s as 250 m/s and V_{app} as 2500 m/s ($V_s/V_{app} \cong 0.1$).

$$H_u = \frac{sin(\frac{a_0}{10})}{(\frac{a_0}{10})} \quad \text{if} \quad a_0 \leq 5\pi \quad \text{or} \quad H_u = \frac{2}{\pi} \quad \text{if} \quad a_0 \geq 5\pi \tag{7}$$

In here, a_0 was used instead of a_0^k and B_e^A was considered in the range of 15-40 m.

Mylonakis's equations are valid in the range of 200-500 m/s for V_s and 2000-3500 m/s for V_{app}. Mylonakis' transfer functions for the V_{app} of 2000, 2500, 3500 and infinite m/s are shown in Fig. 5.

For the study, a foundation-soil system shown in Fig. 1 was modeled assuming a 30 m soft soil

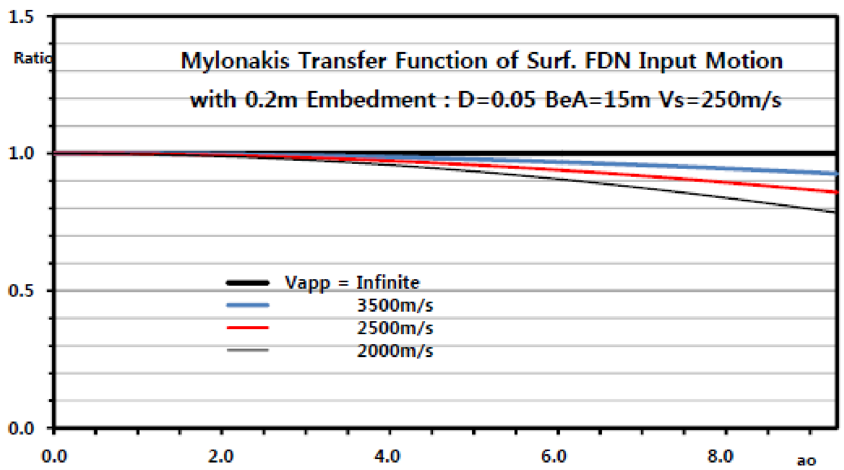


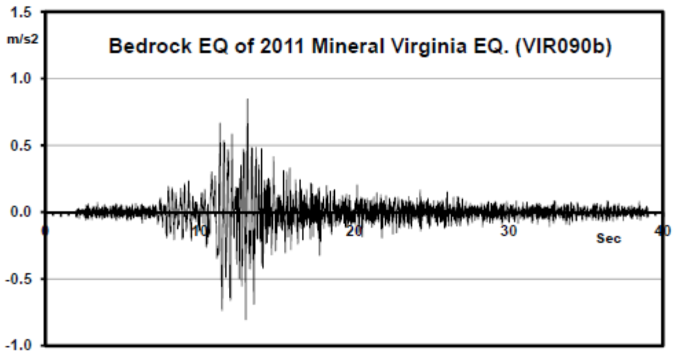
Fig. 5 Mylonakis' transfer functions for V_{app} of 2000, 2500, 3500 and Infinite m/s

layer on the bedrock, and a circular foundation with a radius of 16.926 m which is equivalent to a square foundation with dimensions of 30 m ($2{\cdot}B_e^A$). The radius of an equivalent circular foundation was calculated to have the same area for the swaying motion and the same moment of inertia for the rocking motion, but the difference in radius is negligible with an error of 1.2% for a square foundation. The foundation was assumed to be embedded a small depth of 0.2 m to simulate a practical surface foundation. The lateral boundary was placed at a distance of 25 m (2 times a wave travel distance with an apparent shear wave of 2500 m/s and a wave time step of 0.005 seconds) to simulate the phase shift of input bedrock motions.

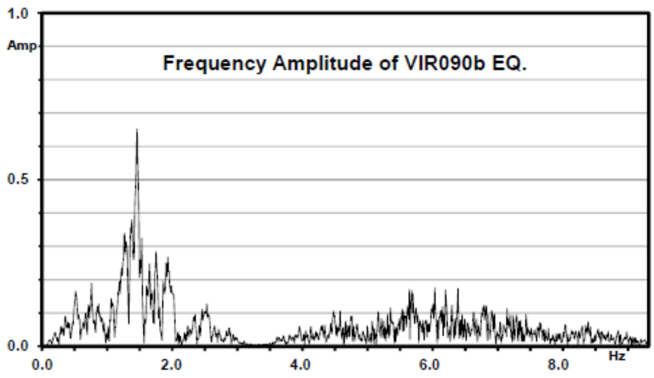
Soil characteristics were assumed as a shear wave velocity (Vs) of 250 m/s, a Poisson's ratio of 0.45, a material damping ratio of 5% and a unit weight of 18 kN/m^3.

