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Unified equivalent frame method for flat plate slab structures under combined gravity and lateral loads – Part 1: derivation

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Abstract. The equivalent frame method (EFM) is widely used for the design of two-way reinforced concrete slab structures, and current design codes of practice permit the application of the EFM in analyzing the flat plate slab structures under gravity and lateral loads. The EFM was, however, originally developed for the flat plate structures subjected to gravity load, which is not suitable for lateral loading case. Therefore, this study, the first part of series research paper, proposed the structural analysis method for the flat plate slab structures under the combined gravity and lateral loads, which is named as the unified equivalent frame method (UEFM). In the proposed method, some portion of rotation induced in the torsional member is distributed to the flexibility of the equivalent columns, and the remaining portion is contributed to that of the equivalent slabs. In the consecutive companion paper, the proposed UEFM is verified by comparing with test results of multi-span flat plate structures. Also, a simplified nonlinear push-over analysis method is proposed, and verified by comparing to test results.

Keywords: flat plate; slab; lateral load; gravity load; combined load; equivalent frame method; torsion

1. Introduction

A flat plate system is a gravity force resisting system (GFRS), in which slabs are directly supported on columns without beams. Thus, compared to conventional moment resisting frame structures, it allows considerable reduction in story heights and great flexibility in plan design, and its casting work is simple, offering superior constructability. (Hwang and Moehle 1993, 2000, Kim and Lee 2005, Lee *et al*. 2009, Park *et al*. 2012, Lee *et al*. 2013) The current design standards

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(ACI Committee 318 2011, ASCE 2010, KCI-M-12) permit the flat plate system to be used as a lateral force resisting system (LFRS) in low to moderate seismic risk regions, and in the design practices, the flat slab system have been generally used with other primary LFRSs, such as the shear walls or conventional moment resisting frames. (Robertson 1997, Park *et al*. 2012) Therefore, the flat plate system subjected to lateral forces, such as wind or seismic loads, should be capable of accommodating the lateral deformation induced in the primary LFRS, and adequate considerations on brittle punching shear failure caused by gravity shear and unbalanced moment should also be made during the design process. (Schwaighofer and Collins 1977, Robertson 1990, Luo and Durrani 1995a,b, Hwang and Moehle 2000, Han *et al*. 2009, Park *et al*. 2009, Park *et al*. 2012, Peiris and Ghlai 2012, Xing *et al.* 2013)

The analysis methods presented in ACI318-11 (2011) and KCI-M-12 (2012), however, cannot adequately estimate the lateral response and deformational capacity of the flat plate slab system. Thus, many existing studies (Vanderbilt and Corely 1983, Cano and Klingner 1987, Hwang and Moehle 2000, Murray *et al*. 2003, Park 2009) pointed it out, and they have made efforts to resolve such issues. The analysis methods for the flat plate system introduced in ACI318-11 (2011) and KCI-M-12 (2012) are the direct design method (DDM), equivalent frame method (EFM). Since these models originally aimed to determine the design flexural moment under gravity loads, they can provide sufficient analysis accuracy using relative stiffness between slabs and columns at connections, rather than the absolute stiffness of slabs and columns. Under lateral loads, however, the absolute stiffness of the slab-column frame is even more important, because both the design moment and the lateral drift of the structure should be examined. (Vanderbilt and Corely 1983, Hwang and Moehle 2000, Ghali and Gayed 2012) On the other hand, ACI318-11 (2011) and KCI-M-12 (2012) recommend that the stiffness degradation of the flat plate structure should be appropriately considered when the EFM or the effective beam width method (EBWM, Pecknold 1975) is used for its lateral analysis. For instance, ACI 318, however, suggests the stiffness reduction factors of 1/4 to 1/2, which is constant even when the flat plate structure experiences large lateral deformation over its elastic range, i.e., nonlinear behavior. The constant stiffness reduction factor makes it difficult to evaluate the lateral drift and design moment accurately, and it also cannot reflect the initial stiffness degradation of the flat slab caused by gravity shear. (Choi and Park 2003)

Therefore, this study proposed the torsional stiffness estimation model that can appropriately evaluate the lateral behavior of the flat plate system subjected to the combined gravity and lateral loads, based on which the unified equivalent frame method (UEFM) was also developed. The UEFM proposed in this study is a modified version of the existing equivalent frame approaches to be suitable for the analysis of the flat plate system under the combined loads. The verification of the proposed model will be performed in a consecutive paper.

2. Review of previous researches

2.1 A Equivalent frame method (EFM) and effective beam width method (EBWM)

The EFM was originally developed as an analysis method for the two-way slab structures under gravity loads, (Corely *et al*. 1961) and various researchers (Corely and Jirsa 1970, Vanderbilt 1981, Vanderbilt and Corely 1983, Murray *et al*. 2003, Hwang and Moehle 2000) have examined its htheoretical rationality and analytical accuracy in detail. Although there are some differences among researchers and design standards, the stiffness of the equivalent column in the EFM is generally determined by summing up the flexibility of the column and the torsional member, so as to convert a 3D slab-column frame system to a 2D planar frame. Thus, the stiffness of an equivalent column (K_{ec}) can be expressed, as follows:

