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Experimental and analytical studies on stochastic seismic response control of structures with MR dampers

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Abstract. The magneto-rheological (MR) damper contributes to the new technology of structural vibration control. Its developments and applications have been paid significant attentions in earthquake engineering in recent years. Due to the shortages, however, inherent in deterministic control schemes where only several observed seismic accelerations are used as the trivial input and in classical stochastic optimal control theory with assumption of white noise process, the derived control policy cannot effectively accommodate the performance of randomly base-excited engineering structures. In this paper, the experimental and analytical studies on stochastic seismic response control of structures with specifically designed MR dampers are carried out. The random ground motion, as the base excitation posing upon the shaking table and the design load used for structural control system, is represented by the physically based stochastic ground motion model. Stochastic response analysis and reliability assessment of the tested structure are performed using the probability density evolution method and the theory of extreme value distribution. It is shown that the seismic response of the controlled structure with MR dampers gain a significant reduction compared with that of the uncontrolled structure, and the structural reliability is obviously strengthened as well.

Keywords: shaking-table test; magneto-rheological damper; random ground motion; probability density evolution method; dynamic reliability

1. Introduction

Various kinds of natural disasters pose a great threat to the safety of civil engineering structures. Since modern control theory was brought into the field of civil engineering by Yao (1972), structural vibration control in modality of passive control, active control, semi-active control and hybrid control has received increasing attention for protection of civil engineering structures against natural hazards such as severe earthquakes and strong winds (Housner *et al.* 1997, Symans and Constantinou 1999, Jung *et al.* 2002, Zhang *et al.* 2009, Hosseini and Farsangi 2012). Among the types of control devices used in structural controls, the magneto-rheological (MR) damper is

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regarded as one of the most promising classes due to its perfect dynamic damping behaviors (Jung *et al.* 2004). So far, MR dampers have been used in full-scale applications. The first implementation of MR dampers into civil engineering applications, as exposed in the reference, appeared in Japan (Spencer and Nagarajaiah 2003). The Tokyo National Museum of Emerging Science and Innovation was mounted with two 30-ton MR dampers between the third and fifth floors towards reducing the seismic responses. The Dongting Lake Bridge in Hunan, China, to the best of our knowledge, constitutes to the first full-scale application of MR dampers into bridge structures, where the rain-wind induced cable vibration is mitigated by the suitably placed MR dampers (Ni *et al.* 2002, Ko *et al.* 2002).

A great number of experimental investigations and numerical simulations on vibration control of MR damping structural systems under earthquake actions have been conducted in recent years. Only several observed seismic accelerations, however, are used as the trivial input in most existing researches (Dyke *et al.* 1996, Nagarajaiah *et al.* 2000, Kim *et al.* 2006, Lee *et al.* 2008, Lin and Loh 2008, Jung *et al.* 2009). The efficiency of vibration mitigation of structures with control is usually presented by the reduction of dynamic responses (particularly referring to peak displacements or peak accelerations) compared with those without control (Spencer *et al.* 1998a, 1998b, 1998c, Ohtori *et al.* 2004). It should be noted that the earthquake shock is a random event in nature and its effect on structures should be assessed using probabilistic methods. Therefore, the randomness involved in the seismic excitations cannot be neglected and the control effect of the attaching dampers to structures should be captured in the sense of probability.

Regarding the vibration control of structural systems under stochastic dynamic excitations, the linear quadratic Gaussian (LQG) control is a widely employed scheme (Ankireddi and Yang 1997, Wu and Yang 2000). While for the stochastic control of nonlinear structural systems with quasi Hamiltonian behaviors, a class of nonlinear stochastic optimal control strategies were proposed and developed by Zhu and his co-workers based on the stochastic averaging method and the dynamical programming principle (Zhu and Ying 1999, Zhu *et al.*2001). It is noted that these stochastic optimal control strategies exclusively hinge upon Itô-type stochastic differential equations, of which the random disturbance term specifying external excitations and measurement noises is mathematically assumed to be Gaussian white noise or filtered Gaussian white noise. The practical dynamic excitations, however, posing upon the civil engineering structure, e.g. earthquake ground motions and strong winds, are usually non-stationary and non-Gaussian white noise process. Due to the shortages inherent in deterministic control schemes and classical stochastic optimal control theory, the derived control policy cannot effectively accommodate the performance of randomly base-excited engineering structures. In order to break through the barriers, a physically informed stochastic optimal control scheme, in context of the theoretical framework of probability density evolution method (PDEM), for linear and nonlinear structural systems subjected to practical random excitations has been systematically developed and numerically studied by Li *et al.* (Li *et al.* 2010, 2011a, 2011b, Peng and Li 2011, Peng *et al.* 2013).

This paper is devoted to investigate the efficiency of seismic response control of structures with MR dampers considering the randomness involved in base-driven excitations. The sections arranged in this paper are distributed as follows. Section 2 is dedicated to introduce a physically based stochastic ground motion model, whereby a series of representative seismic ground acceleration for shaking-table tests are simulated. Section 3 details the experimental setup and experimental program relevant to the shaking-table test of stochastic seismic response control of a six-story steel frame installed with MR dampers. Experimental results are presented and discussed in Section 4 in terms of some probabilistic quantities, as the peak responses and root mean square (RMS) responses. In Section 5 and Section 6, stochastic response analysis and reliability assessment of the experimental structures with and without MR dampers are performed using the probability density evolution method and the theory of extreme value distribution. The concluding remarks are included in the final section.

2. Seismic ground motions for shaking-table tests

Understanding from physical perspectives, the randomness inherent in seismic ground motions derives from three factors: focal mechanism of earthquake, propagation paths of seismic waves, and soil properties of local site (Wang and Li 2011). For a certain engineering site, the ground motion at the bedrock can be described into a random function through the seismic hazard assessment considering the properties of the earthquake source and the propagation paths (Dowrick 2003), whereby a physically based stochastic ground motion model is readily proposed in conjunction with soil properties of the site (Li and Ai 2006).

For the sake of clarity, the engineering site is modeled as a stochastic single-degree-of-freedom (SDOF) system. Using the ground motion at the bedrock as the input, the absolute response of the SDOF system is namely the process of the ground motion at the surface of the engineering site. The equation of motion of the SDOF system can be written as

$$\ddot{x}_{\mathrm{g}} + 2\zeta_0\omega_0\dot{x}_{\mathrm{g}} + \omega_0^2 x_{\mathrm{g}} = 2\zeta_0\omega_0\dot{u}_{\mathrm{b}} + \omega_0^2 u_{\mathrm{b}} \tag{1}$$

where $\ddot{x}_{\mathrm{g}}$, $\dot{x}_{\mathrm{g}}$, x_{g}, denote the absolute acceleration, absolute velocity, and the absolute displacement at the surface of the engineering site, respectively; ζ_0, ω_0 denote the equivalent damping ratio and the predominant frequency of the site soil, respectively; $\dot{u}_{\mathrm{b}}$, u_{b}, denote the velocity and displacement of the ground motion at the bedrock, respectively.

