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Combinatorial continuous non-stationary critical excitation in M.D.O.F structures using multi-peak envelope functions

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Abstract. The main objective of critical excitation methods is to reveal the worst possible response of structures. This goal is accomplished by considering the uncertainties of ground motion, which is subjected to the appropriate constraints, such as earthquake power and intensity limit. The concentration of this current study is on the theoretical optimization aspect, as is the case with the majority of conventional critical excitation methods. However, these previous studies on critical excitation lead to a discontinuous power spectral density (PSD). This paper introduces some critical excitations which contain proper continuity in frequency domain. The main idea for generating such continuous excitations stems from the combination of two continuous functions. On the other hand, in order to provide a non-stationary model, this paper attempts to present an appropriate envelope function, which unlike the previous envelope functions, can properly cover the natural earthquakes' accelerograms based on multi-peak conditions. Finally, the proposed method is developed into the multiple-degree-of-freedom (M.D.O.F) structures.

Keywords: random vibrations; continuous critical excitation; envelope functions; power spectral density

1. Introduction

Based on the previous knowledge of earthquakes, natural earthquake excitations do not follow any known rule for either frequency or time. This explains why an earthquake phenomenon is known as a random and uncertain process. Therefore, future earthquakes are not perfectly predictable. However, in order to estimate the worst possible critical response of a structure, critical excitation methods have been developed. It is believed that considering some properties of earthquakes, such as power and intensity, will enable the maximum response of the structure to be estimated.

The first researcher who introduced the concept of critical excitation was Papoulis (1967). He used this concept in the field of electrical engineering. Then, Drenick (1970) applied the critical excitation method to structures in a given time period in the field of civil engineering. In Drenick's method, the most destructive excitation was found based on the maximum response of the system. In the same year, Shinozuka (1970) expanded this idea into the frequency domain and offered a

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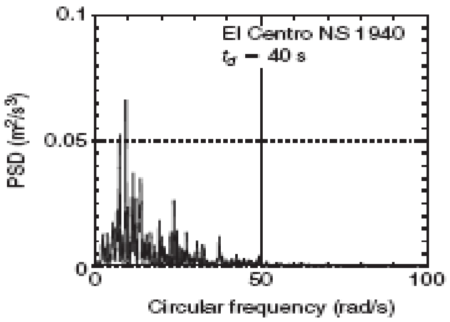


Fig. 1a El-Centro PSD function (Takewaki 2007)

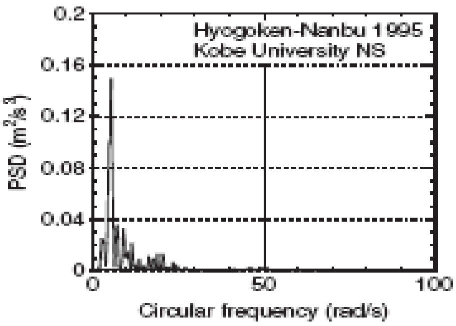


Fig. 1b Hyogoken-Nanbu PSD function (Takewaki 2007)

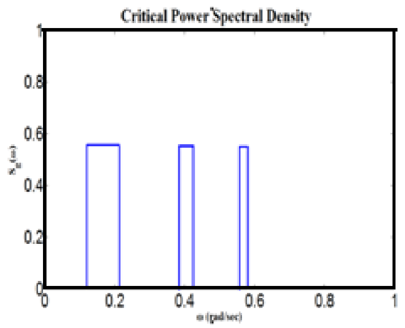


Fig. 1c PSD function of Takewaki's method

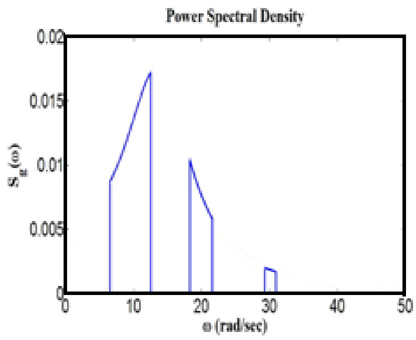


Fig. 1d PSD function of Ashtari's method

narrower upper bound for the maximum response.

In fact, the critical excitation method is an optimization problem to maximize structural response as an objective function subject to constraints. Until now, many people have used different constraints and objective functions. Iyengar (1972), Manohar and Sarkar (1995, 1998) and Takewaki (2001a, 2001b, 2002) deliberately extended the method to stochastic problems to consider the uncertainties of ground motions. Also, Ben-Haim and Elishakoff (1990), and Pantelides and Tzan (1996) presented several interesting convex models.

Furthermore, Takewaki (2007) extended the critical excitation approach when he published his achievements in the form of a comprehensive handbook, briefly, as the optimization methodology, he utilized power's constraint (the area under the power spectral density (PSD) function) and the intensity limit (the magnitude of the PSD function) to figure out critical exaction.

In the last decade, researchers put in noticeable efforts and collaborated in order to reveal the facts and benefits of the critical excitation method. For instance, Abbas M. and Manohar (2002) investigated critical earthquake load models within deterministic frameworks; Moustafa and Takewaki (2009) identified unfavorable earthquake records using the probabilistic and deterministic measures, Moustafa *et al*. (2010) introduced a critical excitation model and damage index for an inelastic structure. In order to enhance the seismic resilience of the structure, in 2012, Takewaki *et al*. proposed the worst case methodology.