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_{tg}} \tag{1}$$

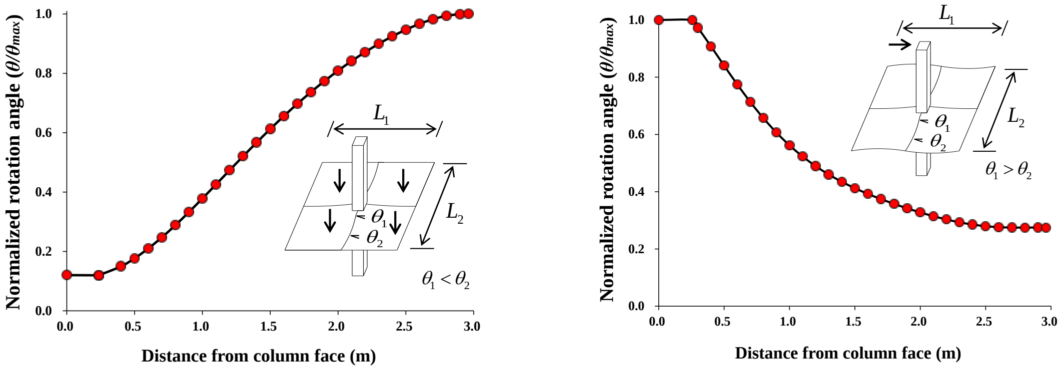
where K_c is the flexural stiffness of the column, and K_{tg} is the rotational stiffness of the torsional member. The current ACI 318-11 (2011) and KCI-M-12 (2012) present the stiffness of a torsional member (K_{tg}), as follows:

$$K_{tg} = \frac{9CE_c}{L_2\left(1-\frac{c_2}{L_2}\right)^3} \tag{2}$$

where C is the torsional constant, E_c is the modulus of elasticity of concrete, L_2 is the slab width perpendicular to the design strip, and c_2 is the column width in the L_2 direction. The stiffness of the torsional member (K_{tg}) presented in Eq. (2) was adjusted by test results so that it can be suitable for determining the design moment of the slab under gravity loads. (Corely *et al*. 1961, Jirsa *et al*. 1969, Corely and Jirsa 1970) Figs. 1(a) and (b) show the finite element analysis results of the flat plate slab-column connections under gravity and lateral loads, respectively. (Park *et al*. 2009) As shown in Fig. 1(a), in the case of the flat slab under the gravity load, the torsional rotation angle decreases as approaching to the column face due to large flexural stiffness of the column. On the contrary, under lateral load, the torsional rotation angle increases as approaching to the column face as shown in Fig. 1(b), because the column is deformed by the direct lateral force before the occurrence of slab rotation. Given this different load-transfer mechanisms between the gravity load and the lateral load, it is unreasonable to adopt the EFM, which was developed as an analysis method for the slabs under gravity loads only, to estimate the lateral behavior of the flat plate slab. (Hwang and Moehle 2000, Park *et al*. 2009)

In the slab-column connection under lateral forces, the rotation of the slab is distributed along the slab width direction as shown in Fig. 2. On this basis, the effective beam width model (EBWM) was developed by simplifying the flat plate structure to a slab-beam member that has a uniform rotation within a specified effective width. (Pecknold 1975, Banchik 1987, Grossman 1997) The EBWM basically simplifies a 3D frame into an equivalent 2D planar frame by introducing the effective beam width factor (α). Pecknold (1975) determined the patterns of load distribution transferred from the column to the slab mathematically by using the Fourier Series, and derived the effective beam width (b_{eff}) of the slab accordingly. Durrani and Luo (1995a,b) proposed the effective beam width factor (α) for exterior and interior connections by modifying the basic equation proposed by Pecknold (1975) based on their collected test results, and they also proposed the slab stiffness reduction factor (χ) as the function of the gravity-shear ratio (V_g/V_c). The effective beam width factor presented in ACSE/SEI 41-06 (ASCE 2007) was determined based on the finite element analysis results performed by Hwang and Moehle (2000), which is similar to the effective beam width model proposed by Banchik (1987). In addition, many other effective beam width models have been proposed, and these can be found elsewhere in existing literatures. (Khan and Sbarounis 1964, Capenter 1965, Alami 1972, Allen and Darvall 1977, Moehle and Diebold 1985, Choi and Song 2005)

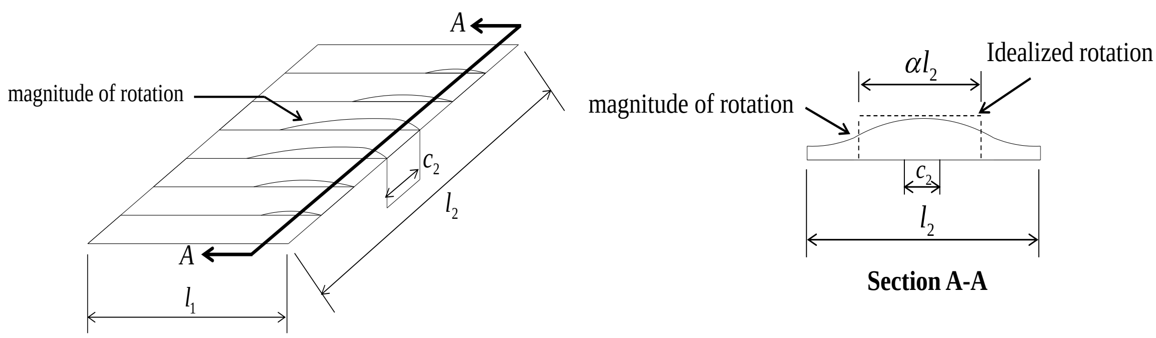
The EBWM is one of the easy and simple analysis models to estimate the lateral behavior of flat-plate structures. However, in the case of flat plate system where the slab width is considerably larger than the column width, the application of effective beam width factor (α) is questionable. In addition, because the EBWM was developed on the basis of the flat plate system where the columns were arranged in a regular pattern, it is not proper to be applied to the building structures with irregular column arrangement. (Khan and Sbarounis 1964, Lee and Kim 2003, Kim and Lee 2005) Furthermore, the EBWM assumes that the stiffness degradation of the flat plate system is entirely contributed by the stiffness degradation of slab members, without any stiffness change of columns, which makes it difficult to evaluate its realistic lateral behavior. Additionally, Cano and



(a) Deformation of flat plate slab under uniformly distributed gravity loads

(b) Deformation of flat plate slab under lateral load at column

Fig. 1 Distribution of rotation angle along attached torsional element (Park *et al*. 2009)



(a) Plate subjected to rotation over slab width

(b) Idealization of slab rotation using effective width

Fig. 2 Concept of effective beam width model (EBWM)

Klingner (1987), and Hwang and Moehle (2000) pointed out that the EBWM cannot appropriately evaluate the moment leakage phenomena of multi-span flat plate structures under the combined gravity and lateral loads.