Operating Fourier transform on both sides of Eq. (1), one could get the frequency-domain equation of the SDOF system in acceleration argument

$$\ddot{X}_{\mathrm{g}}(\omega) = H_{\mathrm{g}}(\mathrm{i}\omega)\times\ddot{U}_{\mathrm{b}}(\omega) = \frac{\omega_0^2 + 2\mathrm{i}\zeta_0\omega_0\omega}{\omega_0^2 - \omega^2 + 2\mathrm{i}\zeta_0\omega_0\omega}\ddot{U}_{\mathrm{b}}(\omega) \tag{2}$$

where $\ddot{U}_{\mathrm{b}}(\omega)$ and $\ddot{X}_{\mathrm{g}}(\omega)$ are the Fourier transforms of $\ddot{u}_{\mathrm{b}}$ and $\ddot{x}_{\mathrm{g}}$, respectively; i denotes the unit length of imaginary number $\sqrt{-1}$. The frequency transfer function of the system can be expressed as

$$H_{\mathrm{g}}(\Theta_{\omega_0}, \Theta_{\zeta_0}, \omega) = \frac{\Theta_{\omega_0}^2 + 2\mathrm{i}\Theta_{\zeta_0}\Theta_{\omega_0}\omega}{\Theta_{\omega_0}^2 - \omega^2 + 2\mathrm{i}\Theta_{\zeta_0}\Theta_{\omega_0}\omega} \tag{3}$$

Due to the random nature of site soil, the predominant frequency of the engineering site ω_0 and the equivalent damping ratio ζ_0 are regarded as random parameters, denoted by $\Theta_{\omega 0}$ and $\Theta_{\zeta 0}$ in Eq. (3), respectively. It is noted that the probability distributions of the two random parameters can be identified by investigating a 1arge number of earthquake ground motion records. Statistical result indicates that $\Theta_{\omega 0}$ and $\Theta_{\zeta 0}$, for a certain type of site soil, admit the lognormal distribution with the mean of 20 rad/sec and 0.7, and with the coefficient of variation of 0.4 and 0.3, respectively (Li and Ai 2006). Through the seismic hazard analysis (Dowrick 2003), the ground motion at the bedrock is described into the random function

$$\ddot{U}_{\mathrm{b}}(\omega) = G_{\mathrm{b}}(\Theta_{\mathrm{b}}, \omega) \tag{4}$$

where Θ_{b} is the random parameter characterizing the randomness involved in the ground motion at the bedrock coming from the earthquake source and the propagation paths. The random parameter Θ_{b} also admits lognormal distribution with the mean of 0.25 $\mathrm{m{\cdot}s^{-1/2}}$ and the coefficient of variation of 0.5, respectively. The random function $G_{\mathrm{b}}(\Theta_{\mathrm{b}}, \omega)$ denotes the ground motion at the bedrock which is mathematically assumed to be band-limited white noise with spectral intensity in the random parameter Θ_{b} (Li and Ai, 2006).

Then, Eq. (2) can be expressed as

$$\ddot{X}_{\mathrm{g}}(\omega) = F(\Theta, \omega) = H_{\mathrm{g}}(\Theta_{\omega_0}, \Theta_{\zeta_0}, \omega) \cdot G_{\mathrm{b}}(\Theta_{\mathrm{b}}, \omega) \tag{5}$$

where $\Theta = (\Theta_{\omega 0}, \Theta_{\zeta 0}, \Theta_{\mathrm{b}})$ denotes the random parameter vector characterizing the randomness involved in the ground motion. The time history of the random ground motion thus could be obtained by the inverse Fourier transform, i.e.

$$\ddot{x}_{\mathrm{g}}(\Theta, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \ddot{X}_{\mathrm{g}}(\Theta, \omega) e^{\mathrm{i}\omega t} \mathrm{d}\omega \tag{6}$$

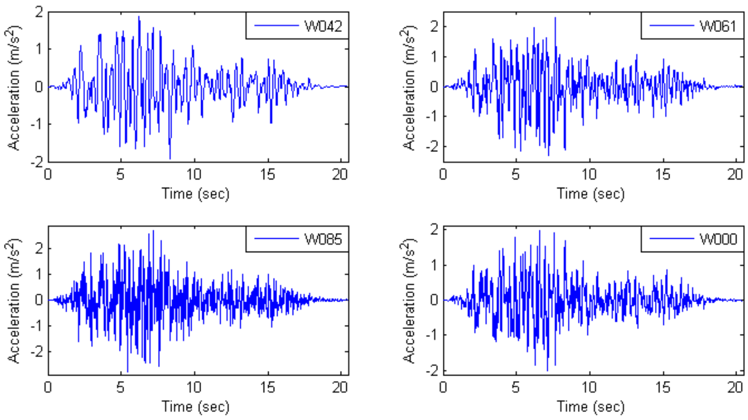
The samples of ground accelerations can be generated by introducing the statistical information into the physically based stochastic ground motion model Eq. (6) and employing the tangent-spheres strategy for selecting representative points (Chen and Li 2008). In this study, 120 representative ground accelerations and the ground acceleration valued by the mean of site parameters, namely, $\Theta_{\omega 0}$ = 20 rad/s, $\Theta_{\zeta 0}$ = 0.7 and Θ_{b} = 0.25 $\mathrm{m{\cdot}s^{-1/2}}$, are included. The peak of the mean-valued ground acceleration is 2.00 $\mathrm{m/s^2}$, which corresponds to the design acceleration of seismic ground motions within the regions of seismic fortification intensity 8, according to the Chinese code for seismic design of buildings (GB 50011-2010).

It is noted that the peak accelerations as well as the frequency spectrum characteristics of these representative ground accelerations differ from each other. The reason for this difference is that for different sample of ground accelerations, the values of the three random parameters; say $\Theta_{\omega 0}$, $\Theta_{\zeta 0}$ and Θ_{b} involved in the stochastic ground motion model are not the same. The statistical properties of peak accelerations of these ground motions are given in Table 1. It is seen that the mean and coefficient of variation (COV) of peak accelerations are 2.18 $\mathrm{m/s^2}$ and 0.26, respectively. The maximum value, moreover, of peak accelerations is 4.61 $\mathrm{m/s^2}$ which features 6 times the minimum

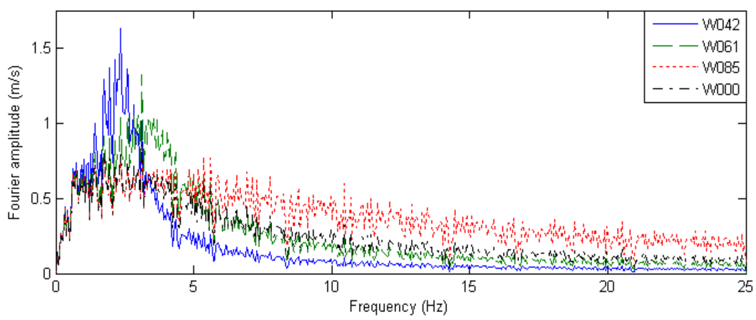
Table1 Statistical values of peak accelerations of sample ground motions

Statistic quantity	Min	Max	Mean	COV
Value ($\mathrm{m/s^2}$)	0.78	4.61	2.18	0.26

Min = minimum; Max = maximum; COV = coefficient of variation



(a)



