

## A simple damper optimization algorithm for both target added damping ratio and interstorey drift ratio

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**Abstract.** A simple damper optimization method is proposed to find optimal damper allocation for shear buildings under both target added damping ratio and interstorey drift ratio (IDR). The damping coefficients of added dampers are considered as design variables. The cost, which is defined as the sum of damping coefficient of added dampers, is minimized under a target added damping ratio and the upper and the lower constraint of the design variables. In the first stage of proposed algorithm, Simulated Annealing, Nelder Mead and Differential Evolution numerical algorithms are used to solve the proposed optimization problem. The candidate optimal design obtained in the first stage is tested in terms of the IDRs using linear time history analyses for a design earthquake in the second stage. If all IDRs are below the allowable level, iteration of the algorithm is stopped; otherwise, the iteration continues increasing the target damping ratio. By this way, a structural response IDR is also taken into consideration using a snap-back test. In this study, the effects of the selection of upper limit for added dampers, the storey mass distribution and the storey stiffness distribution are all investigated in terms of damper distributions, cost function, added damping ratio and IDRs for 6-storey shear building models. The results of the proposed method are compared with two existing methods in the literature. Optimal designs are also compared with uniform designs according to both IDRs and added damping ratios. The numerical results show that the proposed damper optimization method is easy to apply and is efficient to find optimal damper distribution for a target damping ratio and allowable IDR value.

**Keywords:** optimal dampers; target damping ratio; added dampers; optimal passive control; interstorey drift ratio

### 1. Introduction

Damping within a structural system can have various significances for different engineering disciplines. Damping can mean only a reference note on a seismic or wind spectral plot, 5% damped spectra being the most known parameter among the civil engineering community. To the structural engineers, damping means changes in overall stress within a structure subjected to shock and vibration, with frequent arguments considering whether a structure will have 2%, 3%, 4%, but not more than 5% structural damping.

The proper regulation of damping is one of many different ways that have been proposed for supporting a structure subjected to vibration disturbances in order to achieve an optimal

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performance. Conventional methods would suggest that the structure should passively decrease or absorb the effects of transient inputs through a combination of strength, flexibility, deformation capacity and energy absorption. The degree of damping in a conventional building structure is considerably low. Furthermore, the forces on structural elements can exceed the yield points during strong motions such as an earthquake excitation. Therefore, most of the energy dissipated is absorbed by the structure itself through local damages.

The concept of supplemental dampers within a structure suggests that part of the input energy will be absorbed, not by the structure itself, but rather by supplemental damping elements. The usage of the added dampers can increase the damping level of buildings ranging from 20% to 40%. A supplemental damper is an element which can be added to a system to provide a capacity to withstand forces resulting from vibration, therefore bringing out a mean of energy dissipation. Application of supplemental dampers has transitioned from protection related structures to commercial applications on building structures and bridges exposed to seismic or wind loads. Fluid damping technology has been proven to be thoroughly reliable and robust for implementation to structures. A fluid viscous damper is one of the commonly known passive dampers. Fluid viscous devices that use a cylindrical piston immersed in a viscous fluid are extensively used in aerospace and military applications; and recently have been adapted for building applications (Constantinou and Symans 1992). The primary characteristics of these devices for structural applications are linear viscous response achieved over a broad frequency range, insensitivity to temperature and compactness in comparison to stroke and output force. The damper absorbs energy through movement of the piston in the highly viscous fluid. If the fluid is purely viscous, then the output force of the damper is directly proportional to the velocity of the piston.

When this type of passive dampers began to be used in buildings, it was essential to ascertain the appropriate location and number of dampers within the structure. However, there have been a limited numbers of studies about damper allocation in structures. The results of reducing the seismic response of multi-storey shear buildings with first storey damping were presented by an optimization study (Constantinou and Tadjbakhsh 1983). In their study, they remarked that flexibility of structure actually affected the optimal damping of system; and suitable objective functions were proposed for both short and high buildings. Ashour and Hanson (1987) conducted an interesting study on the optimal placement of visco-elastic dampers in relation to seismic excitation. An evaluation of the effect of added visco-elastic dampers on reducing the earthquake response of multi-storey steel frame structures was presented by Zhang *et al.*(1989). The seismic responses of simple building structures were examined in a study carried out by Hahn and Sathiyaveeswaran (1992) to assess the effects of different distributions and magnitudes of damping derived from added visco-elastic dampers. A simple optimal design procedure was proposed by which dimension and number of visco-elastic dampers could be determined and the results of the proposed method were also supported by experimental measurements (Zhang and Soong 1992). Cao and Mlejnek (1995) developed a finite element perturbation method, which provided a simple tool for the prediction of damping in a wide frequency range without the need for repeated analyses. An algorithm was introduced to find the optimum sets of storey stiffness coefficients and damping coefficients of the dampers of an elastic planar shear building with viscous dampers (Tsuji and Nakamura 1996).

Some of the optimal damper procedures based on active control theories were developed to determine damper allocations (Gürgöze and Müller 1992, Gluck *et al.*1996, Agrawal and Yang 2000a, Agrawal and Yang 2000b, Loh *et al.*2000, Hwang *et al.*1995, Yang *et al.*2002, Lavan *et*

*al.*2008, Cimellaro *et al.*2009a). An analytical procedure for redesign of buildings, which are assumed linear, is proposed which optimizes simultaneously both the structure and the structural control system trying to reduce the structural mass (Cimellaro *et al.*, 2009b). An analytical procedure was proposed for the redesign of structural systems with an arbitrary damping system (viscous and/or hysteretic, proportional and/or non-proportional). In this procedure the target transfer functions and the ratios of absolute values of the transfer functions calculated at the fundamental natural frequency of a structural system were taken as controlled quantities together with the undamped fundamental natural frequency (Takewaki 1997a). Takewaki (1997b) devised an efficient method based on minimizing the sum of amplitudes of the transfer functions of interstorey drifts evaluated at the undamped fundamental natural frequency of a structural system together with a constraint on the sum of damping coefficient of the dampers to obtain the optimal damper placement. The effects of variations support member stiffness of dampers upon the optimal damper allocation problem were investigated (Takewaki and Yoshitomi 1998); and Takewaki (1998) also presented a systematic procedure to find the optimal damper positioning to minimize the tip deflection of a cantilever beam. A control strategy of seismic response of multi-storey building frames using optimally placed visco-elastic dampers was investigated using spectral analysis for narrow and broad band stationary random ground motions (Shukla and Datta 1999). An efficient procedure, with the steepest direction search algorithm, was devised to find the optimal damper distribution in a three dimensional shear building model (Takewaki 1999a). A procedure for obtaining the optimal stiffness and damping distributions based upon the optimality criteria was presented by Takewaki (1999b). Different from the conventional critical excitation methods, a new stochastic response index was maximized as an objective function to find the optimal placement of the dampers (Takewaki 1999c). An optimal damper placement method was proposed to minimize the dynamic compliance of a planar building frame (Takewaki 2000a). An incremental inverse problem approach was adopted by Takewaki (2000b) to achieve the stiffness-damping simultaneous optimization of structural system. A gradient-based method was presented to obtain the required amount of viscous and visco-elastic damping (Singh and Moreschi 2001). Recently, several applications of genetic algorithms to optimal damper problems of structural control have appeared (Singh and Moreschi 2002, Bishop and Striz 2004, Wongprasert and Symans 2004, Dargush and Sant 2005, Trombetti and Silvestri 2007, Lavan and Dargush 2009). Uetani *et al.* (2003) presented an application of optimum design to practical building frames with both viscous dampers and hysteretic dampers.

A simplified and practical sequential search algorithm (SSSA) for the optimization of damper configurations was proposed (Lopez-Garcia and Soong 2002). A simultaneous optimization procedure was presented to install both visco-elastic dampers and supporting braces in a structure (Park *et al.* 2004). Lavan and Levy (2005) investigated a method for the optimal design of added viscous damping for a set of realistic ground motion records and a constraint on an energy based global damage index for regular as well as irregular yielding shear frames. Levy and Lavan (2006) investigated fully stressed design of added dampers; they also proposed a simple iterative use of Lyapunov's solution for the linear optimal design of passive devices in framed structures (Lavan and Levy 2009). Lavan and Levy (2006) presented an optimal peripheral drift control of 3D irregular framed structures using supplemental viscous dampers.

For planar building frames, a new objective function considering base shear force transfer function was defined; and the optimal damper's both location and size were determined (Aydin *et al.*2007). Cimellaro (2007) defined top absolute acceleration as an objective function to find optimal damper placement and compared this with other methods previously proposed by

Takewaki (1997b) and Aydin *et al.*(2007). A gradient based evolutionary optimization procedure was proposed for determining the optimal allocation of added visco-elastic dampers and their supporting members to minimize the transfer function of the sum of interstorey drifts (Fujita *et al.*2010a). A new optimal damper placement method using penalty function and first order optimization theory in long span suspension bridges was presented by Wang *et al.*(2010). A historical review was presented on the development of smart or optimal building structural control with passive dampers and some possibilities of structural rehabilitation (Takewaki *et al.*2011, Takewaki 2009). A gradient-based evolutionary optimization methodology is presented for finding the optimal design of viscous dampers to minimize an objective function defined for a linear multi-storey structure (Fujita *et al.*2010b). An optimal damper method was investigated to find optimal seismic design of added viscous dampers in yielding plane frames and the total added damping is minimized for allowable values of local performance indices under the excitation of a set of ground motions in both regular and irregular structures (Lavan and Levy 2010). Applications for the seismic design of building structures equipped with viscous dampers were carried out (Silvestri and Trombetti 2007, Silvestri *et al.*2011). Optimal location and characteristics of TADAS dampers in moment resisting buildings was studied (Yousefzadeh *et al.*2011). Some basic methodologies were also compared with respect to some structural response and usability measures in practice (Whittle 2012). A new objective function for finding optimal size and location of the added viscous dampers was proposed based on the elastic base moment in planar steel building frames (Aydin 2012). Mousavi and Ghorbani-Tanha (2012) developed a systematic procedure for optimal placement and characteristics of different linear velocity-dependent dampers according to modal damping ratios. A practical optimal design method, which was formulated to minimize the maximum interstorey drift or maximum top storey acceleration under design earthquakes for non-linear oil dampers, was proposed (Adachi *et al.*2013). Recently, some meta-heuristic algorithms were proposed to find optimal location and sizes of the added dampers (Sonmez *et al.*2013, Amini and Ghaderi 2013).