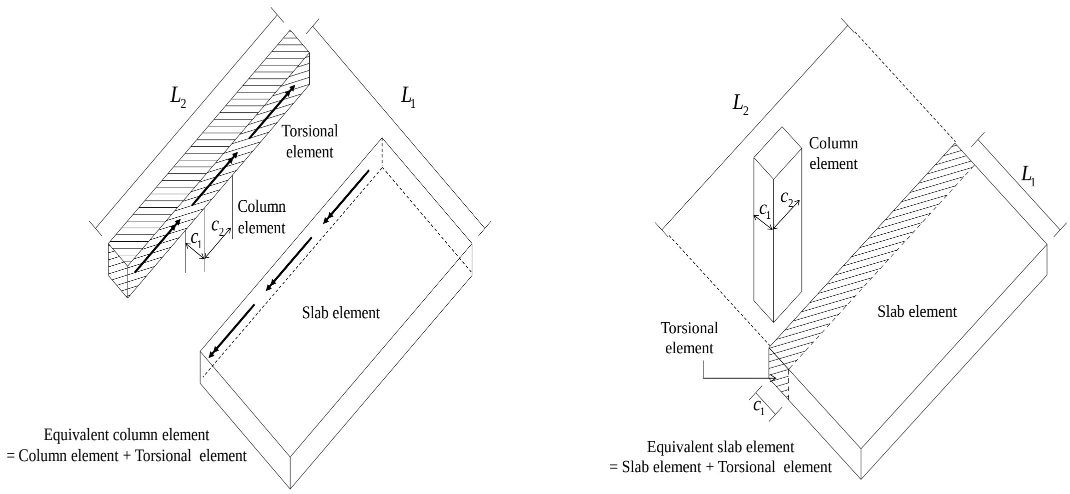
2.2 Modified equivalent frame method (MEFM)

In order to overcome the aforementioned limitations in the application of the EFM, Park *et al.* (2009) proposed the modified equivalent frame method (MEFM). As shown in Fig. 3(a), in the load-transfer mechanism of the flat plate structure under gravity load, the slab member is firstly deformed by gravity load, and the bending moment developed along the end of slab is transferred to the torsional member. Since the torsional member transfers the bending moment from the slab to the column, the column stiffness can be approximated by an equivalent column, as shown in Fig. 3(a). On the contrary, as shown in Fig. 3(b), when lateral loads are dominant, the column is directly deformed by the lateral loads, and the flexural moment is delivered to the slab through the torsional member. In the MEFM, therefore, stiffness of actual slab member was approximated by an equivalent slab, as shown in Fig. 3(b). (Vanderbilt and Corely 1983, Cano and Kligner 1987)

The rotational angle of the equivalent slab is, therefore, the sum of the slab rotational angle and the average rotational angle of the torsional member. Thus, the stiffness of the equivalent slab (K_{es}) can be expressed, as follows: (Vanderbilt 1981)

$$\frac{1}{K_{es}} = \frac{1}{\sum K_s} + \frac{1}{K_{tl}} \tag{3}$$

where K_s is the flexural stiffness of the slab, and K_{tl} is the stiffness of the torsional member under lateral loads. Fig. 4 shows the derivation process of the torsional stiffness in the flat plate structure under gravity loads or lateral loads. Under lateral loads, the torsional moment is assumed



(a) Equivalent column method for gravity resisting system

(b) Equivalent slab method for lateral resisting system

Fig. 3 Load transfer mechanisms in flat plate system subjected gravity and lateral load (Park *et al*. 2009) to be equally distributed along the torsional member, as shown in Fig. 4(b), and thereby the stiffness of the torsional member (K_{tl}) can be derived, as follows: (Park *et al*. 2009)

$$K_{tl} = \frac{6CE_c}{L_2\left(1-\frac{c_2}{L_2}\right)^2} \tag{4}$$

Under the combined gravity and lateral loads, however, the MEFM cannot reflect the initial stiffness degradation of the slab due to the gravity-shear ratio (V_g / V_c). It also cannot properly consider the stiffness change of the column according to the gravity load levels, as in the existing EFM, since it adopted the equivalent slab concept. Furthermore, the limitations of the equivalent slab method, which was adopted in MEFM, have been discussed in detail by Cano and Klingner (1987), and Lee and Lee (1994), and Choi and Park (2003) have reported the critical effects of gravity load on the lateral behavior of flat plate structures. There has also been no clear quantitative analysis on the effect of the combined gravity and lateral loads on the lateral stiffness, which should be considered in the lateral analysis of not only flat plate system but also other types of structures.

