Earthquakes and Structures, Vol. 4, No. 1 (2013) 85-108

DOI: http://dx.doi.org/10.12989/eas.2013.4.1.085

A design procedure of dissipative braces for seismic upgrading structures

A.V. Bergami* and C. Nuti

University of Roma Tre, Department of Structures, Rome, Italy

(Received February 22, 2011, Revised February 8, 2012, Accepted April 12, 2012)

Abstract. The research presented in this paper deals with the seismic protection of existing frame structures by means of passive energy dissipation. A displacement-based procedure to design dissipative bracings for the seismic protection of frame structures is proposed and some applications are discussed. The procedure is based on the displacement based design using the capacity spectrum method, no dynamic non linear analyses are needed. Two performance objective have been considered developing the procedure: protect the structure against structural damage or collapse and avoid non-structural damage as well as excessive base shear. The compliance is obtained dimensioning dissipative braces to limit global displacements and interstorey drifts. Reference is made to BRB braces, but the procedure can easily be extended to any typology of dissipative brace. The procedure has been validated through a comparison with nonlinear dynamic response of two 2D r.c. frames, one bare and one infilled. Finally a real application, on an existing 3D building where dissipative braces available on market are used, is discussed.

Keywords: infilled frames; passive damping; dissipative braces; seismic retrofitting

1. Introduction

Retrofitting of existing concrete buildings aims at reducing risk associated to failure and to damage. Traditional retrofitting strategies increase structural strength to reduce ductility demand, while in the last two decades there has been a large diffusion of new conceptual approaches that can be grouped in two categories: increase of available ductility and reduction of demand. These latter can obtained by increasing energy dissipation or reducing input energy thanks to base isolation.

Base isolation and techniques to increase ductility usually refer to traditional analytical evaluation models: linear elastic analysis for the former and plastic section analysis to evaluate ductility for the latter. Therefore these approaches can be easily adopted and dimensioning can be considered straightforward, at least for what concerns computational aspects.

In particular the use of dissipative bracings, though conceptually clear as general principle, seems to require more complicate procedures. Bracings have, in fact, non linear behavior which can modify behavior of the retrofitted structure and usually requires the evaluation of nonlinear

*Corresponding author, Ph.D., E-mail: alessandro.bergami@uniroma3.it

response; traditional methods applicable to conventional structures, such as the *q* reduction factor of *Eurocode* or the *R* factor of *ACI* become meaningless. Certainly nonlinear dynamic analysis is applicable to evaluate response but, however, it is not a practical tool to dimension the new bracings and check response once bracings have been decided.

The development of a design procedure based on static non linear pushover, simpler to be managed if compared with step by step non linear dynamic analysis, can be seen as a useful design tools to check large number of solutions while giving clear indication to move toward efficient design solutions.

In this work a design procedure to determine the characteristics of dissipative braces *B* to retrofit an existing building *S* is discussed, applied and verified: the retrofitted structure *S+B* would guarantee life safety avoiding collapse and damage of structural and non structural elements. The procedure is based on displacement response control and on the use of the well known non linear static analysis (pushover). It is worth noticing that while in this paper the monomodal pushover is considered, other pushover methods (e.g. multimodal) could be used without any change in the procedure.

The procedure is applied on case studies using a widely diffuse and convenient mechanical type of dissipative brace: the buckling restrained brace (*BRB*). However the extension to any type of dissipative brace, whose characteristics are expressed in terms of elastic stiffness and plastic excursion, is quite straightforward.

In the following, after a brief description of the state of the art of design procedures for bracings, the general aspects of dissipative bracing are discussed: description of *BRB*, principles of dissipative bracing design and the effect of bracing on an existing structure.

Finally the proposed procedure is presented, discussed and applied for validation and feasibility assessment: in appendix some simplification for a quicker version are proposed.

2. State of the art of design methods

No one of the existing codes, with the partial exception of *FEMA,* defines design criteria for dissipative bracing systems: *FEMA 274* (1997) and *FEMA 356* (2000) highlights the variability of design methods accordingly to the different types of existing dissipative devices. In fact dissipative devices can be grouped into two major categories: devices with displacement dependent behaviour (yielding metallic and friction dampers) and devices with velocity dependent behaviour (visco-elastic solids or viscous fluid). Alternatively existing design methods for dissipative braces may be distinguished according to the scope of the design process: optimization of global response parameters such as the dissipated energy, or limiting maximum displacement (performance base design).In the following some representative procedures are briefly described.

Filiatrault and Cherry (1988, 1990) defined a design criteria for dissipative braces, based on nonlinear time history analyses, that aimed at minimizing the difference between seismic input energy and dissipated energy; the existing structure is supposed to remain elastic.

Ciampi *et al*. (1991, 1995) determine a bracing system in order to minimize a cumulative structural damage index (e.g. kinematic ductility or cumulative ductility). The structure is represented by an equivalent elasto-plastic SDOF with one equivalent dissipative brace.

More recently procedures based on the displacement based design have been developed such as those of Vulcano *et al*. (1993, 2010), Kim and Choi (2004) and Ponzo *et al.* (2010).

In Vulcano and Mazza (2002) the authors suggest a distribution of the braces finalized to maintain strength and stiffness distribution of the original structure and consequently, as the authors suggest, guarantee that modal shapes don’t change after the insertion of the braces.

Kim and Choi (2004) assumed all the required additional damping as supplied by the braces whose distribution, and therefore strength and stiffness distribution too, is not discussed.

In Ponzo *et al.* (2009) the characteristics of the bracing systems are determined imposing the equivalence between the energy stored, in case of seismic event, in an equivalent elastic single degree of freedom system (the original structure) and in the elasto-plastic system (the dissipative bracing system); results are verified using the *N2* method proposed by Fajfar (1999) and Fajfar and Gaspersic (2000).

Both procedures of Ponzo *et al.* (2009) and Kim and Choi (2004) are calibrated on the achievement of a target performance point (e.g. the target top displacement) without any consideration of other parameters.

The three latter cited displacement based design procedures (Vulcano and Mazza 2002, Kim and Choi 2004, Ponzo *et al*. 2009) work for design of new buildings, usually conceived regular in plan and elevation and whose seismic response can be controlled by few parameters. However, for the following reasons, these don’t seem sufficiently manageable for interventions on existing buildings.

In fact Vulcano and Mazza (2002) assume that the structure has not to change its modal shapes: therefore irregular structures will remain such. Instead, Kim and Choi (2004) and similarly Ponzo *et al*. (2009) base their evaluation on global parameters, as top displacement, and do not care of significant other ones as interstorey drift, usually relevant for retrofitting design.

It is a matter of fact that existing buildings are usually irregular and characterized by a low plastic limit, the use of dissipative bracings should both regularize the structure and increase dissipation. This way seismic demand is reduced and the evaluation of the seismic response is more reliable with respect to the original irregular structure: this is especially necessary for pushover based methods which make use of nonlinear static procedures.

The methodology presented in the present paper is based on the Capacity Spectrum Method, which take in explicit consideration the energy dissipated by the analyzed structure, and therefore it is suitable for structures with additional dampers. If compared with the one proposed by Kim and Choi (2004), it is more flexible since the contribution to dissipation of the structure *S* and braces *B* are kept distinct and the structure *S* can be non linear as well. In this new approach the computation of the energy dissipated by the devices is evaluated referring to the hysteretic cycle performed by each device of each braced level while, the dissipation offered by the original structure, is computed in a global matter based on pushover curve.

Furthermore, as well as top displacement, also the interstorey drift, a good indicator of irregularity, is kept under control. Finally a criterion is given to dimension *BRBs* at each story. All the cited motivations results in a more accurate dimensioning of the bracings while the application remains simple.

3. General aspects of retrofitting using dissipative bracings

3.1 Buckling restrained braces (BRB)

Buckling restrained braces have become relatively popular among the several typologies of dissipative braces. *BRBs* offer some unquestionable advantages: they are openings adaptive, easy to be installed and provide, with minimum interference with the spaces of the building, a controlled strength increase and significant increment of dissipation.

BRBs consist of a steel core element, endowed with a special coating to reduce friction, encased in a concrete filled steel tube preventing steel core buckling in compression. Axial forces are absorbed by the core only that is free to lengthen and shorten dissipating energy by yielding both in tension and compression. As any metallic damper the behavior of a *BRB* depends on its geometry and mechanical characteristics.

BRBs provide a stable hysteretic energy dissipation with a cyclic response very similar to steel constitutive law (*BRBs* usually exhibit a moderate over strength in compression due to the constraint to expansion given by the encase). The steel core can be realized in various ways according to market availability. The dissipative device can constitute a whole brace element or more frequently, especially in case of very stiff elements working in a very small range of displacements, they can assume the configuration of short devices connected in series to an "elastic" brace (Fig. 3).

3.2 Dissipative bracings positioning: structural effects

The insertion of dissipative braces into the structural frame involves significant effects that can be grouped in two categories: effects on structural response and effects on the architecture of the building. Concerning the former the braces increase both stiffness and strength and consequently, as usually happens, both modal shapes and the capacity curve of the structure are modified. Moreover, for a given top displacement, these improve damp ingand, therefore, reduce demand. In this respect stiffness increase could render less efficient, or even useless, the increase of dissipation. Therefore a careful mix of stiffness and dissipation is requested: this subject is discussed in the following.