In the literature, different structural responses, either as an objective function or a constraint, were chosen to find the optimal damper placement for various damper optimization methods. Some of them were based on active control theory. While formulations of some studies were established on the frequency domain, the others were based on the time domain. While some damper optimization methods are based on indirect optimization methods, some of them use direct optimization methods. There is variability in the solution of the optimal damper distribution problem. The formulations and their solutions on damper optimization must be simple and easy to use as well as the methods cover the structural response. Material cost is one of the most common performance functions in the structural optimization. The supplemental dampers are also both technological and expensive tools. Especially, the use of these supplemental dampers in large scale structures brings on the cost issue. It needs to define cost function of the dampers. Ideally, a damper cost optimization problem should be formulated in terms of its life-cycle cost which includes the costs of materials, fabrication, erection, maintenance and disassembling of the structure at the end of its life cycle.

In this study, a cost function, which is the sum of damping coefficients of the added dampers, is minimized to find optimal damping coefficients of the added dampers under a specified added damping ratio and considering both lower and upper bounds of each damping coefficient of the added dampers. Differential Evolution, Nelder Mead and Simulated Annealing are used to solve the simple numerical minimization problem in this study. After the numerical minimization step in the proposed algorithm, a snap-back test is satisfied to attain the allowable level of the IDRs under

a design earthquake. Moreover, in the numerical examples, the effects of the choice of upper limit of each added damper, the effects of the storey mass and stiffness distribution above the optimal damper designs are investigated. In cases of different storey mass and storey stiffness distribution, the total damping coefficient obtained from the proposed method are taken as an equality constraint in order to find optimal damper distribution applying other existing methods available in the literature (Takewaki 2000a, Cimellaro 2007). The results of the proposed method are compared to the results of these methods. Moreover, the optimal designs are also compared to uniform designs with respect to damping ratio and IDRs. Optimal designs give better results compared to uniform designs in terms of IDRs. In addition to this, while optimal damper design focuses on particular storeys, the total damping obtained from optimal design is distributed to all storeys in a uniform design. This aspect also increases the labour cost in addition to material cost in a uniform design.

## 2. Formulation of problem

Consider an  $n$ -storey shear building model such as linear manufactured viscous dampers that are added to each storey. Two ends of the viscous dampers have different velocity since one end is attached to one building storey and the other end to a different storey. These devices produce damping forces in proportion to relative velocity between each one of the ends. These elements achieve the energy dissipation during an external vibration such as a wind and an earthquake excitation. The damping force of a linear viscous damper is given as

$$F_{ad} = c_{ad} \cdot \dot{u} \quad (1)$$

where  $c_{ad}$ ,  $\dot{u}$  denote the damping coefficient of manufactured viscous damper and relative velocity between each one of the ends of damper, respectively. This type of manufactured damper is considered to add to each one of storeys in a shear building. After the dampers are inserted to the structure subjected to earthquake vibration, the equation of motion can be written as

$$\mathbf{M}\ddot{\mathbf{u}}(t) + (\mathbf{C} + \mathbf{C}_{ad})\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{M}\mathbf{r}\ddot{u}_g(t) \quad (2)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  present mass, structural damping and stiffness matrices, respectively.  $\ddot{\mathbf{u}}(t)$ ,  $\dot{\mathbf{u}}(t)$  and  $\mathbf{u}(t)$  are acceleration, velocity and displacement vectors, respectively. The  $\mathbf{r}$  denotes influence vector that all elements is equal to one.  $\ddot{u}_g(t)$  is defined as ground acceleration. The structural damping matrix,  $\mathbf{C}$  can be calculated in proportion to only mass matrix, only stiffness matrix or linear combination of mass and stiffness matrices. It is given as

$$\mathbf{C} = \alpha\mathbf{M} \quad (3)$$

$$\mathbf{C} = \beta\mathbf{K} \quad (4)$$

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \quad (5)$$

where  $\alpha$  and  $\beta$  are generally calculated in terms of first normal mode of vibration in Eqs. (3) and (4). In general,  $\alpha$  and  $\beta$  in Rayleigh damping matrix, given in Eq. (5), are determined by using the first and second normal modes of vibration. While this is called as proportional damping

matrix,  $C_{ad}$  is the non-proportional damping matrix that should be designed optimally to minimize an objective. The matrix,  $C_{ad}$  can be decomposed into corresponding added viscous dampers and is written as

$$C_{ad} = c_1 C_1 + c_2 C_2 + \dots + c_n C_n \quad (6)$$

where  $c_i$  ( $i = 1, \dots, n$ ) corresponds to the damping coefficient of  $i^{\text{th}}$  added damper; and  $C_i$  ( $i = 1, \dots, n$ ) denotes the location matrix of the  $i^{\text{th}}$  added damper. Moreover, the location matrix is also equal to the partial differential of  $C_{ad}$  with respect to  $i^{\text{th}}$  added damping coefficient of dampers as

$$C_i = \frac{\partial C_{ad}}{\partial c_i} \quad (7)$$

As an example; for values of  $i = 1$  and 2

$$C_1 = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \end{bmatrix}_{n \times n} \quad C_2 = \begin{bmatrix} 1 & -1 & \dots & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & \dots & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \end{bmatrix}_{n \times n} \quad (8)$$

In the fundamental mode, the damping ratio is calculated as follows

$$2\zeta_1 \omega_1 = \frac{\phi_1^T (C + C_{ad}) \phi_1}{\phi_1^T M \phi_1} = \frac{\phi_1^T C \phi_1}{\phi_1^T M \phi_1} + \frac{\phi_1^T C_{ad} \phi_1}{\phi_1^T M \phi_1} \quad (9)$$

where  $\zeta_1$  denotes damping ratio after dampers are inserted to the structure,  $\phi_1$  is the normalized fundamental mode vector and  $\omega_1$  is the undamped natural circular frequency of the model structure. The first term on the right side of Eq. (9) covers proportional damping matrix, and therefore there are no couplings between first mode and any of the other modes. This situation is expressed as

$$\frac{\phi_1^T C \phi_i}{\phi_1^T M \phi_i} = \begin{cases} 2\zeta_s \omega_1 & i = 1 \\ 0 & i \neq 1 \end{cases} \quad (10)$$

where  $\zeta_s$  denotes structural damping ratio for the fundamental mode. The second term on the right side of Eq. (9) include non-proportional damping matrix. However, only for purposes of a simplified design it is convenient to assume that

$$\frac{\phi_1^T C_{ad} \phi_i}{\phi_1^T M \phi_i} = \begin{cases} 2\zeta_{ad} \omega_1 & i = 1 \\ 0 & i \neq 1 \end{cases} \quad (11)$$

where  $\zeta_{ad}$  denotes added damping ratio for the fundamental mode. The Eq. (9) can be rewritten using Eqs. (10) and (11) as follows

$$2\zeta_1 \omega_1 = 2(\zeta_s + \zeta_{ad}) \omega_1 \quad (12)$$

and; therefore

$$\zeta_1 = \zeta_s + \zeta_{ad} \quad (13)$$

Structural damping ratio  $\zeta_s$  is generally assumed to be constant as 0.02 in steel structures or 0.05 in RC structures. The parameter  $\zeta_1$  denotes the desired value of the damping ratio when the dampers are inserted to the structure. The parameter  $\zeta_{ad}$ , which occurs due to the effects of the added dampers, is the added damping ratio. The desired  $\zeta_{ad}$  is determined from Eq. (13), if the structural damping ratio and the desired total damping ratio are known. Therefore, the desired added damping ratio is calculated as

$$\zeta_{ad} = \zeta_1 - \zeta_s \quad (14)$$

The Eq. (9) can be rewritten for only added damping ratio as

$$2\zeta_{ad}\omega_1 = \frac{\phi_1^T c_{ad} \phi_1}{\phi_1^T M \phi_1} = c_1 \frac{\phi_1^T c_1 \phi_1}{\phi_1^T M \phi_1} + c_2 \frac{\phi_1^T c_2 \phi_1}{\phi_1^T M \phi_1} + \dots + c_n \frac{\phi_1^T c_n \phi_1}{\phi_1^T M \phi_1} \quad (15)$$

where the coefficients ( $\mu_i$ ) of the  $c_i$  can be written as follows

$$\mu_i = \frac{\phi_1^T c_i \phi_1}{\phi_1^T M \phi_1} \quad (16)$$

The formula of the desired added damping ratio for fundamental mode is written as below using Eqs. (15) and (16)

$$\zeta_{ad} = \frac{1}{2\omega_1} (\mu_1 c_1 + \mu_2 c_2 + \dots + \mu_n c_n) = \frac{1}{2\omega_1} \sum_{i=1}^n \mu_i c_i \quad (17)$$

### 3. Definition of optimal damper problem for shear buildings

The aim of an optimal design is to minimize or maximize an objective or multiple objectives. Some objective functions appeared such as top displacements, maximum interstorey drifts, sum of interstorey drifts, base shears, top absolute accelerations, overturning moments, a defined damage index, and combinations of some structural performance functions in the previously mentioned literature. Various objective functions can be used in order to solve optimal damper problem and the importance of various cost functions can increase for different types of structures. While the decrease in displacements or inter-storey drifts is important for a displacement-based design, some internal forces and accelerations can be important for a forced-based design. In other words, a defined structural damage index and an energy index may be important for various structures.