3. Proposed model – Unified equivalent frame method

3.1 Effect of gravity and lateral loads on torsional stiffness of equivalent frame

As previously mentioned, according to the EFM presented in ACI 318-11 (2011), when the unit torsional moment induced by gravity load is applied to the torsional member, the torsional moment is distributed as a triangular shape, as shown in Fig. 4(a). This is because the stiffness of column is larger than that of slab, and the effect of column stiffness on the moment distribution is decreased as being away from the column. (Corely and Jirsa 1970) On the contrary, in the MEFM, (Park *et al.* 2009) the torsional moment induced by lateral load is uniformly distributed due to its different load transfer path. As described in Fig. 4(e), and Eqs. (2) and (4), therefore, the EFM and MEFM provide quite different values of torsional stiffness, whose load transfer mechanisms are compatible with the gravity force resisting system (GFRS) and the lateral force resisting system (LFRS), respectively. In other words, while both the EFM and the MEFM are applied to approximate three-dimensional frame to the two-dimensional equivalent frame, the EFM utilizes the concept of the equivalent column for the analysis of GFRS, and the MEFM utilizes the concept of the equivalent slab for the analysis of LFRS. In reality, however, the flat plate structure is generally subjected to both gravity and lateral loads simultaneously. To reflect this combined effect of gravity and lateral forces on the stiffness of the torsional member in the equivalent frame, this study introduced the load ratio factor (λ_c), which indicates the relative ratio of torsional moment caused by lateral loads (T_l) to that caused by gravity loads (T_g), as follows

$$\lambda_c = \frac{T_l}{T_g} \tag{5}$$

As shown in Fig. 5, therefore, if the torsional moment induced by gravity loads is assumed to be the unit value of 1.0, the torsional moment induced by lateral loads can be expressed by the load ratio factor (λ_c). The maximum torsional angle induced by gravity loads ($\theta_{g,\max}$) can be estimated by ACI-EFM (ACI Committee 318-11), as shown in Fig. 4(e), as follows

$$\theta_{g,\max} = \frac{L_2(1-c_2/L_2)^3}{12CG} \tag{6}$$

where G is the shear modulus of concrete. Similarly, the maximum torsional angle induced by lateral loads ($\theta_{l,\max}$) can be expressed, as shown in Fig. 4(e) and Fig. 5(d), by multiplying the maximum torsional angle from the MEFM and the load ratio factor (λ_c), as follows

$$\theta_{l,\max} = \frac{L_2(1-c_2/L_2)^2}{8CG}\lambda_c \tag{7}$$

Assuming that the average rotational angle of the torsional member is one-third of the maximum torsional angle and the shear modulus is $E_c/2$ by neglecting Poisson's effect, the average torsional angle by gravity and lateral loads ($\theta_{g,ave}$ and $\theta_{l,ave}$) can be expressed, respectively, as follows

$$\theta_{g,ave} = \frac{L_2(1-c_2/L_2)^3}{18CE_c} \tag{8}$$

$$\theta_{l,ave} = \frac{L_2(1-c_2/L_2)^2}{12CE_c}\lambda_c \tag{9}$$

The total torsional moment in the equivalent frame is $1+\lambda_c$, and the average torsional angle of the equivalent frame (θ_{tot}) can be calculated by the sum of the average torsional angle induced by gravity loads ($\theta_{g,ave}$) and lateral loads ($\theta_{l,ave}$). Considering the left half of the frame shown in Fig. 5(b) and (d), the average torsional angle (θ_{tot}) and the torsional moment (T_{tot}) of the equivalent frame can be expressed, respectively, as follows

$$\theta_{tot} = \frac{L_2\alpha^2\left(2\alpha+3\lambda_c\right)}{36CE_c} \tag{10}$$

$$T_{tot} = \frac{1+\lambda_c}{2} \tag{11}$$

where α is $1-c_2/L_2$. As shown in Fig. 5(e) and (f), the values of θ_{tot} and T_{tot} under combined loads increase as the load ratio factor (λ_c) increases. Then, the effective stiffness of the torsional member (K_{t,λ_c}), considering the effect of both gravity and lateral loads, can be derived, as follows

$$K_{t,\lambda_c} = \frac{T_{tot}}{\theta_{tot}} = \frac{18CE_c\left(1+\lambda_c\right)}{L_2\alpha^2\left(2\alpha+3\lambda_c\right)} \tag{12}$$

Therefore, once the load ratio factor (λ_c) is determined, the stiffness of the torsional member under the combined gravity and lateral loads (K_{t,λ_c}) can be estimated. For the calculation of the load ratio factor (λ_c), the torsional moment by gravity and lateral loads (T_g and T_l) should be