The bracing system has to be compatible with the architecture of the building: therefore spatial distribution of the braces descents from a compromise between the optimization of the dissipative system and the functionality of the building.

Although braces distribution should be analyzed case by case some general considerations can be made: braces should reduce or eliminate eventual translation-rotation coupling effects, induce constant interstorey drifts, exclude soft storey behavior and maximize damping for a given top displacement.

3.3 Relevant parameters for design ofretrofitting with BRBs

Considering a braced structure, as in Fig. 1, being its capacity curve represented by the curve $S+B$ of Fig. 2, one can assume that this latter is the sum of the capacity curves of the structure (S) and of the bracing system (B): therefore the latter can be obtained subtracting S from $S+B$. This assumption is relatively accurate for design purposes and holds true when the increase of axial forces in the columns is small, in fact the structural behaviour of S does not changes after retrofitting and is kept constant during the design process. In Fig. 2 the capacity curve S is approximated as elasto plastic as well as the capacity curve B: therefore the curve $S+B$ is tri linear.

Given the seismic action in term of response spectrum, for a given capacity curve $S+B$, one can obtain the structural response in term of displacement known the equivalent viscous damping

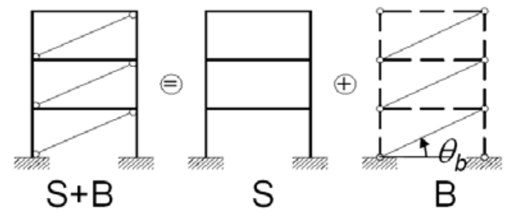

Fig. 1 Scheme of the braced structure (*S*+*B*) as sum of the structure (*S*) and the bracing system (*B*)

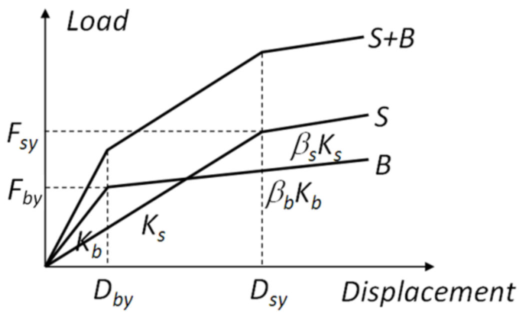

Fig. 2 Interaction between the structure (*S*) and the bracing system (*B*) expressed in terms of horizontal components of the force-displacement relationship

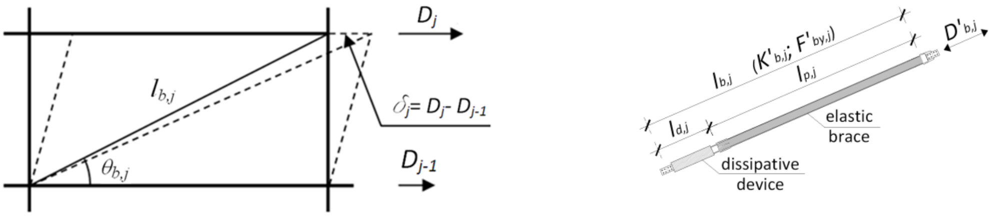

Fig. 3 Deformed shape of a generic single part of the braced frame

$n_{eq,S+B}$ associated to each point of the curve *S*+*B*.

It is well known that the force-displacement behaviour of a *BRB* (with *j* the generic device) can be modelled by a simple bilinear law characterized by the elastic axial stiffness $K'_{b,j}$, the yield strength $F'_{by,j}$ and the hardening ratio $\beta_{b,j}$, as confirmed by numerous experimental studies (Robinsons *et al.* 1976, Whittaker *et al*. 1991, Sakurai *et al*. 1992, Sakurai *et al*. 1992, Hanson *et al*. 2001, Kim *et al*. 2004, Black *et al*. 2004) and as suggested by SEAOC/AISC (2005).

The parameters of the bracing depend on the geometry of the frame and on the characteristics of the device.

$K'_{b,j}$, $F'_{by,j}$, $D'_{by,j}$ e $\beta_{b,j}$ depend on mechanical properties of the selected devices ($D'_{by,j}$ is the axial displacement at yielding) while the length $l_{b,j}$ and the inclination $\theta_{b,j}$ of each brace can be determined referring to both geometric characteristics of the structure and brace distribution (Fig. 3).

Being $K_{b,j}$, $F_{by,j}$, $D_{by,j}$ the horizontal components of stiffness, yield strength and displacement at yield of the bracing system *B* respectively, they can be expressed as follow

$$K_{b,j} = K'_{b,j} \cos^2 \theta_{b,j} \tag{3.1}$$

$$F_{by,j} = F'_{by,j} \cos \theta_{b,j} \tag{3.2}$$

$$D_{by,j} = D'_{by,j} / \cos \theta_{b,j} \tag{3.3}$$

Scope of the design is the definition of the following variables:
1. the plano-altimetric configuration of the bracing system that influences device sizing as it modifies the braced frame deformed configuration both in the linear range as well as beyond the plastic limit;
2. the axial stiffness $K'_{b,j}$ of each brace;
3. the yielding limit of each brace ($D'_{by,j}$, $F'_{by,j}$ in terms of axial components or $D_{by,j}$, $F_{by,j}$ in terms of horizontal components) that is the point beyond which the system B becomes dissipative. It thus influences both resistance and energy dissipation capacity of the braced structure. In Fig. 2 a representation of the cited parameters is given referring, for simplicity, to a bilinear relationship of the horizontal components of load and displacement for both S and B;
4. the hardening ratio $\beta_{b,j}$ of the bracing system that affects both resistance and dissipative capacityof the braced structure.

We can proceed in different manners to determine the stiffness and strength of the braces, to be added to the floors, to reduce maximum response to the intended value in terms of displacements (total and interstorey) and base shear.

It is evident that if the dissipative system yields before the structure itself ($D_{by}<D_{sy}$) the efficiency of the intervention will increase, therefore this should and will be a basic assumption.

Moreover the designer, once defined the desired performance for the structure in terms of top displacement, can decide to avoid or admit plastic deformations of the existing structural elements.

With reference to Fig. 2 three ranges of displacement can be identified on the capacity curve.

The first segment corresponds to a displacement range below the point of first yielding of the bracing system ($D<D_{by}$): in this range both the structure and the braces are elastic and therefore total damping of S+B coincides with the inherent damping ν_I offered by the original structure ($\nu_{tot}=\nu_I$). It is a matter of fact that, in case one uses very stiff braces, total damping could be even smaller than the original inherent damping due to the large increase of elastic energy.

Entering in the second branch, beyond first yielding of B, the structure S is still elastic ($D_{by}<D\leq D_{sy}$) and the bracing system dissipate energy: therefore total damping is the sum of the inherent plus the one due to braces dissipation ($\nu_{tot}=\nu_I+\nu_{eq,B}$). This latter displacement range can be assumed as acceptable at least for frequent earthquakes.

Finally, if it is accepted that also the structure yields ($D>D_{sy}$), total damping of S+B is the sum of the inherent damping and the damping offered by both the bracing system and the structure itself ($\nu_{tot}=\nu_I+\nu_{eq,B}+\nu_{eq,S}$). This latter situation is often the case: many existing structures have been designed to resist to vertical loads only or, at most, to very small horizontal forces. In general yielding of S can be accepted for rare earthquakes and excluded for frequent earthquakes in order to limit damage.

It is now useful to express each limit state of interest in terms of displacement D^*. The same D_i^* can be obtained adopting different retrofitting combinations of stiffness, strength and consequently dissipation.

The first parameter to be determined is the stiffness of the braces (additional stiffness).

Different criteria to distribute the additional stiffness are proposed in scientific literature: constant at each story, proportional to story shear, proportional to interstorey drifts of the original structure. In this work the latter is assumed and therefore, given the interstorey drift δ_j, the stiffness $K'_{b,j}$ corresponding to each storey of the bracing system is

$$K'_{b,j} = K_{global} c_{b,j} \tag{3.4}$$

where

$$c_{bj} = \frac{\delta_j}{\max_j \{\delta_j\}} \tag{3.5}$$

Each brace is a composite element realized coupling an elastic element (usually a steel profile) with a dissipative device in series. The latter will determines the desired yielding force whereas the former will be designed to assure the desired stiffness of the series.

