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Lateral stiffness of reinforced concrete flat plates with steps under seismic loads

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Abstract. The purpose of this study is to propose a modification factor to reflect the lateral stiffness modification when a step is located in flat plates. Reinforced concrete slabs with steps have different structural characteristics that are demonstrated by a series of structural experiment and nonlinear analyses. The corner at the step is weak and flexible, and the associated rotational stiffness degradation at the corner of the step is identified through analyses of 6 types of models using a nonlinear finite element program. Then a systematic analysis of stiffness changes is performed using a linear finite element procedure along with rotational springs. The lateral stiffness of reinforced concrete flat plates with steps is mainly affected by the step length, location, thickness and height. Therefore, a single modification factor for each of these variables is obtained, while other variables are constrained. When multiple variables are considered, each single modification factor is multiplied by the other. Such a method is verified by a comparative analysis. Finally, a complex modification factor can be applied to the existing effective slab width.

Keywords: lateral stiffness; slab with step; flat plate, modification factor; effective slab width method

1. Introduction

Apartments have been widely favored as a solution for housing market problem due to rapid urbanization and over-population. Additionally, they have been popular with urban dwellers, who prefer more comfortable and convenient housing. However, for those dwellers, the noise problem is a trade-off as it is both a nuisance and a social concern. In particular, the flushing of apartment toilets causes not only a noise problem but also a privacy invasion.

Typically, drainage systems for an apartment bathroom of a given floor have been installed in the ceiling of the lower floor. A method (KLHC 2012) of installing the drainage within the same floor by placing a step of 230 mm height in the slab is considered to deal with this problem (Fig. 1). However, this design proposal involves step installation, which in turn causes the change of the slab height. Previous design limited the step height only to 60 mm, but this recent proposal will

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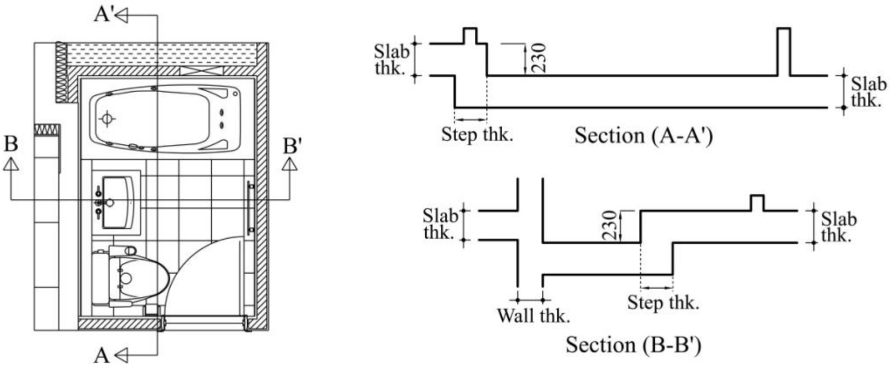


Fig. 1 Plan and section of proposal

increase it to 230 mm, which greatly affects the structural performance of the slab as the slab-column frames are quite sensitive to geometric, loading and reinforcing conditions (Grossman 1997, Song *et al*. 2012, George and Tian 2012, Kaynia 2012, Ellouse *et al*. 2013).

The flat plates with steps should have different structural characteristics compared to the normal flat plates in terms of bending strength, stiffness, deflection, cracking, etc. It is difficult to apply an existing flat plate design method equally to flat plates with steps (ACI 2012). In particular, the larger the moment increase by seismic load, the greater the decrease in stiffness of slabs with steps. Unlike the lateral bending stiffness of the normal flat plate, consideration of the step should be included in the seismic design of reinforced concrete flat plates with steps.

This study aims to determine the lateral bending stiffness of flat plates with steps under seismic loads, and to propose a modification factor to reflect a change of lateral bending stiffness as compared to normal flat plates without steps. The influence of steps was analyzed using the effective slab width method that is generally used as an analysis method of flat plates. A modification factor is proposed for the case where steps were installed in the flat plates.

2. Analysis of flat plate with step

2.1 Method of research

The research was divided into two steps to determine a modification factor for lateral stiffness of flat plates with steps under seismic loads. The first step is a process of defining the rotational stiffness of the steps through the nonlinear analysis. The value of the elastic rotational stiffness is close to that of rigid connection because the load is relatively small and damage of material is thus minor. However, the rotational stiffness decreases after damage occurs to the concrete member, cracks in the steps rapidly progress, and the rotational stiffness of the steps rapidly decreases compared to normal flat plates.

Because the value of the rotational stiffness in a finite element program should be constant, nonlinearity of the rotational stiffness is not directly considered. Therefore, it is necessary to determine average rotational stiffness values (decreased stiffness values reflecting damage of steps) depending on the analysis condition. Thus, a change in the value of the rotational stiffness

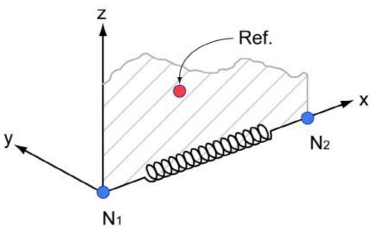


Fig. 2 Element coordinates

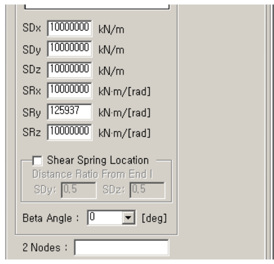


Fig. 3 Input form for rotational stiffness

of each step corner was analyzed using the nonlinear analysis program ABAQUS.

By defining the rotational stiffness values through a nonlinear analysis program, it is possible to improve the accuracy of elastic frame analysis. Commercial finite element analysis programs (SAP2000, ETABS, MIDAS Gen) use nodes and elements, and these programs typically assume completely rigid or completely hinged condition at the node or use a constant stiffness value. Because a change of rotational stiffness value with increasing applied load at the step varies, it is reasonable that a defined rotational stiffness value can be input in the elastic stiffness analysis.