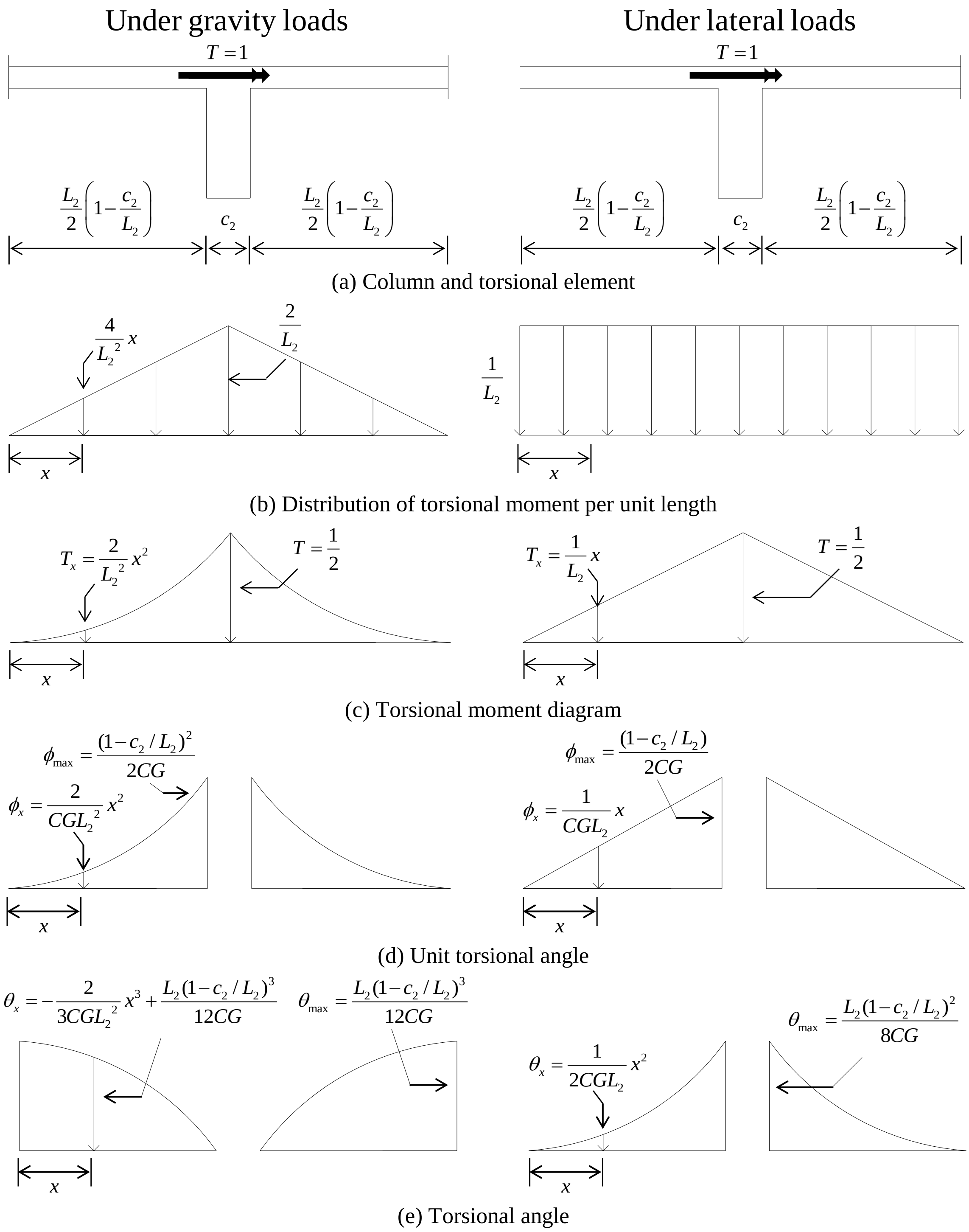


Fig. 4 Rotational stiffness of torsional member (Park *et al*. 2009)