(b)

Fig.1 Time histories and Fourier amplitude spectra of typical ground accelerations: (a) time histories of ground accelerations; (b) Fourier amplitude spectra of ground accelerations

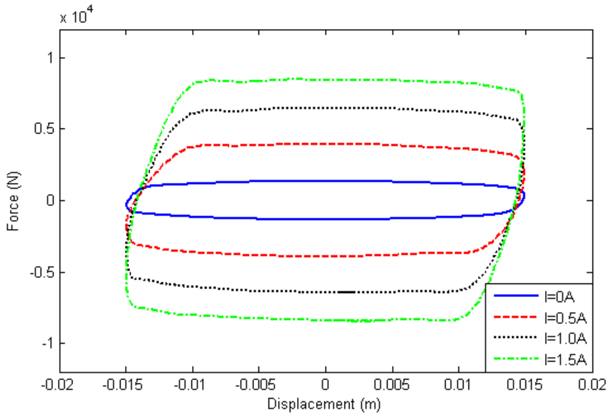
value 0.78 m/s^2. Three typical ground accelerations, labeled W042, W061, W085, and their Fourier amplitude spectra are respectively shown in Fig. 1. It is indicated that their time- and frequency-domain characteristics are quite different. Besides, the mean-valued ground acceleration, labeled W000, and its Fourier amplitude spectrum are also pictured in Fig. 1.

3. Shaking-table tests on structural model

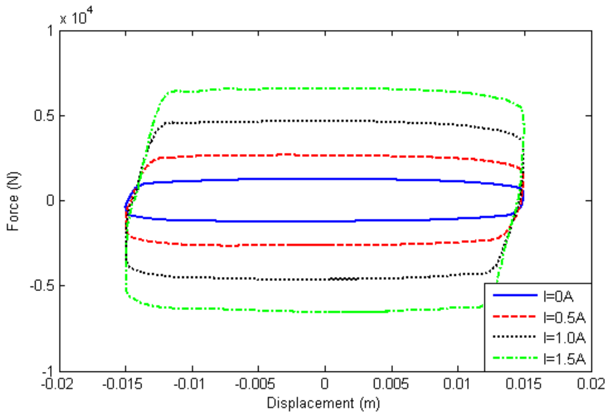
3.1 Experimental setup

Experimental investigations of shaking-table test on a structural model with MR dampers were performed in the State Key Laboratory of Disaster Reduction in Civil Engineering at Tongji University, China. The experimental facility includes a 4.0 m × 4.0 m MTS shaking table with a capacity of 2.5×10^4 kg. The motion of the shaking table involves *X*, *Y*, *Z* three spatial dimensions and six degree-of-freedoms. In case of bearing 1.5×10^4 kg specimen, the maximum accelerations exerted on the horizontal direction of the table, *X* and *Y*, are up to 1.2 g and 0.8 g, respectively.

Fig. 2 Photograph of the tested structure with MR dampers



(a)



(b)

Fig. 3 Typical hysteresis curves of MR dampers with type of MRD-100-10: (a) MRD-A and (b) MRD-B

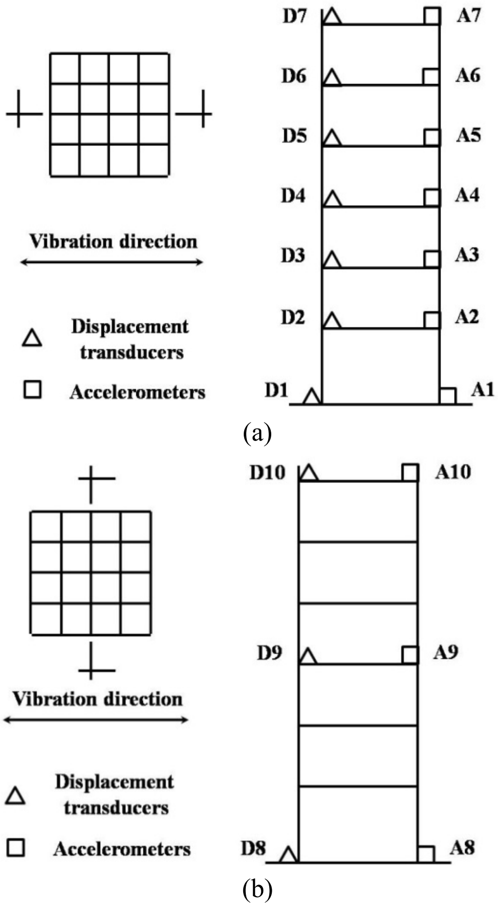


Fig. 4 Setup of transducers on the tested structural model: (a) along the direction of table motion; (b) across the direction of table motion

The tested model used in this experiment is a six-story and single-bay steel structure, as shown in Fig. 2, which is designed to be a 1/5-scale model of a prototype steel structure. The structural model is of 1.6 m × 1.6 m in projection of plane, and is of 1.0 m height in the first story and 0.8 m height in the other stories. Its total mass is 10.0 ton including 7.2 ton artificial mass distributed evenly on the six floors, allowing for the similar dynamic behaviors to the prototype structure. The beam member consists of channel steel with section height 63 mm, and the pillar member consists of channel steel with section height 80 mm. The material both of beams and pillars is of Q345 steel of which the nominal yield strength is 345 MPa. The floor member consists of steel plate with thickness 10.0 mm while its material is of Q235 steel of which the nominal yield strength is 235 MPa. The model-to-prototype ratios of time, force, mass, displacement and acceleration are 0.4472, 0.04, 0.04, 0.2 and 1, respectively.

Through a numerical optimization analysis, using the commercial software ANSYS, of tested structural model exposed to the representative seismic excitations, the parameters of MR damper including its maximum output and stroke range are scheduled. Two dampers with the same design parameters, marked by MRD-A and MRD-B, in type of MRD-100-10 are employed, as shown in

Fig. 2. The force output that each damper could provide is up to 10 kN. The damper is of 72.5 cm in length, and is of 10.0 cm in diameter of the main cylinder. It has a ±5.5 cm stroke and the input current is in the range of 0-2A. The force outcome of the device is stable in the range of –40-60 ° C, and its total weight is approximately 20 kg. The typical hysteresis curves of the two MR dampers are shown in Fig. 3, respectively. It is seen that the hysteretic curves are very plump indicating that both MRD-A and MRD-B exhibit a perfect energy-dissipation capability. Simultaneously, it is readily observed that the maximum damping force increases as the applied current increases in the range of 0 to 1.5A. That is to say, the damping force of the two MR dampers can be regulated by adjusting the input current.

In the shaking-table test, the data are recorded by an automatic data acquisition system. Two accelerometers and two displacement transducers are employed to measure the motion of the shaking table in the crossing direction of plane. Simultaneously, six accelerometers and six displacement transducers are mounted to measure the dynamic responses of each floor along the direction of table motion, and two accelerometers and two displacement transducers are mounted to measure the dynamic responses of the third and sixth floors across the direction of table motion. The setup of these transducers is shown in Fig. 4.