In 2006, Ashtari tried to introduce the flexible critical excitation method as his doctorate dissertation. Then, Ghasemi continued his work to present the continuous critical excitation method (2010a, 2010b, 2013a, 2013b).

Fig. 1 shows the power spectral density of El-Centro earthquake (Fig. 1a [Takewaki (2007)]) and Kobe earthquake (Fig. 1b [Takewaki (2007)]). As it can be seen, the natural earthquake ground motion continuously covers its frequency domains. However, previously proposed critical excitations like Takewaki (Fig. 1c) and Ashtari (Fig. 1d) suffer discontinuities in their power spectral density; therefore, these artificial critical excitations are not likely probable.

This paper intends to introduce a new method to obtain continuous critical excitations with respect to the maximization of the mean square of the story drifts. Herein, the power spectral density (PSD) of the excitation is assumed to be a linear combination of two functions: one known as Kanai-Tajimi (1957, 1960) function and the other assumed as square of the dynamic-response function ($F(\omega)$).

The Kanai-Tajimi function is a spectral density function of the ground motion, and $F(\omega)$ depends on the dynamic characteristics of the structure. Both of these functions are continuous. Therefore, the combinatorial PSD of them leads to a continuous critical excitation. The optimization problem will be solved using the Lagrangian method.

Moreover, besides the continuity of the power spectral density, as an independent study in time domain, the authors intend to introduce some envelope functions that are capable of following the earthquakes' accelerograms at various times since the previous envelope functions are not accurate enough to describe severe earthquake movements, particularly at the final moments. The proposed envelope functions are trying to cover up this limitation in the time domain. Finally, the results are separately compared with the previous methods proposed by Ashtari (2006) and Takewaki (2007).

2. Non-Stationary critical excitation theory

According to the theory of random vibrations by Takewaki (2007), the sum of the mean square of story drifts is:

$$f(t) = \sum_{k=1}^{n} \sigma_{Dk}(t)^2 = \int_{-\infty}^{\infty} H_M(t;\omega) S_w(\omega) d\omega \tag{1}$$

where $S_w(\omega)$ denotes the PSD function of a stationary Gaussian process with zero mean ($w(t)$), and $H_M(t;\omega)$ can be expressed by:

$$H_M(t;\omega) = \sum_{k=1}^{n} \begin{bmatrix} \left\{\sum_{j=1}^{n} \Gamma_j \left(\varphi_k{}^{(j)} - \varphi_{k-1}{}^{(j)}\right) A_{Cj}(t;\omega)\right\}^2 + \\ \left\{\sum_{j=1}^{n} \Gamma_j \left(\varphi_k{}^{(j)} - \varphi_{k-1}{}^{(j)}\right) A_{Sj}(t;\omega)\right\}^2 \end{bmatrix} \tag{2}$$

where Γ_j is the j^{th} participation factor, m_i is the mass of the j^{th} story, $\varphi_k{}^{(j)}$ is the kth component in the j^{th} eigenvector$\varphi^{(j)}$, and A_{Cj}, and A_{Sj} are respectively equal to:

$$\Gamma_j = \frac{\sum_{i=1}^{n} m_i \varphi_i{}^{(j)}}{\sum_{i=1}^{n} m_i \varphi_i{}^{(j)2}} \tag{3}$$

$$A_{Cj}(t;\omega) = \int_0^t e(\tau) g_j(t-\tau) cos\omega\tau d\tau \tag{4}$$

$$A_{Sj}(t;\omega) = \int_0^t e(\tau) g_j(t-\tau) sin\omega\tau d\tau \tag{5}$$

which $e(t)$ is given deterministic envelope function and $g_j(t) = H_e(t)\left({}^{1}/_{\omega_{dj}}\right) e^{-\xi\omega_j t} \sin(\omega_{dj} t)$ (Eq. (4) and Eq. (5), $H_e(t)$ is the Heaviside step function, which is $\omega_{dj} = \sqrt{1-\xi^2}\omega_j$ and $\omega_j$is the natural frequency of the j^{th} mode)

2.1 Critical excitation optimization problem

Regarding the previous section, the problem of non-stationary critical excitation consists of double maximization procedures that have been described mathematically by Takewaki (2007):

$$Max_{S_w(\omega)}\, Max_t\{f(t; S_w(\omega))\} \tag{6}$$

subjected to

$$\int_{-\infty}^{\infty} S_w(\omega)d\omega \le \bar{S}_w \ (\bar{S}_w: given\ power\ limit) \tag{7}$$

and,

$$\sup \mathrm{S_w}(\omega) \le \bar{\mathrm{s}}_\mathrm{w} \quad (\bar{s}_w: given\ PSD\ amplitude\ limit) \tag{8}$$

The first maximization of Eq. (6) is performed with respect to time, and the second maximization is given with respect to the spectral density function.

2.2 Continuous critical excitation

With respect to the previously proposed, discontinuous critical excitations, it is clear that the obtained critical excitations are not able to fully cover the frequency range of an earthquake, and they only have been concentrated on the main frequency of the structures. Herein, in addition to some necessary constraints (power and intensity), shape similarity between critical excitations function and recorded real PSD of an earthquake is being considered. One of the important aspects of the shape similarity is the continuity and discontinuity behavior.