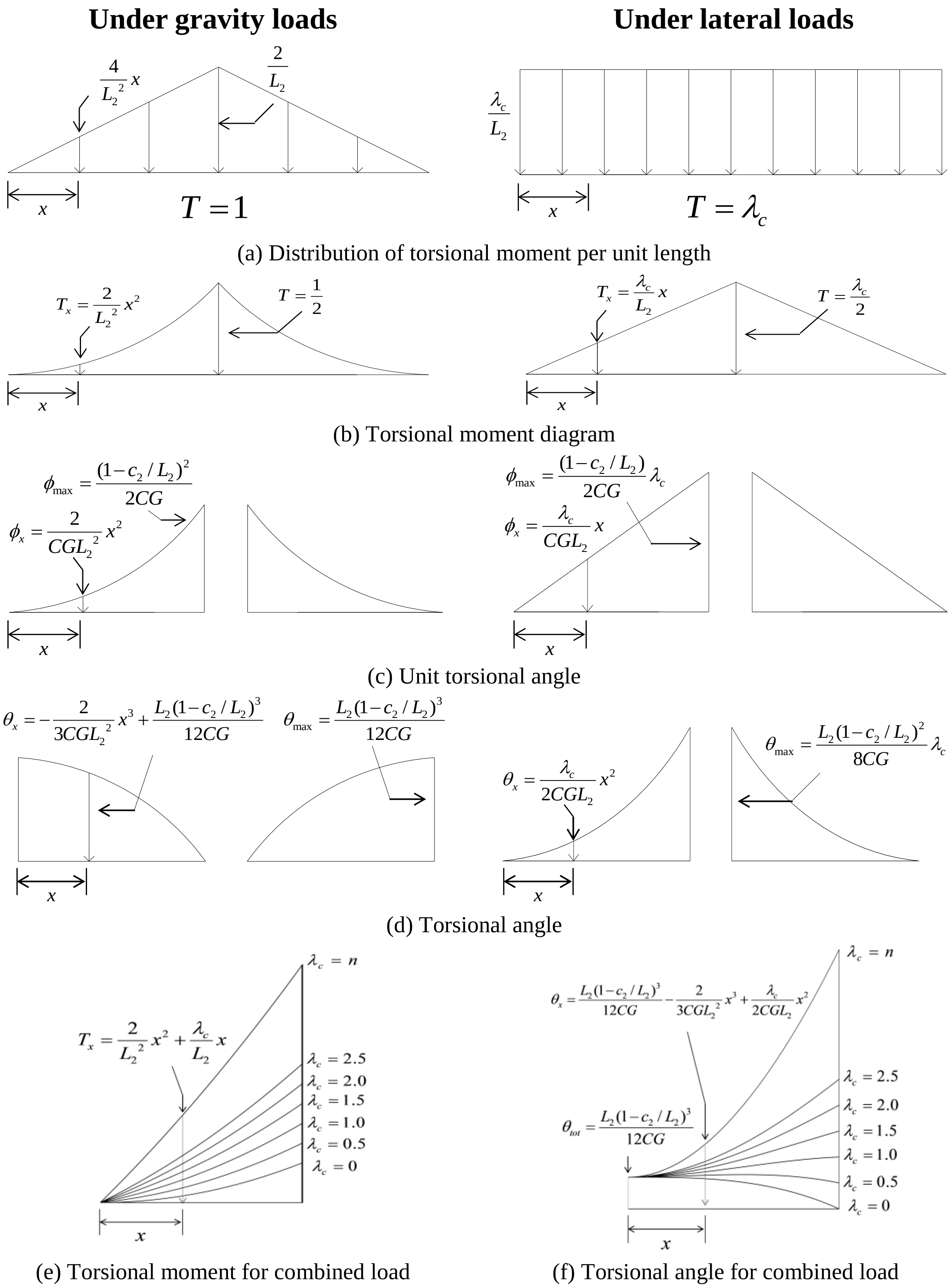


Fig. 5 Concept of load ratio factor

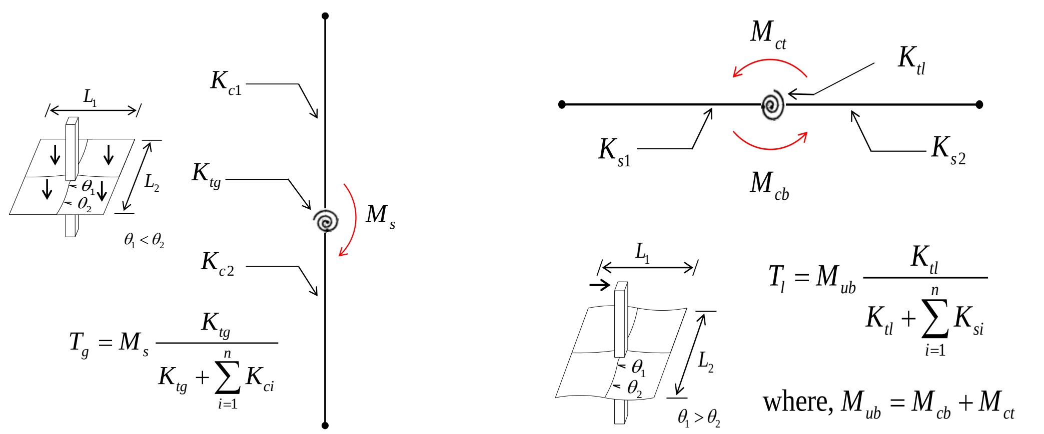
determined as well. As shown in Figs. 6(a) and (b), based on the aforementioned load-transfer mechanisms, the torsional moment by gravity loads (T_g) and lateral loads (T_l) can be approximated, utilizing the concept of equivalent column and the equivalent slab, respectively, as follows:

$$T_g = M_s \frac{K_{tg}}{K_{tg} + \sum_{i=1}^{n} K_{ci}} \tag{13}$$

$$T_l = M_{ub} \frac{K_{tl}}{K_{tl} + \sum_{i=1}^{n} K_{si}} \tag{14}$$

where, M_s is the flexural moment at the end of slab by gravity loads (i.e., the fixed-end moment of slab member), and M_{ub} is the unbalanced moment at the slab-column connection by lateral loads, which is identical to the sum of bending moments in the upper and the lower columns developed at joint ($M_{cb} + M_{ct}$). Here, it is again noted that, since the load ratio factor (λ_c) is merely the ratio between the torsional moments due to gravity and lateral loads, the relative ratio is more important than their absolute values. For this reason, the approximated methods presented in Eqs. (13) and (14) would be applicable for the estimation of the torsional moment ratio.

Fig. 7 shows the comparison of the stiffness of the torsional member (i.e., K_{tg} and K_{tl}) presented in the EFM (ACI Committee 318-11) and the MEFM (Park *et al.* 2009) with that proposed in this study considering the combined effect of gravity and lateral loads (K_{t,λ_c}) according to the load ratio factor (λ_c). Under the gravity load only (i.e., λ_c is 0), the torsional stiffness obtained by the proposed model is identical to that of ACI-EFM. Also, when the effect of lateral load is significantly greater than gravity load, i.e., λ_c is infinitely large, it converged to the torsional stiffness of the MEFM. This graph showed that proposed model can express the changes



(a) Torsional moment in gravity resisting system (b) Torsional moment in lateral resisting system

Fig. 6 Determination of moment in torsional member

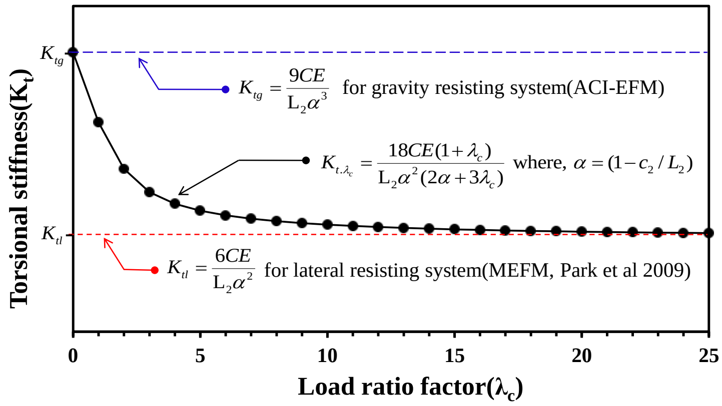


Fig. 7 Determination of torsional stiffness using the load ratio factor

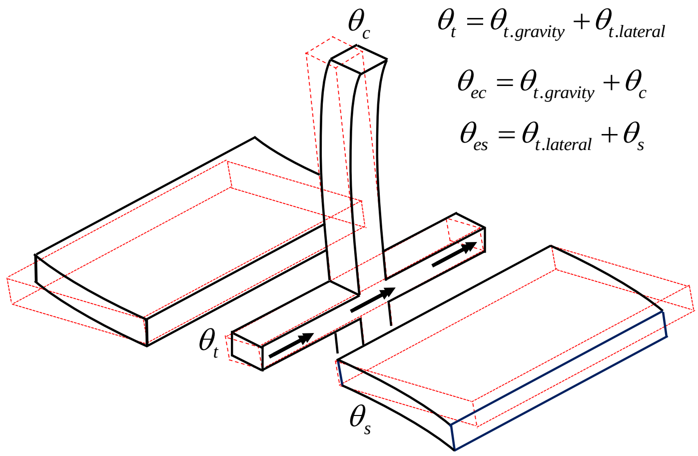


Fig. 8 Rotational contribution of torsional member to equivalent column and slab

in the stiffness of the torsional member under the combined gravity and lateral loads by the unified equation. On this basis, the proposed model was, therefore, named as the unified equivalent frame method (UEFM).