3.2 Experimental program

The time histories of representative ground accelerations detailed in Section 2 are used as the one-dimensional input of shaking table. These ground accelerations must be reproduced as the time-scale factor since the tested structure under consideration is a scaled model. It should be noted that the intensities of the representative ground accelerations are not be valued by sufficiently high peak accelerations so that the mechanical characteristics of the structural model remain unchanged in the process of shaking-table tests. The purpose of this treatment is to expose the influence of the randomness involved in seismic excitations by excluding other possible influential factors. Although nonlinear mechanics of structural components are usually allowed in the traditional shaking-table tests, e.g., the shaking-table test conducted by Chang and his colleagues where the inelastic behavior of the tested model was permitted (Chang *et al.* 2008), this feature is not included in the present investigation since the coupling effect between nonlinearity and randomness would result in extremely complicated dynamic behaviors of structures, and distort the rationality of experimental investigations. Therefore, the inelastic behavior is not allowed in the tested structure. Besides, dynamic responses of the tested model should not be too small to gain an obvious reduction of structural vibrations in case of control with MR dampers. Thus trade-off between the minimum nonlinear response of the structure and the maximum control gain of the MR dampers is achieved whereby the peak accelerations of the input ground motions are designed; see Table 1.

There are total 242 operating cases as shown in Table 2. In the uncontrolled cases (Cases 1 and 242), the ground acceleration W000 serves as the seismic input, and the experimentally measured frequency response functions (FRFs) are used for the parameter identification of the tested structure so as to validate the invariant dynamical behaviors of the structural model. In the passive-on control cases (Cases 2-121), the MRD-A is placed between the ground and first floor, and the MRD-B is placed between the second and third floors of the model structure. The current applied to the two MR dampers are held fixing at 1.5A. It is noted that the arrangements of MR dampers are determined through an optimization scheme (Li *et al.* 2010), in which the optimization objective is the minimization of the mean value of peak inter-story drifts of the tested

Table 2 Cases of experimental program

Case number	Seismic input	Amplitude (m/s^2)	Remark
1	W000	1.00	Without control
2-121	W001-W120	Statistical values as shown in Table 1	Passive-on control (with MR dampers)
122-241	W001-W120	Half of the amplitudes in Cases 2-121, respectively	Without control
242	W000	1.00	Without control

Table 3 The first six natural frequencies of tested structure without control

Case number	Natural frequencies of first six modes (Hz)					
	1	2	3	4	5	6
1	1.460	4.624	8.365	12.452	16.803	20.277
242	1.453	4.605	8.338	12.385	16.745	20.184

structure. In the uncontrolled cases (Cases 122-241), the steel structural model without MR dampers is tested. One might realize that the amplitudes of seismic inputs in Cases 122-241 are just half of those in Cases 2-121, respectively. This treatment is to secure the time-independent mechanical characteristics of the structural model in the process of shaking-table tests. Comparing with the dynamic responses of the experimental structure with and without MR dampers, the control effectiveness of the dampers can be gained.

3.3 Validation of linear elastic state of tested structure

Using the FRFs integrated from the data measured in the experiment, the dynamic characteristics of the tested structure without control can be identified. In the two cases of uncontrolled structures subjected to the mean-valued ground accelerations W000, the experiment is carried out with relatively small peak ground acceleration whereby the slightly visual vibration is observed.

According to the measured FRFs, the first six natural frequencies of the tested structure without control are identified and the results are listed in Table 3. It is seen that the first six natural frequencies is only slightly changed throughout all the cases of the experiment. For instance, the first natural frequency is changed from 1.460 Hz to 1.453 Hz; see the natural frequencies of Cases 1 and 242 in Table 3, only decreases by about 0.48%. As a result, it has reason to believe that the tested structure itself still remains within the range of linear elastic state in the process of shaking-table tests.

As mentioned previously, the amplitudes of seismic inputs in the shaking-table test for Cases 122-241 (without control) are only half of those for Cases 2-121 (passive-on control) so as to bypass the situation that the overlarge response of uncontrolled structure leads to an unexpected nonlinear structural behavior. In order to investigate the efficiency of vibration control with the same loading level, the experimentally measured data of the tested structure without MR dampers is doubled on its value based on the fact that the structural model remains in the linear state.

Table 4 Statistical values of peak responses of controlled and uncontrolled structures

Peak responses			Story level					
			1	2	3	4	5	6
Inter-story drift ratio	Mean	Unc.	0.014	0.011	0.010	0.009	0.008	0.006
		Con.	0.006	0.006	0.005	0.005	0.004	0.003
		Eff. *	57.1%	45.4%	50.0%	44.4%	50.0%	50.0%
	COV	Unc.	0.15	0.14	0.16	0.20	0.17	0.16
		Con.	0.08	0.08	0.14	0.09	0.10	0.13
		Eff. *	46.7%	42.9%	12.5%	55.0%	41.2%	18.8%
Story accelerations	Mean (m/s^2)	Unc.	3.82	3.53	3.54	3.54	3.94	5.17
		Con.	3.55	3.12	3.52	3.80	3.73	4.65
		Eff. *	7.1%	11.6%	0.6%	-7.3%	5.3%	10.1%
	COV	Unc.	0.23	0.18	0.20	0.24	0.17	0.17
		Con.	0.18	0.16	0.26	0.16	0.17	0.17
		Eff. *	21.7%	11.1%	-30.0%	33.3%	0.0%	0.0%

* Efficiency is defined as (Unc.-Con.)/Unc.; unc. = uncontrolled; con. = controlled; eff. = efficiency; COV = coefficient of variation.

4. Experimental data based control efficiency analysis

4.1 Peak responses

The statistical values of peak responses of the controlled (Con.) and uncontrolled (Unc.) structures are given in Table 4, including those of peak inter-story drift ratio that is defined as the difference in lateral displacements between two consecutive floors normalized by the inter-story height, and of peak story acceleration. It is seen that the mean value of peak inter-story drift ratio is averagely reduced by 48.8%. Especially for the first and third floors where the MR dampers mounted, the mean values are cut down by 59.0% and 51.5%, respectively. The coefficient of variation of peak inter-story drift ratio of each story in the controlled cases is also decreased to some extent, compared to those in the uncontrolled cases. The mean and coefficient of variation, meanwhile, of peak story acceleration of most stories are reduced in the controlled cases. However, the mean value of peak story acceleration of the fourth floor with control is even greater than that without control, and that is also true for the coefficient of variation of peak story acceleration of the third floor. It is understood that the stiffness of supports, with which the MR dampers are connected, has a negative impact on the acceleration responses of the controlled structure. This influence would be significant in case of small inter-story drifts where the damper-support systems mainly serve as brace components providing a stiffness contribution to the structural dynamics. Moreover, it is explained that the optimal placement of MR dampers just hinges upon the minimization of mathematical expectation of peak inter-story drifts of tested structure, which might have disadvantageous influence upon the acceleration responses of the structure. The control mode, however, results in that the story acceleration varies more smoothly along with the height of the structure, meeting with the thrust of performance control of structures (Peng *et al.* 2013).

