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Earthquake time-frequency analysis using a new compatible wavelet function family

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Abstract. Earthquake records are often analyzed in various earthquake engineering problems, making time-frequency analysis for such records of primary concern. The best tool for such analysis appears to be based on wavelet functions; selection of which is not an easy task and is commonly carried through trial and error process. Furthermore, often a particular wavelet is adopted for analysis of various earthquakes irrespective of record's prime characteristics, e.g. wave's magnitude. A wavelet constructed based on records' characteristics may yield a more accurate solution and more efficient solution procedure in time-frequency analysis. In this study, a low-pass reconstruction filter is obtained for each earthquake record based on multi-resolution decomposition technique; the filter is then assigned to be the normalized version of the last approximation component with respect to its magnitude. The scaling and wavelet functions are computed using two-scale relations. The calculated wavelets are highly efficient in decomposing the original records as compared to other commonly used wavelets such as Daubechies2 wavelet. The method is further advantageous since it enables one to decompose the original record in such a way that a clear time-frequency resolution is obtained.

Keywords: time-frequency analysis; wavelet function; continuous wavelet transform; multi-resolution analysis

1. Introduction

In recent decades, wavelets have been used widely in analysis of signals obtained from physical systems, such as seismic, mechanical and electrical systems. They are successfully applied to earthquake engineering problems such as time-frequency analysis (Bazrafshan and Bagheripour 2011), soil-structure interaction (Bagheripour *et al.* 2010), signal de-noising (To *et al.* 2009, Liu and Tang 2011) and generation of artificial accelerograms (Giaralis and Spanos 2009, Das and Gupta 2011). The reason for popularity of wavelets is their effectiveness in emulating nonstationary (transient) signals. Since most natural and human-made signals are transient in nature, varieties of wavelets have been used to represent the broad range and class of signals; far more than Fourier Transformation (FT) of stationary signals. Unlike Fourier-based analysis, which basically uses global (nonlocal) sine and cosine functions (Takewaki and Tsujimoto 2011), wavelet analysis uses various bases that are localized both in time and frequency to represent nonstationary signals in more efficiently process. Hence, a wavelet representation is much more compact and easier process to implement than is the FT. Using the powerful multiresolution analysis, one can represent a signal

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by a finite sum of components at different resolutions so that each component can be processed adaptively based on the objectives of a given application. This capability to represent signals compactly and at several levels of resolution is the major strength of wavelet analysis.

However, it is important to note that most wavelets have been developed primarily from a mathematical point of view, usually by mathematicians, without any direct link to physical systems that are to be analyzed. Because of this, most practicing engineers and engineering researchers are not familiar with wavelet analysis tools; particularly with their capabilities and/or limitations in solving earthquake engineering problems. Such conditions motivate researchers to develop new approaches in designing wavelets particular to the physical phenomenon being studied, while satisfying the mathematical requirements in wavelet construction.

In this regard, Gupta *et al.* (2005) designed biorthogonal and semi-orthogonal wavelets from a particular signal by maximizing its projection onto successive scaling subspaces while minimizing signal's energy in the wavelet subspace. However, their proposed method is rather complicated for earthquake engineers to apply; it involves solving sets of simultaneous equations such as those from error energy functionals as well as those generated to minimize the functionals for perfect reconstruction which again entails a complicated and time consuming process.

Yan and Gao (2010) proposed an impulse wavelet particular to ball bearing systems rooted in the actual structural response of a bearing; so to enable more effective defect signature extraction from bearing's vibration signals. Dick *et al.* (2012) proposed a formula for calculating the scaling function coefficients based on two-scale equations. The signals they analyzed were limited to impulse responses from a single degree-of-freedom under-damped linear oscillator and a cantilevered aluminum beam. However, efficiency of the proposed formula was not demonstrated for high-transient signals such as earthquakes.

As a common practice in geotechnical earthquake engineering, a uniquely selected wavelet, such as Daubechies2 wavelet, is used for time-frequency analysis of various records having different characteristics. Extensive investigations into time-frequency analysis of strong ground motion records conducted by researchers have revealed that the use of a unique wavelet for analysis of most or all the records could lead to significant inefficiency. While an alternative wavelet based on particular earthquake characteristics could be a better analysis tool.

With the objective of improving the efficiency of wavelets in time-frequency analysis of earthquake records, a new method is proposed in this study for calculating the scale and wavelet functions. The method is based on multi-resolution decomposition technique. The method benefits low-pass filter; which is assigned for each earthquake record and is introduced as the normalized version of the last approximation component with respect to its magnitude. The scaling and wavelet functions are computed using two-scale relations. The merit of work presented here is that the calculated wavelets are very efficient in decomposing the original earthquake records when compared with commonly used wavelets, such as Daubechies2 wavelet. Further advantage of the wavelet function family presented in this study is that they are able to decompose and reconstruct the original records with much clearer frequency resolution.