In this study, design variables are considered as the damping coefficients of the added dampers. Optimal damper problem is based on minimization of total cost of the dampers that is expressed as the sum of damping coefficients of the added dampers which is given as

$$\text{Min. } f = \sum_{i=1}^n c_i \quad (18)$$

The cost function to be minimized in Eq. (18) indicates the total damping coefficient of the added dampers. Eq. (17) can be rewritten as an equality constraint in terms of the added damping ratio

$$\zeta_{ad} = \frac{1}{2\omega_1} (\mu_1 c_1 + \mu_2 c_2 + \dots + \mu_n c_n) = \frac{1}{2\omega_1} \sum_{i=1}^n \mu_i c_i \quad (19)$$

where  $\zeta_{ad}$  is a fixed damping ratio that can be given as a desired damping ratio. The fundamental natural circular frequency and  $\mu_i$  is known parameter from the vibration characteristics of the structure. Both objective function and equality constraint are the linear function of the design parameters.

In practice, building design codes and guides do not assign a specific method for optimal damper design. However, in the NEHRP (2003) a technique for obtaining a total damping value is provided to achieve a required effective damping ratio. Another way of determining the total damping is to calculate a specified value to satisfy a response in linear elastic region for a design earthquake (Whittle *et al.* 2012). The desired effective damping ratio can be converted into a total damping coefficient which corresponds to the sum of damping coefficients of added dampers.

Taking into account the inequality constraints on the upper and lower bounds of the damping coefficients of each added damper gives the following

$$0 \leq c_i \leq \bar{c}_i \quad (i=1,2,\dots,n) \quad (20)$$

where  $\bar{c}_i$  is the upper bound of damping coefficient of the damper in  $i^{\text{th}}$  storey. In practical applications, a damper capacity and size which corresponds to the upper bound of the added damper should be restricted because of commercial and manufacturing limitations.

#### 4. Numerical minimization methods

There are many optimization tools for solving the proposed damper optimization problem in the literature. The solution of the proposed optimization problem is easy; because the objective function and the constraint functions are simple and are linear functions of the design variables. In this optimal damper problem, the numerical minimization module in Mathematica 5.0 (Wolfram Research 2003) is used to calculate the optimal damper coefficients under the mentioned constraints to minimize the total damping cost. The three various numerical minimization methods such as Differential Evolution, Nelder Mead and Simulated Annealing, which are well known in the optimization literature, are used to solve the optimization problem. These methods present the good agreement between them according to the numerical results in this problem. The aim of using these three optimization methods is to verify the results obtained from a method with the other methods. The used optimization methods in the numerical minimization module of the Mathematica 5.0 (Wolfram Research 2003) are expressed in the following paragraph.

Differential Evolution is a genetic algorithm that maintains a population of specimens,  $x_1, \dots, x_n$ , represented as vectors of real numbers (“genes”). Every iteration, each  $x_i$  chooses random integers  $a$ ,  $b$ , and  $c$  and constructs the mate  $y_i = x_i + \gamma(x_a + (x_b - x_c))$ , where  $\gamma$  is the value of Scaling Factor. Then  $x_i$  is mated with  $y_i$  according to the value of Cross Probability, giving us the child  $z_i$ . At this point  $x_i$  competes against  $z_i$  for the position of  $x_i$  in the population. Search Points is  $\text{Min}[10*d, 50]$ , where  $d$  is the number of variables. Differential Evolution is quite robust, but generally slower than other methods due to the relatively large set of points it maintains.

Nelder Mead method is an implementation of the Nelder-Mead simplex algorithm. For simplicity, we assume here that minimization is being done. We start with the highest vertex,  $v_h$ , of a simplex, and reflect it across the centroid,  $v_c$ , of the remaining points to a new point,  $v_a$ , such that

$\frac{\|v_a - v_c\|}{\|v_h - v_c\|}$  equals the Reflect Ratio. If  $v_a$  is lower than all other vertices, we expand reflection by finding the point  $v_b$  such that  $\frac{\|v_b - v_a\|}{\|v_a - v_c\|}$  equals the Expand Ratio. Then the lower of  $v_a$  and  $v_b$  replaces  $v_h$  in the simplex and the process starts over. If  $v_a$  is lower than the highest vertex but not lower than the lowest vertex,  $v_a$  replaces  $v_h$  and the process starts over. Finally, if  $v_a$  is the highest vertex, we construct  $v_b$  so that  $\frac{\|v_c - v_b\|}{\|v_c - v_h\|}$  equals the Contract Ratio. If  $v_b$  is lower than  $v_h$ , we replace  $v_h$  with  $v_b$  and start over; otherwise, we move each of the simplex vertices,  $v_i$ , toward the lowest vertex  $v_l$ , giving new vertices,  $v'_i$ , where  $\frac{\|v_i - v'_i\|}{\|v_i - v_l\|}$  matches the value of Shrink Ratio, and start over.

Simulated Annealing is an implementation of a biased random-walk search method. It generates a random set of starting points, and for each starting point picks a random direction in which to move. If the move is a better point, it is accepted. If the move is not a better point, we calculate a probability  $\rho$  and compare it to a random value  $r \in (0, 1)$ , and if  $r < \rho$  we accept the new point despite it not being an improvement. The probability  $\rho$  is given by  $\rho = e^{b[i, \Delta f, f_0]}$  where  $b$  is the function defined by Boltzmann Exponent,  $i$  is the current iteration,  $\Delta f$  is the change in objective function value, and  $f_0$  is the value of the objective function from the previous iteration. For each starting point, this is repeated until the maximum number of iterations is reached, the method converges to a point, or the method stays at the same point consecutively for the number of iterations given by Level Iterations. The default value of Search Points is also  $\text{Min}[2*d, 50]$ , where  $d$  is the number of variables. Simulated Annealing is generally a bit faster than Differential Evolution but may not do as well on some problems (Wolfram Research 2003).

## 5. Proposed algorithm

The procedure for the optimal placement of added dampers in a shear-building frame is given as follows:

**Step 1.** Read the input data to construct the stiffness matrix (K), mass matrix (M), calculate the first natural circular frequency of the structure ( $\omega_1$ ), the first mode vector and calculate the structural damping matrix (C). Choose a design earthquake for the linear time history analyses. Select an upper limit of the design variables,  $\bar{c}_i$ .

**Step 2.** Iteration number=1 in the beginning of the algorithm.

**Step 3.** Calculate a new target added damping ratio as  $\zeta_{ad}^{new} = \zeta_{ad}^{old} + 0.01$ .

Assume  $\zeta_{ad}^{old} = 0$  in the first iteration.  $\zeta_{ad}$  is increased by 1% for each iteration in this study.

**Step 4.** Minimize the cost function defined in Eq. (18) considering the constraints in Eqs. (19) - (20). Use the numerical minimization module of Mathematica 5.0 to solve the linear optimization problem with three different methods which are Differential Evolution, Nelder Mead and Simulated Annealing. Find a candidate optimal damper design.

**Step 5.** Test the candidate optimal damper design obtained in Step 4 by performing the time history analysis and calculating the  $i^{\text{th}}$  interstorey drift ratios as  $IDR_{i+1} = \frac{\{\delta_{i+1}(t) - \delta_i(t)\}^{peak}}{h_{i+1}}$  for upper storeys and  $IDR_1 = \frac{\{\delta_1(t)\}^{peak}}{h_1}$  for the first storey, where  $\delta_i(t)$  denotes the interstorey drift

of  $i^{\text{th}}$  storey and  $h_i$  is the  $i^{\text{th}}$  storey height. If all IDRs calculated in this step is below the allowable level (assumed to be 1% in this study), stop the iteration. Otherwise, return to Step 3, increase the iteration number (as iteration number = iteration number+1) and continue for a new target added damping ratio.

If all the design variables attain to the upper limit in Step 4 and any one of IDRs (calculated in Step 5) is not below the allowable level, optimization will not satisfy convergence in Step 4. In this case, one should return to Step 1 and to increase upper limit of the design variables,  $\bar{c}_i$ .

## 6. Numerical examples

### 6.1 The effects of upper limit of design variables on optimal damper problem and comparison with uniform designs

Six-storey shear building model is considered here as a numerical example as shown in Fig. 1. Each one of storey masses is equal to  $8.0 \cdot 10^4$  kg and each one of storey stiffnesses is equal to  $2.0 \cdot 10^7$  N/m. Structural damping in the fundamental mode is assumed to be 0.02 and damping matrix is proportional to mass matrix. The height of the storeys is taken as 3 m. The fundamental period of the model structure is calculated to be 1.65 s. Different upper values of damping coefficients are chosen in the optimization problem and the effects of various upper limits of the added dampers are investigated. The upper limit values,  $\bar{c}_i$  are specified to be  $0.8 \cdot 10^6$  Ns/m,  $0.9 \cdot 10^6$  Ns/m,  $1.0 \cdot 10^6$  Ns/m,  $2.0 \cdot 10^6$  Ns/m,  $3.0 \cdot 10^6$  Ns/m and  $4.0 \cdot 10^6$  Ns/m, respectively. El Centro (NS) earthquake record is chosen as the design earthquake in this example.