The reason behind stressing the shape similarity, which at least can be mitigated to the continuity, refers to the randomness of the ground motions. It is believed that critical excitation is a random phenomenon. That randomness is concluded from observed sets of ground motion. However, regarding the stochastic sampling, the random process should represent the same trend with the same constraints. Therefore, the shape similarity is the most controversial characteristic of the random phenomenon.

Based on the formulation of the random vibrations in Clough and Penzien (1975), the spectral density function of random phenomenon is Fourier’s transform of the autocorrelation function of the random process. Hence, as long as the earthquake’s record is continuous alongside the time domain, it is expected that Fourier’s transform works as a continuous process in the frequency domain. From the stochastic point of view, the continuity of the records can represent one of the significant parameters of the sampling.

It is clear that if the PSD function is the Dirac’s delta function at the peak of $H_M(t;\omega)$ without any amplitude limit, the structural response will be maximized. However, because of the limitation of an amplitude the excitation generates the rectangle(s) as high as $\bar{s}_w$, and the frequency range $\Delta\omega$ is determined with respect to the limitation of the power.

To solve this problem and achieve a continuous spectral density of the excitation, a linear combination of $F(\omega)$ and the Kanai-Tajimi function ($S_{K.T.}(\omega)$) is considered to generate a continuous critical excitation, where, α and β are constant coefficients, which are being determined with respect to the power and intensity constraints of the earthquakes

$$S_g(\omega) = \propto F(\omega) + \beta S_{K.T.}(\omega) \tag{9}$$

where $F(\omega) = |H(\omega)|^2$ is a dynamic system characteristic, which simply can be represented in forms of the structural dynamic properties $H(\omega) = 1/\{M[({\omega_n}^2 - \omega^2) + 2i\xi\omega_n\omega]\}$ which were used for the same intent by Ghasemi *et al*. (2013b), and $S_{K.T.}(\omega)$ represents the power spectral density of the ground motion of the previous natural earthquakes:

$$S_{K.T.}(\omega) = S_0 \frac{{\omega_g}^4 + 4{\xi_g}^2{\omega_g}^2\omega^2}{\left(\omega^2 - {\omega_g}^2\right)^2 + 4{\xi_g}^2{\omega_g}^2\omega^2} \tag{10}$$

where ω_g is the fundamental frequency of the ground's motion, ξ_g is the ground damping, which represents the sharpness of the power spectral density function $S_{K.T.}(\omega)$, and S_0 is a white noise constant parameter. Considering β as a comprehensive coefficient, therefore, it is not necessary to precisely obtain S_0.

At $\omega=0$ the Kanai-Tajimi function goes to S_0. Therefore, Eq. (11) must be filtered to achieve $S = 0$ at $\omega = 0$ to get similar results to the natural earthquake. There are several proposed filters to cope with this difficulty, one of them is Lai's filter (1982).

$$S_{K.T-L}(\omega) = S_0 \frac{{\omega_g}^4 + 4{\xi_g}^2{\omega_g}^2\omega^2}{\left(\omega^2 - {\omega_g}^2\right)^2 + 4{\xi_g}^2{\omega_g}^2\omega^2} \left(\frac{\omega^2}{\omega^2 + {\omega_c}^2}\right) \tag{11}$$

where ω_c is a variable that determines the low-frequency content of ground motion.

As it was mentioned earlier, in order to determine the constant coefficients (α and β) in Eq. (9), it is necessary to consider the power and intensity limits. Using the Lagrangian optimization methods, α and β can be extracted from Eq. (12).

$$\begin{cases} \bar{S} = \int_{-\infty}^{+\infty} \propto F(\omega)d\omega + \int_{-\infty}^{+\infty} \beta(S_{K.T.}(\omega))d\omega \\ \bar{s} = Max[\propto F(\omega) + \beta S_{K.T.}(\omega)] \end{cases} \tag{12}$$

It should be noted that in the second equation of Eq. (12), the methodology of finding Eq. (12) will be represented in the next section. .

It is clear that if the ω_g of ground motion is closed to ω_n , a stronger critical excitation will be generated; however, it is rare for ω_g to be equal to ω_n. Therefore, this excitation represents rare event, which is here called the rare continuous critical excitation. Thus, substituting $\omega_g = \omega_n$ in the Kanai-Tajimi equation, Eq. (11) becomes Eq. (13) to describe the rare continuous critical excitation. Eq. (12) would be a solution for assumed critical excitation in form Eq. (9), which this solution just presented as mathematical interpretation without any mathematically optimization effort, therefore we called Eq. (12) a simplified mathematical solution.

$$S_{K.T-L}^{Rare}(\omega) = S_0 \frac{{\omega_n}^4 + 4{\xi_g}^2{\omega_n}^2\omega^2}{(\omega^2 - {\omega_n}^2)^2 + 4{\xi_g}^2{\omega_n}^2\omega^2} \left(\frac{\omega^2}{\omega^2 + {\omega_c}^2}\right) \tag{13}$$

$S_{K.T-L}^{Rare}(\omega)$ is the PSD of the rare critical continuous excitation using Lai's filtering. In the proposed method, the fluctuations of earthquake excitation in frequency domain can be simulated by multiplying the absolute value of the sine function by the second part of Eq. (9):

$$S_g(\omega) = \propto F(\omega) + \beta(S_{K.T-L}^n(\omega))|\sin(\omega)| \tag{14}$$

This sine function generates an artificial critical earthquake, which is more similar to critical earthquake's excitation.