The exciting input bedrock motions were applied block-wise with a distance interval of 12.5 m for two different patterns. One is a time-delayed incoming exciting motion traveling from the far field to the core region, and the other one is a time-delayed outgoing exciting motion traveling from the core region to the far field.

In this study, an earthquake time history recorded at the fire station #25 in Reston during the 2011 Mineral Virginia Earthquake (VIR090) and discretized in the interval of 0.005 seconds was utilized to consider the time delay even for the case of a small foundation. The original record downloaded from the USGS data base was amplified to 0.133 g (considering 0.2 g level earthquake) and de-convoluted for a bedrock earthquake record of VIR090b at the 30 m depth



(a) Time history of bedrock earthquake



(b) Fourier amplitude of bedrock earthquake

Fig. 6 Bedrock input earthquake

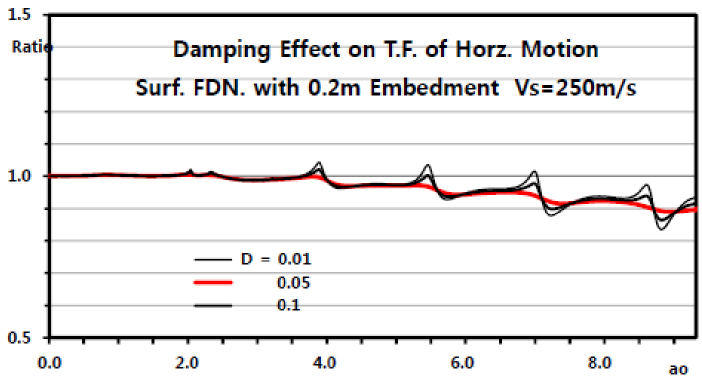
below the free field surface assuming an engineering bedrock having a shear wave velocity of 1050 m/s. Time history of a de-convoluted bedrock earthquake is shown in Fig. 6(a), and Fourier amplitude of a bedrock earthquake is shown in Fig. 6(b).

Following study results are plotted in terms of a_0 ($\omega \cdot B_e^A / Vs$) which is a dimensionless frequency depending on the shear wave velocity of a soil layer. In this study, a_0 up to 9.425, which corresponds to the frequency range up to 25 Hz with B_e^A of 15 m and soil shear wave velocity of 250 m/s, was considered.

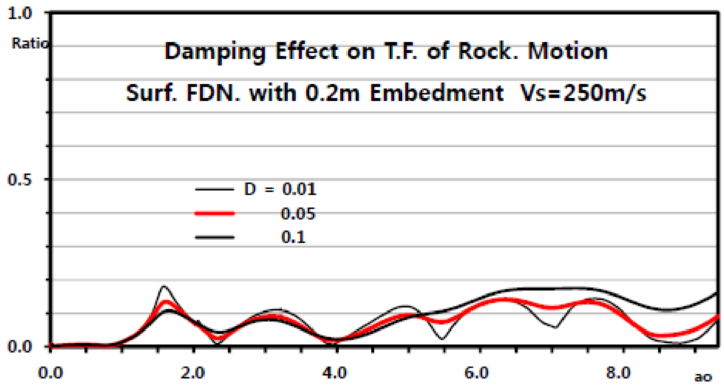
4. Effect of damping ratio on transfer functions of foundation motions

The effect of a damping ratio on the transfer functions of horizontal and rocking foundation motions was investigated with three different damping ratios of 0.01, 0.05 and 0.1 for the study.

Study results shown in Fig. 7(a)-(b) indicate that damping ratio does not change the frequency contents of the transfer functions very much, but the fluctuation of rocking transfer function is smoothed flattening the curves more in the high frequency range due to higher damping ratio. Therefore the effect of a damping ratio on the transfer functions of horizontal and rocking motions is negligible.