Proposed Algorithm presented in Section 5 is applied to find optimal damper designs for each one of the upper limit of added dampers,  $\bar{c}_i$ . The variation of cost function is plotted in the numerical minimization stage (Step 4) according to three different numerical minimization methods given in Fig. 2 which are plotted only for  $\bar{c}_i = 2.0 \cdot 10^6$  Ns/m. It can be seen in Fig. 2 that the target added damping ratio is taken as 0.01 in Step 3 in the first iteration; and the candidate optimal values of the design variables are evaluated in Step 4. Then, the optimal design is tested in terms of peak IDRs in Step 5. Some of them satisfy an IDR value below 1% in the first iteration. The target added damping ratio is gradually increased to 1% iteration by iteration until all of the IDRs become less than the allowable level. All of the peak IDRs become less than the allowable level at the eighth iteration and the optimal design is reached at this iteration for  $\bar{c}_i = 2.0 \cdot 10^6$  Ns/m. The minimum value of cost function and target added damping ratio in the last iteration are calculated as  $2.86645 \cdot 10^6$  Ns/m and 0.08, respectively. The same procedure is achieved for other values of  $\bar{c}_i$  such as  $0.8 \cdot 10^6$  Ns/m,  $0.9 \cdot 10^6$  Ns/m,  $1.0 \cdot 10^6$  Ns/m,  $3.0 \cdot 10^6$  Ns/m and  $4.0 \cdot 10^6$  Ns/m, respectively. In cases of  $\bar{c}_i = 0.8 \cdot 10^6$  Ns/m,  $\bar{c}_i = 0.9 \cdot 10^6$  Ns/m and  $\bar{c}_i = 1.0 \cdot 10^6$  Ns/m, the optimal designs, which satisfy the allowable level for all of the IDRs, are determined at the seventh iteration. The target added damping ratio is calculated as 0.07 at the last iteration in these cases. In cases of  $\bar{c}_i = 3.0 \cdot 10^6$  Ns/m and  $\bar{c}_i = 4.0 \cdot 10^6$  Ns/m, the iteration number reach to 10 and 12, respectively to provide the limit of IDR. These iteration numbers correspond to target damping ratios of 0.10 and 0.12, respectively. The calculated optimal values of the design variables, corresponding to cost value and target added damping ratio for different upper limits of damping coefficient in 6-storey building ( $T_1=1.65$  s) are shown in Table 1. The optimal distribution of the design variables according to storey level and corresponding uniform design are plotted in Fig. 6. The uniform designs are evaluated by uniformly distributing the total damping

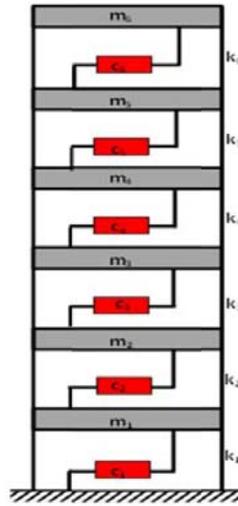


Fig. 1 6-storey shear building model with supplemental dampers

Table 1 The variation of optimal designs according to upper limit of damping coefficient of dampers for 6-storey building ( $T_1=1.65s$ )

Upper limit of damping coefficient Ns/m ( $10^6$ )	Optimal damping coefficient Ns/m ( $10^6$ )						Minimum value of cost function Ns/m ( $10^6$ )	Target added damping ratio %
$\bar{c}_i$	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$\sum_{i=1}^6 c_i$	$\zeta_{ad}$
0.80	0.80	0.80	0.80	0.80	0.03	0.00	3.23	7
0.90	0.90	0.90	0.90	0.24	0.00	0.00	2.94	7
1.00	1.00	1.00	0.78	0.00	0.00	0.00	2.78	7
2.00	2.00	0.87	0.00	0.00	0.00	0.00	2.87	8
3.00	3.00	0.35	0.00	0.00	0.00	0.00	3.52	10
4.00	4.00	0.17	0.00	0.00	0.00	0.00	4.17	12

coefficient obtained from the proposed algorithm. Both Figs. 6 and 7 state that the obtained optimal designs for different upper limits of design variables give a better performance than the uniform design in terms of both damping ratios and peak IDRs. If Fig. 6(f) is examined according to the damping ratio and the optimal design is compared with the uniform design, the optimal design ( $\zeta_{ad}=0.12$ ) provides a better response than the uniform design ( $\zeta_{ad}=0.0662$ ) in terms of added damping ratio. While the optimal designs focus on the lower storeys, the uniform designs are allocated at all storeys in which case the labour cost increases when it is compared with the optimal case. The labour cost is not taken into consideration in this study, however it can be said that the placement of dampers to optimally specified storeys also provides advantages instead of the placement to all storey in terms of labour cost. The variations of the IDRs according to iteration number are plotted in Fig. 3 in cases of all upper limit values of added damping coefficients. As can be seen in Fig. 3, all of them fall below the allowable level at the end of the algorithm. Fig. 4 presents the variations of the peak absolute accelerations ( $A_i$ ) of each storey with

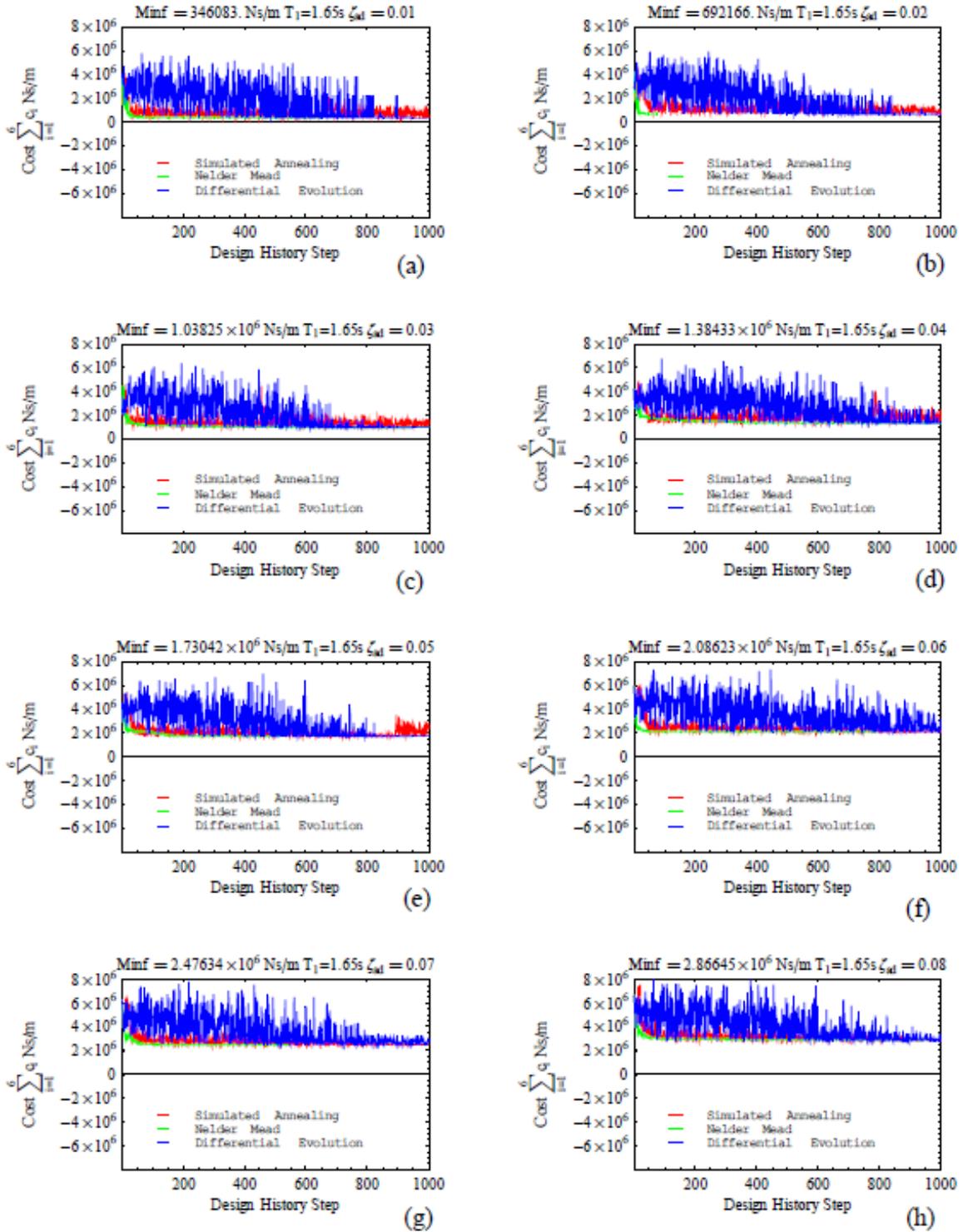


Fig. 2 The variation of the cost function for each iteration of 6-storey shear building model ( $T_1=1.65s$ ) in the numerical minimization stages in case =  $\bar{c}_i = 2.0 \cdot 10^6 \text{ Ns/m}$

respect to iteration number for all upper limit values. It can be seen in Fig. 4 that the peak absolute acceleration of each storey presents a downtrend according to the iteration number for different upper limits of design variables. The peak value of IDRs, their allowable levels (Fig. 5(a)) and the peak absolute accelerations (Fig. 5(b)) according to storey levels are plotted in Fig. 5 for the model building. All of the optimal designs give good performance in terms of the IDR and the absolute acceleration. If Fig. 5(a) is examined carefully, it is observed that all optimal designs exhibit close performance to each other in terms of IDR response. Designer can choose an optimal design among the optimal solution sets in cases of different selection of the upper limit for the dampers. When the optimal designs are examined in Table 1, it is observed that if a selection according to cost function is desired, the optimal design in case of  $\bar{c}_i = 1.0 \cdot 10^6 \text{ Ns/m}$  gives the best performance among the optimal design sets. If a choice according to the added damping ratio is desired, the optimal design in case of  $\bar{c}_i = 4.0 \cdot 10^6 \text{ Ns/m}$  gives the best performance among the optimal designs. The higher upper limit selection provides that the optimal dampers are placed to lower storeys. The decrease of the upper limit value for the damping coefficients provides the placement of the dampers to more storeys.