Table 5 Statistical values of RMS responses of controlled and uncontrolled structures

RMS responses			Story level					
			1	2	3	4	5	6
Inter-story drift ratio	Mean	Unc.	0.006	0.005	0.004	0.003	0.002	0.002
		Con.	0.002	0.002	0.001	0.001	0.001	0.001
		Eff.*	66.7%	60.0%	75.0%	66.7%	50.0%	50.0%
	COV	Unc.	0.15	0.14	0.14	0.14	0.13	0.19
		Con.	0.08	0.09	0.12	0.08	0.08	0.07
		Eff.*	46.7%	35.7%	14.3%	42.9%	38.5%	63.2%
Story accelerations	Mean (m/s^2)	Unc.	0.82	0.92	1.01	1.12	1.19	1.38
		Con.	0.72	0.70	0.74	0.74	0.81	0.98
		Eff.*	12.2%	23.9%	26.7%	33.9%	31.9%	29.0%
	COV	Unc.	0.18	0.14	0.15	0.15	0.13	0.14
		Con.	0.13	0.10	0.09	0.05	0.06	0.07
		Eff.*	27.8%	28.6%	40.0%	66.7%	53.8%	50.0%

* Efficiency is defined as (Unc.-Con.)/Unc.; unc. = uncontrolled; con. = controlled; eff. = efficiency; COV = coefficient of variation.

4.2 RMS responses

The statistical values of RMS responses of the controlled and uncontrolled structures along height of the structure are listed in Table 5. It is seen that the mean and coefficient of variation of RMS inter-story drift ratio are averagely reduced by 62.6% and 40.2%, respectively, and those of the first floor are reduced by 69.0% and 46.7%, respectively. A similar capacity towards the mitigation of RMS story accelerations is also clearly exposed: the mean value of RMS story acceleration of each story decreases by 12.2%-33.9%, and the coefficient of variation decreases by 27.8%-66.7%.

Compared with the peak responses, the reduction of RMS responses is more remarkable. It is explained that the qualification of structural responses might be different from the defined norms. Peak responses are calculated in the sense of the ∞-norm while RMS responses are calculated in the sense of the 2-norm. The former is appropriate for the problems related to first-passage reliability, whereas the latter is appropriate for the problems related to failure reliability with accumulated damage.

4.3 Damping characteristics of controlled structure

In order to investigate the damping characteristics of the controlled and uncontrolled structural systems, typical responses of the tested structure subjected to the ground acceleration W085 are presented. Time histories of inter-story drift of the first floor and story acceleration of the top floor of the structure with and without MR dampers are pictured in Fig. 5. It is clearly seen that the control action in passive modality is capable of mitigating both inter-story drift and story acceleration. The peak value of inter-story drift of the first floor, for instance, is reduced by 64.7%, as shown in Fig. 5(a), which owes to the additional damping provided by the two MR dampers.

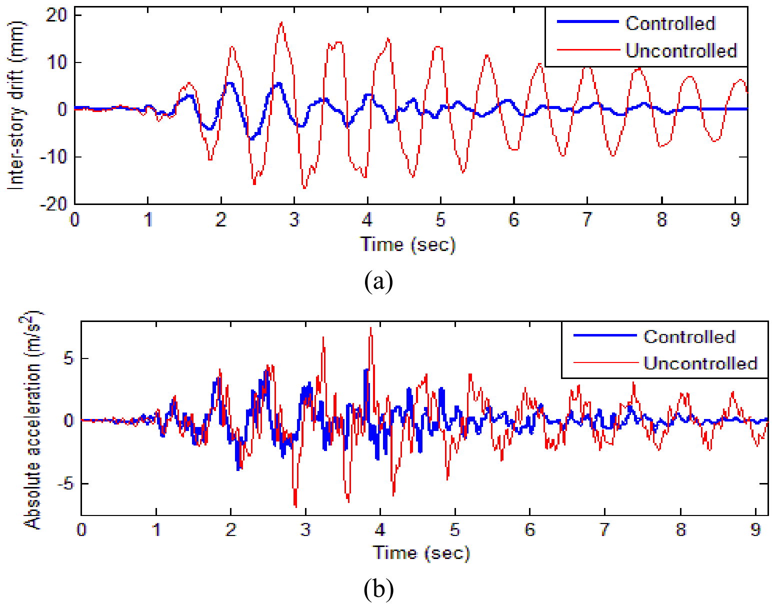


Fig. 5 Typical responses of tested structure subjected to ground acceleration W085: (a) inter-story drift of the first floor and (b) story acceleration of the top floor

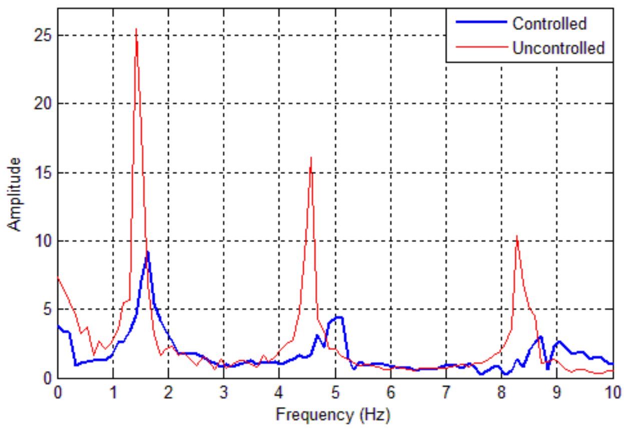


Fig. 6 Amplitude-frequency curves of story acceleration of the top floor of tested structure

The amplitude-frequency characteristics of story acceleration of the tested structure subjected to the ground acceleration W085 is presented for providing a quantitative description of the damping characteristics of the controlled and uncontrolled structural systems, as shown in Fig. 6. The damping ratios of the first two vibrational modes of the structure with and without dampers can be identified by using the half-power-point method (Clough and Penzien 1993). The results show that the damping ratios of the first two vibrational modes of the controlled structure are 11.6 % and 5.3%, respectively, while those of the uncontrolled structure are only 1.2% and 0.7%, respectively. One might realize that the involvement of MR dampers results in a remarkable strengthening upon structural capacity of dissipation energy. The damping ratio, moreover, of the first vibrational mode deduced from the experimental response is very close to the design value 12.0%, indicating that the damping system can achieve the expected control gain.

5. Stochastic response analysis using probability density evolution method

In Section 4, statistical values of peak and RMS responses of the tested structure with and without control are presented, while the probability density distributions for complete description of stochastic responses of structures are still unsolved, which might result in an inaccurate assessment with regard to the structural safety. In this section, stochastic response analysis of the controlled and uncontrolled structures is carried out using the probability density evolution method (PDEM), of which the kernel is the generalized density evolution equation. The advantages of PDEM expose to be that it can readily provide the instantaneous probability density function of system response in high dimensions, and its computational effort, in case of the same calculation accuracy, is far less than that of Monte Carlo simulations (Li and Chen 2009).