The second step is a process of constituting the effective slab width model for lateral load analysis to determine a modification factor by examining story drift ratios of flat plates with elastic rotational springs. Therefore, the MIDAS Gen ver.8.0.0 program was used in order to embody a stiffness reduction due to damage of the steps. In the program, the elastic connection element (elastic link) was applied. Two nodes can be connected with the spring (elastic link) of desired stiffness, and this spring can manifest flexible rotational stiffness at the node. This study applied the rotational spring to carry out stiffness analysis with a variety of variables. Then, the result of the stiffness analysis was used to determine the step modification factor (γ), which can be additionally considered in existing effective width models.

Typically, stiffness of an elastic link can be analyzed by three coordinates and rotational stiffnesses. In other words, it can be expressed in terms of force per unit length (kN/m) in three directional coordinates (x, y, z) and moment per rotational angle (kN・m/[rad]) (Fig. 2). Tensile component and compressive component can be attributed to elastic link, and in this case stiffness in the axial direction of the element coordinates only is necessary. Additionally, rigid characteristics can be attributed to the elastic link to connect two nodes.

This study considered rotational stiffness at the node at which two members interface. In particular, the lateral stiffness of flat plates was investigated in consideration of the rotational stiffness for SRy. Values for the three stiffnesses in axial directions (SDx, SDy, SDz) and two rotational stiffnesses (SRx, SRz) were given as infinite (∞) (Fig. 3).

2.2 Determination of rotational stiffness at step corner using nonlinear analysis

Nonlinear analysis was performed to define the rotational stiffness that reflects the damage of the concrete at the step using ABAQUS. The concrete damage plasticity model (Fig. 4) was used for concrete, which can accurately model or consider the plastic strain and damage; the increase in compression strength by the lateral confinement; the difference in compression-tension behavior due to deviatoric plane; and the non-associated flow rule (ABAQUS, 2012). The behavior of

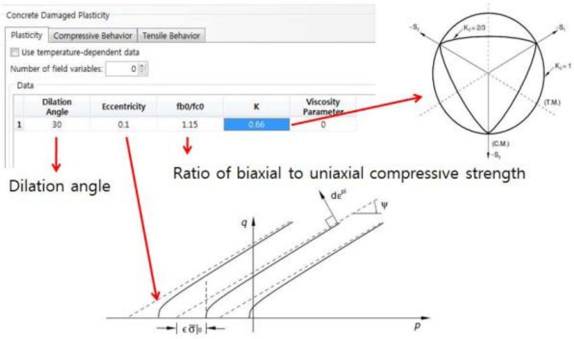


Fig. 4 Input form for concrete damage plasticity model

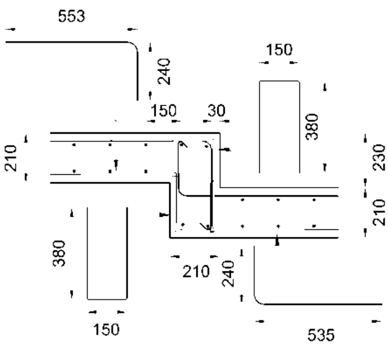


Fig. 5 Typical reinforcing details in the step

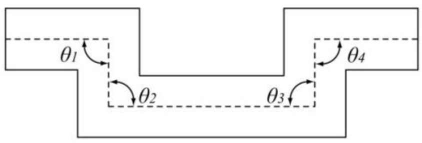


Fig. 6 Definition of rotational angles at four step corners

reinforcing steel was assumed to be perfectly elasto-plastic. The process of determining the bending moment-rotational angle of the step was performed using a nonlinear analysis, with typical reinforcing details in the step as shown in Fig. 5.

Since the rotational stiffness at a joint is defined by an elastic link with a certain stiffness value in elastic finite element analysis, it is not possible to directly apply the complete bending moment-rotational angle relationship from nonlinear analysis to the elastic analysis program. In this study, 2 × 3 frame models were analyzed to determine the representative rotation angle in the step at a given story drift ratio (SDR). The SDRs of 0.0025 and 0.005 were used for the serviceability and strength design stages, respectively, following the previous design experience (Grossman, 1997). Accordingly, the representative secant stiffness value was obtained at each location and denoted as θ_1, θ_2, θ_3, or θ_4 (see Fig. 6).