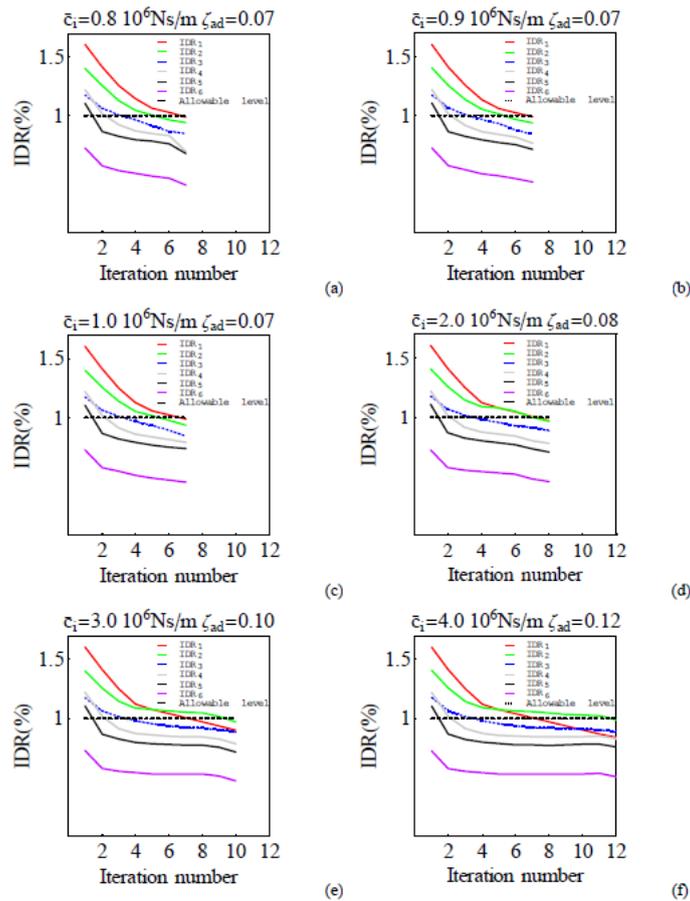


Fig. 3 The variation of IDRs for each iteration number in 6-storey shear building model ( $T_1=1.65s$ ) in case of different upper limit of the added dampers

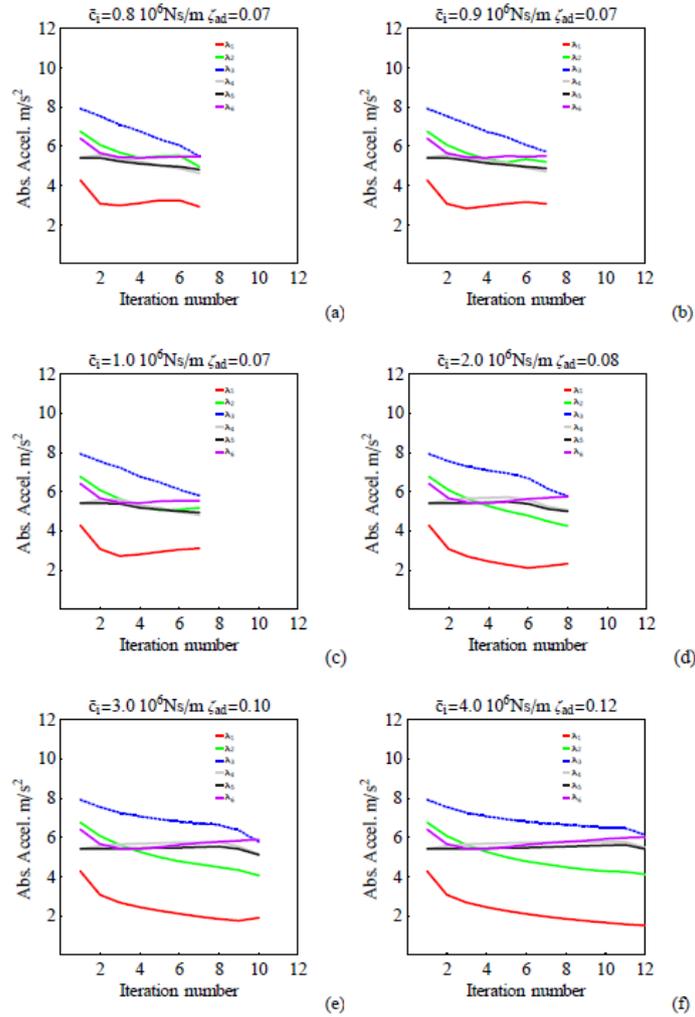


Fig. 4 The variation of absolute accelerations for each iteration number in 6-storey shear building model ( $T_1=1.65s$ ) in case of different upper limit of the added dampers

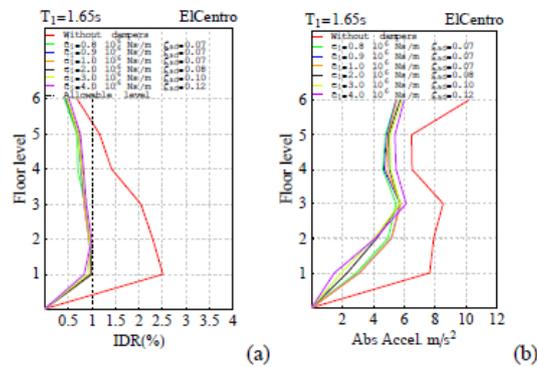


Fig. 5 Peak IDR and absolute acceleration values for optimal designs of 6-storey shear building model ( $T_1=1.65s$ ) in case of different upper limit of the added dampers

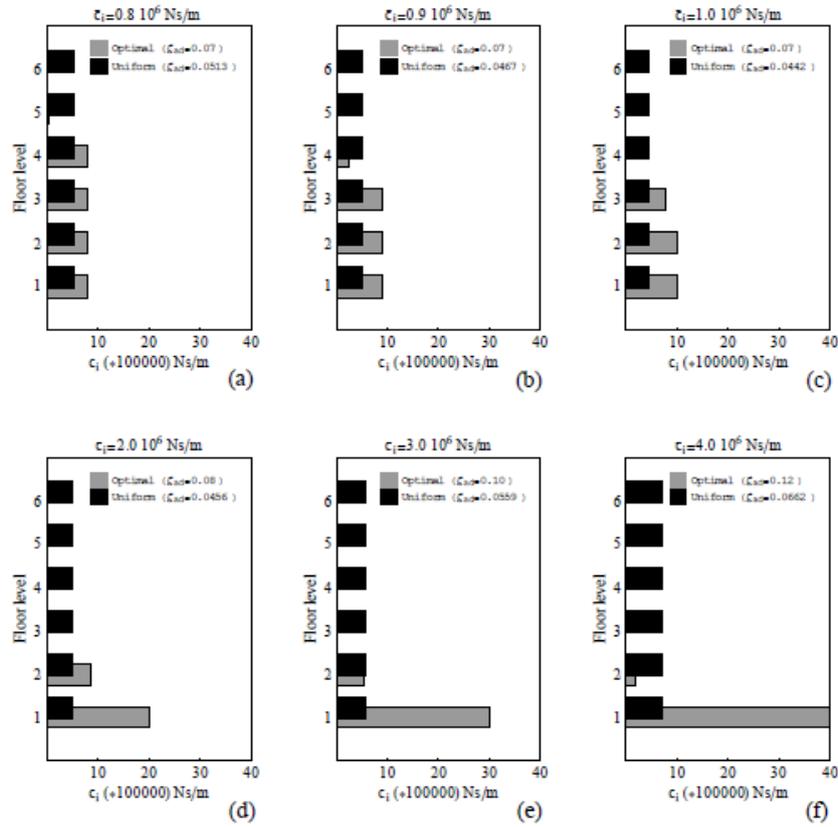


Fig. 6 Optimal and uniform designs for 6-storey shear building model ( $T_1=1.65s$ ) in case of different upper limit of the added dampers

This example presents the effect of different upper limits for the design variables above the proposed optimal damper problem. While this method minimizes the total damping coefficient of added dampers, at the same time, it satisfies an allowable boundary for IDRs and also attains a target value for an added damping ratio. The effects above these structural responses of the upper limit value of design variables and the effects above distribution of the added dampers of this limit value are discussed in this section.