2. Formulation

In this research, the decomposition and reconstruction filters are estimated using the idea of multiresolution analysis (MRA) method. MRA forms the most important building block for the construction of filter banks and hence, the scaling and wavelet functions. In multiresolution analysis, a signal is decomposed into various levels of approximations or resolutions. By increasing the level of decomposition, the signal becomes simpler, having fewer data points.

In MRA, it is possible to decompose any square-integrable function in L^2 as the sum of base functions that characterize two orthogonal complementary subspaces V_j and W_j (with $V_j = V_{j-1} \oplus W_{j-1}$, where $\oplus$ is direct sum operator). The V_j spaces are referred to as the approximation spaces and are defined by the scale function family $\{\phi_{j,k}(t) = 2^{j/2}\phi(2^j t - k), j, k \in \mathbb{Z}\}$, where j and k are referred to as level and translation, respectively. The W_j spaces are called detail spaces and are defined by the Wavelet function family $\{\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k), j, k \in \mathbb{Z}\}$. In the above relations, $\phi(t)$ and $\psi(t)$ are mother scaling and wavelet functions, respectively. $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ are the dilated and translated versions of mother scaling and wavelet functions, respectively. The relation between $\phi(t)$ and $\psi(t)$ is expressed by two-scale relations (Boggess and Narcowich 2001)

$$\begin{aligned} \phi(t) &= \sum_k g_0[k]\,\phi(2t-k) \\ \psi(t) &= \sum_k g_1[k]\,\phi(2t-k) \end{aligned} \tag{1}$$

where $\{g_0[k]\}$ is called the low-pass reconstruction filter and $\{g_1[k]\}$ is called the high-pass reconstruction filter. The relation between these two sequences is given as

$$g_1[k] = (-1)^k g_0[1-k] \tag{2}$$

A function (or a signal) $v_j(t) \in V_j$ can be represented as a linear combination of $\phi_{j,k}(t)$, i.e.,

$$v_j(t) = \sum_k a_{j,k}\phi_{j,k}(t) \tag{3}$$

Similarly, a function (or a signal) $w_j(t) \in W_j$ can be written as

$$w_j(t) = \sum_k d_{j,k}\psi_{j,k}(t) \tag{4}$$

where $a_{j,k}$ and $d_{j,k}$ are called scale and wavelet coefficients, respectively. V_j spaces generate nested sequences and are embedded as (Goswami and Chan 1999)

$$\{0\} \leftarrow \cdots \subset V_{j-1} \subset V_j \subset V_{j+1} \subset \cdots \rightarrow L^2$$

Every function (or signal) in L^2 can be approximated using the space V_J where J is a sufficiently large number such that space V_J tends to L^2 and W_J tend to $\{0\}$. In this way, every function $f(t) \approx f_J(t) \in V_J$ can be decomposed as the sum of two functions which belong to V_{J-1} and W_{J-1}, respectively. Using Eqs. (3) and (4)

$$\begin{aligned} f_J(t) &= \sum_k a_{J,k}\phi_{J,k}(t) \\ &= \sum_k a_{J-1,k}\phi_{J-1,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) \\ &= A_1(t) + D_1(t) \\ &= \sum_k a_{J-2,k}\phi_{J-2,k}(t) + \sum_k d_{J-2,k}\psi_{J-2,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) \\ &= A_2(t) + D_2(t) + D_1(t) \\ &= \cdots \end{aligned} \tag{5}$$

Here, $A_i(t)$ and $D_i(t)$ are the approximation and detail functions (or components), respectively. If the first decomposition level is denoted as 0, and the decomposition process is continued up to level s, the original function (or signal) can be expressed as

$$f(t) = A_0(t) = A_s(t) + \sum_{i=1}^{s} D_s(t) \tag{6}$$

As can be seen from Eq. (1), at each level, the number of data points is decimated by two from the previous level. In this research, s is chosen to be the smallest power of two which is less than the number of data points representing an input signal. For instance, in an earthquake record which contains 1000 sample points, the number of levels, s, for decomposition is equal to 9, i.e., $2^9 = 512 < 1000 < 2^{10} = 1024$. Fig. 1 shows the wavelet decomposition tree as well as the number of samples for approximation and detail signals for each level. Hence, by performing successive decomposition analysis over a given signal, we can obtain a rough approximation of the input signal at the latest level. For the sample signal shown in Fig. 1, this latest level is presented by A9.

