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Robust passive damper design for building structures under uncertain structural parameter environments

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Abstract. An enhanced and efficient methodology is proposed for evaluating the robustness of an uncertain structure with passive dampers. Although the structural performance for seismic loads is an important design criterion in earthquake-prone countries, the structural parameters such as storey stiffnesses and damping coefficients of passive dampers are uncertain due to various factors or sources, e.g. initial manufacturing errors, material deterioration, temperature dependence. The concept of robust building design under such uncertain structural-parameter environment may be one of the most challenging issues to be tackled recently. By applying the proposed method of interval analysis and robustness evaluation for predicting the response variability accurately, the robustness of a passively controlled structure can be evaluated efficiently in terms of the so-called robustness function. An application is presented of the robustness function to the design and evaluation of passive damper systems.

Keywords: robustness; earthquake response; passive damper; uncertain parameter; interval analysis; structural control

1. Introduction

In the usual seismic structural design procedure, the maximum dynamic responses have to be evaluated in a structure subjected to wind or earthquake excitation in order to quantify its structural safety. For suppressing structural dynamic responses to a lower level to satisfy performance criteria, various vibration control systems, e.g. passive or active dampers and base-isolation systems, have been developed and applied to buildings so far. However, it is well known that there exist a lot of uncertainties in structural parameters caused by various sources, e.g. material-property variability, initial manufacturing errors, aging deterioration of performance. These uncertainties may lead to various unexpected situations where the structural response may exceed the performance limit. For this reason, it is needed to evaluate the degree of uncertainty of structural responses more accurately and efficiently for the sake of the robust and reliable design under various uncertain environments of structural parameters.

A number of studies on uncertainty analysis methods have been accumulated which can be used

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to investigate the upper bound of the structural responses considering the uncertainties of structural parameters (for example see Ben-Haim and Elishakoff 1990, Ben-Haim 2001, 2006, Takewaki and Ben-Haim 2005, Takewaki 2006, Kanno and Takewaki 2006, Elishakoff and Ohsaki 2010). The interval analysis is well known as a representative of the reliable uncertainty analysis methods. The concept of interval analysis was introduced by Moore (1966). Alefeld and Herzberger (1983) have then done the pioneering work. They treated linear interval equations, nonlinear interval equations and interval eigenvalue analysis by developing interval arithmetic. Since their innovative achievements, various interval analysis techniques based on the interval arithmetic algorithm have been proposed by many researchers (for example Dong and Shah 1987, Koyluoglu and Elishakoff 1998, Qiu 2003).

More recently, some interval analyses using Taylor series expansion have been proposed by Chen *et al*. (2003), Chen and Wu (2004), Chen *et al*. (2009) and Fujita and Takewaki (2011). In the early stage of the interval analysis using Taylor series expansion, first-order Taylor series expansion was investigated for the problems of static response and eigenvalue. Chen *et al.* (2009) developed a matrix perturbation method using second-order Taylor series expansion and obtained an approximation of the bounds of the objective function without interval arithmetic. They pointed out that the computational effort can be reduced from the number of calculation 2^N (N: number of interval parameters) to $2N$ by neglecting the non-diagonal elements of the Hessian matrix of the objective function with respect to interval parameters. Furthermore, Fujita and Takewaki (2011) have proposed the so-called Updated Reference-Point (URP) method where the critical uncertain structural parameters can be obtained by the approximation of second-order Taylor series expansion and the upper bound of the structural responses can be evaluated by reanalyzing the structural response using critical uncertain structural parameters.

In the structural design procedure, the robustness of building structures should be taken into account under various uncertainties of structural parameters and inputs. Ben-Haim (2001) has proposed an index, called the robustness function, for measuring the robustness based on the info-gap decision theory. In the info-gap model, the uncertainty of structural parameters is assumed to be given by a non-probabilistic model, e.g. an interval model used in the interval analysis. According to the definition of the robustness function, it can be regarded as a quantitative index of the robustness of the building structure (Takewaki and Ben-Haim 2008).