3.4 *Evaluation of the equivalent viscous damping*

As mentioned in the previous section, a specific energy dissipated by the structure and the braces corresponds to each deformation reached by the structure, be it with or without dissipative braces; the dissipated energy can be expressed in terms of equivalent viscous damping.
Referring to the formula proposed by Chopra (2001), the equivalent viscous damping of the structure $\nu_{eq,S}$ at the generic displacement D can be expressed as follows:

$$\nu_{eq,S} = \frac{1}{4\pi} \frac{E_{D,S}}{E_{S,S}} \tag{3.6}$$

All the parameters of the Eq. (3.6) can be easily determined from the capacity curve: $E_{D,S}$ is the energy dissipated in a single cycle of amplitude D and $E_{S,S}$ is the elastic strain energy corresponding to the displacement D. Referring to an equivalent bilinear capacity curve (it can be determined from the capacity curve using one of the methods available in literature) terms of Eq. (3.6),considering an ideal elasto-plastic hysteretic cycle, can be determined as follow:

$$E_{D,S}^{bilinear} = 4\left(F_{sy} D - D_{sy} F_s\left(D\right)\right) \tag{3.7}$$

$$E_{S,S} = \frac{1}{2} D F_s\left(D\right) \tag{3.8}$$

with

D	the displacement reached from the structure
$F_s(D)$	the force corresponding to D (the force is the base shear)
D_{sy}	displacement at yielding
F_{sy}	the yielding force (base shear at yielding)

It is well known that the hysteretic cycle of a real structure differs from the ideal cycle, therefore this difference can be taken into account adopting a corrective coefficient c_S for the structureand c_B for the braces (c=1 for the ideal elasto-plastic behaviour). Therefore

$$E_{D,S} = \chi_S E_{D,S}^{bilinear} \tag{3.9}$$

$$E_{D,B} = \chi_B E_{D,B}^{bilinear} \tag{3.10}$$

with $E_{D,B}^{bilinear}$ the energy dissipated by the ideal hysteretic cycle of the dissipative brace.

For the applications discussed in this paper the parameter c_S has been determined referring to the provisions of *ATC40* (1996). For the braces the assumption of $c_B \approx 1$ has been considered reasonable: in fact, according to *AISC/SEAOC –Recommended Provisions for Buckling-Restrained Braced Frames* (2005), the force-displacement relationship of a *BRB* can be idealized as a bilinear curve. However different values can be adopted, if the case, with no difference in the procedure. Authors have assumed a bilinear curve characterized by a yielding force equal to the yielding traction force (the maximum compressive strength of *BRBs* is slightly larger than the maximum tensile strength due to the confining effect of the external tube): the hysteretic cycle obtained is elasto-plastic but precautionary smaller than the real one. Than the evaluation of the equivalent viscous damping of the braced structure $\nu_{eq,S+B}$, to be added to the inherent damping ν_I (usually ν_I=5% for r.c. structures and ν_I=2% for steel ones), can be obtained using the following expression

$$\nu_{eq,S+B} = \frac{1}{4\pi}\frac{E_{D,S+B}}{E_{S,S+B}} = \frac{1}{4\pi}\left[\frac{\chi_S E_{D,S}^{bilinear}}{E_{S,S+B}} + \frac{\chi_B \sum_j E_{D,B,j}^{bilinear}}{E_{S,S+B}}\right] \tag{3.11}$$

$$\nu_{eq,S} = \chi_S \frac{1}{4\pi}\frac{E_{D,S}^{bilinear}}{E_{S,S+B}};\nu_{eq,B} = \chi_B \frac{1}{4\pi}\frac{\sum_j E_{D,B,j}^{bilinear}}{E_{S,S+B}} \tag{3.12}$$

where $E_{D,B,j}^{bilinear}$ is the energy dissipated by the dissipative braces placed at level j.

Eq. (3.11) can be generalized assuming that $E_{D,B,j}^{bilinear} = \sum_i E_{D,B,i}^{bilinear}$ with $E_{D,B,i}^{bilinear}$ the energy dissipated by the i braces placed at level j.

Note that $\nu_{eq,S}$ and $\nu_{eq,B}$ are obtained dividing the dissipated energy, determined from the capacity curve of S or B respectively, by the elastic strain energy of the braced structure, determined from the curve of $S+B$.

4. Proposed design procedure

In paragraph 3 we discussed the main aspect of the evaluation of seismic response of a structure with *BRBs*; in this paragraph the proposed procedure is detailed.

The procedure is based on the capacity spectrum method: the target is expressed in terms of displacement. Iteration is required since the addition of braces modifies structural response and the capacity curve has to be updated as long as the characteristics of the new braces are defined. Moreover, the energy dissipated by the braces is considered additional to the dissipative capacity of the structure, computed on the capacity curve of the original one.

Structural response is obtained reducing the design spectrum on the base of the damping of the braced structure ν_{tot}.

$$\nu_{tot} = \nu_I + \nu_{eq,S+B} \tag{4.1}$$

In a displacement based design perspective the performance desired is selected at first as the displacement (target displacement) corresponding to a selected limit state for a given seismic action. Than the required total effective damping needed to make the maximum displacement not larger than the target one is determined. The additional damping, due to bracing, is estimated as the difference between total damping and hysteretic damping of the structure without braces. The characteristics of the braces to guarantee the required additional damping are finally determined. The procedure is iterative but it converges in few iterations: the main steps follow.

1. **Define the seismic action**: the seismic action is defined in terms of elastic response acceleration spectrum (T-S_a).

2. **Select the target displacement**: the target displacement is selected (for example the top displacement D_t^*) according to the performance desired (limit state).

3. **Define the capacity curve**: the capacity curve of the braced structure $S+B$, in terms of top displacement and base shear (D_t-V_b), is determined via pushover analysis. The pushover analysis can be easily performed using a software for structural analysis: many different force distributions can be adopted selecting the best option for the specific case (e.g. modal shape load profile).

If a modal shape load profile has been selected it is important to underline that the modal shape is influenced by the bracing system and consequently, at each iteration, the load profile has to be updated to the modal shape of the current braced structure.

Notice that, at the first iteration, the structure without braces is considered and therefore the capacity curve obtained will be fundamental for the evaluation of the contribution offered by the existing structure to the braced structure of the subsequent iterations.

4. **Define the equivalent bilinear capacity curve**: the capacity curve is approximated by a simpler bilinear curve D_t-F_{s+b} that is completely defined by the yielding point ($D_{s+b,y}$, $F_{s+b,y}$) and the hardening ratio β_{s+b} (at the first iteration the parameters correspond to $D_{s,y}$, $F_{s,y}$, β_s of the existing building).

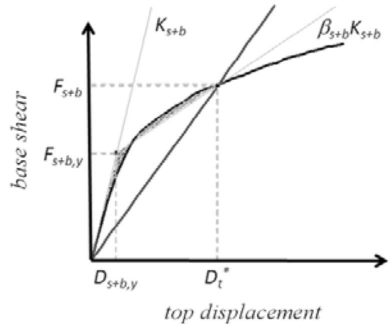

Fig. 4 Evaluation of the equivalent bilinear capacity curve

5. **Define equivalent single degree of freedom**: MDOF system is converted in a SDOF system by transforming the capacity curve into the capacity spectrum (S_{dt}-S_{ab})

$$S_{dt} = \frac{D_t}{\Gamma\phi_t};S_a = \frac{F_{S+B}}{\Gamma \cdot L} \tag{4.2}$$

where Γ is the participation factor of the modal shape ϕ($\Gamma=(\phi^TMI)/(\phi^TM\phi)$)and $L=\phi^TMI$.

The modal characteristics of the braced structure may change at every iteration due to new brace characteristics. Therefore ϕ,Γ and L have to be updated with the current configuration

6. **Evaluate the required equivalent viscous damping**: the equivalent viscous damping $v^*_{eq,S+B}$ of the braced structure to meet the displacement of the equivalent SDOF system and the target spectral displacement $S_{dt}^*=D_t^*/(\Gamma\phi^T)$ is determined.

According to the Capacity Spectrum Method the demand spectrum is obtained reducing the 5% damping response spectrum by multiplying for the damping correction factor h that is function of v_{tot}

$$\eta = \sqrt{\frac{10}{5 + v_{tot} \cdot 100}} = \frac{S_{v_{eff}}}{S_{5\%}} \tag{4.3}$$

From Eq. (4.3) one obtain v^*_{tot} the damping needed to reduce displacement up to the target S_{dt}^*.

$$v^*_{tot} = 0.1\left(\frac{S_{5\%}}{S^*_{dt}}\right)^2 - 0.05 \tag{4.4}$$

7. **Evaluate the equivalent viscous damping contribution due to the naked structure**: the contribute to damping of the structure $v^*_{eq,S}(D_t^*)$ can be determined from Eq. (3.12) being D_t^* the top displacement corresponding to $E_{D,S}^{bilinear}$ and $E_{S,S+B}$ that are the energy dissipated by $S$and the elastic strain energy of S+B($E_{D,S}^{bilinear}$ and $E_{S,S+B}$ are determined from the capacity curve of S and S+B respectively).

8. **Evaluate the additional equivalent viscous damping contribution due to braces**: given v^*_{tot} from Eq. (4.4) the equivalent viscous damping needed to be supplied by the braces $v^*_{eq,B}(D_t^*)$ is evaluated from Eq. (3.11) and Eq. (4.1) as follows

$$v^*_{eq,B}(D_t^*) = v^*_{tot}(D_t^*) - v^*_{eq,S}(D_t^*) - v_I \tag{4.5}$$

9. **Dimensioning of the braces**: once the required equivalent viscous damping $v^*_{eq,B}(D_t^*)$ has been evaluated from Eq. (4.5),axial stiffness and yielding strength required to achieve the desired additional damping can be determined with the same procedure previously adopted for the structure (step 7).