In this research, the decomposition process is carried by Daubechies2 (*db*2) wavelet using

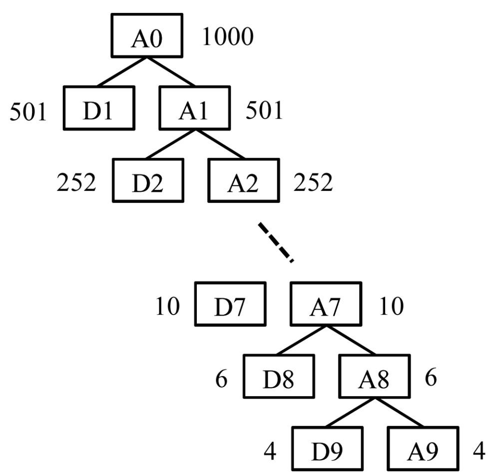


Fig. 1 Wavelet decomposition tree for a 1000 sample point signal

MATLAB (2011) program. For each acceleration, $\{g_0[k]\}$ sequence is taken to be the last approximation component and is normalized by its magnitude. Using Eq. (2), $\{g_1[k]\}$ can be computed for each $\{g_0[k]\}$ sequence. An iterative process is utilized to compute the scale and wavelet functions. Rewriting Eq. (1) for levels m and m+1 one obtains

$$\phi_{m+1}(n) = \sum_k g_0[k]\phi_m(2n-k) \quad m = 0, 1, 2, 3,... \tag{7}$$

In the first iteration, $\phi_0(n) = \delta(n) = 1$ and after upsampling by 2, the sequence is convolved with the $g_0[k]$ sequence to yield $\phi_1(n)$. This sequence is upsampled and convolved with $g_0[k]$ again to give $\phi_2(n)$ and so on. This procedure converges when two successive scaling functions have negligible difference. Once the scaling function has been obtained, the associated wavelet function can be computed using the two-scale relation

$$\psi(t) = \sum_k g_1[k]\phi(2t-k)$$

In order to plot $\phi(t)$ and $\psi(t)$, the abscissa needs to be divided by 2^m for each iteration m to get the corresponding position along time axis (Goswami and Chan 1999).

3. Daubechies2 wavelet

The Daubechies wavelet family is attributed to Ingrid Daubechies (1992) who invented the compactly supported orthonormal wavelets, making wavelet analysis in discrete time possible. The Daubechies wavelet family is denoted as *dbN* where *N* represents its order. The first order Daubechies wavelet is also known as the Haar wavelet; for which the wavelet function resembles a

Table 1 Two-scale sequences for Daubechies2

$\{g_0[k]\}$	0.683	1.183	0.317	-0.183
$\{g_1[k]\}$	-0.183	-0.317	1.183	-0.683

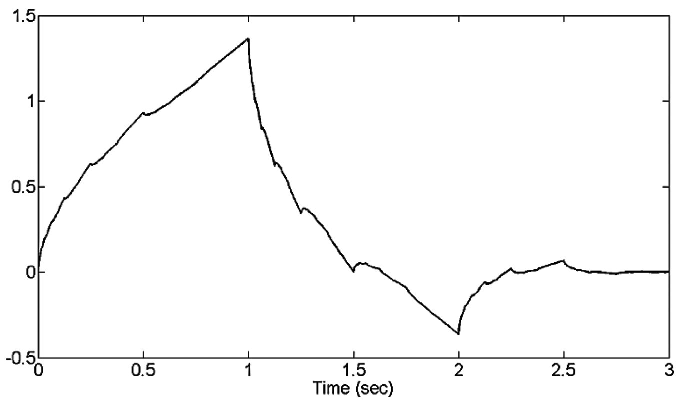


Fig. 2 Daubechies2 scaling function

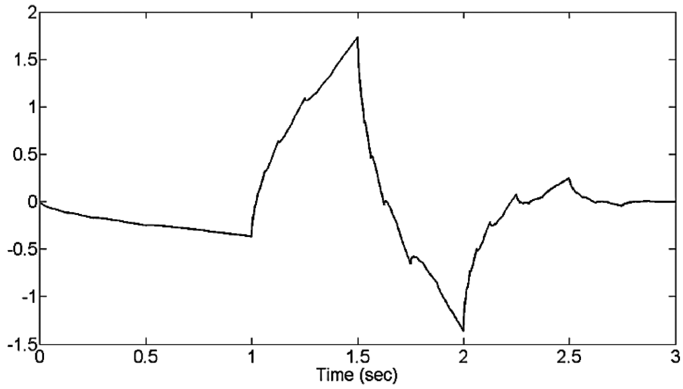


Fig. 3 Daubechies2 wavelet function

step function. Daubechies showed that for a given order N, there will be $2N$ nonzero, real, scaling coefficients resulting in a scaling and a wavelet function that are supported on the interval $0 < t < 2N\text{-}1$ (Boggess and Narcowich 2001). Therefore, for $db2$ the two-scale sequences, $\{g_0[k]\}$ and $\{g_1[k]\}$, have 4 coefficients which are provided in Table 1. It is worthy to note that in some texts there is a factor of $\sqrt{2}$ for these two sequences (Goswami and Chan 1999). Daubechies wavelet family doesn't have explicit mathematical expression and the wavelets are given in terms of two-scale sequences. Therefore, in order to graphically display the scale and wavelet functions, an iteration method is used which is fully described by Goswami and Chan (1999). Figs. 2 and 3 show the scale and wavelet functions for $db2$, respectively.

4. Analysis and results

To validate the proposed method, 7 different earthquakes and their corresponding records are adopted from Pacific Earthquake Engineering Research Centre (PEER) database and are listed in Table 2.