(a) Damping effect on horizontal transfer function



(b) Damping effect on rocking transfer function

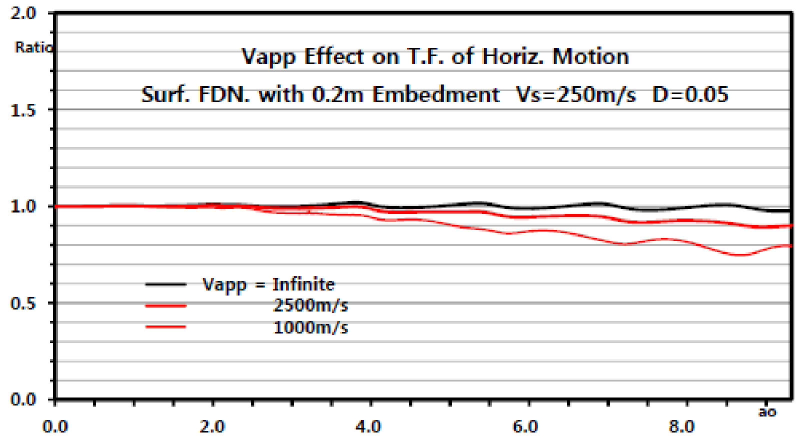
Fig. 7 Damping effect on transfer function

5. Effect of apparent shear wave velocity on transfer functions of foundation motions

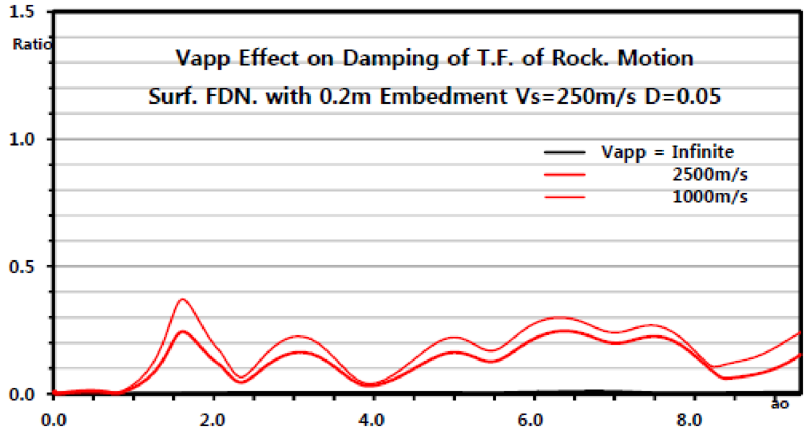
The effect of an apparent shear wave velocity on the transfer functions of horizontal and rocking foundation motions was investigated with the soil shear wave velocity of 250 m/s and the soil damping ratio of 0.05, assuming engineering bedrock beneath the soft soil layer for three different apparent shear wave velocities of 1000, 2500 and infinite m/s for the study.

Study results shown in Fig. 8(a)-(b) indicate that the apparent shear wave velocity does change the frequency contents of the transfer functions in the high frequency range. Slower apparent shear wave velocity reduces the horizontal transfer functions with some fluctuations above the mid-frequency range. However the rocking transfer function shows larger ratio with slower apparent shear wave velocity showing deep fluctuations in all frequency ranges. It also can be seen that negligible rocking foundation motions are produced with an apparent shear wave velocity of infinite.

The study result indicates that small apparent shear wave velocity of the bedrock beneath a soft soil layer may affect on the seismic response of a building structure.



(a) V_{app} effect on horizontal transfer function



(b) V_{app} effect on rocking transfer function

Fig. 8 V_{app} effect on transfer function

6. Results of wave passage analysis

Wave passage analysis to verify the reliability of P3DASS program was also performed comparing the analysis results of P3DASS with Mylonakis' study results.

In the case of no time lag due to the infinite apparent shear wave velocity, the transfer function (H_u=U_f /U_g) between foundation motion (U_f) and free surface motion (U_g) in Fig. 8(a) shows almost unity except some fluctuations in the high frequency range. However, the transfer function with an apparent shear wave velocity of 2500 m/s in Fig. 9 shows a gradual reduction of less than 15.1% in the high frequency range, which is similar to the Mylonakis' transfer function.