The energy dissipated by the braces inserted at each j_{th} level can be expressed as

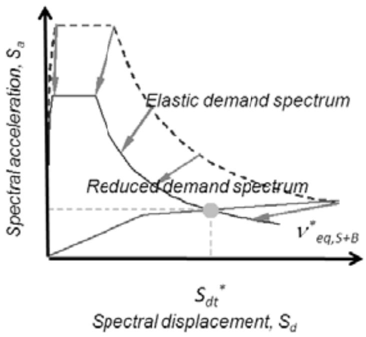

Fig. 5 Evaluation of the equivalent viscous damping needed to achieve the target performance point

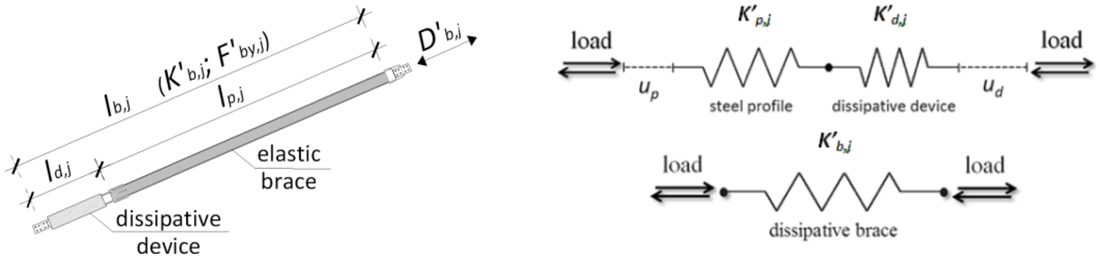

Fig. 6 Dissipative device "j" assembled in series with an extension element (e.g. a steel profile): equivalent model of springs in series ($K'_{d,j}$; $K'_{p,j}$) and equivalent single spring model ($K'_{b,j}$)

$$E_{D,B}^{bilinear} = \sum_{j=1}^{n} 4\left(F'_{by}\delta'_j - \delta'_{y,j}F'_{b,j}(\delta'_j)\right) \tag{4.6}$$

being δ'_j the component of the interstory drift δ_j at j_{th} of the n floors along the axe of the brace ($\delta'_{y,j}$ is the axial displacement corresponding to yielding of the device).

The axial displacement of the damping brace at the j_{th}-floor δ'_{bj} can be determined from its inclination angle $\theta_{b,j}$ and interstorey drift $\delta_j=D_j-D_{j-1}$: therefore $\delta'_{b,j}=\delta_j \cos\theta_{bj}$.

The dissipative brace is usually constituted by a dissipative device (e.g. the *BRB*) assembled in series with an extension element (e.g. realized with a steel profile) in order to connect the opposite corners of a frame (Fig. 6).

Therefore, being $K'_{b,j}$ and $K'_{by,j}$ the equivalent stiffness of the spring series in the elastic and plastic range respectively, $a=K_{e,j}/K_{d,j}{}'$ the ratio between elastic stiffness of the steel profile and of the device and $\beta_{d,j}$ the ratio between stiffness after and before yielding of the dissipative device, the following expression can be derived

$$K'_{b,j} = \frac{K'_{d,j}}{\dfrac{1}{\alpha_j}+1};\;\; K'_{by,j} = \frac{\beta_{b,j}K'_{d,j}}{\dfrac{\beta_{b,j}}{\alpha_j}+1};\;\; \alpha_j = \frac{K'_{p,j}}{K'_{d,j}} \tag{4.7}$$

Therefore

$$F'_{b,j} = F'_{by,j} + (\delta'_j - \delta'_{y,j}) \frac{\beta_{b,j} K'_{d,j}}{\frac{\beta_{b,j}}{\alpha_j} + 1} \tag{4.8}$$

$$\delta'_{y,j} = \frac{F'_{by,j}}{K'_{b,j}} = \frac{F'_{by,j}}{K'_{d,j}} (\frac{1}{\alpha_j} + 1) \tag{4.9}$$

Consequently, if there is one brace per direction and per floor, substituting Eq. (4.8) into Eq. (4.6), $v^*_{eq,B}(D^*_t)$ can be expressed in the following way

$$v^*_{eq,B}(D^*_t) = \chi_B \frac{2}{\pi} \frac{\sum_{j=1}^{n} \left\{ F'_{by,j} \delta'_j - \delta'_{y,j} \cdot \left[F'_{by,j} + (\delta'_j - \delta'_{y,j}) \frac{\beta_{d,j} K'_{d,j}}{\frac{\beta_{d,j}}{\alpha_j} + 1} \right] \right\}}{F_{S,S+B}(D^*_t) \cdot D^*_{S,S+B}} \tag{4.10}$$

δ'_j are determined from the pushover analysis for the top displacement D_t and $\delta'_{y,j}$, that is the yielding displacement of devices, can be reasonable assumed as $\delta'_{y,j} \le \delta'_j / 4$.

$F'_{y,j}$ is, for each direction, the yielding force of the floor brace: once $\delta'_{y,j}$ has been defined $F'_{y,j}$ is consequently determined Eq. (4.9).

Thus, remembering Eq. (3.4) and according to (4.7), $K'_{d,j}$ can be expressed as follows

$$K'_{d,j} = K_{global} \cdot c_{b,j} \cdot (\frac{1}{\alpha_j} + 1) \tag{4.11}$$

Therefore substituting Eq. (4.11) into Eq. (4.10), K_{global} can be determined as follows

$$K_{global} = \frac{\pi \cdot v^*_{eq,B}(D^*_t) \cdot F_{S,S+B}(D^*_t) \cdot D^*_{S,S+B}}{2 \cdot \chi_B \cdot C_1} \tag{4.12}$$

with

$$C_1 = \sum_{j=1}^{n} c_{b,j} \left\{ \delta'_{y,j} \cdot \delta'_j - \delta'_{y,j} \left[\delta'_{y,j} + (\delta'_j - \delta'_{y,j}) \frac{\beta_{b,j} (\frac{1}{\alpha_j} + 1)}{\frac{\beta_{b,j}}{\alpha_j} + 1} \right] \right\} \tag{4.13}$$

A value of a_j>3 is usual in applications, therefore $K'_{b,j}>3/4K'_{d,j}$, while the steel profile must be stronger (neither yielding nor buckling) than the device: for a given interstorey drift the larger is a_j the larger are device displacements and hysteretic cycles.

At this point all terms of Eq. (4.12) are known so, from Eq. (4.11) and Eq. (4.7) , the floor brace stiff nesses $K'_{b,j}$ can be defined (the yielding force $F'_{by,j}$ can be directly derived since the stiffness $K'_{b,j}$ and the yielding displacement $\delta'_{y,j}$ have been defined).

Though in this paper the procedure is discussed referring to Eq. (4.10) it is important to underline that, in a general case, one can have *m* different braces for each level *j.* In fact, at the same level, each brace *i* can be characterized by its specific properties as a consequence, for example, of the geometry of the bays of the structural frame. Consequently Eq. (4.10) can be generalized as follows.

$$v^*_{eq,B}(D^*_t) = \frac{2}{\pi} \frac{\sum_{j=1}^{n}\sum_{i=1}^{m} \chi_{B,i} \left\{ F'_{by,j,i}\delta'_j - \delta'_{y,j,i} \cdot \left[F'_{by,j,i} + (\delta'_j - \delta'_{y,j,i}) \frac{\beta_{d,j,i} K'_{d,j,i}}{\frac{\beta_{d,j,i}}{\alpha_{j,i}} + 1} \right] \right\}}{F_{S,S+B}(D^*_t) \cdot D^*_{S,S+B}} \tag{4.14}$$

A simplified approach of this step is presented in Appendix A:this simplified procedure is useful to get a first dimension of the bracing system.

10. **Check convergence**: one must repeat steps from 3 to 9 until the performance point of the braced structure converges to the target displacement with adequate accuracy.

11.**Impose the simultaneous yielding of the braces (optional):** Though after convergence (step 10) the bracing system is finally defined, it is quite unlikely that braces will yield simultaneously since some have larger elastic limits with respect to the displacement corresponding to the configuration at which the first brace yields (Fig. 7). Thus, keeping the same stiffness, the yielding forces can be reduced to impose that all the braces yield at the same top displacement. Global elastic stiffness is the same while total force is reduced.

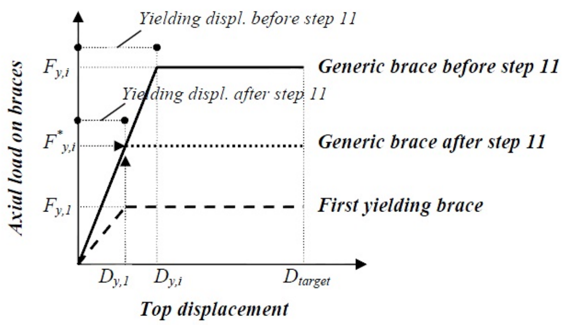

Fig. 7 Determination of the yielding force of the braces at step 11. $F_{y,i}$ = yielding force before step 11; $F^*_{y,i}$ = yielding force after step 11; $F_{y,i}$ = yielding force of the first yielding brace

Off course, this operation, will modify the equivalent viscous damping offered by the bracing system. In Eq. (3.12) the reduction of elastic strain energy (denominator) due to reduction of forces in the devices is usually smaller than the reduction of the energy dissipated by braces (numerator). As an example, referring to an elastic portal frame with a single dissipative brace (defining $A=F^*_{y,i}/F_{y,i}$,$B=D_{target}/D_{y,i}$and $R=(K_S D_{target})/\ F_{y,i}$, with K_S elastic stiffness of the naked structure)Fig. 8 shows $\nu_{eq,B}$ as a function of the brace yielding force (reduction of A). This graph has been drown for different ratios of forces absorbed by the portal frame and by the brace (R=4÷8) and for a given ratio of target displacement and yield displacement (B=0.3).