### 6.2 The effects of mass distribution on the proposed optimal damper problem

The effects of storey mass distribution on the proposed optimal damper design are investigated in this section. Three six-storey shear building models accompanied with three different mass distributions are considered as shown in Fig. 8 and Table 2. The total mass is equal for all of these three cases. While the storey mass values increase through the upper storeys in Case-M-1 (Fig. 8(a)), they are chosen in decreasing mass quantity in Case-M-2 (Fig. 8(b)). The other mass distribution model is selected as a uniformly distributed mass model as shown in Fig. 8(c). All storey stiffnesses are equal to  $k_1 = 2.5 \cdot 10^7 N/m$  for all models. Structural damping in the fundamental mode is assumed to be 0.02 and structural damping matrix is selected to be

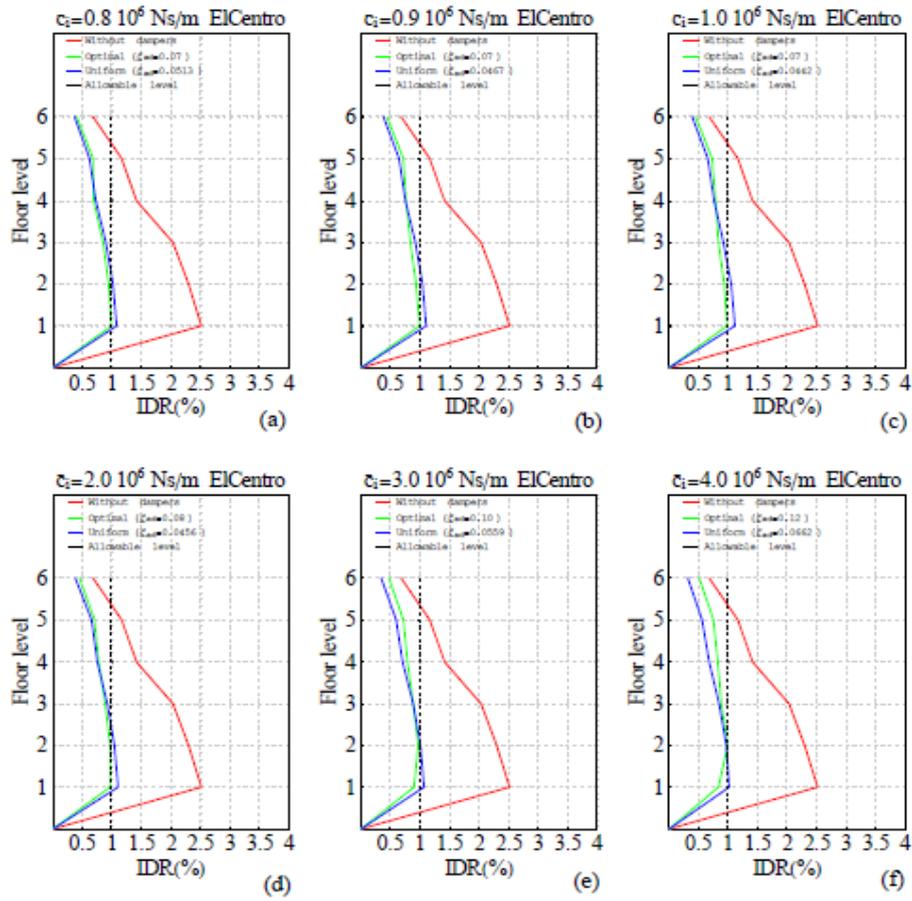


Fig. 7 Comparison of optimal and uniform designs in terms of IDRs

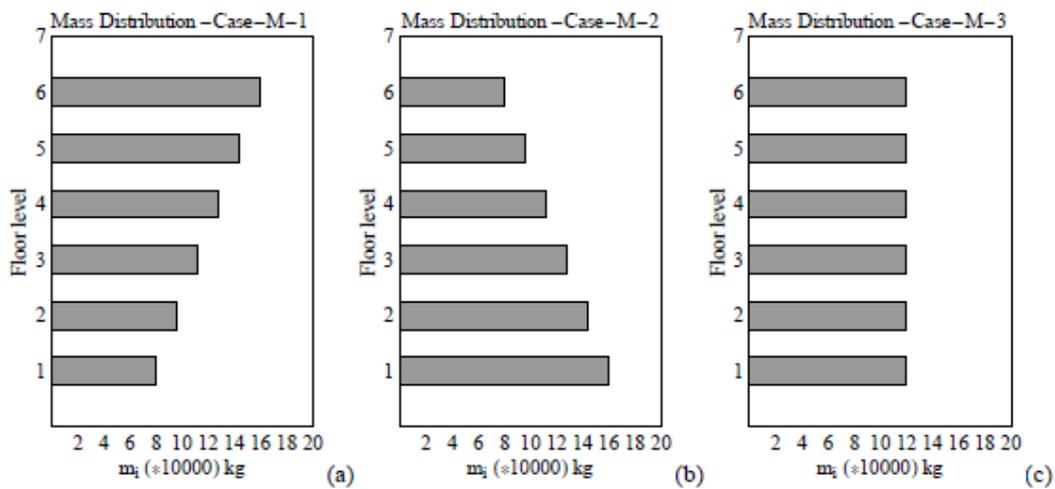


Fig. 8 Different mass distribution models (a) Case-M-1  $T_1 = 1.93s$  (b) Case-M-2  $T_1 = 1.68s$  (c) Case-M-3  $T_1 = 1.81s$

Table 2 Different mass design and corresponding fundamental period and used upper limit of damping coefficient of added dampers

Mass design case	Mass kg ( $10^4$ )						Fundamental period	Upper limit of damping coefficient Ns/m ( $10^6$ )
	$m_1$	$m_2$	$m_3$	$m_4$	$m_5$	$m_6$	$T_1$ (s)	$\bar{c}_i$
Case-M-1	8.0	9.6	11.2	12.8	14.4	16.0	1.93	6.0
Case-M-2	16.0	14.4	12.8	11.2	9.6	8.0	1.68	6.0
Case-M-3	12.0	12.0	12.0	12.0	12.0	12.0	1.81	6.0

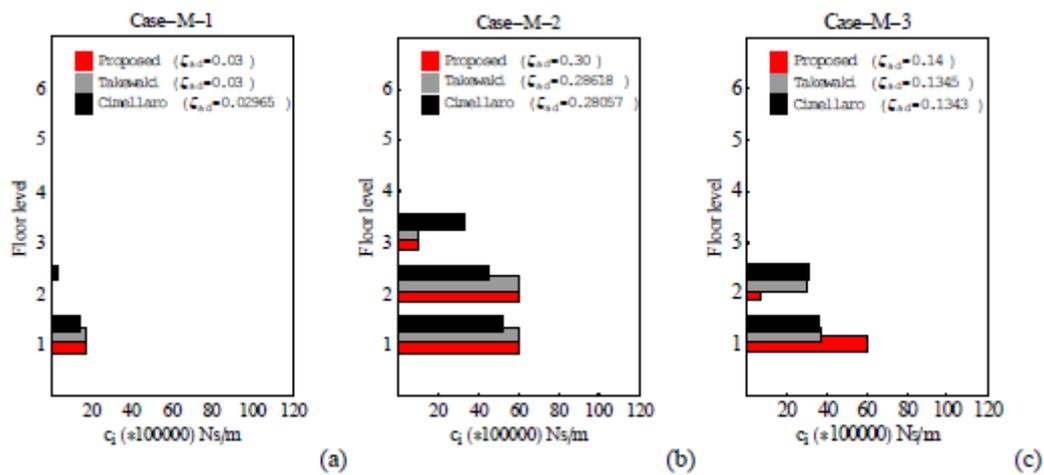


Fig. 9 Optimal damper distributions: (a) Case-M-1, (b) Case-M-2 and (c) Case-M-3

proportional to mass matrix. The corresponding fundamental periods for model structures are calculated as  $T_1=1.93s$  in Case-M-1,  $T_1=1.68s$  in Case-M-2,  $T_1=1.81s$  in Case-M-3. The upper limit value of added damping coefficients is fixed at  $\bar{c} = 6.0 \cdot 10^6 Ns/m$  in the optimization stage for all models in this section.

After the proposed optimization method is carried out to find the optimal design, the variation of optimal damping coefficients is plotted in Fig. 9. In order to make a comparison with some existing methods in literature (Takewaki 2000a, Cimellaro 2007), the total damping coefficient obtained from the proposed method is used to find optimal damper placement using two different methods. The sum of the damping coefficients, which is calculated according to the proposed method, is optimally distributed to minimize both the sum of the interstorey drifts (Takewaki 2000a) and the top absolute acceleration (Cimellaro 2007). Optimal designs and corresponding target damping ratios can be seen in Fig. 9 and Table 3 for three different mass distribution models. As can be seen in Fig. 9(a) and Table 3, placement of a little optimal damper to the first storey is sufficient to attain the allowable level of IDRs in Case-M-1. The target added damping ratio (3%) and the minimum value of objective function ( $1.698 \cdot 10^6 Ns/m$ ) in Case-M-1 is lower than the other cases. While one optimal damper according to proposed method and Takewaki's method (2000a) is placed to the first storey, a major part of the total damping coefficient is added to the first storey with a little portion to the second storey according to Cimellaro's method (2007). The optimal

damper design is obtained for Case-M-2 and is plotted in Fig 9(b). Moreover, the optimal values of damping coefficients, corresponding target damping ratios and the minimum values of the cost function are presented in Table 3. The dampers are optimally distributed to the first three storeys in Case-M-2. The decrease trend of the storey mass causes a higher value than the others in terms of cost function value and the corresponding added damping ratio. The optimal allocations of the proposed method are compatible with the placement of Takawaki's method in Case-M-2 as shown in Fig 9 (b), while it is different from the Cimellaro's method in terms of the magnitude of optimal damping coefficient. Optimal damper design obtained in case of uniform mass design (Case-M-3) represents a distribution that is approximately between Case-M-1 and Case-M-2. Dampers are placed to the first-two storeys in Case-M-3. While optimal damper distribution according to Takewaki's method is close to the placement according to Cimellaro's method as shown in Fig 9(c), the proposed method gives different magnitudes for the damping coefficients. In this case, the target damping ratio and the minimum value of the cost function for the proposed method attain to 14% and  $6.715 \cdot 10^6$  Ns/m, respectively as shown in Table 3. The increase of the storey mass through the upper storeys causes the increase of the fundamental period of the building. It is observed that the fundamental periods of these structures increase while the cost and the corresponding added damping ratio decrease. If Figs. 9 are examined in detail, optimal designs of the proposed method give a better response in terms of the added damping ratio compared to other methods. Proposed method is different from these methods. The suggested method minimizes the total damping coefficient under a target damping ratio and both lower and upper bounds of the design variable. It also includes a time history analysis to test the candidate optimal design. The other methods (Takewaki 2000a, Cimellaro 2007) are based on minimizing the transfer functions of both top displacement and top absolute acceleration evaluated at the fundamental frequency.