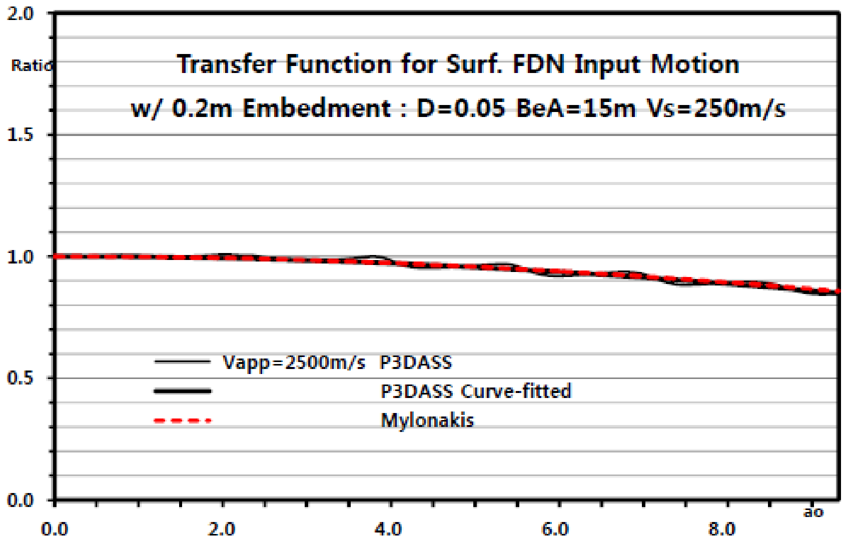


Fig. 9 Transfer Functions for V_{app}=2500 m/s

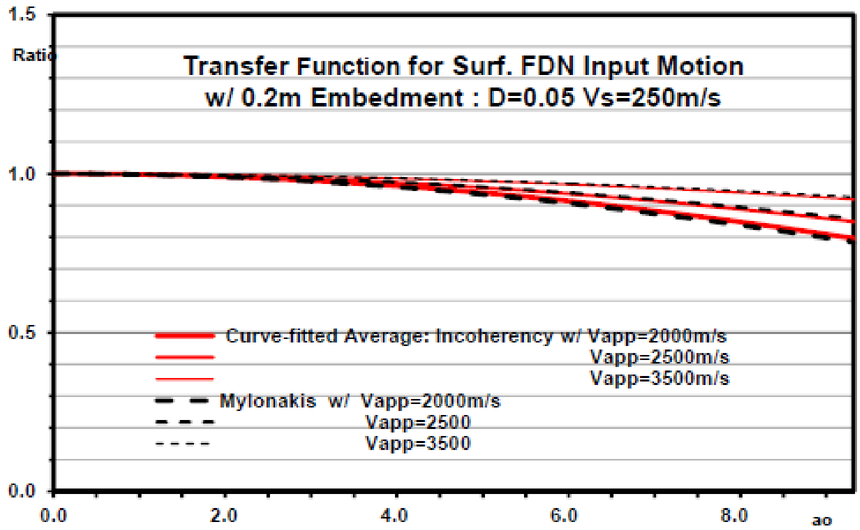


Fig. 10 Transfer Functions for V_{app}=2000, 2500 and 3500 m/s

The curve-fitted transfer function can be written using Mylonakis' expression as follows

$$H_u = \frac{sin(0.103a_0)}{0.103a_0} = \frac{sin\left(\frac{a_0}{9.71}\right)}{\left(\frac{a_0}{9.71}\right)} \quad (8)$$

And using Kim's expression (Roesset and Kim 1987), it can be also expressed as

$$TF(U) = cos(0.059a_0) = cos(\frac{a_0}{16.9}) \quad (9)$$

Also, additional transfer functions for different apparent shear wave velocities (V_{app}) of 2000 m/s and 3500 m/s are shown in Fig. 10 with the shear wave velocity (V_s) of 250 m/s.

The transfer functions by P3DASS analyses for the shear wave velocities of 2000 and 3500 m/s show very similar trend with the transfer function for a V_{app} of 2500 m/s, showing some reductions of less than 20.4% and 8.1% respectively in the high frequency range. And they are almost the same as the transfer functions of Mylonakis in all cases.

The lateral boundaries for the P3DASS analyses were placed at distances of 16.9, 18.75 and 26.25 m considering wave traveling distances with apparent shear waves of 2000, 2500 and 3500 m/s and a wave time step of 0.005 seconds to simulate time-delayed bedrock input motions. The lateral boundary distances were determined through the parametric studies, which gives the best fits to the Mylonakis' study results.

Curve-fitted transfer functions for V_{app} of 2000 and 3500 m/s with R of 16.926 m (B_e^A of 15 m) and H of 30 m can be expressed as follows using Kim's expression

$$TF(U) = cos(0.069a_0) = cos(\frac{a_0}{14.5}) \quad \text{for} \quad V_{app}\text{=2000 m/s} \quad (10)$$

$$TF(U) = cos(0.043a_0) = cos(\frac{a_0}{23.3}) \quad \text{for} \quad V_{app}\text{=3500 m/s} \quad (11)$$

7. Wave passage effect on response spectrum of a SDOF system

Effect of a wave passage on the response spectrum of a SDOF system shown in Fig. 11 was investigated by comparing the response spectra, which were found applying 0.08 g consistent and inconsistent bedrock input motions. Response analyses were performed in the case of an apparent