2.3 Solving continuous critical excitation problem using the lagrangian methodOLVING

In this section, using the Lagrangian's optimization method, continuous critical excitation problem is solved, and the results are compared with the outcome of Eq. (12). The given conditions of this problem for multi-degree-of-freedom (MDOF) structures are showing in Eq. (15-17) which has been presented by Takewaki (2007):

$$Max:\ f = \sum_{i=1}^{n} {\delta_{Di}}^2 = \int_{-\infty}^{\infty} F(\omega) S_g(\omega) d\omega \tag{15}$$

subjected to

$$\int_{-\infty}^{\infty} S_g(\omega) d\omega \le \bar{S} \tag{16}$$

$$S_g(\omega) \le \bar{s} \tag{17}$$

Now, it is possible to apply the Lagrangian's derivation to find out the coefficient of α and β. In order to use Lagrangian's method choosing the following functions as a basic function can be helpful:

$$f = \sum_{i=1}^{n} {\delta_{Di}}^2 = \int_{-\infty}^{\infty} F(\omega) S_g(\omega) d\omega \tag{18}$$

$$g = \int_{-\infty}^{\infty} S_g(\omega) d\omega - \bar{S} \tag{19}$$

$$h = S_g(\omega) - \bar{s} \tag{20}$$

Firstly, by substituting Eq. (11) into Eq. (9), then plugging that result into Eqs. (18-20), the following equations can be achieved.

$$f = \sum_{i=1}^{n} {\sigma_{Di}}^2 = \int_{-\infty}^{\infty} F(\omega)(S_{K.T-L}^{n}(\omega))\, d\omega \tag{21}$$

$$g = \int_{-\infty}^{\infty} \{\propto F(\omega) + \beta\left(S_{K.T-L}^{n}(\omega)\right)\} d\omega - \bar{S} \tag{22}$$

$$h = \{\propto F(\omega) + \beta\left(S_{K.T-L}^{n}(\omega)\right)\}_{Max} - \bar{s} \tag{23}$$

Eq. (21-23) can be reduced to the following forms.

$$f = \propto \mathrm{c}_1 + \beta \mathrm{c}_2 \tag{24}$$

$$g = \propto \mathrm{c}_3 + \beta \mathrm{c}_4 - \bar{S} \tag{25}$$

$$h = \propto \mathrm{c}_5 + \beta \mathrm{c}_6 - \bar{s} \tag{26}$$

where c_i constitutes constant values. Using the Lagrangian derivation lead to

$$\begin{cases} \frac{\partial f}{\partial \alpha} - (\lambda \frac{\partial g}{\partial \alpha} + \gamma \frac{\partial h}{\partial \alpha}) = 0 \\ \frac{\partial f}{\partial \beta} - (\lambda \frac{\partial g}{\partial \beta} + \gamma \frac{\partial h}{\partial \beta}) = 0 \end{cases} \tag{27}$$

As it can be seen the derivations of the functions of f, g, and h respecting the α and β are going to be : $\frac{\partial f}{\partial \alpha} = \mathrm{c}_1$, $\frac{\partial f}{\partial \beta} = \mathrm{c}_2$, $\frac{\partial g}{\partial \alpha} = \mathrm{c}_3$, $\frac{\partial g}{\partial \beta} = \mathrm{c}_4$, $\frac{\partial h}{\partial \alpha} = \mathrm{c}_{35}$, and $\frac{\partial h}{\partial \beta} = \mathrm{c}_6$. Then by plugging c_i's into the Eq. (27) we will have:

$$\begin{cases} c_1 - c_3\lambda - c_5\gamma = 0 \\ c_2 - c_4\lambda - c_6\gamma = 0 \end{cases} \tag{28}$$

By solving the two-unknown, two-equation system (Eq. 28), λ and γ can be found. These two equations are independent of α and β. Therefore, α and β are only obtained based on the power and intensity constraints.

$$\begin{cases} \bar{S} = \int_{-\infty}^{+\infty} \propto F(\omega)d\omega + \int_{-\infty}^{+\infty} \beta S_{K.T-L}^{n}(\omega)d\omega \\ \bar{s} = \{\propto F(\omega) + \beta(S_{K.T-L}^{n}(\omega))\}_{Max} \end{cases} \tag{29}$$

Eq. (29) shows that the Lagrangian's optimization problem is independent of Lagrangian's coefficients. Therefore, it means we still can use Eq. (12) to find continuous critical excitation.

3. Envelop function study

This section is devoted to propose some new envelope functions. Regardless of the previous section, this section can be considered as an independent effort to represent the non-stationary behavior of the earthquake in the time domain. However, the authors also desired to compare the modification of the envelope function and comprehend all differences in one article.

Simplification of envelope function leads to the more efficient simulation of the earthquake's motion. In this section new envelope functions are going to be presented, which they have an ability to cover more than one significant peak of the earthquake's accelerogram. Indeed, the main goal of introducing these envelope functions is to propose a new multi-peak envelope function in time domain.