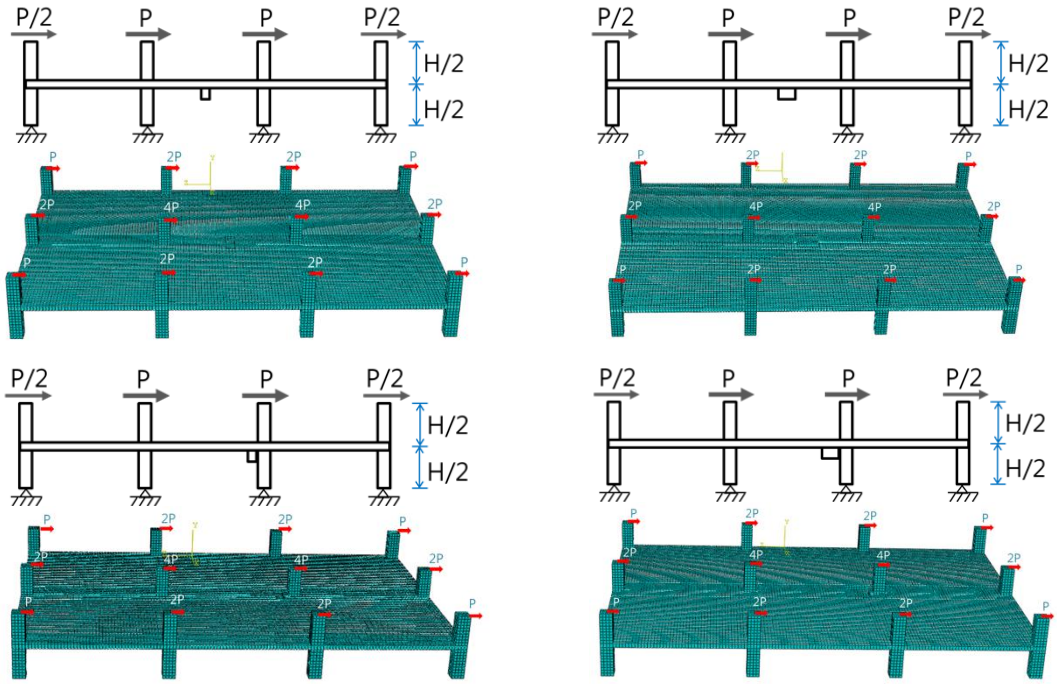


Fig. 7 Nonlinear analysis of 2 bay x 3 bay flat plate system models

Four prototype models with 2 x 3 frames were classified based on the size of the step and the location of the step as shown in Fig 7. Based on many apartment plans in Korea, the average span length between vertical elements such as columns and walls is 7000 mm and thickness of the slab is 210 mm. The concrete strength is given as 24 MPa. Longitudinal reinforcement in slab was laid using SD500 D13 at 150 mm spacing according to concrete design codes (AIK 2009 and KCI 2012), and additional reinforcing bars were provided in the step as shown in Fig 5. In the disturbed region, the addition of reinforcing bars would be a promising solution (ACI 2012, Lu *et al*. 2012).

Based on the prototype apartment plans, the step slab is typically located in the column strip transversely. Note that the column strip definition is the same as that of flat plates without steps. The steps are located at mid-span or near the vertical member in the longitudinal direction. Thus, the first two prototype models have a step centered at mid-span, whereas one side of the step connects to the vertical member for the other two prototype models. Two different lengths of step (1,500 and 700 mm) were used, although the width (2,000 mm) was the same for all models. To analyze using story drift ratio under seismic lateral loads, half the height (H / 2 = 1.6 m) of the upper and lower columns was modeled with a pin at the bottom of each column and with a free end at the top of the column. A concentrated lateral load was applied to the top column.

The columns were assumed to behave elastically, neglecting the effects of local damage of the column, and lateral force assigned to a column is proportional to the tributary area of the slab supported by the column. Concrete slabs and columns were modeled using the cube element of 150 mm^3 (default size), and the reinforcing bar was modeled using a wire element similar to a truss element with tension and compression. The bending moment-rotational angle relationship

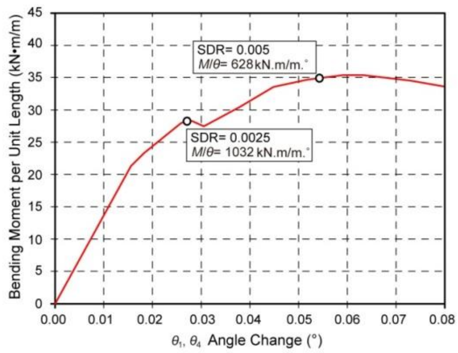


Fig. 8 Bending moment-rotational angle per unit length (+)

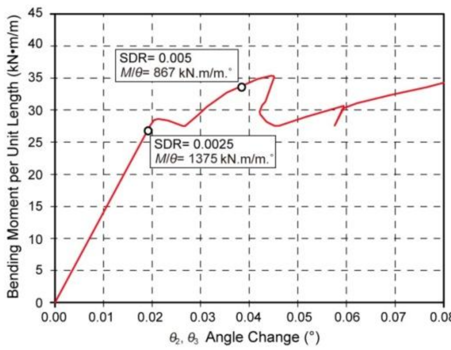


Fig. 9 Bending moment-rotational angle per unit length (-)

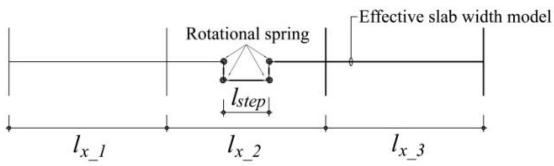


Fig.10 Frame modelling

from the nonlinear analysis results are shown in Figs 8 and 9. The graphs indicate that the bending moment-rotational angle relationship is different according to the four step corners. As a result, the rotational stiffness was analyzed separately for positive angle (θ_1 and θ_4 in Fig. 8) and for negative angle (θ_2 and θ_3 in Fig. 9).

The range of rotational stiffness for the given story drift ratio is as follows:

(1) Story drift ratio of 0.0025: 1,032 kN/° (positive angle) ~ 1,375 kN/° (negative angle).

(2) Story drift ratio of 0.005: 628 kN/° (positive angle) ~ 867 kN/° (negative angle).

Therefore, the minimum value was conservatively taken as representative rotational stiffness: 1,032 kN/° for SDR of 0.0025 [for serviceability design] and 628 kN/° for SDR of 0.005 [for strength design].

2.3 Application of rotational stiffness

The column strip width, which is similar to the effective slab width or a little larger, was regarded as the length of the area of influence of rotational stiffness in the width direction. The width of the step differs by design. Even though the width of the step changes every time, in general it is probably less than the width of the column strip or effective slab width. Therefore, applying the rotational stiffness per unit length (M/θ), which was computed by nonlinear analysis, to the design strip would cause an imprecise representation, because it is flat slab outside the step. Even if it is to be applied to column strip, of which the width would probably be larger than the step width, it would underestimate the rotational stiffness a little. Nevertheless, the column strip width was regarded as the length of the area of influence of rotational stiffness in the width direction in order to conservatively compute the lateral displacement, i.e., to make the rotational stiffness unfavorable. Then, the rotational stiffness, which was computed in consideration of the effective slab width, was reflected onto the design of slab reinforcement (Fig. 10).