For a small reduction of yielding force, say 20% reduction (A=0.8), one can assume to have a negligible damping reduction. Moreover, since usually not all the devices will be significantly redefined, damping reduction can be even smaller. However this matter must be carefully evaluated case by case.

12. **Possible optimization:** the calibration of the mechanical characteristics between the extension element (stiffness) and the device (stiffness and strength) allows to optimize the dissipative brace: a stiffer profile implies smaller elastic deformations u_p and therefore the same global displacement of the series implies larger plastic excursion of the device (Fig. 6).This way the device can perform the same dissipation with lower strength.

5. Case Studies

5.1 Application on a bare/infilled plane frame: validation of the procedure

The proposed design procedure has been applied to retrofit an existing structure designed to resist vertical loads only. The structure is a 2*D* r.c. regular frame with three bays (5.0 m span length) and six stories (2.85 m interstorey height). Two configurations have been considered: the bare frame of Fig. 10a and the infilled frame of Fig. 10b. Masonry infill panels have been modelled by means of an equivalent single strut element (Fig. 9). The constitutive model of the equivalent truss inserted in the frame is the one proposed by *Combescure* (1996). This model is characterized by four

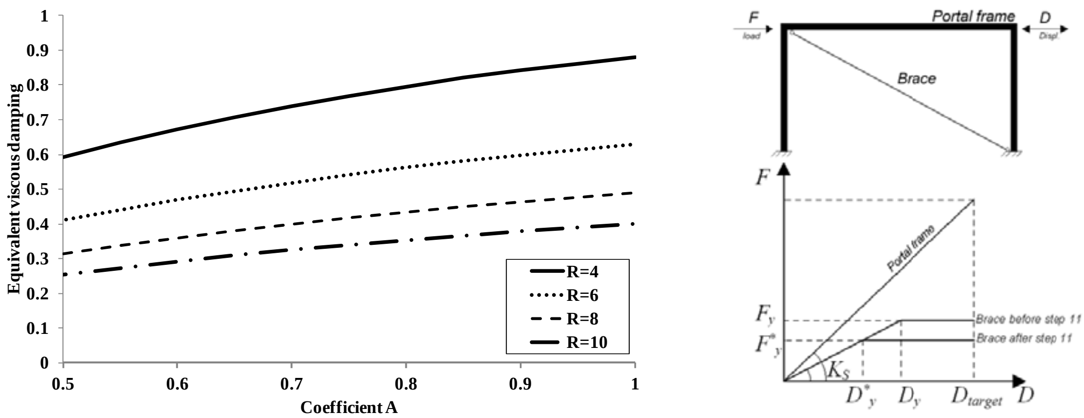

Fig. 8 Variation of the equivalent viscous damping after step 11(left): simple case of an elastic portal frame with a single dissipative brace with B=0.3 (right)

branches (Fig. 9) describing the axial force–axial displacement relationship.

The first one represents the non cracked behaviour and the second one the progression of cracks (stiffness decay). The third branch, characterized by a constant load, simulates failure of the paneland is followed by the fourth softening branch simulating collapse: a residual strength remains after collapse. Both the constitutive law and the cyclic behaviour (the cyclic behaviour of masonry is characterized by a cyclic stiffness decay) of the strut have been calibrated on the basis of the experimental activity performed by the authors in other studies (Bergami 2008) considering single panel walls realized with half full bricks (compression strength of the wall: parallel to holes f_{wp}=5.28 MPa and orthogonal to holes f_{wh}=2.69MPa). According to the proposed approach, pushover analyses have been carried out to define the capacity curves and to evaluate the structural response of both existing and braced frames. First mode proportional load profiles have been applied.

The structure has been analyzed referring to a seismic action characterised by a response spectrum given by Eurocode 8 with a p.g.a. of 0.35*g* and a soil type B (Fig. 15). The performance points of the existing structures in terms of base shear and top displacement are V_{Sbare}=227 kN and $D_{t,Sbare}$=83 mm for the bare frame and V_{Sinf}=573 kN and $D_{t,Sinf}$=70 mm for the infilled frame.

Then, the performance objective is to reduce maximum displacements in order to avoid damage inr.c. elements, for the bare frame, and damage in the masonry panels for the infilled one. For the design seismic event the target displacement mentioned has been selected adopting the following parameters: for the bare structure about 2.5‰ of total height ($D_{t,Sbare,targ}$=46 mm) and, for the infilled frame, a maximum interstorey drift of 5‰ at whichever level of the frame (target top displacement of the infilled frame has been updated at each iteration as drift distribution is modified by the bracing system; in the last iter the target was $D_{t,Sinf,targ}$=41 mm).

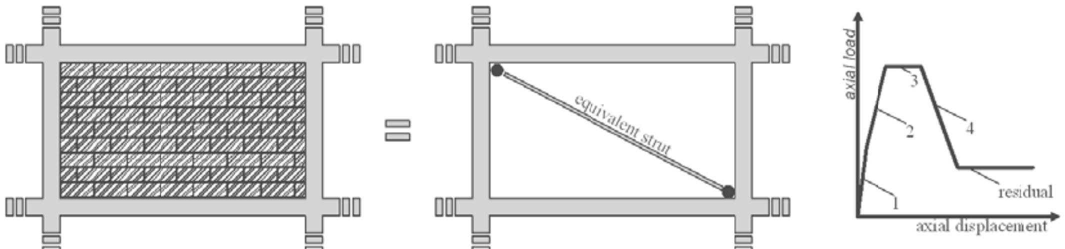

Fig. 9 One infill panel from the frame: equivalent single strut model and its multilinear constitutive law proposed by (Comberscure 1996) (right)

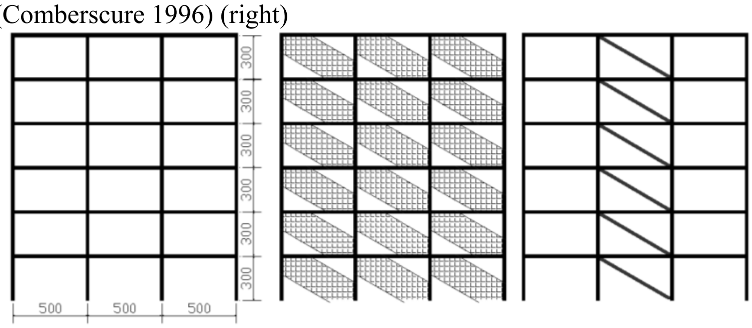

Fig. 10 2D r.c. frames selected as case study [cm]: (1)3x6 r.c.bare frame, (2) 3x6 r.c. infilled frame (infills replaced by the equivalent single strut) and (3) distribution of *BRBs* adopted

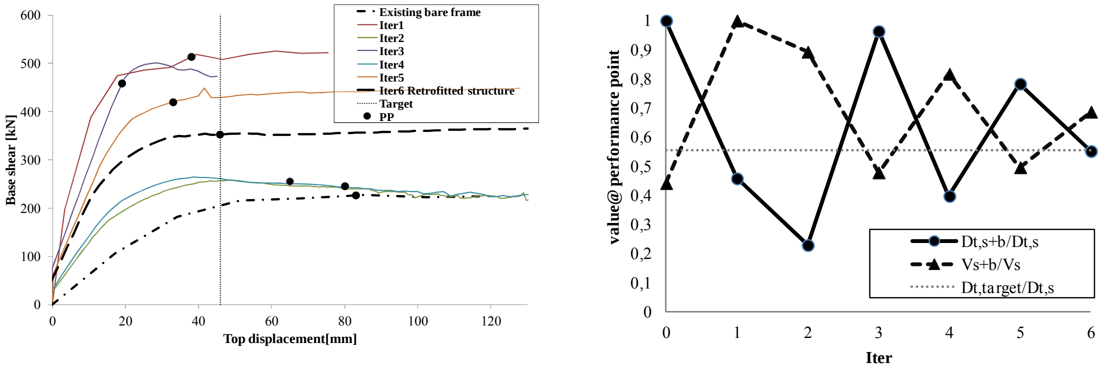

Fig. 11 Bare frame. Variation of response with iterations of the procedure and target displacement $D_{t,target}$. Capacity curve and relative performance point at each iter (left); top displacement $D_{t,s+b}$ and base shear V_{s+b} at performance point of each iter normalized with respect to the maximum corresponding value (right)