The most popular used envelope function is Bolotin's envelope function (1960). Apart from all convenient aspects, Bolotin's function could not cover multi peaks of the accelerogram at a different time. For instance, imagine there is a peak at the beginning of the ground motion and there is another significant peak at the very final moments. In order to resolve this issue, a new envelope function is presented, and this new envelope function is established based on a single harmonic wave. Applying a sine wave helps to cover the rest of the possible severe excitations at the end of the motion.

$$e(t) = \begin{cases} (e_o.e^{-at} - e_1.e^{-bt}), & 0 \le t < t_o \\ e_2.\sin\left(\frac{2\pi t}{c.T}\right) & , \ t_o \le t < T \end{cases} \tag{30}$$

where T is the total duration of a natural earthquake, t_o is the desired time depending on the natural earthquake, which it can be achieved by trial and error efforts to find the best adjustment. e_o and e_1 and e_2 are constant coefficients representing the shape of the envelope function, and *a* and *b* and *c* are constant values, which show the position of the peaks in an earthquake excitation.

The most important properties of the new proposed envelope function is the ability to model the final movements of the accelerogram's peaks. Therefore, we called it a multi-peak envelope function, which it can even be considered as a more exact function in comparison with Bolotin's function.

This envelope function is sufficiently flexible to perfectly cover peaks. In addition, to simulate the fluctuation of the non-stationary behavior, here the other types of the envelope function can be proposed. It is worth mentioning the inspiration for this envelope function stems from the similarity between the heart beats in cardiograph and functional behavior of the beat function.

Therefore, the other proposed envelope functions can be demonstrated based on the beat function.

$$e(t) = \left| e_o . \sin\left(\left(\frac{2\pi}{b.T} + c\right).t\right) + e_1 . \sin\left(\frac{2\pi.t}{b.T}\right) + d \right| \tag{31}$$

One of the fantastic aspects of the beat function is the ability to model several peaks of the earthquake's accelerogram. However, the amplitude of the beat function is constant during in test sampling. Therefore, to solve this problem, the partition function is recommended. Regarding the number of considerable peaks the constant parameters of e_o and e_1, d ,c will be modified.

4. Comparison of the results of the combinatorial continuous critical excitation with precious discontinuous critical excitation

This section was designed to manifest the comparison of the non-stationary continuous critical excitation and discontinuous critical excitations, which can be found in the previous studies such as Takewaki (2007) and Ashtari (2006). Also, the effects of different envelope functions are studied to figure out the envelope functions' influence on continuous critical excitation.

For this purpose, a two-degree-of-freedom structure with the following mass, stiffness and damping ratio equal to 0.02 was subjected to the El-Centro earthquake.

$$k_1 = 1.2 \times 10^6 \; KN, \qquad k_2 = 6 \times 10^5 \; KN.$$
$$m_1 = 6 \times 10^3 \; Ton, \qquad m_2 = 3 \times 10^3 \; Ton.$$

Fig. 2 shows the real accelerogram of the El-Centro earthquake whose peak ground acceleration (PGA) was scaled to $0.3g$.

The Bolotin's envelope function for El-Centro is:

$$e(t) = e^{-0.14t} - e^{-0.33t}$$

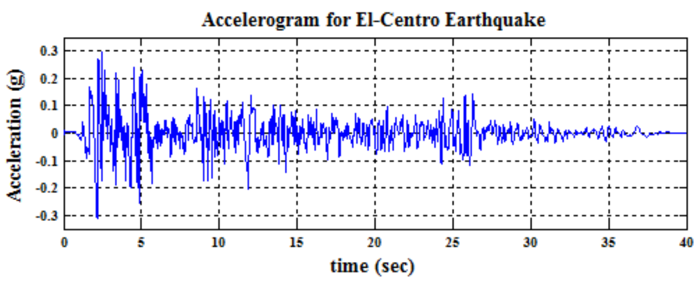


Fig. 2 Scaled accelerogram of El-Centro earthquake

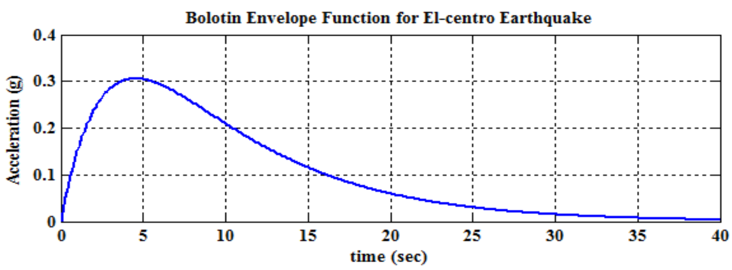


Fig. 3 Bolotin's envelope function

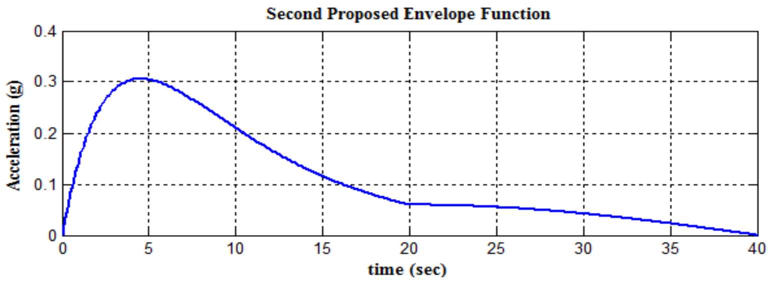


Fig. 4 the proposed envelope function using Eq. (30)