3. Modification factor

3.1 Research method for step modification factor

The structural design of flat plate building generally uses the effective slab width method in which the slab is assumed as the effective slab including lateral bending stiffness. The existing effective slab methods consider the decrease in the lateral bending stiffness of the flat plates with a story drift ratio. However, the reduction in lateral bending stiffness of the flat plates with large steps over slab`s thickness occurs largely as compared with the normal flat plate due to rapid decrease in rigidity in the steps. In this study, in order to consider the reduction in lateral bending stiffness due to the step, by multiplying the appropriate modification factor to existing effective slab width methods, the concept of the step modification factor for increasing or decreasing the effective slab width is introduced. The research is conducted with two parts such as a strength design and a serviceability design, the procedure of research to determine modification factor is as follows.

a) Composition of the following three models for lateral load analysis:

Model 1: Model of flat plate without step, with the effective slab width (Step 1 of Fig. 11).

Model 2: Model of flat plate with step, with the same effective slab width as Model 1 and rotational springs from Section 2.2 (Step 2 of Fig. 11).

Model 3: Model of flat plate without step, with new effective slab width (Step 3 of Fig. 11).

Although any available effective slab width models can be used, in this research the effective slab width is calculated using Grossman's equation (1997), as follows:

$$\alpha l_2 = K_d[0.3l_1 + \frac{C_1 l_2}{l_1} + \frac{C_2 - C_1}{2}](\frac{d}{0.9h})(K_{FP}) \quad (1)$$

$$(0.2)(K_d)(K_{FP})l_2 \le \alpha l_2 \le (0.5)(K_d)(K_{FP})l_2$$

where K_d is the factor considering degradation of stiffness of slabs at various lateral load levels; l_1 is the length of span in the direction parallel to lateral load (average of two spans at interior supports); l_2 is the length of span in the direction transverse to lateral load (average of two spans at interior supports), C_1 is size of support in direction parallel to lateral load; C_2 is the size of support in the direction transverse to lateral load; d is the effective depth of slab; h is the slab thickness; K_{FP} is the factor adjusting αl_2 at edge exterior and corner supports (1.0 for interior supports, 0.8 for exterior and edge supports and 0.6 for corner supports).

b) Compute lateral force P for Model 1 when lateral displacement, δ_i, does not surpass (height)/200 and the closest to (height)/200 [for strength design]. Then continue to compute lateral force P for Model 1 when δ_i does not surpass (height)/400 and the closest to (height)/400 [for serviceability design] (Step 1 of Fig. 11).

c) Compute new lateral displacement, δ_k, taking place when lateral force P is applied to Model 2 (Step 2 of Fig. 11).

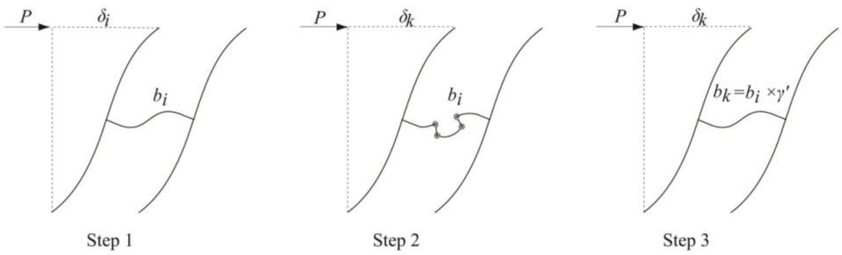


Fig. 11 Determination of modification factor through stiffness analysis

Table 1 Basic parameter of stiffness analysis modeling

Column size	600 × 600 mm	Step height	230 mm	Column spacing	7000 mm
Slab thickness	210 mm	Step thickness	210 mm	Loads (kN/m^2)	D.L.: 8; L.L.: 3

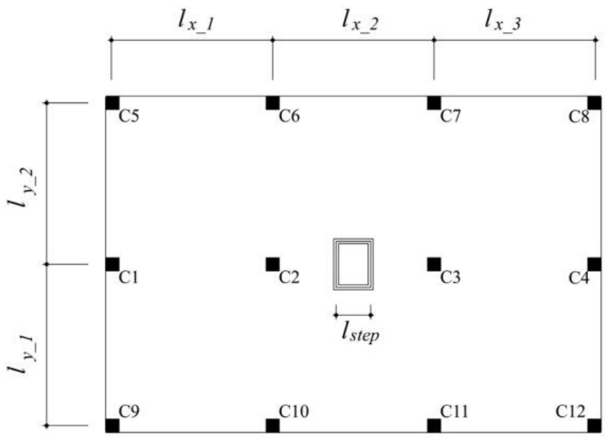


Fig. 12 Basic plan for stiffness analysis

d) Exert lateral force P on Model 3 and compute new effective slab width, b_k, which results in the equivalent lateral displacement δ_k (Step 3 of Fig. 11).

e) Define single modification factor γ' (= b_k/b_i) by using the new effective slab width (b_k) from Model 3 relative to the effective slab width (b_i) of the flat plate without step from Model 1.

It is difficult to predict which variable will influence the stiffness of the slab with step. Thus, this section varies only one condition and all other conditions are set to be the same in order to find out a single modification factor (γ'). Then, a step modification factor (γ), which considers a variety of variables, can be finally obtained, if the single modification factors, which considered only one variable at a time, are multiplied as expressed in Eq. (2).

$$\gamma = \prod \gamma'_n = \gamma'_1 \times \gamma'_2 \times \gamma'_3 \times \cdots \times \gamma'_n \tag{2}$$

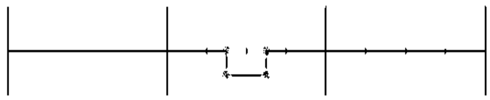
The prototype model for stiffness analysis and basic specification are presented in Table 1 and Fig. 12. In this given condition, the rotational stiffness has the values of 206,952 kN-m/radian and 125,937 kN-m/radian when the story drift ratios are 0.0025 and 0.005, respectively.