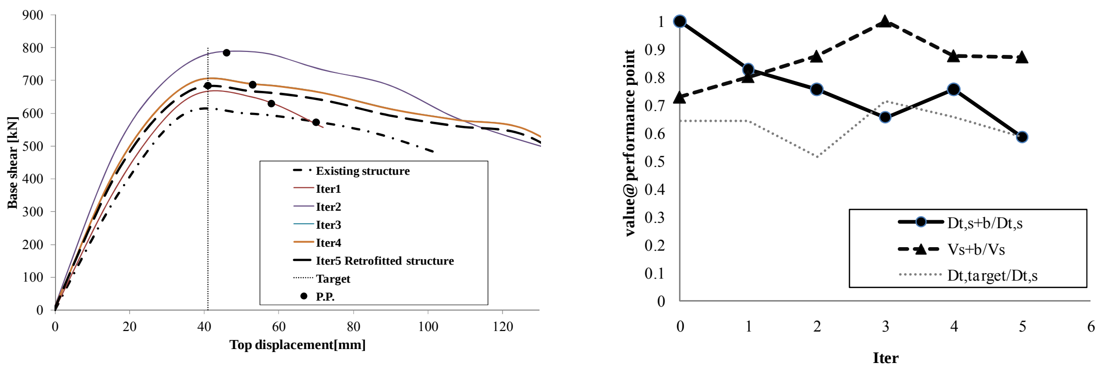

Fig. 12 Infilled frame: same as Fig. 11. Note that the target displacement is update at each iteration because it depends on the interstorey drift limit

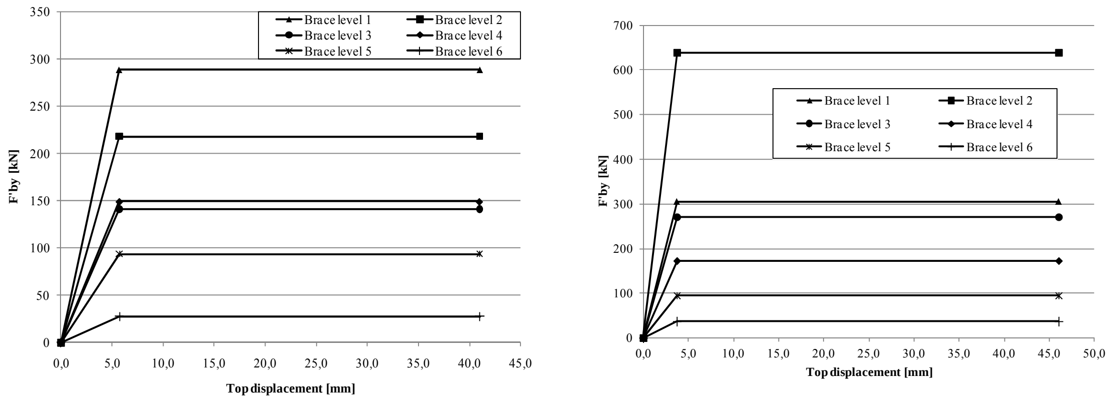

Fig. 13 Axial load on the braces at different stories when the target displacement is reached: bare frame (left), infilled frame (right)

Results of the iterative procedure are shown in Figs. 11-12 for the bare and the masonry infilled

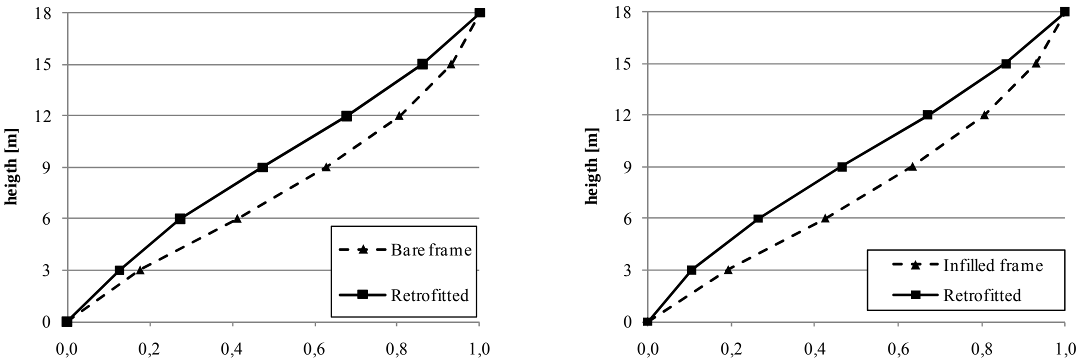

Fig. 14 Modal shapes before and after the retrofitting: bare frame $T_{bare,s}$=1.180 sec., $T_{bare,s+b}$= 0.610 sec. (left), infilled frame $T_{inf,s}$=0.880 sec., $T_{inf,s+b}$=0.528 sec. (right)

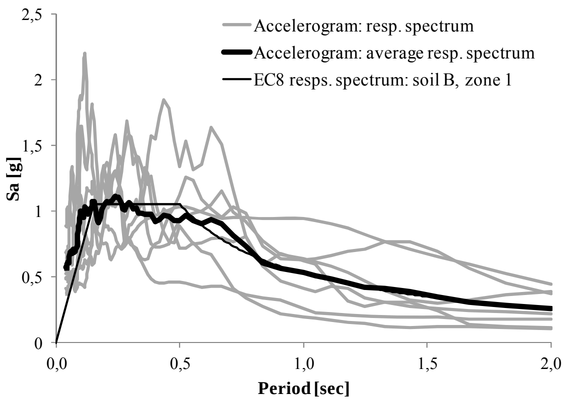

Fig. 15 Response spectrum of the accelerograms used together with their average response spectrum and the Eurocode 8 spectrum used to design the dissipative braces

frame respectively. For every iteration (every stiffness and strength distributions of *BRBs*) the pushover curve and the performance point are given. Convergence to the intended result is obtained in six iterations for the bare frame and five for the infilled one.

The result (performance point, iter6) of the braced bare frame gives a base shear $V_{S+B,bare}$=353 kN, with a 55% increase with respect to the original frame (the existing structure reaches the maximum base shear in correspondence to its performance point), and a top displacement $D_{t,S+B,bare}$=45.8 mm (practically coincident with the target one). The performance point of the braced infilled frame with *BRBs*(iter5) have a base shear $V_{S+B,inf}$=685 kN, with 10% increase with respect to the maximum base shear reached by the original structure, and atop displacement of$D_{t,S+B,inf}$=41 mm (practically coincident with the target one). The *BRBs* obtained for the two cases analyzed are totally different. In the case of the infilled frame, stiffness and strength of *BRBs* are relevantly smaller in the first than in the second storey. In both cases modal shapes, from the existing to the retrofitted structure, changes. Final modal shapes are quite linear and characterized by smaller interstorey drifts (Fig. 14). The comparison of a nonlinear dynamic analysis of the retrofitted structure with pushover results, using first mode or mass proportional force distribution,

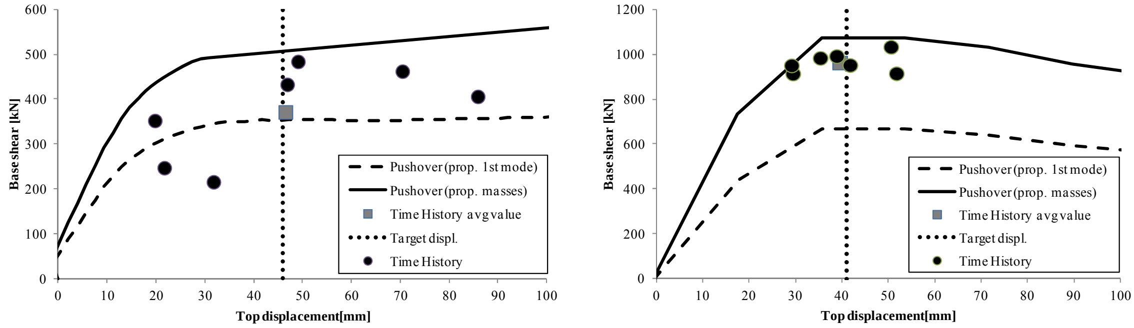

Fig. 16 Comparison between pushover (load distribution prop. to masses and to 1st mode) and incremental nonlinear dynamic analyses (response to each T.H. and their average value) of the retrofitted structures: bare frame with bracing (left), infilled frame with bracing (right)

allows a significant validation of the procedure.

Non linear dynamic analyses have been carried out using a set of seven real record (Iervolino *et al.* 2008) compliant with the response spectrum used for pushover and for the design procedure (Fig. 15).

The mean top displacement from dynamic analyses is 46.5 mm for the braced bare frame and 40 mm for the braced infilled frame: those values are both minor than the relative target displacement.

It can be noted (Fig. 16) that the pushover curve with first mode force distribution is the most accurate for the bare frame whereas, for the infilled frame, the curve relative to mass proportional forces is the most accurate.

One can conclude that, at the moment, the design procedure is efficient in terms of displacements whereas an evaluation of base shear using both a proportional to masses and to 1st mode force distribution seems to be appropriate.

5.2 Application to a real structure: feasibility assessment of the procedure

The proposed design procedure has been applied to retrofit a real building designed according to Italian Code in 1964, to be retrofitted with buckling restrained braces (*BRB*). This application is a test on a real case characterized by real materials, real geometry and moreover using devices available on the market.