These records differ in magnitude as well as in PGA; and have different number of data points. Hence, level of decomposition for each earthquake is set according to the criteria introduced in preceding section. Using the proposed method, scale functions and wavelet functions for these earthquakes are calculated and are graphically presented in Figs. 4 and 5, respectively.

Table 2 List of earthquakes used for validation

Location	Date	Magnitude	PGA (g)
Cape Mendocino, USA	1992	7.01	0.549
Coyote Lake, USA	1979	5.74	0.113
El-Centro, USA	1940	6.95	0.313
Imperial Valley, USA	1979	6.53	0.588
Loma Prieta, USA	1989	6.93	0.442
Tabas, Iran	1978	7.35	0.836
Victoria, Mexico	1980	6.33	0.621

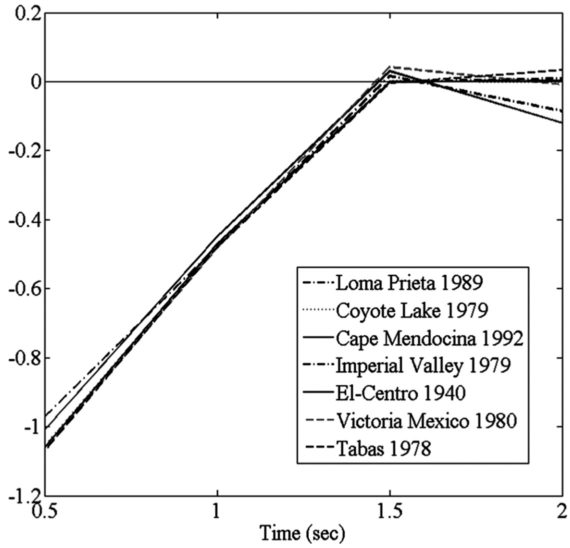


Fig. 4 Calculated scaling functions

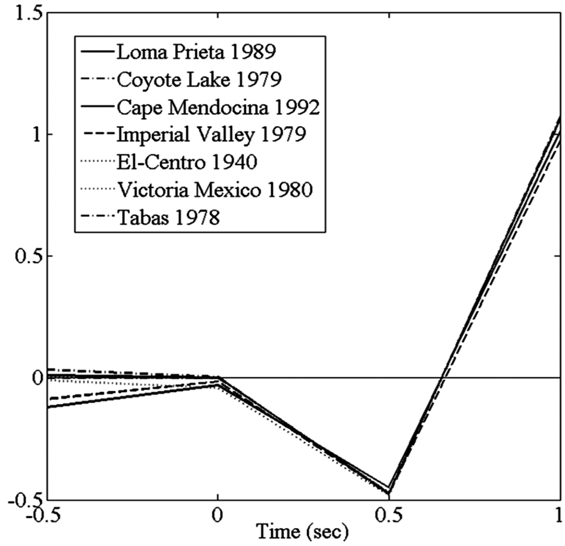


Fig. 5 Calculated wavelet functions

For time-frequency analysis of an earthquake record, the Integral Wavelet Transform is used; which can decompose signals into various frequency components while the related time information is maintained. It is defined as

$$W_\psi f(a,b) = \int_{-\infty}^{\infty} f(t)\,\overline{\psi_{a,b}(t)}\,dt \tag{8}$$

where $\psi_{a,b} = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$, $a > 0$ is the translated and dilated version of mother wavelet while a and b are scale and translation parameters, respectively. For reconstruction of the original signal, Inverse Wavelet Transform is used; which is expressed as

$$f(t) = \frac{1}{C_\psi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{a^2}[W_\psi f(a,b)]\,\psi_{a,b}(t)\,da\,db \tag{9}$$

where C_ψ is known as the admissibility condition and is given by (Radunovic 2009)

$$C_\psi = \int_{-\infty}^{\infty}\frac{|\hat{\psi}(\omega)|^2}{|\omega|}d\omega < \infty \tag{10}$$

here ^ is the Fourier Transform operator.

The admissibility condition implies that all wavelets must have $\hat{\psi}(0) = 0$. Since $\hat{\psi}(\omega) = \int_{-\infty}^{\infty}\psi(t)e^{-i\omega t}dt$; then, $\hat{\psi}(0) = \int_{-\infty}^{\infty}\psi(t)dt = 0$. This implies that the mean value of $\psi(t)$ in time domain is zero. As can be seen in Fig. 5, the admissibility condition is satisfied for all calculated wavelet functions. Inspecting Figs. 4 and 5 also shows that inner product of the scale and wavelet functions vanishes; implying that these functions are orthogonal, i.e.,

$$\int_{-\infty}^{\infty} \phi(t)\,\psi(t-k)\,dt = 0 \tag{11}$$

In Fig. 6, the original and reconstructed earthquake records are shown. As can be seen from this figure, the calculated wavelets can reconstruct the input earthquake records with high accuracy such that the computational error is negligible (Table 3).