3.2 Analysis results of step modification factor

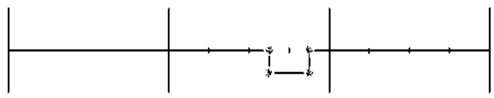
3.2.1 Relationship between variables and single modification factor

Firstly, the modification factor by the location of step and change of the step length was investigated. It was analyzed with the simultaneous variables of step length (l_{step}) and location as discussed in Table 1 of the basic condition. Since the step location and step length together affected the stiffness of slab, the two variables were controlled simultaneously for the analysis. The locations of step were designated at eight points (Fig. 13): 2/4 (center), 3/4 and 4/4 of the inner span, and 0/4, 1/4, 2/4, 3/4 and 4/4 of outer span. The step length (l_{step}) was increased by 10% of the slab span to vary between 0% ～ 90%.

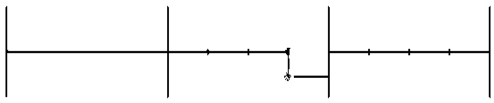
The new effective slab width (b_k), which manifested lateral displacement equivalent to the frame of the flat plate with step by step location and step length, was divided by the effective slab width (b_i) of the flat plate without step to compute the single modification factor ($\gamma' = b_k/b_i$) (Tables 2 and 3). Examining the values of strength design and serviceability design, lateral stiffness was affected by the combination of step location and step length. The following three cases of the step location exhibited the same trend: the vertical member connected to one side of the step (0/4 and 4/4 location); the center of the step being the center of slab (2/4 location); and the center of the step not being the center of slab (1/4 and 3/4 location).



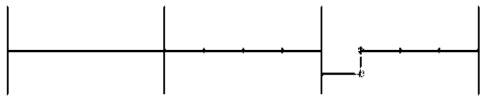
(a) Center of the step (x) is also the center of inner span (l/2)
(Inner 2/4)



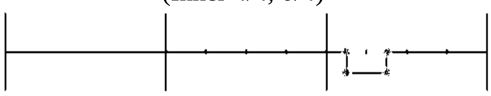
(b) Center of the step (x) is at
$0 < x < l/2$
(Inner 3/4)



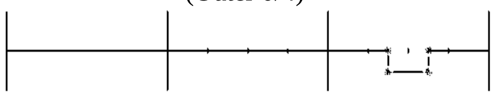
(c) The step is on inner span and one side is connected to inner column
(Inner 4/4, 0/4)



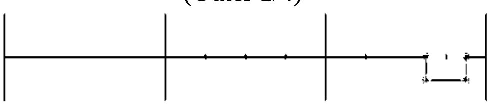
(d) The step is on outer span and one side is connected to inner column
(Outer 0/4)



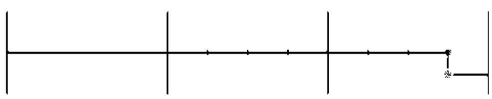
(e) Center of the step (x) is at
$0 < x < l/2$
(Outer 1/4)



(f) Center of the step (x) is also the center of outer span ($l/2$) (Outer 2/4)



(g) Center of the step (x) is at
$l/2 < x < l$
(Outer 3/4)



(h) The step is on outer span and one side is connected to outer column
(Outer 4/4)

Fig. 13 Location of step

Table 2 Single modification factor by step length and location for strength design

Outer 4/4				Outer 3/4				Outer 2/4				Outer 1/4			
Length change	b_i	b_k	γ'	Length change	b_i	b_k	γ'	Length change	b_i	b_k	γ'	Length Change	b_i	b_k	γ'
10%	1784	1583	0.89	10%	1784	1546	0.87	10%	1784	1767	0.99	10%	1784	1552	0.87
20%	1784	1521	0.85	20%	1784	1506	0.84	20%	1784	1732	0.97	20%	1784	1510	0.85
30%	1784	1621	0.91	30%	1784	1254	0.7	30%	1784	1677	0.94	30%	1784	1263	0.71
40%	1784	2017	1.13	40%	1784	951	0.53	40%	1784	1607	0.9	40%	1784	967	0.54
50%	1784	2612	1.46	50%	1784	710	0.4	50%	1784	1524	0.85	50%	1784	731	0.41
60%	1784	2175	1.22	60%	1784	554	0.31	60%	1784	1433	0.8	60%	1784	578	0.32
70%	1784	1730	0.97	70%	1784	542	0.3	70%	1784	1315	0.74	70%	1784	571	0.32
80%	1784	1608	0.9	80%	1784	714	0.4	80%	1784	1216	0.68	80%	1784	746	0.42
90%	1784	1569	0.88	90%	1784	-	-	90%	1784	1119	0.63	90%	1784	-	-
Outer 0/4				Inner 0/4 (4/4)				Inner 3/4				Inner 2/4			
Length change	b_i	b_k	γ'	Length change	b_i	b_k	γ'	Length change	b_i	b_k	γ'	Length change	b_i	b_k	γ'
10%	1784	1570	0.88	10%	1982	1763	0.89	10%	1982	1722	0.87	10%	1982	1967	0.99
20%	1784	1536	0.86	20%	1982	1731	0.87	20%	1982	1669	0.84	20%	1982	1924	0.97
30%	1784	1625	0.91	30%	1982	1860	0.94	30%	1982	1404	0.71	30%	1982	1856	0.94
40%	1784	2018	1.13	40%	1982	2316	1.17	40%	1982	1086	0.55	40%	1982	1768	0.89
50%	1784	2615	1.47	50%	1982	2997	1.51	50%	1982	831	0.42	50%	1982	1667	0.84
60%	1784	2096	1.17	60%	1982	2452	1.24	60%	1982	664	0.34	60%	1982	1558	0.79
70%	1784	1698	0.95	70%	1982	1967	0.99	70%	1982	646	0.33	70%	1982	1446	0.73
80%	1784	1594	0.89	80%	1982	1819	0.92	80%	1982	816	0.41	80%	1982	1335	0.67
90%	1784	1514	0.85	90%	1982	1722	0.87	90%	1982	-	-	90%	1982	1229	0.62