5.1 Generalized density evolution equation

With discretization techniques such as the finite element method, the equation of motion of the damped structure subjected to random seismic ground motions reads

$$M\ddot{X}+f(\dot{X},X)=-ME\ddot{x}_{\mathrm{g}}(\Theta,t)\ ,\ \dot{X}_0=\dot{x}_0, X_0=x_0 \tag{7}$$

where $X=(X_1,X_2,\cdots,X_n)^{\mathrm{T}}$ denotes n by 1 displacement vector; n denotes the number of degrees of freedom of the structural system considered after discretization; overdots denote differentiation with regard to t; M denotes n by n mass matrix; f denotes internal forces of the structures including the damping and restoring forces; E denotes a n-order column vector with all the components being 1. The joint probability density function (PDF) of Θ is identified as $P_\Theta(\theta)$. $\dot{x}_0$, x_0 denote the initial velocity and initial displacement vectors, respectively.

It is evident that any arbitrary response of interest $X(t)$ depends upon Θ and could be written in the form of a function as

$$X=H(\dot{x}_0,x_0,\Theta,t) \tag{8}$$

Using the principle of preservation of probability, one can deduce the generalized density evolution equation governing the joint PDF of (X,Θ) (Li and Chen 2008, 2009)

$$\frac{\partial p_{X\Theta}(x,\theta,t)}{\partial t}+\dot{X}(\theta,t)\frac{\partial p_{X\Theta}(x,\theta,t)}{\partial x}=0 \tag{9}$$

where $p_{X\Theta}(x,\theta,t)$ indicates joint PDF of (X,Θ);$\dot{X}(\theta,t)$ indicates the velocity of $X(t)$ under the condition $\{\Theta=\theta\}$.

The initial condition for Eq. (9) reads

$$p_{X\Theta}(x,\theta,t)\big|_{t=0}=\delta(x-x_0)\,p_\Theta(\theta) \tag{10}$$

where $\delta(\cdot)$ denotes the Dirac's delta function.

The joint PDF $p_{X\Theta}(x,\theta,t)$ could be given by solving the initial-value partial differential Eqs. (9) and (10) using the numerical procedures (Li and Chen 2008). The instantaneous PDF of $X(t)$ could be obtained by

$$p_X(x,t) = \int_{\Omega_\Theta} p_{X\Theta}(x,\theta,t)d\theta \tag{11}$$

where Ω_Θ indicates the distribution domain of Θ. It is indicated that Eq. (11) really denotes the marginal distribution of the random variable $X(t)$.

5.2 Numerical procedure solving generalized density evolution equation

To get instantaneous PDFs and other probabilistic indices of responses, a collection of equations consisting of the motion Eq. (7), the generalized density evolution Eq. (9) and the integration Eq. (11) are to be solved. The numerical implementation involves the following steps:

Step 1: Select representative points $\theta_q(q = 1,2,\cdots,N_{\mathrm{sel}})$ in the domain Ω_Θ, where N_{sel} is the total number of the selected points;

Step 2: For the prescribed $\Theta = \theta_q$, solve Eq. (7) with a deterministic time integration method to evaluate the value of $\dot{X}(\theta_q,t_m)$, where $t_m = m\mathrm{D}t(\mathrm{m} = 0,1,2,\cdots)$, $\mathrm{D}t$ is the time step;

Step 3: Introduce $\dot{X}(\theta_q,t_m)$ into the generalized density evolution Eq. (9), and solve Eq. (9) under the initial condition Eq. (10) using the finite difference method whereby the component of numerical solution $p_{X\Theta}(x,\theta,t)$, denoted by $p_{X\Theta}(x_j,\theta_q,t_k)$ is obtained, here $x_j = x_0 + j\mathrm{D}x(j = 0,\pm1,\pm2,...)$; $\mathrm{D}x$ is the space step; $t_k = k\mathrm{D}t_s\,(k = 0,1,2,...);\, \mathrm{D}t_s$ is the time step in the finite difference method;

Step 4: Repeat steps 2 and 3 running over $q = 1,2,..., N_{\mathrm{sel}}$, and then take numerical integration in Eq. (11) to calculate the numerical solution of $p_X(x,t)$.

5.3 Stochastic response of structure with and without control

Employing the PDEM outlined above, the stochastic response of the tested structure with and without control is analyzed. The time histories of mean and standard deviation of inter-story drift of the first floor and those of story accelerations of the top floor are shown in Fig. 7, respectively. It is seen that the mean and standard deviation of inter-story drift and story acceleration with and without control are all non-stationary processes. The standard deviation process of inter-story drifts without control, for instance, possesses a stage of larger amplitude (the time interval from 2 sec to 5 sec) and then tends to a stage with smaller amplitude. The amplitude level of the mean, meanwhile, is much smaller than that of corresponding standard deviation. It is explained that the tested structure in the experiment responses to the random seismic ground motions with near-zero mean.

One might see that the inter-story drift of the controlled structural system is reduced significantly compared with that of the uncontrolled structural system, especially in the time interval with stronger variability, i.e. the time interval from 2 sec to 5 sec. However, the mitigation of story acceleration of the top floor is much smaller than that of inter-story drift of the first floor, as presented in Fig. 7. Meanwhile, the story acceleration with control is even amplified compared

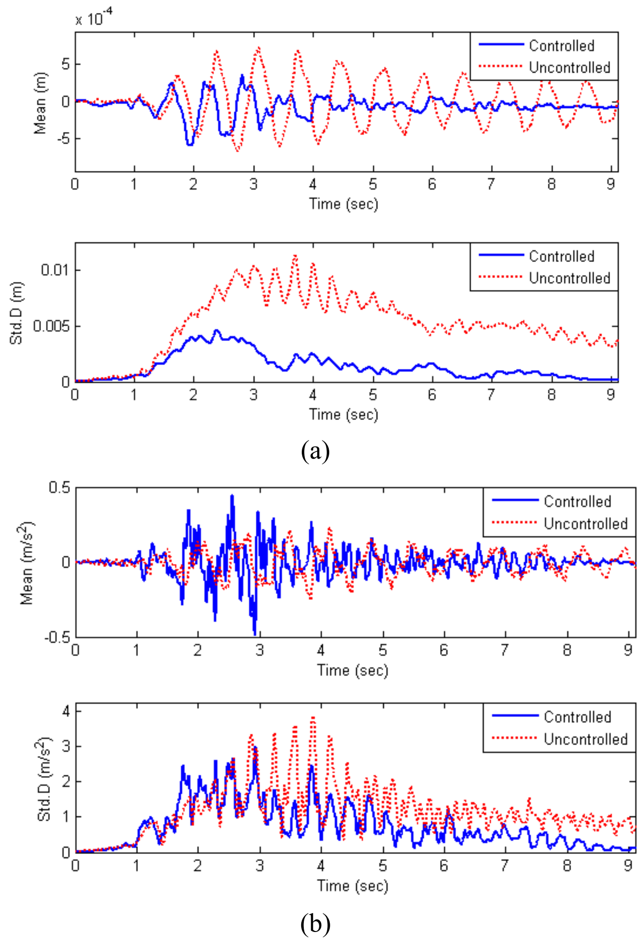
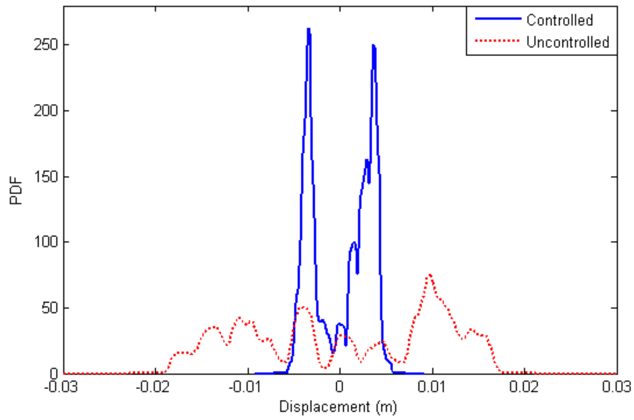