Daubechies2 wavelet offers acceptable resolution in both time and frequency domains; hence, it is routinely utilized in time-frequency analysis of nonstationary signals (Al-Badour *et al.* 2011, Chang and Shi 2010, Iyama 2005). Because of this acceptability and applicability, the proposed method benefits from Daubechies2 wavelet in development of low-pass reconstruction filters. Hence, comparisons of the obtained results are made primarily with those given by Daubechies2 wavelet in terms of decomposition tree and Continuous Wavelet Transform (CWT) representation.

In Fig. 7, the Loma Prieta earthquake is decomposed up to 4 levels using filters obtained from itself according to the proposed method. In Fig. 8 this earthquake record is decomposed by Daubechies2 wavelet. Comparing Figs. 7 and 8, it can be seen that the highest amplitude of the forth approximation component in Fig. 7 is nearly 0.1, whereas in Fig. 8 it is around 0.4. Hence, the proposed method can decompose the original signal faster and approximation components decay more rapidly with respect to Daubechies2 wavelet. This is due to the fact that, in the proposed method, the decomposition is made using filters that are originated from the input signal and hence they can analyze the earthquake record more efficiently.

Since wavelet analysis works with time and scale parameters, therefore, in order to plot the results

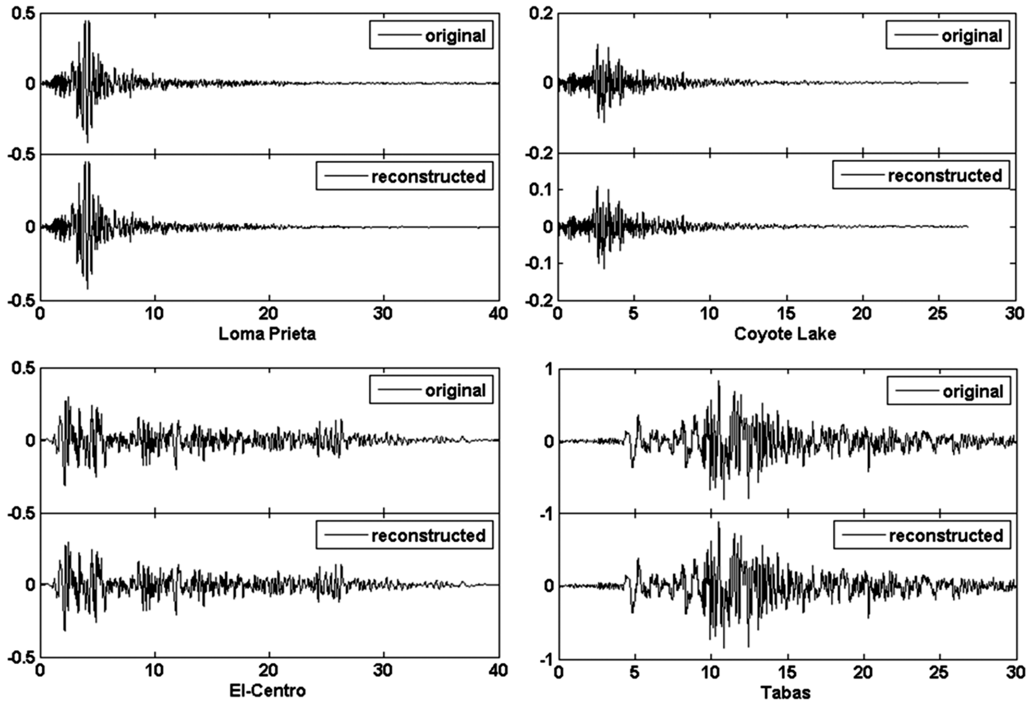


Fig. 6 Original and reconstructed earthquakes using proposed method

Table 3 Values of computational error in reconstruction process

Earthquake		Min acceleration	Max acceleration	Error (-) %	Error (+) %	Mean error %
Loma Prieta 1989	Original	-0.418	0.442	1.9	1.5	1.7
	Reconstructed	-0.426	0.449			
Coyote Lake 1979	Original	-0.113	0.109	0.6	0.6	0.6
	Reconstructed	-0.114	0.110			
Cape Mendocina 1992	Original	-0.479	0.549	1.8	2.2	2.0
	Reconstructed	-0.470	0.537			
Imperial Valley 1979	Original	-0.588	0.510	0.5	2.2	1.4
	Reconstructed	-0.585	0.499			
El-Centro 1940	Original	-0.313	0.296	0.8	0.9	0.9
	Reconstructed	-0.316	0.298			
Victoria Mexico 1980	Original	-0.621	0.291	0.8	1.7	1.2
	Reconstructed	-0.626	0.296			
Tabas 1978	Original	-0.800	0.836	2.2	1.2	1.7
	Reconstructed	-0.817	0.826			