Table 3 Single modification factor by step length and location for serviceability design

Outer 4/4				Outer 3/4				Outer 2/4				Outer 1/4			
Length change	b_i	b_k	γ'	Length change	b_i	b_k	γ'	Length change	b_i	b_k	γ'	Length change	b_i	b_k	γ'
10%	2230	1990	0.89	10%	2230	1907	0.86	10%	2230	2205	0.99	10%	2230	1912	0.86
20%	2230	1633	0.73	20%	2230	1851	0.83	20%	2230	2171	0.97	20%	2230	1876	0.84
30%	2230	1635	0.73	30%	2230	1395	0.63	30%	2230	2119	0.95	30%	2230	1468	0.66
40%	2230	2137	0.96	40%	2230	887	0.4	40%	2230	2047	0.92	40%	2230	1003	0.45
50%	2230	3060	1.37	50%	2230	508	0.23	50%	2230	1960	0.88	50%	2230	650	0.29
60%	2230	2416	1.08	60%	2230	290	0.13	60%	2230	1864	0.84	60%	2230	442	0.2
70%	2230	1793	0.8	70%	2230	310	0.14	70%	2230	1705	0.76	70%	2230	469	0.21
80%	2230	1761	0.79	80%	2230	661	0.3	80%	2230	1590	0.71	80%	2230	797	0.36
90%	2230	1931	0.87	90%	2230	-	-	90%	2230	1475	0.66	90%	2230	-	-
Outer 0/4				Inner 0/4 (4/4)				Inner 3/4				Inner 2/4			
Length change	b_i	b_k	γ'	Length change	b_i	b_k	γ'	Length change	b_i	b_k	γ'	Length change	b_i	b_k	γ'
10%	2230	1960	0.88	10%	2478	2220	0.9	10%	2478	2130	0.86	10%	2478	2463	0.99
20%	2230	1642	0.74	20%	2478	1860	0.75	20%	2478	2061	0.83	20%	2478	2418	0.98
30%	2230	1621	0.73	30%	2478	1874	0.76	30%	2478	1597	0.64	30%	2478	2346	0.95
40%	2230	2139	0.96	40%	2478	2463	0.99	40%	2478	1076	0.43	40%	2478	2254	0.91
50%	2230	3125	1.4	50%	2478	3551	1.43	50%	2478	681	0.27	50%	2478	2144	0.87
60%	2230	2335	1.05	60%	2478	2740	1.11	60%	2478	450	0.18	60%	2478	2024	0.82
70%	2230	1813	0.81	70%	2478	2082	0.84	70%	2478	475	0.19	70%	2478	1899	0.77
80%	2230	1800	0.81	80%	2478	2036	0.82	80%	2478	822	0.33	80%	2478	1772	0.72
90%	2230	1830	0.82	90%	2478	2111	0.85	90%	2478	-	-	90%	2478	1647	0.66

Secondly, the modification factor by the increase in step thickness was investigated. The step length of 3,500 mm and step location at the inner 2/4 point were fixed to analyze the analysis result with step thickness as the only variable. The step thickness increased from 210 mm, the same as the thickness of slab, by 10% of the slab thickness, i.e. to 230 mm, 250 mm, etc., in order to attain the effective slab width (b_k) in consideration of the step. Then, its ratio (b_k/b_i) to the effective slab width of flat plate without step (b_i) was computed. Then, the ratio of the new value (b_k/b_i) to the (b_k/b_i) for the standard step thickness of 210 mm, i.e. the step thickness of 0% in Table 4, was used to define a single modification factor (γ') by the change of step thickness. Examining the analysis result, it was found that the lateral stiffness of the slab with step increased with increased thickness of the step.

Thirdly, the modification factor by the increase in step height was investigated. The step length of 3,500 mm and step location at the inner 2/4 point and the first variable analysis were again fixed to analyze the analysis result with step height as the only variable. The step height also increased

Table 4 Single modification factor by step thickness

Thk. change	Strength design				Serviceability design			
	b_i	b_k	b_k/b_i	γ'	b_i	b_k	b_k/b_i	γ'
0%	1982	1667	0.84	1.00	2478	2144	0.87	1.00
10%	1982	1695	0.86	1.02	2478	2167	0.87	1.01
20%	1982	1704	0.86	1.02	2478	2183	0.88	1.02
30%	1982	1712	0.86	1.03	2478	2195	0.89	1.02
40%	1982	1717	0.87	1.03	2478	2203	0.89	1.03
50%	1982	1721	0.87	1.03	2478	2210	0.89	1.03
60%	1982	1724	0.87	1.03	2478	2215	0.89	1.03
70%	1982	1726	0.87	1.04	2478	2219	0.90	1.03
80%	1982	1728	0.87	1.04	2478	2222	0.90	1.04
90%	1982	1730	0.87	1.04	2478	2225	0.90	1.04
100%	1982	1731	0.87	1.04	2478	2227	0.90	1.04

Table 5 Single modification factor by step height

Height change	Strength design				Serviceability design			
	b_i	b_k	b_k/b_i	γ'	b_i	b_k	b_k/b_i	γ'
0%	1982	1667	0.84	1.00	2478	2144	0.87	1.00
10%	1982	1660	0.84	1.00	2478	2135	0.86	1.00
20%	1982	1653	0.83	0.99	2478	2126	0.86	0.99
30%	1982	1646	0.83	0.99	2478	2117	0.85	0.99
40%	1982	1640	0.83	0.98	2478	2108	0.85	0.98
50%	1982	1633	0.82	0.98	2478	2099	0.85	0.98
60%	1982	1626	0.82	0.98	2478	2091	0.84	0.98
70%	1982	1620	0.82	0.97	2478	2082	0.84	0.97
80%	1982	1613	0.81	0.97	2478	2073	0.84	0.97
90%	1982	1607	0.81	0.96	2478	2065	0.83	0.96
100%	1982	1600	0.81	0.96	2478	2056	0.83	0.96