As previously explained, ACI-EFM adopted the concept of equivalent column for the gravity load-dominant flat plate system, whereas the MEFM utilized the concept of equivalent slab for the flat plate system governed by lateral loads. As shown in Fig. 8, in the UEFM proposed in this study, the rotation angle of the torsional member ($\theta_{t,gravity}$) by gravity loads is considered to contribute to the rotation of the equivalent column, and the rotation angle of the torsional member ($\theta_{t,lateral}$) by lateral loads is considered to contribute to the rotation of the equivalent slab. Therefore, the flexibility of the equivalent column and the equivalent slab can be expressed, respectively, as follows:

$$\frac{1}{K_{ec}}=\theta_{t,gravity}+\theta_c=\frac{\sum K_c}{\sum K_c+\sum K_s}\frac{1}{K_{t,\lambda_c}}+\frac{1}{\sum K_c} \tag{15}$$

$$\frac{1}{K_{es}} = \theta_{t,lateral} + \theta_s = \frac{\sum K_s}{\sum K_c + \sum K_s}\frac{1}{K_{t,\lambda_c}} + \frac{1}{\sum K_s} \tag{16}$$

where, K_{t,λ_c} is the stiffness of the torsional member considering the combined gravity and lateral loads, expressed in Eq. (12).

4. Conclusion

In this study, the existing methods for the lateral behavior analysis of the two-way flat plate structures were thoroughly reviewed, and it was found that they are not properly applicable to the flat plate structure subjected to combined gravity and lateral loads. It is because the stiffness of the torsional member in the equivalent frame subjected to combined gravity and lateral loads is largely affected by the relative ratio of the gravity and lateral loads, in which quite a different load-transfer mechanism is expected compared to the cases under only either gravity loads or lateral loads. To overcome such shortcomings, this study proposed the lateral analysis method for the two-way flat plate structures, introducing the concept of unified equivalent frame method (UEFM). In the proposed UEFM model, the rotation angle of the torsional member was considered to contribute to the rotation of both the equivalent column and the equivalent slab according to the ratio of the gravity and lateral loads as well as the relative stiffness of the slab and the column.

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References

ACI Committee 318 (2011), "Building code requirements for structural concrete and commentary (ACI 318M-11)", American Concrete Institute, Farmington Hills, 503.

Alami, B. (1972), "Moment-rotation relation between column and slab", *ACI J., Proc.*, **69**(5), 263-269.

Allen, F.H. and Darvall, P. (1977), "Lateral load equivalent frame", *ACI J., Proc.*, **74**(7), 294-299.

ASCE (American Society of Civil Engineers) (2007), ASCE/SEI 41-06: Seismic rehabilitation of existing buildings. ASCE, Reston, VA.

Banchik, C.A. (1987), "Effective beam width coefficients for equivalent frame analysis of flat-plate structures", ME Thesis, University of California, Berkley, CA.

Cano, M.T. and Kilngner, R.E. (1987), "Comparison of analysis procedures for tow-way slabs", *ACI Struct. J.*, **85**(6), 597–608.

Carpenter, J.E. (1965), "Flexural characteristics of flat plate floors in buildings subjected to lateral loads", Ph.D. Thesis, Purdue University, West Lafayette, IN.

Choi, K.K. and Park, H.G. (2003), "Moment-rotation relationship and effective stiffness of flat plates under lateral load", *J. Korean Concrete Inst.*, **15**(6), 856-865.

Choi, J.W. and Song, J.G. (2005), "A method for effective beam widths of slabs in flat plate structures under