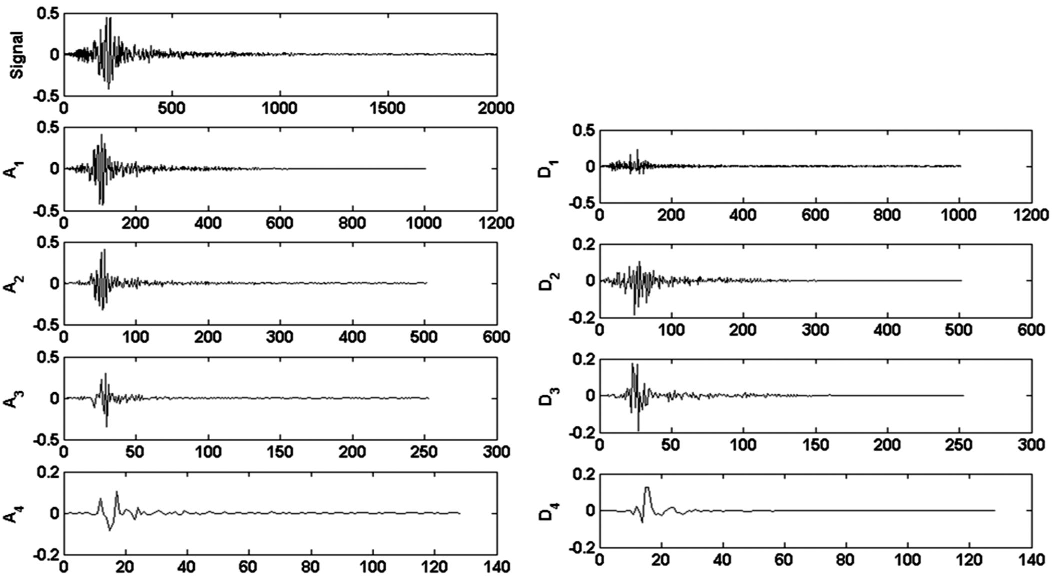


Fig. 7 Decomposition tree for Loma Prieta earthquake using the proposed method

in terms of time and frequency, relationship between scale and frequency for the calculated wavelets should be defined. A rational choice is to relate the wavelet to a purely periodic signal of frequency f_C and to compute the center frequency f_C and then use the following relation (Abry 1997)

$$frq(Hz) = \frac{f_C}{a.\Delta t} \tag{12}$$

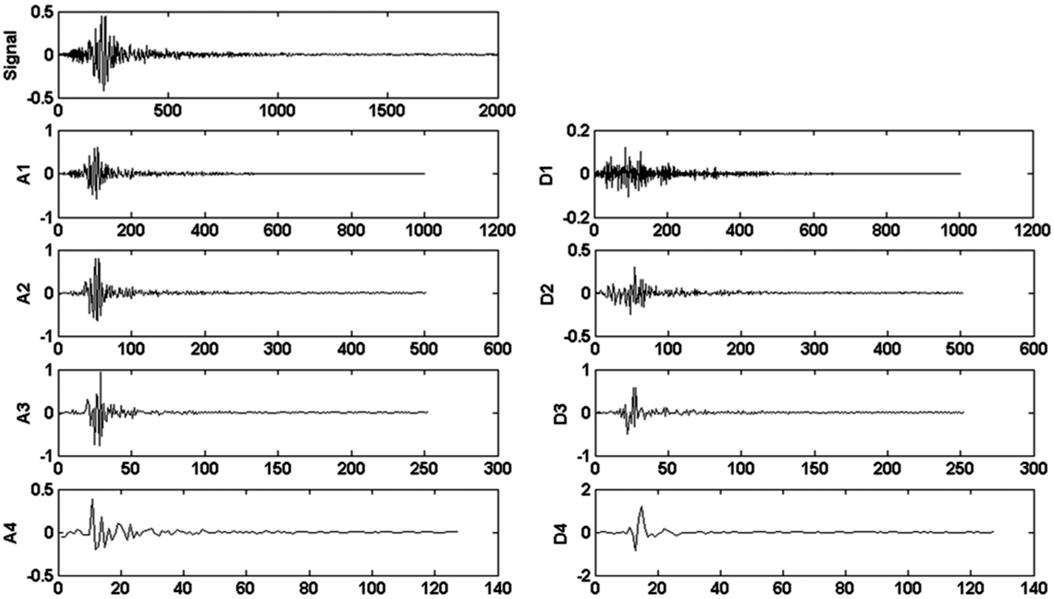


Fig. 8 Decomposition tree for Loma Prieta earthquake using Daubechies2

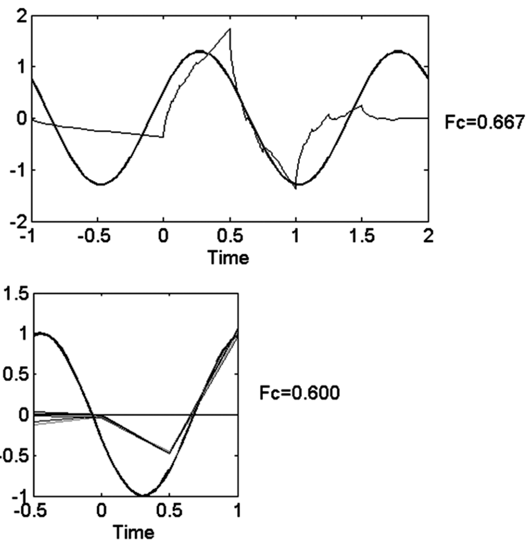


Fig. 9 Center frequency for Daubechies2 and the new wavelets

where *frq* is frequency related to scale a and Δt is the sampling period.