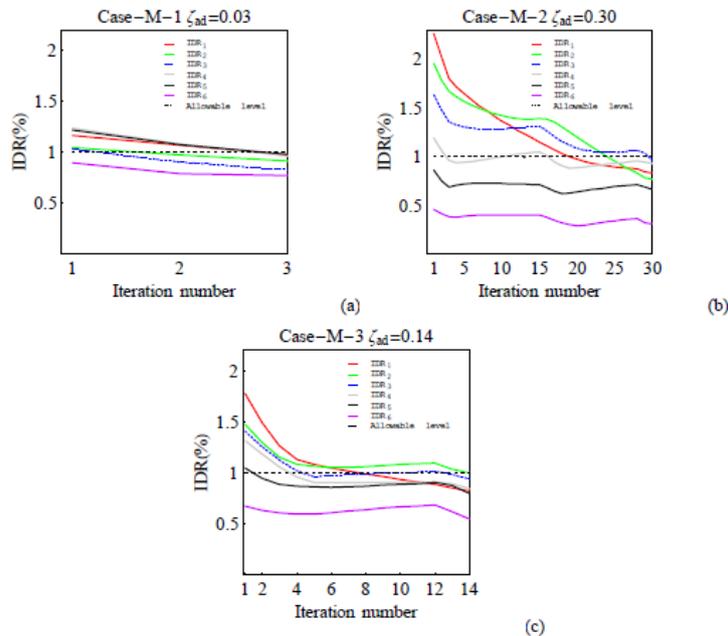


Fig. 10 The variation of IDRs for each iteration number in 6-storey shear building models with different mass distributions

Table 3 Optimal designs and corresponding cost function values, target added damping ratios for various mass distributions

Mass Design case	Optimal design method	Optimal damping coefficient Ns/m ( $10^6$ )						Minimum value of cost function Ns/m ( $10^6$ )	Target added damping ratio %
		$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$\sum_{i=1}^6 c_i$	$\zeta_{ad}$
Case-M-1	Proposed	1.698	0.00	0.00	0.00	0.00	0.00		3
	Takewaki	1.698	0.00	0.00	0.00	0.00	0.00	1.698	3
	Cimellaro	1.40368	0.2943	0.00	0.00	0.00	0.00		2.965
Case-M-2	Proposed	6.00	6.00	0.989	0.00	0.00	0.00		30
	Takewaki	6.00	6.00	0.989	0.00	0.00	0.00	12.989	30
	Cimellaro	5.1523	4.5028	3.3338	0.00	0.00	0.00		28.057
Case-M-3	Proposed	6.00	0.715	0.00	0.00	0.00	0.00		14
	Takewaki	3.6933	3.0218	0.00	0.00	0.00	0.00	6.715	13.45
	Cimellaro	3.6037	3.1113	0.00	0.00	0.00	0.00		13.43

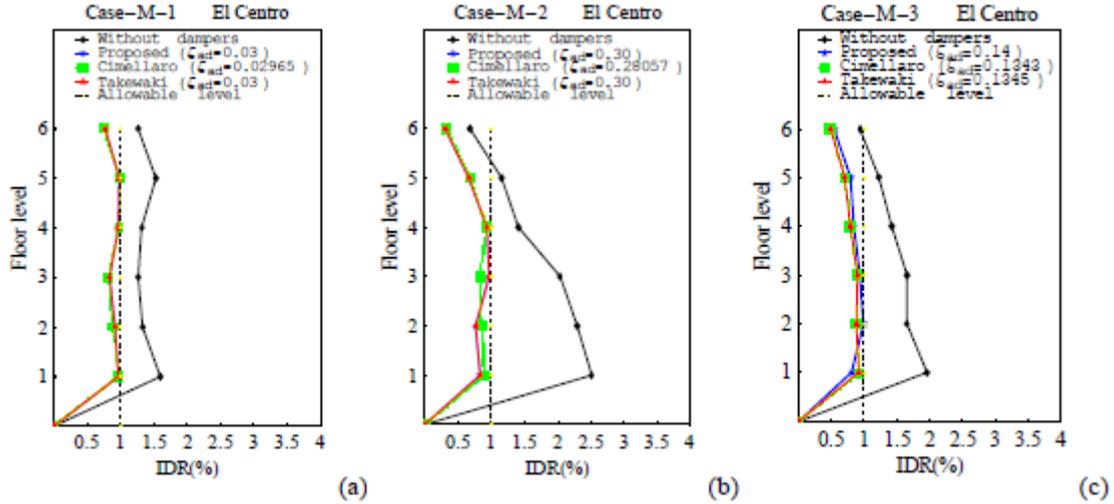


Fig. 11 The effect of different mass distribution on IDRs

### 6.3 The effects of stiffness distribution on the proposed optimal damper problem

The effects of storey stiffness distribution on the optimal damper problem are examined here as similar to the effects of storey mass distribution. Six-storey shear building models with three different stiffness distributions are considered as given in Fig. 12 and Table 4. The total stiffness is same for all three design cases. While the storey stiffness values increase through the upper storeys in Case-S-1 (Fig. 12(a)), they are given in decreasing quantities in Case-S-2 (Fig. 12(b)). The total stiffness is uniformly distributed to all storeys in a uniform stiffness model as shown in Fig. 12(c). Each one of storey masses is taken as  $m_i = 1.2 \cdot 10^5 \text{ kg}$  for all models. Structural damping in the fundamental mode is assumed to be 0.02 and structural damping matrix is proportional to mass matrix. The corresponding fundamental periods for model structures are calculated as  $T_1=1.56\text{s}$  in Case-S-1,  $T_1=1.70\text{s}$  in Case-S-2 and  $T_1=1.62\text{s}$  in Case-S-3. The upper limit value of the added damping coefficients is also fixed at  $\bar{c} = 6.0 \cdot 10^6 \text{ Ns/m}$  in the optimization stage for all models in this section.

If the optimal damper allocations are examined in Fig. 13, dampers are placed to first two storeys for both Case-S-1 and Case-S-2, and one damper is inserted to the first storey in Case-S-3, according to the proposed method. If Case-S-1 is compared with Case-S-2, while the higher quantity of the damping coefficient is focused to second storey in Case-S-1, it is focused to the first storey in Case-S-2 in opposition to Case-S-1. Optimal locations of the dampers are compared according to different optimization methods as shown in Fig 13. While a major part of total damping coefficient is added to the 2<sup>nd</sup> storey in Fig. 13(a), a little part is placed on the first storey in case of using the proposed method. While the total damping coefficients are distributed to the first two storeys according to Takewaki's methods, it is placed to the first three storeys in case of Cimallaro's method. Fig. 13(b) presents that all optimal designs give us a good agreement with respect to the location of the dampers in Case-S-2. While the magnitude of the optimal dampers presents a decreasing trend from the proposed method to other methods in the first storey as shown in Fig. 13(b), there is an opposite situation for the second storey. As can be seen in Fig.13(c) there

Table 4 Different stiffness design and corresponding fundamental period and used upper limit of damping coefficient of added dampers

Stiffness design case	Stiffness coefficient N/m ( $10^7$ )						Fundamental period	Upper limit of damping coefficient Ns/m ( $10^6$ )
	$k_1$	$k_2$	$k_3$	$k_4$	$k_5$	$k_6$	$T_1$ (s)	$\bar{c}_i$
Case-S-1	3.75	3.50	3.25	3.00	2.75	2.50	1.56	6.0
Case-S-2	2.50	2.75	3.00	3.25	3.50	3.75	1.70	6.0
Case-S-3	3.125	3.125	3.125	3.125	3.125	3.125	1.62	6.0

Table 5 Optimal designs and corresponding cost function values, target added damping ratios for various stiffness distributions

Stiffness design case	Optimal design method	Optimal damping coefficient Ns/m ( $10^6$ )						Minimum value of cost function Ns/m ( $10^6$ )	Target added damping ratio %
		$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$\sum_{i=1}^6 c_i$	$\zeta_{ad}$
Case-S-1	Proposed	0.883	6.00	0.00	0.00	0.00	0.00	6.883	10
	Takewaki	3.4415	3.4415	0.00	0.00	0.00	0.00		9.89
	Cimellaro	2.5467	2.5467	1.7896	0.00	0.00	0.00		9.71
Case-S-2	Proposed	6.00	1.288	0.00	0.00	0.00	0.00	7.288	18
	Takewaki	5.1016	2.1864	0.00	0.00	0.00	0.00		17.35
	Cimellaro	4.1299	3.1581	0.00	0.00	0.00	0.00		16.65
Case-S-3	Proposed	5.298	0.00	0.00	0.00	0.00	0.00	5.298	10
	Takewaki	3.1435	2.1545	0.00	0.00	0.00	0.00		9.54
	Cimellaro	3.0022	2.2958	0.00	0.00	0.00	0.00		9.51

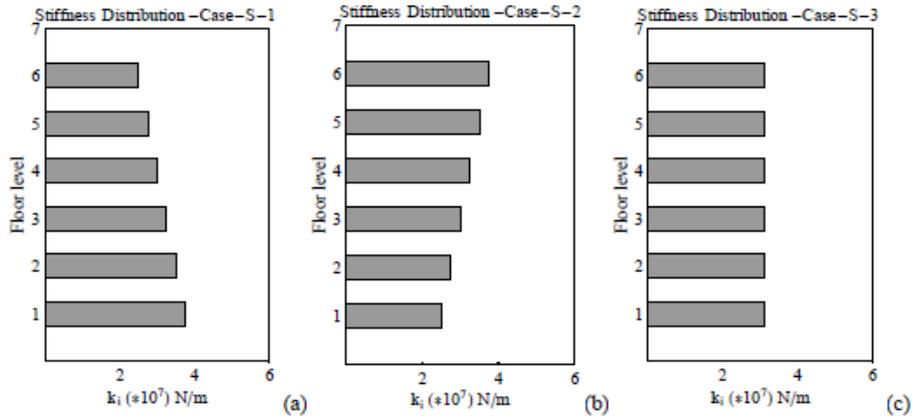


Fig. 12 Different stiffness distribution models (a) Case-S-1  $T_1=1.56s$  (b) Case-S-2  $T_1=1.70s$  (c) Case-S-3  $T_1=1.62s$