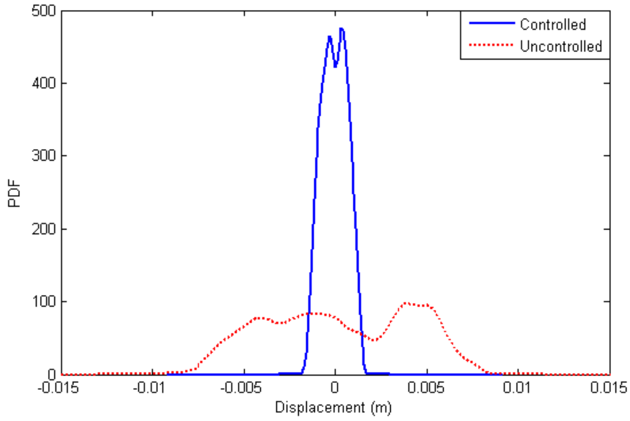
Fig. 7 Time histories of mean and standard deviation of responses: (a) inter-story drifts of the first floor; (b) story accelerations of the top floor

with that without control during the time interval from 1.0 sec to 3.0 sec. Besides, the control efficiency of MR dampers acting on the tested structure indicated in Fig. 7 exposes a consistent result with that included in the analysis of statistical values of experimental data; see Tables 4 and 5, which verifies the application of the PDEM experimentally.

The PDFs of the structural responses with and without control at typical instants of time (3 sec and 8 sec) are given in Figs. 8 and 9. It is seen that the PDFs are mostly irregular, quite different from widely-used regular probability distributions. Moreover, the PDFs vary greatly against time, which could be examined in conjunction with Fig. 7. For instance, the distribution width of the PDF at 3 sec is larger than that at 8 sec in the controlled cases, which could be verified from Fig. 7(a) by the time history of the standard deviation, and meanwhile, the shapes of the two PDFs are obviously different. Comparing the PDFs at the same instant of time, as shown in Fig. 8, the stochastic fluctuation (quantified by the distribution width of the PDF) of inter-story drift of the first floor is significantly reduced in the controlled cases. That is to say, the variation of inter-story drift of the first floor is obviously decreased. Similarly, the distribution range of the PDFs of story



(a)



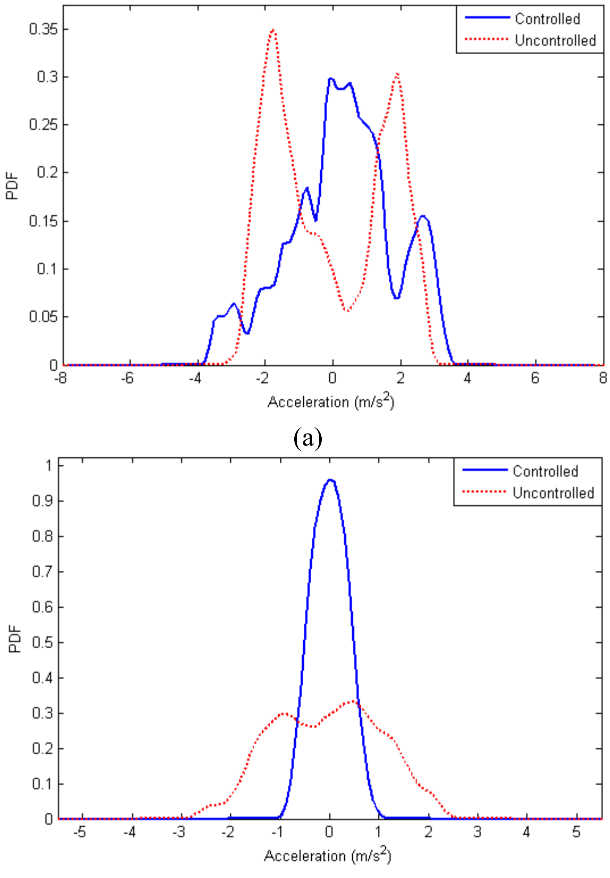
(b)

Fig. 8 PDFs of inter-story drifts of the first floor at typical instants of time: (a) 3 sec and (b) 8 sec

acceleration of the top floor at the typical instants of time, as shown in Fig. 9, becomes narrower to some extent. Moreover, the shape of these PDFs gives rise to be more regular in the controlled cases, indicating that the seismic performance of the tested structure is significantly enhanced with MR dampers in passive-on control modality.

6. Extreme value distribution based structural reliability assessment

Statistical moments of structural responses serve as the critical component for representing the efficiency of vibration mitigation. While it is the reliability that really quantifies the safety of structures especially in case of disaster actions such as earthquakes and strong winds. A controlled structural system having smaller responses might still not meet the requirement of safety or serviceability in the sense of acceptable probability. Thus, reliability analysis of controlled structures should be carried out. In this section, the reliabilities of the controlled and uncontrolled structures are evaluated through integrating the probability density evolution method and the



(b)

Fig. 9 PDFs of story accelerations of the top floor at typical instants of time: (a) 3 sec and (b) 8 sec

theory of extreme value distribution.

6.1 Formulation of dynamic reliability evaluation

According to the criterion of first-passage failure, the dynamic reliability of the structural system defined in Eq. (7) could be denoted by

$$R = \Pr\{X(t) \in \Omega_s, t \in [0, T]\} \tag{12}$$

where $\Pr\{\cdot\}$ indicates the probability of the random event; Ω_s denotes the safe domain.

The expression of Eq. (12) could be re-written into

$$R = \Pr\{W(\Theta, T) \in \Omega_s\} \tag{13}$$

where $W(\Theta, T)$ denotes extreme value of *X(t)* over time interval [0, *T*] corresponding to the failure criterion, i.e.

$$W(\Theta,T)=\underset{t\in[0,T]}{\operatorname{ext}} X(\Theta,t) \tag{14}$$

If the symmetric double boundary criterion is used in Eq. (12), Eq. (13) has the specified form as follows

$$R=\Pr\{|X(t)|\le x_{\mathrm{B}},t\in[0,T]\} \tag{15}$$

in which x_{B} denotes the symmetric boundary, then Eq. (14) becomes

$$W(\Theta,T)=\max_{t\in[0,T]}|X(\Theta,t)| \tag{16}$$

As a random variable, the PDF of the extreme value defined by Eq. (14) or Eq. (16) is available through employing the PDEM by introducing the concept of virtual stochastic process (Chen and Li 2007)

$$Z(\tau)=\varphi(W(\Theta,T),\tau)=\phi(\Theta,\tau) \tag{17}$$

which satisfies the conditions

$$Z(\tau)\big|_{\tau=0}=0\text{ , }Z(\tau)\big|_{\tau=\tau_c}=\varphi(W(\Theta,T),\tau_c)=\phi(\Theta,\tau_c)=W(\Theta,T) \tag{18a,b}$$

where τ_c denotes a prescribed value. One might realize that Eq. (17) is in a form similar to Eq. (8) and therefore the joint PDF of the arguments (Z,Θ), denoted by $p_{Z\Theta}(z,\theta,\tau)$, is governed by the following generalized density evolution equation

$$\frac{\partial p_{Z\Theta}(z,\theta,\tau)}{\partial\tau}+\dot{\phi}(\theta,\tau)\frac{\partial p_{Z\Theta}(z,\theta,\tau)}{\partial z}=0 \tag{19}$$

where $\dot{\phi}(\theta,\tau)=\partial\phi(\theta,\tau)/\partial\tau$.