Fig. 9 shows the center frequency for Daubechies2 and the calculated wavelets. As mentioned in a paper by Misiti *et al.* (2011), the center frequency for Daubechies2 wavelet is $f_C = 0.667$ Hz while for the calculated wavelets, a sinusoid with center frequency f_C=0.6 Hz is fairly matched with the shape of wavelets calculated here. Therefore, the relation between scale and frequency for the new wavelets becomes

$$frq(\mathrm{Hz}) = \frac{0.6}{a.\Delta t} \tag{13}$$

As for continuous wavelet transform representation, Loma Prieta earthquake is transformed by the related wavelet as well as Daubechies2 wavelet and comparison is made in Fig. 10. In Figs. 10(a) through 10(d), the acceleration time-history, Fourier spectrum and 3-dimensional CWT for the proposed method and Daubechies2 wavelet are shown. It can be observed that the calculated wavelet can perform accurate time-frequency analysis of the earthquake record; and that in comparison with Daubechies2 wavelet, it demonstrates more sensitivity to frequency spikes and the time of their occurrence. In particular, in Fig. 10(d), Daubecies2 wavelet is seen to be unable to clearly show frequency spikes in frequency range of 5-15 Hz while the calculated wavelet proposed here can represent even the low frequency content of the signal. It further demonstrates the accuracy and efficiency of the method proposed in this paper.

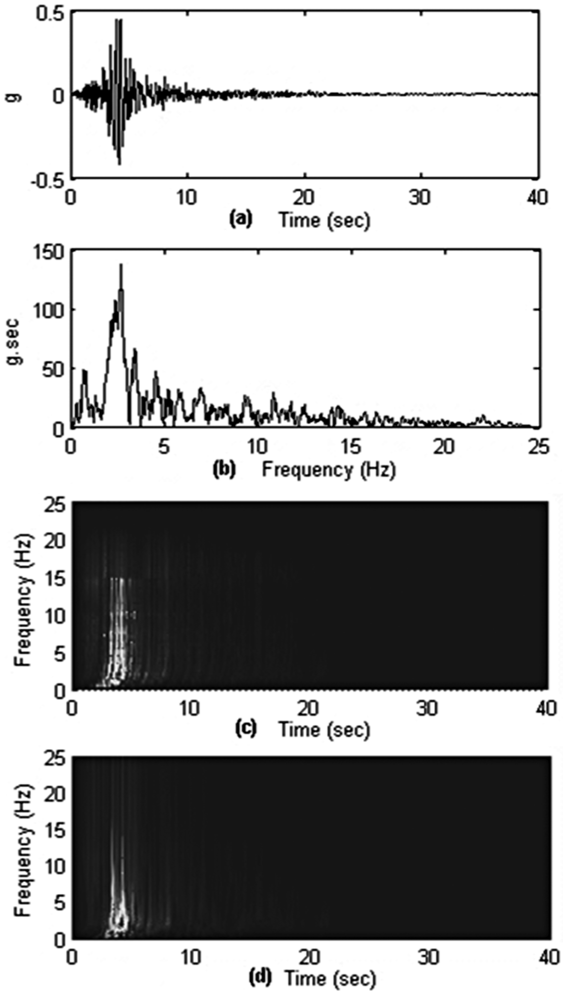
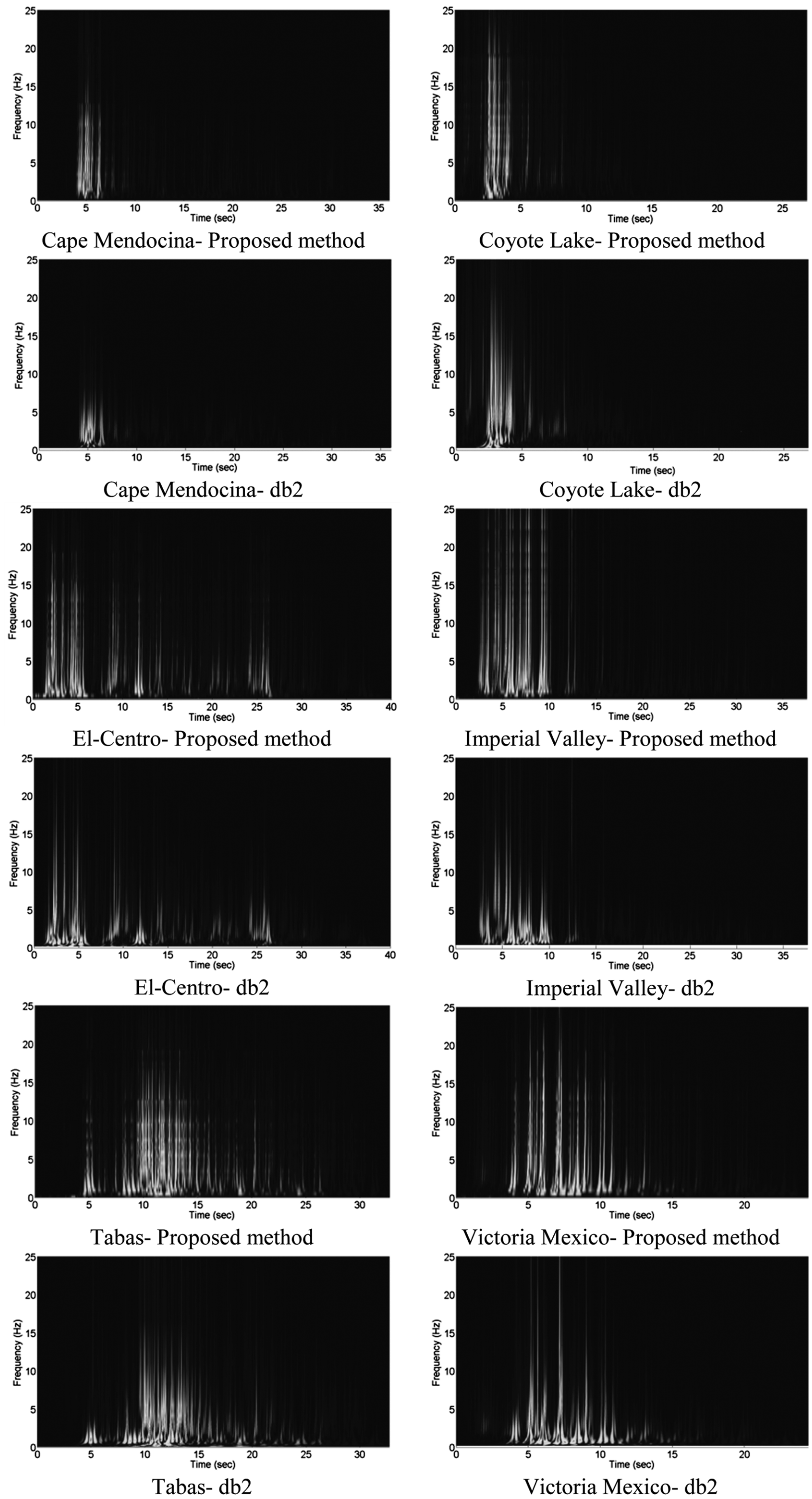