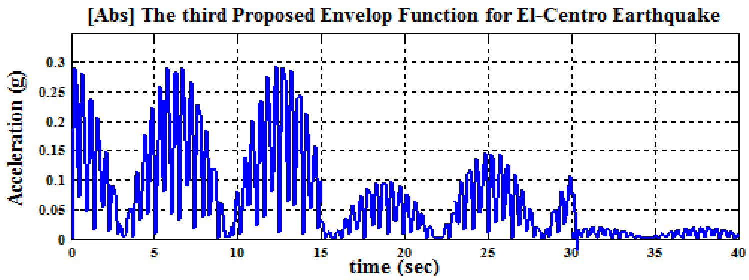
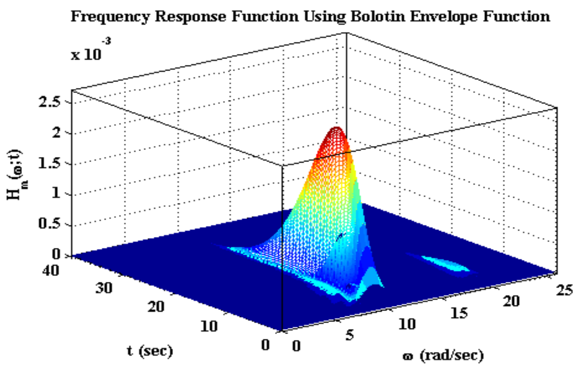
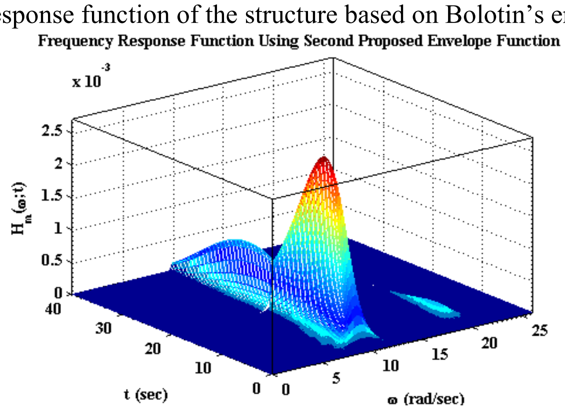


Fig. 5 absolute value of envelope function of Eq. (31)