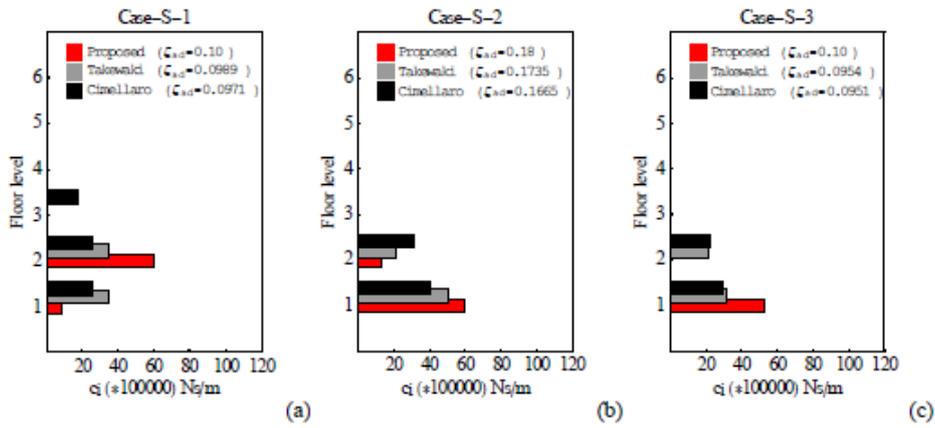


Fig. 13 Optimal damper distributions: (a) Case-S-1, (b) Case-S-2 and (c) Case-S-3

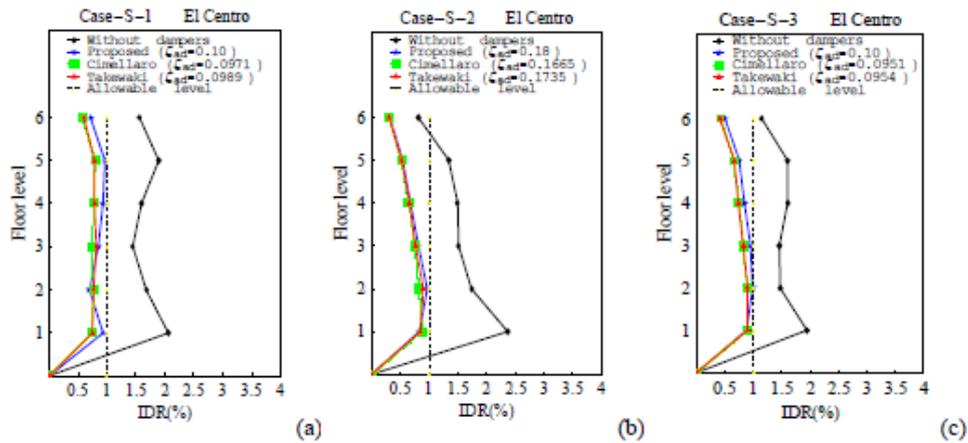


Fig. 14 The effect of different stiffness distribution on IDRs

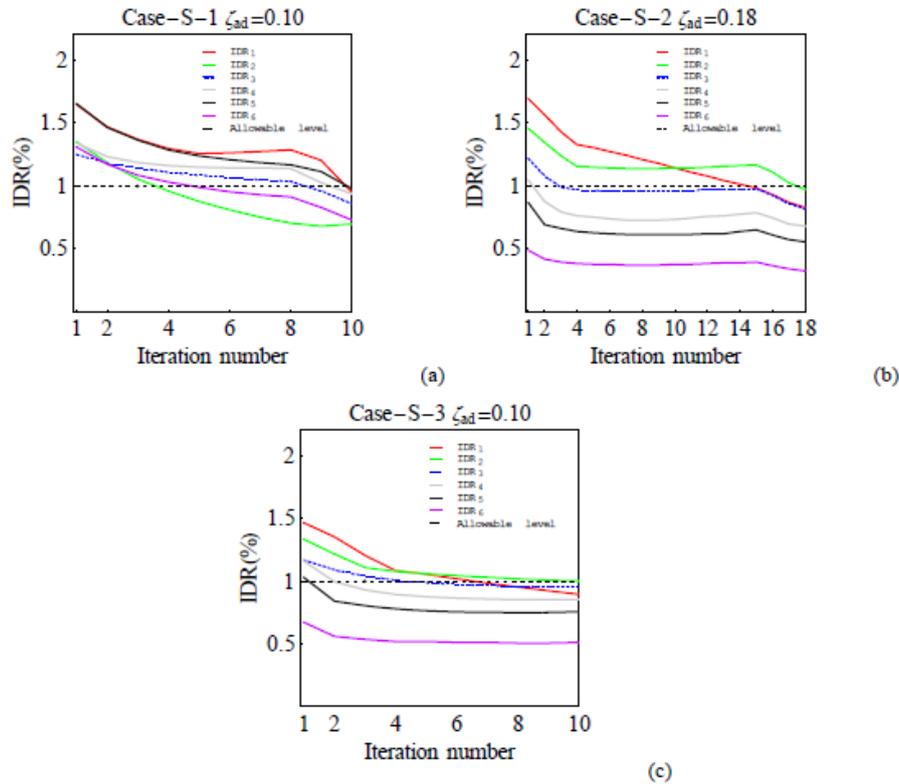


Fig. 15 The variation of IDRs for each iteration number in 6-storey shear building models with different stiffness distributions

is an optimal damper, which is obtained from the proposed method, placed on the first storey. The total damping coefficient is distributed to the first two storeys in the other methods.

All optimal designs satisfy the allowable level for IDRs as shown in Figs. 14 and 15. While iteration number and target added damping ratio attain to 10 and 10%, respectively in Case-S-1 and Case-S-3, iteration number and corresponding target added damping ratio are obtained as 18 and 18%, respectively in Case-S-2. Case-S-3 gives a lower value of cost function than the others as shown in Table 5, while its added damping ratio is equal to the added damping ratio of Case-S-1. If the dynamic characteristic of the model structure is changed with variation of the stiffness distribution, this results in the change of the optimal design. In case of the decrease of the storey stiffness through the upper storeys, an optimal damper with a higher damping coefficient value is focused to the second storey. In case of the increase of the storey stiffness through the upper storeys, an optimal damper with higher damping coefficient value is focused to the first storey to attain a target damping ratio and a boundary level of IDR. It can be seen in Fig. 14 that both the proposed design and the others present a good response with respect to the allowable IDR level. While the calculated optimal designs give better results in terms of the added damping ratio compared to the others, the optimal designs obtained from the other existing methods present a better IDR response than the proposed method. One design earthquake is used in this paper; however, the number of design earthquakes can be increased in a practical application.

## 7. Conclusions

A simple optimization method is proposed to find optimal damper placement in shear buildings. The optimization problem is constructed on minimizing the cost (the sum of the damping coefficients of the added dampers) under a target added damping ratio in the first mode and both upper and lower bounds of the added dampers. Proposed algorithm covers both a numerical minimization to find a candidate optimal damper design under constraints and time history analyses to test the candidate optimal designs obtained in the numerical minimization step. After the algorithm satisfies numerical optimization, the candidate optimal designs are tested with a time history analyses in each one of iterations. If all IDRs calculated from time history analyses are below the allowable level, the algorithm is stopped; otherwise, it continues to satisfy the desired IDR level. The formulation of proposed optimal design method is simple and the solution of this optimization problem by using any numerical minimization method is easy. Both the cost function and the constraint functions are linear function of the design variables. The numerical optimization methods employed in this study can be changed with the available one which can solve the linear optimization problems.

The following conclusions can be drawn from the numerical analyses results:

- 1) The effects of upper limit of the added damping coefficient are investigated in the proposed damper optimization algorithm. The increase of the upper limit of the design variables results in the distribution of the dampers to lower storeys. The decrease of the upper limit value for the damping coefficients provides the placement of the dampers to more storeys. All optimal designs with different upper limits of dampers exhibit close performances to each other in terms of IDR response. Designers can choose an optimal design among the optimal solution sets.
- 2) The obtained optimal designs for different upper limits of design variables give a better performance than the uniform design in terms of damping ratio and IDRs. While the optimal designs focus on the lower storeys, the uniform designs are allocated to all storeys in which case the labour cost will also increase when it is compared with the optimal case. The labour cost is not taken into consideration in this study; however it can be said that the placement of dampers to optimally specified storeys also gives the advantages compared to their placement to all storeys in terms of labour cost.
- 3) The increase of the storey mass through the upper storeys causes the increase of the fundamental period of the building. The increase of the period results in the decrease of both cost and corresponding added damping ratio. The decrease trend of the storey mass causes a higher cost function value and a corresponding added damping ratio than the others.
- 4) In case of both decrease and increase of the storey stiffness through the upper storeys, optimal dampers are focused on different storeys. If the dynamic characteristic of the model structure is changed with a variation of the stiffness distribution, this results in the change of the optimal design.

The practicality of the proposed method can be associated with simplicity of the proposed formulation and usability of basic numerical optimization methods. The numerical results state explicitly that the proposed method is effective in order to minimize the total damping coefficient, to attain a desired damping ratio and to fall below the allowable level of IDRs.

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