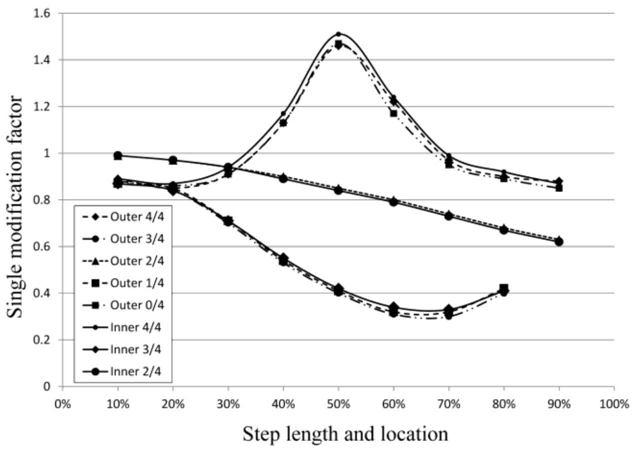


Fig. 14 Trends of single modification factor according to step length and location (strength design)

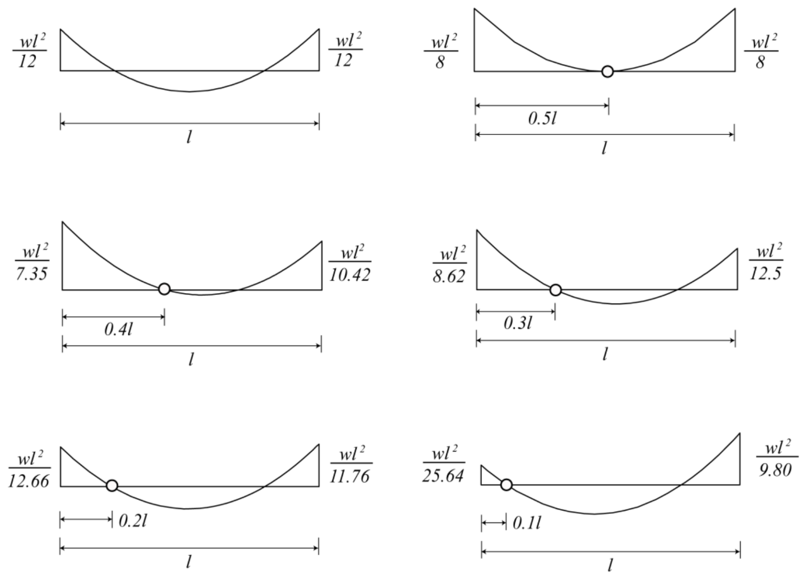


Fig. 15 Moment distribution depending on true hinge location

from 230 mm by 10%, i.e. to 250 mm, 275 mm, etc., in order to attain new effective slab width (b_k) in consideration of the step. Subsequently, the ratio of the new value (b_k/b_i) to the (b_k/b_i) for the standard step height of 230 mm, i.e. the step height of 0% in Table 5, was used to define a single modification factor (γ') by the change of step thickness. Based on the analysis result, it was found that the lateral stiffness of the slab with step decreased with increased height of the step.

It is noted that the values of single modification factor (γ') are sensitive to the variables of step length and location, and there are certain trends regarding these variables as shown in Fig. 14. The large sensitivity is mainly due to the large difference of moment distribution depending on the hinge location, as shown in Fig. 15. The sensitivity of the stiffness to the step thickness and height is not as much as that to the step length and location, as the former is related to the flexural stiffness (*EI*) value while the latter is related to the moment distribution, which is significantly affected by the hinge location and the distance between hinges.

3.2.2 Single modification factors

When several variables such as step length, step location, step thickness and step height are considered in combination, the final modification factor for the step can be computed based on the single modification factor, which was computed in consideration of only one variable at a time. If the single modification factor by step location and length (Table 6), step thickness (Table 7), and step height (Table 8) are denoted as γ'_1, γ'_2, and γ'_3, respectively, then multiplying each of these single modification factors would yield the final modification factor for the step in consideration of these variables as expressed in the following Eq. (3).

$$\gamma = \prod \gamma'_n = \gamma'_1 \times \gamma'_2 \times \gamma'_3 \tag{3}$$

Table 6 Summary of single modification factor by step length and location

Design type	Location of step		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Strength Design	One side of the step connects to the vertical member	1	0.89	0.86	0.92	1.14	1.48	1.21	0.97	0.90	0.87
	Center of the step is also the center of the slab	1	0.99	0.97	0.94	0.90	0.85	0.79	0.73	0.68	0.62
	Center of the step is NOT the center of the slab	1	0.87	0.84	0.71	0.54	0.41	0.32	0.32	0.41	
Serviceability Design	One side of the step connects to the vertical member	1	0.88	0.73	0.73	0.96	1.37	1.05	0.80	0.79	0.82
	Center of the step is also the center of the slab	1	0.99	0.97	0.95	0.91	0.87	0.82	0.76	0.71	0.66
	Center of the step is NOT the center of the slab	1	0.86	0.83	0.63	0.40	0.23	0.13	0.14	0.30	

Table 7 Summary of single modification factor by step thickness

Step thickness / slab thickness	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
Strength design	1	1.02	1.02	1.03	1.03	1.03	1.03	1.04	1.04	1.04	1.04
Serviceability design	1	1.01	1.02	1.02	1.03	1.03	1.03	1.03	1.04	1.04	1.04

Table 8 Summary of single modification factor by step height

Step height / standard height	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
Strength design	1	1	0.99	0.99	0.98	0.98	0.98	0.97	0.97	0.96	0.96
Serviceability design	1	1	0.99	0.99	0.98	0.98	0.98	0.97	0.97	0.96	0.96

Table 9 Complex analysis result in simultaneous consideration of several variables