The structure is a regular five storey r.c. frame with one additional underground floor. Scope of the retrofitting is to prevent damage in both structure and infills in case of a severe seismic event with a p.g.a.=0.284*g* (Italian technical code D.M. 2008). According to the proposed approach, pushover analyses have been carried out to obtain the capacity curves and to evaluate the structural response for both longitudinal and transverse directions (first mode proportional load profile has been applied): in the paper only the longitudinal analysis, that is the most significant, is presented for brevity.

The adopted *BRB*s longitudinal distribution are shown in Fig. 17; in order to guarantee a uniform load redistribution on the foundation, the basement frame has been retrofitted using r.c. infills.

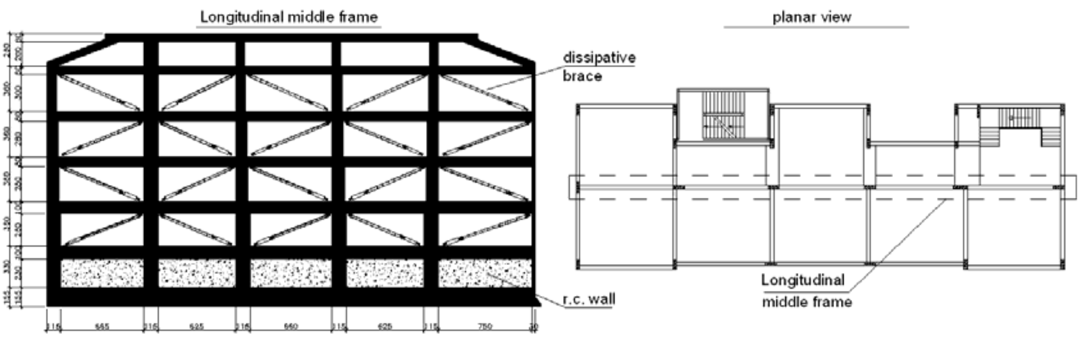

Fig. 17 *BRB*s longitudinal distribution inside the structure: longitudinal braced frame (right), planar view (left)

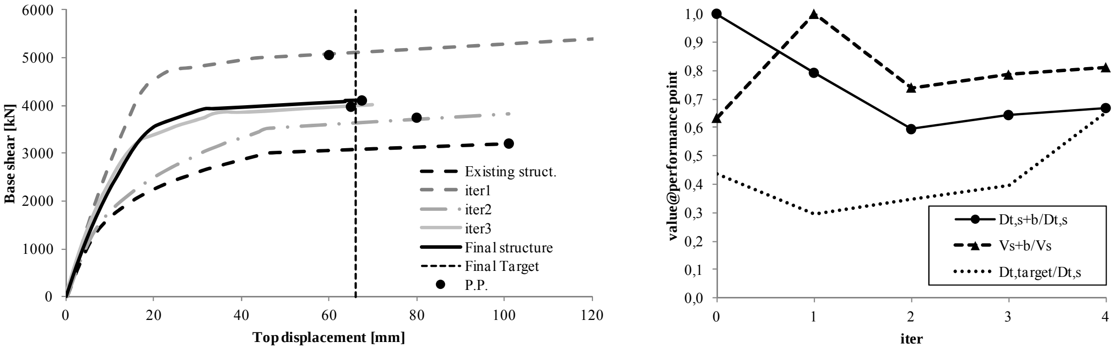

Fig. 18 Real structure. Variation of response with iterations of the procedure and target displacement $D_{t,target}$. Capacity curve and relative performance point at each iter (left); top displacement $D_{t,s+b}$ and base shear V_{s+b}at performance point of each iter normalized with respect to the maximum corresponding value (right). $D_{t,target}$ corresponds to the achievement of the predetermined interstorey drift limit.

The selected target displacement D^*, adopted in the *BRBs* design procedure, corresponds to the achievement, at any level, of an interstorey drift not larger than 0.005 h_i (in the following $D_{0.005}$ with h_i intersorey height): this interstorey limit occurs before that collapse top displacement $D_{s,u}$ is reached ($D^*=D_{0.005}<D_{s,u}$=70 mm).

The procedure converged at third iteration; afterwards the theoretical mechanical parameters of the designed bracing system have been switched with what available on the market. A fourth analysis has than been carried out to check the real solution (commercial *BRBs* were chosen with the aim of obtaining a bracing system as closer as possible to what was previously obtained at iter 3). As shown in Fig. 18 the performance point before retrofitting is $D_{S,pp}$=100 mm (while collapse displacement for is $D_{s,u}$ = 70 mm) with a base shear $V_{S,pp}$ = 3200 kN. Instead, for the retrofitted structure (iter 3), the performance point corresponds to $D_{S+B,pp,3}$ = 65 mm and $V_{S+B,pp,3}$ = 3970 kN.

It is therefore clear that the dissipative braces are able to provide supplemental damping: the equivalent viscous damping for the existing and the retrofitted frame are ν_S=0,21 and $\nu_{S+b,3}$=0,43 respectively). The performance with the real adopted *BRBs* (iter 4) is: $D_{S+B,pp,ex}$ = 67 mm, $V_{S+B,pp,ex}$ = 4100 kN, $\nu_{S+b,ex}$=0,32. Therefore a light increase in terms of both displacement and base shear with respect to the theoretical solution (3% for both) is obtained, but with a substantial reduction in displacement with respect to the original structure.

6. Conclusions

In this work a displacement based procedure to design dissipative braces for the seismic rehabilitation of existing r.c. frames has been presented and applied. The first objective is to obtain a defined target displacement, or interstorey drift, increasing both stiffness and dissipation; a second objective is the limitation of base shear increase.

In the presented applications the target displacement has been differentiated for the case of a bare r.c. frame (collapse prevention) or an infilled frame (infill damaging prevention): the goal is reached adding a system of *BRBs* of appropriate stiffness and yielding force.

The proposed procedure, which determines stiffness and yielding force of the *BRBs*, although relatively simple as it is based on static (non linear) analysis, proves to be effective and efficient requiring few iterations to converge. Moreover it is adaptable to different situations that can be found working with existing structures: irregularity in plane and elevation, low plastic limit and other negative characteristics. Differently from other existing proposal the procedure takes into account, although in a simple way (from the original pushover curve), dissipation coming from the original structure while the exact contribution to dissipation offered by every single brace is considered as well.

A development of the procedure is suggested in order to obtain the needed global energy dissipation with a limited increase of base shear (reducing the yielding force of the dissipative system) and thus reducing the need of foundation strengthening.

The effectiveness of the procedure based on the non linear static analysis has been shown through applications to two case studies of plane frame: one bare and one infilled with brick masonry. Fast convergence to very stringent requirements has been obtained. The final design has been checked with a comparison with dynamic non linear analysis: the comparison has shown that the evaluation of the mean expected displacement is very accurate while, depending on the force distribution adopted in the pushover analysis, base shear can be underestimate: in this latter case the upper bound of base shear can be found with forces proportional to masses instead that to first mode.

The final application of the procedure on a real building shows the significant impact on the solution of complex cases, typical of interventions on existing structures.

To conclude the procedure represent a substantial improvement of displacement based design for retrofitting using dissipative braces. It proves to be simple, though it permits to account the non linear behaviour of the original structure, and determines stiffness and strength of all braces to be added to the structure. Some possible optimization of the final design of the braces are discussed as well.

References

Albanesi, T., Bergami, A.V. and Nuti, C. (2008), “Design procedure for buckling restrained braces to retrofit existing r.c. frames”, *Proceedings of 14th World Conference on Earthquake Engineering*, Beijing, China.

Applied Technology Council. (1996), *Seismic evaluation and retrofit of concrete buildings, Report ATC-40*, Redwood City, California.

Bergami, A.V. (2008), *Implementation and experimental verification of non linear models for masonry infilled r.c. frames*, *Ph.D Thesis*. Università degli Studi Roma Tre, Rome, Italy.

Bergami, A.V. (2011), *Masonry infilled r.c. frames*, LAP LAMBERT Academic Publishing GmbH & Co. KG, ISBN: 978-3-8465-0324-9.