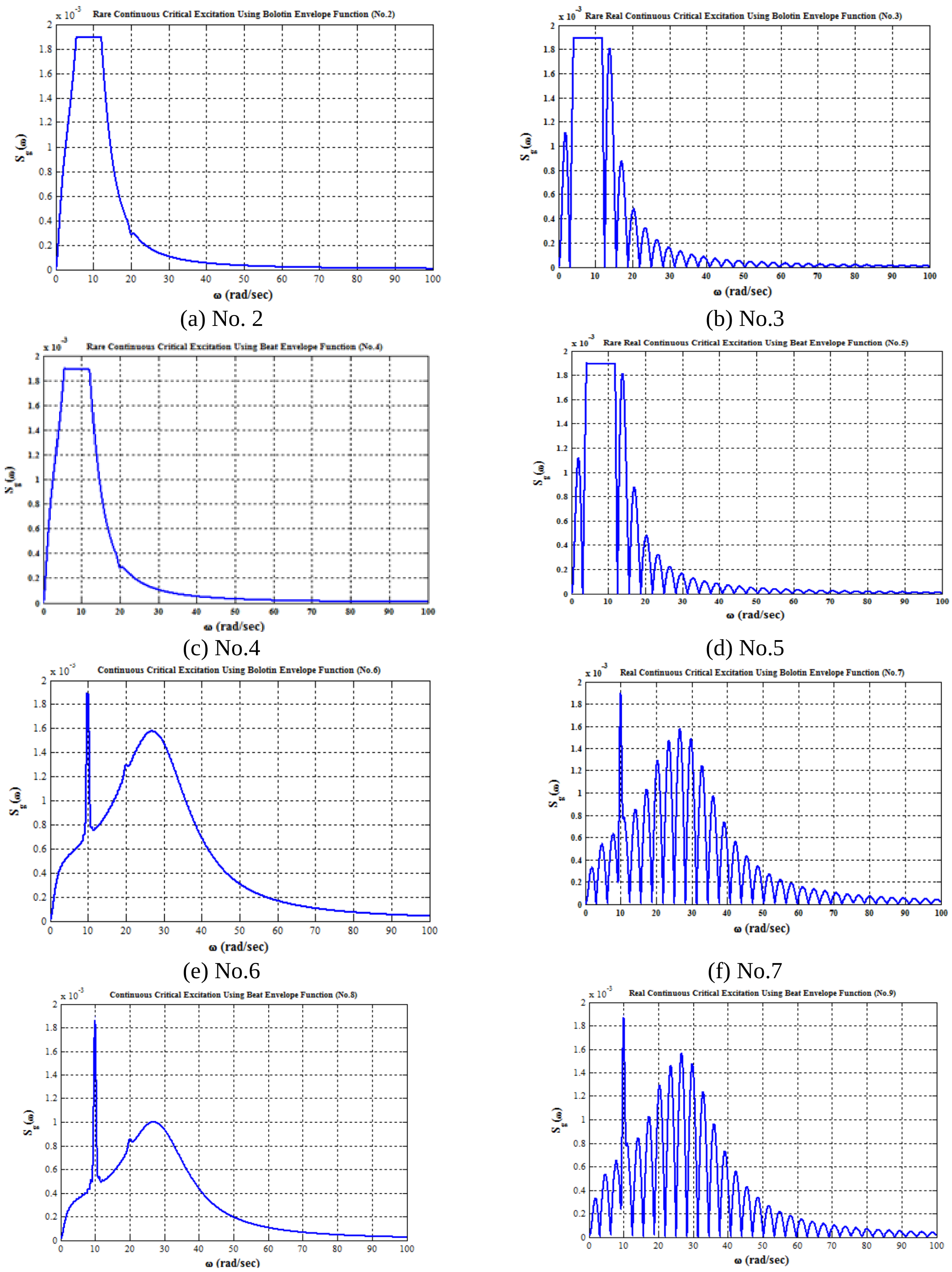


(a) Frequency response function of the structure based on Bolotin's envelope function



(b) Frequency response function based on the Eq. (30)

Fig. 6 Frequency response function using Bolotin's envelope function and the proposed envelope functions



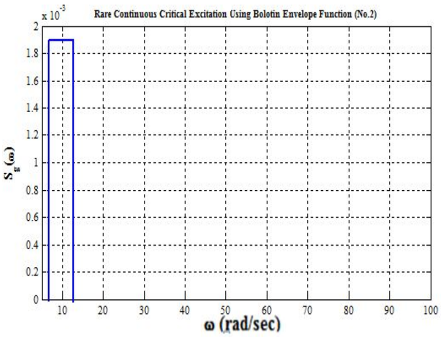
(a) No. 2 (b) No.3

(c) No.4 (d) No.5

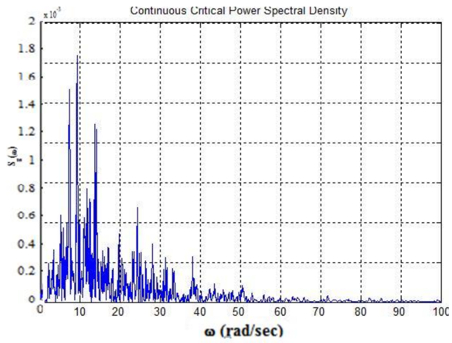
(e) No.6 (f) No.7

(g) No.8 (h) No.9

Fig. 7 Critical power spectral density of different proposed methods (see Table 1)



(i) No.10



(j) Natural PSD of El-Centro

Fig. 7 Coninued

As it can be seen in Figs. (2) and (3), Bolotin's function could not cover the final considerable peaks after $t = 20$ seconds. On the other hand, it is notable that this function sometimes is more conservative especially during the initial motions. Therefore, Fig. (4) shows the first proposed envelope function based on Eq. (30).

$$\mathrm{e(t)} = \begin{cases} (\mathrm{e}^{-0.14\mathrm{t}} - \mathrm{e}^{-0.33\mathrm{t}}) & 0 \leq \mathrm{t} < 20 \\ 0.0594 \sin\left(\frac{2\pi.\mathrm{t}}{80}\right) & 20 \leq \mathrm{t} < 40 \end{cases}$$

where e_2 was achieved at $t = 20$ second.

$$a \sin\left(\frac{2\pi * 20}{80}\right) = (e^{-0.14*20} - e^{-0.33*20}) \quad \Rightarrow \quad a = 0.0594$$

The other proposed envelope function can be obtained as follows (see Fig. 5).

$$f(t) = \begin{cases} e(t) = 0.15\left(\sin\left(\left(\frac{2\pi}{1} + 1\right).t\right) + \sin\left(\frac{2\pi.t}{1}\right)\right), & 0 < t < 15 \\ e(t) = 0.05\left(\sin\left(\left(\frac{2\pi}{1} + 1\right).t\right) + \sin\left(\frac{2\pi.t}{1}\right)\right), & 15 < t < 23 \\ e(t) = 0.075\left(\sin\left(\left(\frac{2\pi}{1} + 1\right).t\right) + \sin\left(\frac{2\pi.t}{1}\right)\right), & 23 < t < 30 \\ e(t) = 0.01\left(\sin\left(\left(\frac{2\pi}{1} + 1\right).t\right) + \sin\left(\frac{2\pi.t}{1}\right)\right), & 30 < t < 40 \end{cases}$$

Fig. (6) shows the frequency response function of the structures based on the different type of envelope functions.

Fig. (6): Frequency response function using Bolotin's envelope function and the proposed envelope functions

Based on the proposed envelope functions, now, it could be possible to present the most exact estimation of non-stationary behavior. As it can be seen, Fig. (6a) is the frequency response of the structure based on Bolotin's envelope function. This envelope function covers the considerable peaks of the ground motion at the end of ground movements, and it decently covers two main peaks. Therefore, it could be considered as a most flexible and coverable envelope function for non-stationary modeling.