Fig. 10 Loma Prieta earthquake: (a) acceleration, (b) Fourier transform, (c) CWT by proposed method and (d) CWT by *db*2



Cape Mendocina- Proposed method
Coyote Lake- Proposed method
Cape Mendocina- db2
Coyote Lake- db2
El-Centro- Proposed method
Imperial Valley- Proposed method
El-Centro- db2
Imperial Valley- db2
Tabas- Proposed method
Victoria Mexico- Proposed method
Tabas- db2
Victoria Mexico- db2

Fig. 11 CWT comparison of proposed method with Daubechies2 wavelet

The same comparison is made for other six earthquake records shown in Fig. 11. Each earthquake record is transformed by its own calculated wavelet and compared with Daubechies2 CWT. It can be observed that the results obtained from the proposed method show more sensitivity for low frequencies than does of the Daubechies2 wavelet, i.e., nonzero points in the proposed method results exceed the *db*2 results.

5. Conclusions

In wavelet analysis, there is no unique representation of a given signal, as there are many different wavelets available to accomplish this task. Not all these wavelets work well for all signals (some may work very well for a specific signal and may not work at all for another signal), therefore, the choice of wavelet becomes difficult for earthquake engineers. In this paper, a new and easy method is introduced for obtaining efficient scaling and wavelet functions for time-frequency analysis of earthquake motions. Using Daubechies2 wavelet, each earthquake signal is decomposed up to levels where the approximation component in the last level contains 4 elements. This component is a rough representation of the input signal. The low-pass filter is assigned as the normalized version of this component with respect to its magnitude. Using the relation between low-pass and high-pass filters, the high-pass filter is calculated. Therefore, the scale and wavelet functions are computed using two-scale relations. Moreover, the relation between scale and frequency for calculated wavelets are presented.

Seven well-known earthquake records were adopted for analysis and utilized to validate the proposed method. For each earthquake record, the scaling and wavelet functions were calculated. It was observed that the wavelets obtained here can decompose the signals efficiently and more rapidly as compared to Daubechies2 wavelet; and can reconstruct the original signals perfectly with negligible computational error. A good representation of time-frequency resolution is obtained from the calculated wavelets. They show a better sensitivity to low frequency spikes than Daubechies2 wavelet.

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