Albanesi, T., Bergami, A.V. and Nuti, C. (2009), “Una procedura progettuale per la protezione sismica di telai in c.a. mediante controventi dissipativi”, *Proceedings of XIII Convegno ANIDIS L' ingegneria Sismica in Italia*. Bologna, Italy.
Bergami, A.V. and Nuti, C. (2009), “Discussione, applicazione e validazione di una procedura progettuale per la protezione sismica di telai tamponati mediante controventi dissipativi”, *Proceedings of I convegno di ingegneria forense IF CRASC ’09*, Naples, Italy.
Berman, J.W. and Brruneau, M. (2009), “Cyclic testing of buckling restrained braced frame with uncostrained gusset connections”, *J. Struct. Eng.-ASCE*, **135**(12), 1499-1510.
Black, C.J., Makris, N. and Aiken, I.D. (2004), “Component testing, seismic evaluation and characterization of buckling restrained braces”, *J. Struct. Eng.-ASCE*, **130**(6), 880-894.
Ciampi, V. (1991), “Use of energy dissipating devices, based on yielding of steel, for earthquake protection of structures”, *Proceedings of Protezione sismica degli edifici*, Ancona, Italy.
Ciampi, V., De Angelis, M. and Paolacci, F. (1995), “Design of yelding or friction-based dissipative bracings for seismic protection of buildings”, *Eng. Struct.*, **17**(5), 381-391.
Constantinou, M.C., Soong, T.T. and Dargush, G.F. (1998), “Passive energy dissipation systems for structural design and retrofit”, *MCEER* - State University of New York at Buffalo.
Dolce, M., Cardone, D. and Marnetto, R. (2000), “Implementation and testing of passive control devices based on shape memory alloys”, *Earthq. Eng. Struct. D.*, **29**(7), 945-968.
Dolce, M., Cardone, D. and Ponzo, F.C. (2001), “Retrofitting of R/C framed structures through SMA-based energy dissipating and re-centering braces”, *Proceedings of 7th International Seminar on Seismic Isolation, Passive Energy Dissipation and Active Control of Vibrations of Structures*, Assisi, Italy.
Di Sarno, L. and Manfredi, G. (2010), “Seismic retrofitting with buckling restrained braces: Application to an existing non-ductile RC framed building”, *Soil Dyn. Earthq. Eng.,* **30**(11), 1279-1297.
Earthquake Engineering Research Institute (2009), “The Mw 6.3 Abruzzo, Italy, Earthquake of April 6, 2009”, Report available on line at: *<http://www.reluis.it/doc/pdf/Aquila/EERI_L_Aquila_report.pdf>*.
Fajfar, P. (1999), “Capacity spectrum method based on inelastic spectra”, *Earthq. Eng. Struct. D.,* **28**, 979-993.
Fajfar, P. and Gaspersic, P. (2000), “The N2 method for the seismic damage analysis for RC buildings”, *Earthq. Eng. Struct. D.,* **25**(1), 23-67.
FEMA-274 (1997), “NEHRP Commentary on the guidelines for the seismic rehabilitation of buildings”, *Federal Emergency Management Agency Publication*, U.S.A., 274.
FEMA – ASCE 356 (2000), “Prestandard and commentary for the seismic rehabilitation of buildings”, Washington, DC, USA.
Filiatrault, A. and Cherry, S. (1988), “A simplified seismic design procedure for friction damped structures”, *Procedings of Fourth U.S. National Conference on Earthquake Engineering*, **3**, Palm Springs, U.S.A.
Filiatrault, A. and Cherry, S. (1990), “Seismic design spectra for friction damped structures”, *J. Struct. Eng.*, **116**(5), 1334-1355.
Iervolino, I., Maddaloni, G. and Cosenza, E. (2008), “Eurocode 8 compliant real record sets for seismic analysis of structures”, *J. Earthq. Eng.*, **12**(1), 54-90.
Kim, J. and Choi, H. (2004), “Behavior and design of structures with buckling-restrained braces”, *Eng. Struct.,* **26**(6), 693-706.
Mazzolani, F.M. (2006), “Seismic upgrading of RC buildings by advanced techniques”, *The ILVA-IDEM Research Project, Polimerica Publisher,* Italy.
Mazzolani, F.M. (2008), “Innovative metal systems for seismic upgrading of RC structures”, *J. Constr. Steel Res.,* **64**(7-8), 882-895.
Palazzo, G., López-Almansa, F., Cahís, X. and Crisafulli, F. (2009), “A low-tech dissipative buckling restrained brace. Design, analysis, production and testing”, *Eng. Struct.,* **31**(9), 2152-2161.
Ponzo, F.C., Dolce, M., Vigoriti, G., Arleo, G. and Di Cesare, A. (2009), “Progettazione di controventi dissipativi a comportamento dipendente dagli spostamenti”, *Proceedings of XIII Convegno ANIDIS L' ingegneria Sismica in Italia,* Bologna, Italy.
Soong, T.T. and Spencer Jr, B.F. (2002), “Supplemental energy dissipation: state-of-the-art and

state-of-the-practice", *Eng. Struct.*, **24**(3), 243-259.
Sabelli, R., Mahin, S. and Chang, C. (2003), "Supplemental energy dissipation: state-of-the-art and state-of-the-practice", *Eng. Struct.*, **25**(5), 655-666.
SEAOC/AISC (2005), "Recommended provisions for buckling-restrained braced frames", *Struct. Eng. Assoc. of California/American Inst. of Steel Const.*
Vulcano, A. and Mazza, F. (2002), "A simplified procedure for the seismic design of framed buildings with dissipative braces", *Proceedings of 12th European Conference on Earthquake Engineering*, London, UK.
Wen-Hsiung, L. and Chopra, A.K. (2003), "Eartquake response of elastic single-degree-of-freedom systems with nonlinear visco-elastic dampers", *J. Eng. Mech.-ASCE*, **129**(6), 597-606.
Whittaker, A.S., Bertero, V., Alonso, J. and Thompson, C. (1989), "Earthquake simulator testing of steel plates added damping and stiffness elements", *Technical Report EERC- 89/02,* University of California, Berkeley, CA.
Xie, W. (2005), "State of the art of buckling-restrained braces in Asia", *J. Constr. Steel Res.*, **61**(6), 727-748.
Zhang, R.H. and Soong, T.T. (1992), "Seismic design of viscoelastic dampers for structural applications", *J. Struct. Eng.,* **118**(5), 1375-1392.

SA

APPENDIX A

Step 9 of the procedure (Chapter 4) can be simplified as in the following.

Assuming that the dissipative device is an elasto-plastic truss element with cross section $A_{d,j}$ and yielding strength $F'_{dy,j}(F'_{by,j}= F'_{dy,j})$, the following expression can be derived

$$F'_{by,j} = f_{dy,j} A_{d,j} = f_{dy,j} \frac{K'_{d,j} \cdot l_{d,j}}{E_{d,j}} \tag{A.1}$$

$$A_{d,j} = K'_{b,j} \left(\frac{1}{\alpha_j} + 1\right) \frac{l_{d,j}}{E_{d,j}} = \frac{K'_{d,j} l_{d,j}}{E_{d,j}} \tag{A.2}$$

$$\delta'_{y,j} = \frac{F'_{by,j}}{K'_{d,j}} \left(\frac{1}{\alpha_j} + 1\right) = \frac{f_{dy,j} \cdot l_{d,j}}{E_{d,j}} \left(\frac{1}{\alpha_j} + 1\right) \tag{A.3}$$

$$K'_{b,j} = \frac{K'_{d,j}}{\frac{1}{\alpha_j} + 1}; \quad K'_{d,j} = \frac{E_{d,j} \cdot A_{d,j}}{l_{d,j}} \tag{A.4}$$

Therefore Eq. (4.10) can be expressed as the following Eq. (A.5)

$$v^*_{eq,B}(D^*_t) = \chi_B \frac{2}{\pi} \frac{\sum_{j=1}^{n} \left\{ \frac{f_{dy,j} K'_{b,j} l_{d,j} \delta'_j}{E_{d,j}} - \delta'_{y,j} \left[\frac{f_{dy,j} K'_{b,j} l_{d,j}}{E_{d,j}} + (\delta'_j - \delta'_{y,j}) \frac{\beta_{d,j} K'_{b,j}}{\frac{\beta_{d,j}}{\alpha_j} + 1} \right] \right\}}{F_{S,S+B}(D^*_t) \cdot D^*_{S,S+B}} \tag{A.5}$$

where $f_{dy,j}$, $E_{d,j}$ and $l_{d,j}$ are respectively the yielding stress, the elastic modulus and the length of the device. Consequently the coefficient C_1 of Eq. (4.13) can be expressed as follows (this expression of C_1 is named C^*_1)

$$C^*_1 = \sum_{j=1}^{n} c_{b,j} \left\{ f_{dy,j} \frac{l_{d,j}}{E_{d,j}} \delta'_j - \delta'_{y,j} \left[f_{dy,j} \frac{l_{d,j}}{E_{d,j}} + (\delta'_j - \delta'_{y,j}) \frac{\beta_{b,j}}{\frac{\beta_{b,j}}{\alpha_j} + 1} \right] \right\} \tag{A.6}$$

Moreover with a further simplification the dissipative braces can be considered as a one piece devices all realized with the same material (the device is directly connected with both the corners of the frame; f_{dy}, β_d and E_d are known as the material has been selected and $l_{d,j}$ is known from the geometry of the structure), thus Eq. (A.6) can be further simplified as follows (this simplified expressions of C^*_1 is named $C^*_{1,S}$)

$$C_{1,S}^{*} = \sum_{j=1}^{n} c_{b,j} \left\{ f_{dy} \frac{l_{d,j}}{E_d} \delta_j^{'} - f_{dy} \frac{l_{d,j}}{E_d} \left[f_{dy} \frac{l_{d,j}}{E_d} + (\delta_j^{'} - f_{dy} \frac{l_{d,j}}{E_d}) \beta_d \right] \right\} \tag{A.7}$$

Since as commercial devices are characterized by a narrow range of lengths (usually: 1000 mm < l_d < 1400 mm) an approximate evaluation of l_d (e.g. l_d =1200 mm) can be made without influence results.

Therefore, remembering Eqs. (A.1)-(A.2) and similarly to the standard procedure, $K'_{b,j}$ and $F'_{by,j}$ can be immediately determined.

This simplified expression can be useful for predimensioning the bracing system.