Variables				Strength Design							Serviceability Design						
Step length	Step location	Step thk.	Step height	γ'_1	γ'_2	γ'_3	γ	b_{cal}	b_k	b_{cal}/b_k	γ'_1	γ'_2	γ'_3	γ	b_{cal}	b_k	b_{cal}/b_k
40%	Outer 4/4	250	230	1.14	1.02	1.00	1.16	2074	2033	1.02	0.96	1.02	1.00	0.98	2184	2163	1.01
40%	Outer 2/4	315	230	0.90	1.03	1.00	0.93	1654	1639	1.01	0.92	1.03	1.00	0.95	2113	2090	1.01
60%	Innter 4/4	355	230	1.21	1.04	1.00	1.26	2494	2485	1.00	1.11	1.03	1.00	1.14	2833	2793	1.01
60%	Outer 3/4	210	275	0.31	1.00	0.99	0.31	548	485	1.13	0.13	1.00	0.99	0.13	229	157	1.46
30%	Outer 1/4	210	345	0.71	1.00	0.98	0.70	1241	1235	1.01	0.66	1.00	0.98	0.65	1442	1357	1.06
30%	Innter 2/4	210	460	0.94	1.00	0.96	0.91	1794	1825	0.98	0.95	1.00	0.96	0.91	2260	2310	0.98
70%	Outer 0/4	250	250	0.95	1.02	1.00	0.97	1729	1726	1.00	0.81	1.02	1.00	0.83	1842	1820	1.01
20%	Inner 2/4	420	460	0.97	1.04	0.96	0.97	1728	1745	0.99	0.97	1.04	0.96	0.97	2160	2190	0.99
50%	Inner 3/4	315	300	0.42	1.03	0.99	0.43	849	952	0.89	0.27	1.03	0.99	0.28	682	833	0.82

3.2.3 Complex analysis and proposal of step modification factor

Using the values of the modification factor computed from Eq. (3) and the research method as described in Section 3.1, the values in consideration of the model with simultaneous multiple variables could be obtained (Table 9). In the table, values of the step modification factor (γ) are presented by multiplying all single modification factors, which were attained by controlling for only one variable at a time. The effective slab width of b_{cal} is given as (b_i x γ) as indicated in Table 9, when b_i is the effective slab width for the slab without step (see Step 1 of Fig. 11). The value of (b_{cal}/b_k) in Table 9 compares the effective slab width b_{cal} with the effective slab width (b_k), which was obtained by applying all variables simultaneously. They were very similar. Nonetheless, the two values differed somewhat noticeably for the cases of very small effective slab width and excessively large step length or height, though these cases are seldom encountered in actual practice. Thus, it is deemed that the method of computing step modification factor for several variables as expressed in Eq. (3) is appropriate.

4. Conclusions

In this study, a modification factor is proposed to reflect the lateral stiffness modification when a step is located in flat plates. The proposal was developed through a systematic analysis of stiffness changes using a linear finite element procedure along with rotational springs, whose stiffness was determined from a series of 3D nonlinear finite element analyses. Based on the analysis results, the following conclusions were drawn.

1) Reduction of stiffness due to the presence of a step in the flat plate can be analyzed with rotational stiffness around the step, and the lateral bending stiffness can be designed by a simplified method of multiplying appropriate step modification factor to existing effective slab width models.

2) The effect of the step is mainly influenced by step length and location, step thickness and step height. This study suggested a single modification factor, which can reflect the influence of each variable. Additionally, one step modification factor (γ) can be computed by multiplying each single modification factor for the case of considering the influence of a combination of several variables.

3) Applying the step modification factor, which was developed and suggested in this study, the result was very similar to the linear analysis result in consideration of several variables simultaneously. Therefore, the reliability of the proposed step modification factor could be ascertained.

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References

ABAQUS (2012), "4.5.2 *Damaged Plasticity Model for Concrete and Other Quasi-Brittle Materials*", ABAQUS Theory Manual, Version 6.12, Dassault Systèmes Simulia Corp., Providence, RI, USA.

American Concrete Institute (ACI) (2012), "Detailing corner: steps in beams", *Concrete Int.*, **34**(6), 41-44.

Architectural Institute of Korea (AIK) (2009), *Korean Building Code and Commentary 2009*, Kimoondang Publishing Company, Korea.

Ellouse, A., Ouezdou, M.B. and Karray, M.A. (2013), "Experimental Study of Steel Fiber Concrete Slabs Part I: Behavior under Uniformly Distributed Loads", *Int. J. Concrete Struct. Mater.*, **4**(2), 181-196.

George, S.J. and Tian, Y. (2012), "Structural Performance of Reinforced Concrete Flat Plate Buildings Subjected to Fire", *Int. J. Concrete Struct. Mater.*, **6**(2), 111-121.

Grossman, J.S. (1997), "Verification of Proposed Design Methodologies for Effective Width of Slabs in Slab-Column Frame", *ACI Struct. J.*, **94**(2), 181-196.

Kaynia, A.M. (2012), "Dynamic response of pile foundations with flexible slabs", *Earthq. Struct.*, **3**(3), 495-506.

Korea Concrete Institute (KCI) (2012), *Concrete Design Code*, Kimoondang Publishing Company, Korea.

Korea Land & Housing Corporation (KLHC), Bureau of Mechanical Design (2012), *Establishment of Design Standard for Drainage Layout at the Floor for the Noise Reduction in the Bathroom (Proposal)*, Report of Korea Land & Housing Corporation, Korea.

Lu, X., Urukap, T.H., Li, S. and Lin, F. (2012), "Seismic behavior of interior RC beam-column joints with additional bars under cyclic loading", *Earthq. Struct.*, **3**(1), 37-57.

Song, J.K., Kim, J., Song, H.B. and Song, J.W. (2012), “Effective punching shear and moment capacity of flat plate-column connection with shear reinforcements for lateral loading”, *Int J. Concrete Struct. Mater.*, **6**(1), 19-29.