Considering Eq. (18a), the initial condition reads

$$p_{Z\Theta}(z,\theta,\tau)\big|_{\tau=0}=\delta(z)p_{\Theta}(\theta) \tag{20}$$

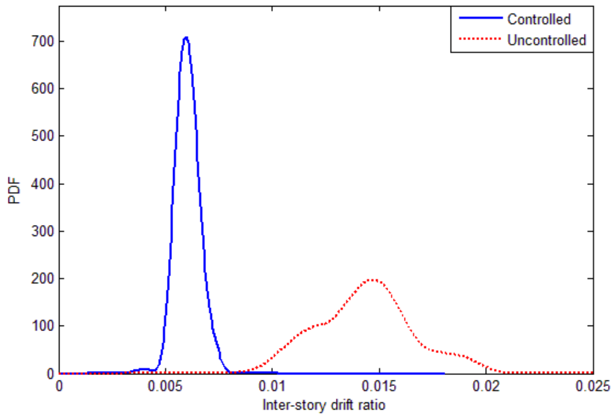
One can then obtain the PDF of $Z(\tau)$

$$p_Z(z,\tau)=\int_{\Omega_\Theta}p_{Z\Theta}(z,\theta,\tau)\mathrm{d}\theta \tag{21}$$

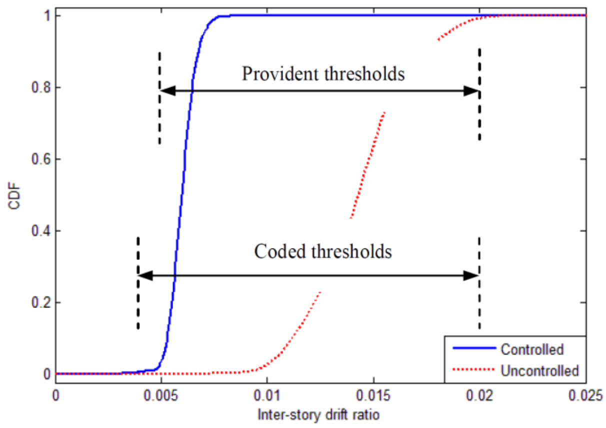
Noting Eq. (18b) one gets the PDF of W

$$p_W(w)=p_Z(z=w,\tau)\big|_{\tau=\tau_c} \tag{22}$$

The reliability denoted by Eq. (12) could then be evaluated by



(a)



(b)

Fig. 10 PDFs and CDFs of extreme values of inter-story drift ratios: (a) PDFs and (b) CDFs

$$R = \Pr\{W(\Theta, \tau) \in \Omega_s\} = \int_{\Omega_s} p_W(w)dw \tag{23}$$

One might see that the dynamic reliability evaluation becomes a problem of one-dimensional integration on the extreme value distribution. This avoids the main difficulties encountered in traditional dynamic reliability theories.

6.2 Reliability assessment of tested structure

The extreme value of inter-story drift ratio serves as the performance index for reliability assessment of the structural system, where the theoretical principles of the extreme value distribution and of the equivalent extreme value event (Li *et al.* 2007) are employed. The PDFs and corresponding cumulative density functions (CDFs) of the equivalent extreme value of inter-story drift ratio of tested structure with and with MR dampers are shown in Figs. 10(a) and 10(b), respectively. It is seen from Fig. 10(a) that the value domain of inter-story drift ratio of the controlled structure is much smaller than that of the uncontrolled structure. Moreover, the distribution range of the equivalent extreme value of inter-story drift ratio in the controlled cases is narrower than that in the uncontrolled cases. It is indicated that the system reliability of the tested structure with control is larger than that without control, especially where, as shown in Fig. 10(b), the threshold of inter-story drift ratio is in the range from 0.005 rad to 0.02 rad. Assuming that the threshold of inter-story drift ratio is 0.01 rad for the tested structure, the system reliability of the controlled structure is 1.000 while the system reliability of the uncontrolled structure is just 0.027. It thus can be remarked that the system reliability of the structure is obviously strengthened with the application of MR dampers.

One might realize that minimum design requirement needs to be satisfied allowing for an effective control gain ensuring structural safety, and meanwhile, an additive control gain would be improvident if the sufficient safe design is provided (here the structural safety is a generalized concept, which relies upon the decision maker and is quantified by the threshold of the performance index). The control effort, for example, would be fruitless if the threshold of inter-story drift ratio is less than 0.005 rad or more than 0.02 rad. While a provident control gain would be reached if the threshold of inter-story drift ratio varies from 0.005 rad to 0.02 rad, which just locates in the range of the elastic-plastic inter-story drift ratio (0.004 rad to 0.02 rad) defined in Chinese code for seismic design of buildings (GB 50011-2010); see Figure 10(b). It is indicated that the control action would operate effectively if the structural system is designed in rule of acceptable coded thresholds. Besides, the most significant control gain will be implemented in case that the threshold of inter-story drift ratio is 0.008 rad.

7. Conclusions

We investigate the stochastic seismic response control of structures with MR dampers through complete shaking-table tests and numerical analysis using probability density evolution method. The structural reliability evaluation based on the theory of extreme value distribution is also performed. The seismic ground motions acting on the shaking-table are represented by the physically based stochastic ground motion model. Experimental and analytical results show that the controlled structure with MR dampers in passive-on control modality gain a significant response reduction. The control efficiency included in the numerical analysis exposes a consistent result with that included in experimental investigations. Due to the stiffness contribution of control devices and minimum inter-story drift criterion of damper-placement optimization, the inter-story drift of the controlled structure is obviously reduced in the sense of probability; while the story acceleration with control does not receive such improvement where the acceleration at some stories arises to be even greater than that without control.

Reliability assessment of the tested structure reveals that the structural safety is obviously enhanced with application of MR dampers, indicating that the control scheme reaches a desirable structural performance. Besides, the minimum design requirement needs to be satisfied allowing for an effective control gain ensuring structural safety, and an additive control gain would be improvident if the sufficient safe design is provided. The control modality, meanwhile, would exert itself effectively if the structural system is designed in rule of acceptable coded thresholds.

It is noted that the two MR dampers mounted in the tested structure only operate as their passive-on modality though they are effective semi-active control devices. The semi-active control of structure with MR dampers subjected to random seismic ground motions is expected to be experimentally investigated in the next stage of the research.

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