gravity and lateral loads", *Struct. Eng. Mech.*, **21**(4), 451-468
Corely, W.G., Sozen, M.A. and Siess, C.P. (1961), "The equivalent frame analysis for reinforced concrete slabs", Structural Research Series No. 218, Department of Civil Engineering, University of Illinois, Urbana, IL.
Corley, W.G. and Jirsa, J.O. (1970), "Equivalent frame analysis for slab design", *ACI J., Proc.*, **67**(11), 875-884.
Ghali, A. and Gayed, R.B. (2012), "Requirements for seismic-resistant flat plates : Strength and ductility", *ACI Special Publication*, **287**, 1–19.
Grossman, J.S. (1997), "Verification of proposed design methodologies for effective width of slabs in slab–column frames", *ACI Struct. J.*, **94**(2), 181–196.
Han, S.W., Park, Y.M. and Kee, S.H. (2009), "Stiffness reduction factor for flat slab structures under lateral loads", *J. Struct. Eng. - ASCE*, **135**(6), 743-750.
Hwang, S.J. and Moehle, J.P. (1993). "An experimental study of flat-plate structures under vertical and lateral loads", University of California, Berkeley, CA.
Hwang, S.J. and Moehle, J.P. (2000), "Models for laterally loaded slab–column frames", *ACI Struct. J.*, **97**(2), 345-352.
Jirsa, J.O., Sozen, M.A. and Siess, C.P. (1969), "Pattern loadings on reinforced concrere floor slabs", *J. Struct. Div.- ASCE*, **96**(ST5), 1117-1137.
KCI-M-12 (2012), *Design Specifications for Concrete Structures*. Korea Concrete Institute, Korea.
Khan, F.R. and Sbarounis, J.A. (1964), "Interaction of Shear Walls and Frames", *J. Struct. Div.- ASCE*, **90**(ST3), 285-335.
Kim, H.S. and Lee, D.G. (2005), "Efficient analysis of flat slab structures subjected to lateral loads", *Eng. Struct.*, **27**(2), 251-263.
Lee, D.H., Kim, K.S., Kim, S.S., Kim, Y.N. and Lim, J.H. (2009), "Applicability of partial post-tension method for deflection control of reinforced concrete slabs", *J. Korean Concrete Inst.*, **21**(3), 347-358.
Lee, S.J., Lee, D.H., Kim, K.S., Oh, J.Y., Park, M.K. and Yang, I.S. (2013), "Seismic performances of RC columns reinforced with screw ribbed reinforcements connected by mechanical splice", *Comput. Concr.*, **12**(2), 131-149
Lee, S.Y. and Lee, D.G. (1994), "Effect of gravity load on seismic response of multistorey buildings", *Eng. Struct.*, **16**(6), 445-454.
Lee, Y.W. and Kim, J.W. (2003), "An analysis of the flat plate structure with simplified equivalent beam model", *J. Archit. Insti. Korea*, **19**(11), 57-64.
Luo, Y.H. and Durrani, A.J. (1995a), "Equivalent beam model for flat slab buildings – part I: Interior connection", *ACI Struct. J.*, **92**(1), 115-124.
Luo, Y.H. and Durrani, A.J. (1995b), "Equivalent beam model for flat slab buildings – Part II: Exterior connection", *ACI Struct. J.*, **92**(2), 250-257.
Moehle, J.P. and Diebold, J.W. (1985), "Lateral load response of flat-plate frame", *J. Struct. Eng.*- ASCE, **111**(10), 2149-2164.
Murray, K.A., Cleland, D.J., Gilbert, S.G. and Scott, R.H. (2003), "Improved equivalent frame method for flat plate structures in vicinity of edge column", *ACI Struct. J.*, **100**(4), 454-464.
Park, Y.M. (2009), "Modified equivalent frame method for flat plate systems under lateral loads", Ph.D. thesis, Department of Architectural Engineering, Hanyang University, Seoul, Korea.
Park, Y.M., Han, S.W. and Kee, S.H. (2009), "A modified equivalent frame method for lateral load analysis", *Mag. Concrete Res.*, **61**(5), 359-370.
Park, Y.M., Rew, Y.H. and Han, S.W. (2012), "Stiffness reduction factor for post-tensioned flat plate slabs", *Mag. Concrete Res.*, **64**(1), 83-92.
Pecknold, D.A. (1975), "Slab effective width for equivalent frame analysis", *ACI J., Proc.*, **72**(4), 135-137.
Peiris, C. and Ghali, A. (2012), "Flexural reinforcement essential for punching shear resistance of slabs", *ACI Special Publication*, **287**, 83-97.
Robertson, I.N. (1990), "Seismic response of connections in indeterminate flat-slab subassemblies", Ph.D.

thesis, Department of Civil Engineering, Rice University, Houston, TX.
Robertson, I.N. (1997), “Analysis of flat slab structures subjected to combined lateral and gravity loads”, *ACI Struct. J.*, **94**(6), 723-729.
Schwaighofer, J. and Collins, M.P. (1977), “Experimental study of the behavior of reinforced concrete coupling slabs”, *ACI J., Proc.*, **74**(3), 123-127.
Vanderbilt, M.D. (1981), *The Equivalent Frame Analysis for Reinforced Concrete Slabs*, Structural Research Series No. 36, Civil Engineering Department, Colorado State University, Fort Collins, CO.
Vanderbilt, M.D. and Corley, W.G. (1983), “Frame analysis for concrete buildings”, *Concrete Int.: Design construct.*, **5**(12), 33-43.
Xing, G.H., Wu, T., Niu, D.T. and Liu, X. (2013), “Seismic behavior of reinforced concrete interior beam-column joints with beams of different depths”, *Earthq. Struct.*, **4**(4), 429-449.

Notations

c_1 = width of column in loading direction
c_2 = width of column in perpendicular to loading direction
d_{ave} = average effective depth of slab
C = torsional constant
E_c = modulus of elasticity of concrete
K_c = flexural stiffness of column
K_{ec} = stiffness of equivalent column
K_{es} = stiffness of equivalent slab
K_s = flexural stiffness of slab-beam
K_{tg} = stiffness of torsional member in flat plate system subjected to gravity load
K_{tl} = stiffness of torsional member in flat plate system subjected to lateral load
K_{t,λ_c} = effective stiffness of torsional member in flat plate system subjected to gravity and lateral load
L_1 = slab width of the design strip
L_2 = slab width perpendicular to the design strip
M_s = flexural moment at the end of slab by gravity loads
M_{ub} = unbalanced moment at the slab-column connection by lateral loads
T_g = torsional moment by gravity loads
T_l = torsional moment by lateral loads
T_{tot} = total torsional moment by gravity and lateral loads
V_g = external gravity shear force
V_c = punching shear strength
λ_c = load ratio factor
ν = Poisson's ratio
χ = slab stiffness reduction factor
$\theta_{gravity}$ = rotational contribution of torsional member to equivalent column
$\theta_{lateral}$ = rotational contribution of torsional member to equivalent slab
$\theta_{g,\max}$ = maximum torsional angle induced by gravity loads
$\theta_{l,\max}$ = maximum torsional angle induced by lateral loads
$\theta_{g,ave}$ = average torsional angle induced by gravity loads
$\theta_{l,ave}$ = average torsional angle induced by lateral loads
θ_{tot} = total torsional angle induced by gravity and lateral loads