Table 1 Comparison of responses of different critical excitation methods

Case	Method	$S(\omega)$	Envelope function	t max (sec)	Response (cm)
No. 1	Ashtari	Ashtari's Method	Bolotin	8	9.23
No. 2	Rare continuous critical excitation	$\propto F(\omega)+\beta S_{K.T.}(\omega)$	Bolotin	9.5	10.76
No. 3	Rare real continuous critical excitation	$\propto F(\omega)+\beta S_{K.T.}(\omega)\lvert\sin(\omega)\rvert$	Bolotin	9.5	7.85
No. 4	Rare continuous critical excitation	$\propto F(\omega)+\beta S_{K.T.}(\omega)$	Beat: sum(sin)	7.5	9.31
No. 5	Rare real continuous critical excitation	$\propto F(\omega)+\beta S_{K.T.}(\omega)\lvert\sin(\omega)\rvert$	Beat: sum(sin)	7.5	6.82
No. 6	Continuous critical excitation	$\propto F(\omega)+\beta S_{K.T.}(\omega)$	Bolotin	9.5	6.65
No. 7	Real continuous critical excitation	$\propto F(\omega)+\beta S_{K.T.}(\omega)\lvert\sin(\omega)\rvert$	Bolotin	9.5	4.99
No. 8	Continuous critical excitation	$\propto F(\omega)+\beta S_{K.T.}(\omega)$	Beat: sum(sin)	7.5	4.93
No. 9	Real continuous critical excitation	$\propto F(\omega)+\beta S_{K.T.}(\omega)\lvert\sin(\omega)\rvert$	Beat: sum(sin)	7.5	3.74
No. 10	Takewaki	Takewaki's Method	Bolotin	8	10.89

Table (1) compares the results of conventional methods with the obtained results by presenting combinatorial continuous non-stationary critical excitation using the proposed envelope functions.

As it is shown, the results of the rare continuous critical excitation methods are almost close to Takewaki's method (2002, 2007). However, other proposed methods have a thought-provoking advantage for presenting more possible PSD functions. For instance, when it is utilized $S_g(\omega) = \alpha F(\omega) + \beta S_{K.T.}(\omega)\lvert\sin(\omega)\rvert$ as critical PSD, then it may generate some fluctuations on the PSD in frequency domain, but for attaining both critical and more possible PSD it is better to use Real Rear Continuous Critical Excitation.

Fig. 6 shows PSD of different proposed critical excitations. As shown in Fig. 6, all of critical power spectral density functions are continuous.

As Fig. 7 shows, regarding the comparison among all methods (see Table 1), the rare continuous critical excitation coerces extra congestion near natural frequency of the structure, which concludes a dramatic increase of the responses. However, the most significant advantage of the proposed method "combinatorial continuous critical excitation" is that all of the generated PSDs, unlike previous critical excitations, are fully continuous in their frequency domain. At first glance, the discrete PSD does not account for a significant issue especially in the scope of critical responses. Nonetheless, it is a clear axiom that the possibility of occurrence of an earthquake with discontinuous spectral density is equal to zero, because there is no recorded earthquake with a discontinuous spectral density function. Therefore, the absence of continuity of PSD of the ground motion is felt.

Another outcome of Fig. 7 can be the tangible fluctuations in PSD, which is related to the behavior of the attendance of harmonic function in proposed continuous critical excitation, (terms sin(x)) $S_g(\omega) = \propto F(\omega) + \beta S_{K.T.}(\omega)|\sin(\omega)|$, thus the shape of PSD tends to the actual PSD.

And the final fact which may be concealed in Fig. 7, is that multi-peak envelope functions have little reduction effects on the final response of the structure. The reason behind this reduction was predictable, because Bolotin's envelope function is doing an overestimation of the earthquake's motion in the time domain. Therefore, the maximum response in this example tends to the maximum acceleration of El-Centro, for instance the maximum response occurs at 9.5 sec for Bolotin's envelope function, the maximum response for the second proposed envelope function happens at 7.5 sec and the maximum acceleration of El-Centro happened at 9.5 sec (see Table 1).

5. Conclusions

The main objective of this paper was to provide an overview of a new generation of critical excitation methods. The basic controversial aspect of the previous critical excitation method was the discontinuity of their power spectral density functions in the frequency domain. Therefore, in order to attain a critical excitation with a continuous frequency, the continuous critical excitation method was created. This study applied a combinatorial continuous non-stationary critical excitation method for multi-degree-of-freedom structures. This method was established based on a linear combination of the characteristic function ($F(\omega)$) and the Kanai-Tajimi spectral density. Using merely two constraints of the earthquake (intensity and power), it is possible to calculate the maximum response of the structure. The main reason behind generating continuous critical excitations stems from the probability of occurrence of a critical earthquake.

To investigate the reliability of the result, several types of continuous critical excitations were considered. Finally, the rare continuous critical excitation method can be suggested as the most practical solution because it provides the worst possible response. The results of applying the rare continuous critical excitation in this study are almost the same as Takewaki method (2007).

As an independent study, in terms of the non-stationarity, this paper proposed new envelope functions which could simulate severe motions of any earthquake. They may be particularly useful for the earthquakes with significant accelerations at the end of the ground motions. In other words, it helps us to cover up the several peaks of the earthquake's accelerogram at any time of the ground motion.

Eventually, it was observed that the responses of the continuous critical excitation are not quite as critical as the regular critical excitation; however, they are more plausible, and in shapes of power spectral density function, they are continuous in their frequency domain.

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