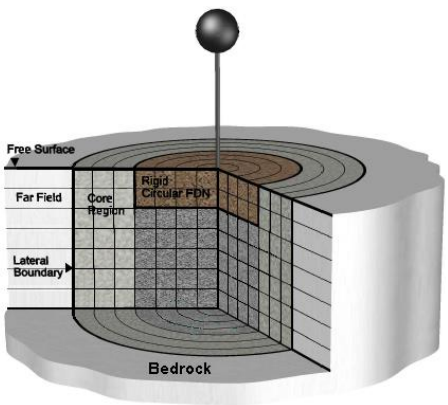


Fig. 11 Model of P3DASS

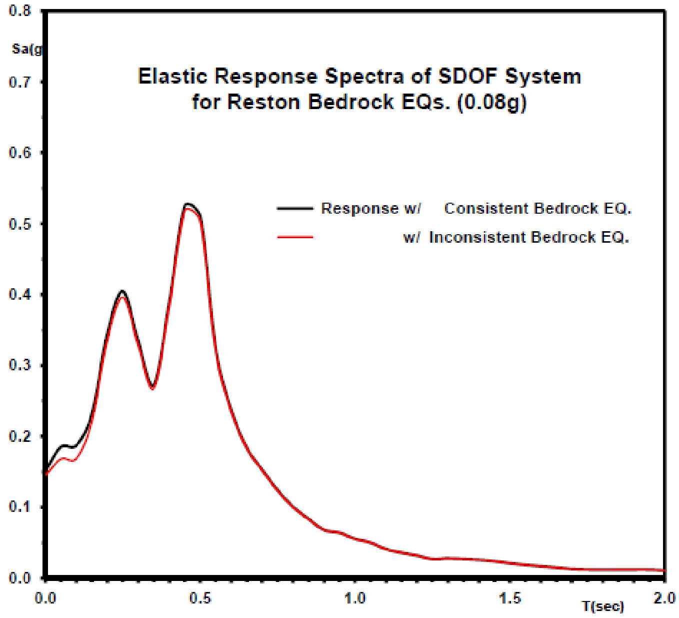


Fig. 12 Comparison of Responses with V_{app}=2000 m/s for Wave Passage Effect

Table 1 Acceleration responses for V_{app}=2000 m/s

T (sec)	**Acceleration Response (g)**		**Difference (%)**
	Consistent Input Motion	**Inconsistent Input Motion**	
0.00	0.152	0.145	-4.6
0.05	0.185	0.168	-9.2
0.10	0.188	0.169	-10.1
0.15	0.232	0.221	-4.8
0.20	0.346	0.336	-2.9
0.25	0.405	0.396	-2.3
0.30	0.334	0.327	-2.1
0.35	0.273	0.268	-1.9
0.40	0.390	0.384	-1.6
0.45	0.525	0.518	-1.4
0.50	0.510	0.503	-1.4

shear wave velocity of 2000 m/s. Shear wave velocity of a 30 m soil layer was assumed to be 250 m/s, and the radius of a solid surface foundation was assumed as 16.926 m equivalent to a square foundation having a dimension of 30 m with a lateral boundary placed at a distance of 20 m from the center.

Study results shown in Fig. 12 and Table 1 indicate that the elastic response spectrum with the inconsistent bedrock earthquake is reduced approximately 10% in the short period range (high frequency range) due to the wave passage effect on the input motion in the high frequency range.

8. Conclusions

In this research, wave passage effect on the response spectrum of a building structure built on a soft soil layer was investigated utilizing a finite element program of P3DASS (Pseudo 3-dimensional Dynamic Analysis of a Structure-soil System). Wave passage analysis was performed with the axisymmetric P3DASS program developed originally in the cylindrical coordinate and modified to apply anti-symmetric input earthquake motions to the bedrock beneath the soil site. Seismic input motions time-delayed each other were applied to the load points located along the bedrock surface.

Study results of P3DASS analyses were compared with the Mylonakis' study results, which are valid for the shear wave velocities of 200-500 m/s and the apparent shear wave velocities of the bedrock of 2000-3500 m/s, to verify the reliability of P3DASS program. Studied transfer functions of seismic responses between surface mat foundation and free ground surface were agreed well to experimental ones with a small difference in all frequency ranges, showing some reductions of the transfer function in the high frequency range.

Also wave passage effect on the elastic response spectrum of a SDOF system was investigated by comparing the response spectra with consistent and inconsistent bedrock input motions. Study results show some reduction (approximately 10% in the case of this study) of the elastic seismic response of a SDOF system in the short period range due to the wave passage.

Finally, it is concluded that a finite element program coded in the axisymmetric coordination system like P3DASS can be modified for the inconsistent seismic bedrock input motion to analyze the wave passage effect. Also, the modified program can be utilized for the analysis of the wave scattering problem if seismic bedrock input motions taking into account the wave scattering effect are prepared by a seismologist.

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