In this paper, an efficient evaluation method using the robustness function with respect to the constraint on seismic performance is presented by taking advantage of the proposed uncertainty analysis method called the URP method. A planar shear building model with passive viscous dampers is used for the robustness analysis. By comparing the robustness functions for various damper distributions, a preferable damper distribution is investigated to enhance the robustness under various uncertainties of structural parameters.

2. Robustness function for seismic performance

In the structural design of buildings in earthquake-prone countries, the design constraints on dynamic responses for earthquake loadings should be taken into account. In these design constraints, the dynamic responses such as maximum horizontal displacement and member stress evaluated by a reliable time history response analysis are required to check the satisfaction of the performance criteria. Even if all the design constraints are satisfied at the initial construction stage, some responses to external loadings during service life may violate such constraints due to various factors resulting from randomness, material deterioration, temperature dependence etc. To overcome such difficulty, the introduction of the robustness function which represents the degree of robustness of the objective building structure may be one of effective solutions for the design of more robust building structures under uncertainties.

In this section, a definition of the robustness of building structures for the seismic performance is introduced based on the info-gap model (Ben-Haim 2001). According to the info-gap model, the uncertainty of structural parameters is defined as a non-probabilistic model. In this paper, uncertain structural parameters are assumed to be given by an interval model. The interval parameter $\mathbf{X}^I$ is defined by

$$\mathbf{X}^I = \{X_i^I \big| [X_i^c - \Delta X_i, X_i^c + \Delta X_i], i=1, \cdots, N\} \tag{1}$$

In Eq. (1), $(\ )^I$ and $[a, b]$ denote the definition of an interval parameter where a and b are the lower and upper bounds of the interval parameter, respectively. Furthermore, $(\ )^c$, $\Delta(\ )$ and N denote the nominal value of an interval parameter, half the varied range of the interval parameter and the number of interval parameters respectively. When the uncertainty of structural parameters is given by the interval vector, it means that the feasible domain of interval parameters is constrained into an N-dimensional rectangle.

In the info-gap model, the level of uncertainty is defined by a single uncertain parameter α. Based on the definition of an uncertain parameter α in the info-gap model, the feasible domain of the interval parameter $\mathbf{X}^I$ can be regarded as an uncertainty set $\mathbf{X}^I(\alpha) \in \mathbf{R}$ described by

$$\mathbf{X}^I(\alpha) = \{X_i^I \big| [X_i^c - \alpha\Delta X_i, X_i^c + \alpha\Delta X_i], i=1, \cdots, N\} \tag{2}$$

In Eq. (2), ΔX_i is regarded as a prescribed value of half the varied range of the interval parameters. Therefore, it can be mentioned that the uncertainty level of the uncertainty set $\mathbf{X}^I(\alpha)$ varies according to the variation of uncertain parameter α. Fig. 1 shows the variation of 2-dimensional interval model with an uncertain parameter α. When $\alpha = 0$, the uncertainty set $\mathbf{X}^I(0)$ corresponds to a nominal vector of structural parameters.

The robustness function $\hat{\alpha}$ for the design constraint of the seismic performance can be defined as

$$\hat{\alpha}(\mathbf{X}^c, f_c) = \max\{\alpha | f \le f_c, f \in U(\mathbf{X}^c, \alpha)\} \tag{3}$$

where f, f_c and $U(\mathbf{X}^c, \alpha)$ denote the objective function, the performance criterion value and the set of the possible structural responses in the domain of the uncertainty set $\mathbf{X}^I(\alpha)$. In Eq. (3), the robustness function $\hat{\alpha}$ is the maximum value of the uncertain parameter α which satisfies the performance criterion. If the nominal value $f(\mathbf{X}^c)$ of the objective function violates f_c or just coincides with f_c without considering a safety factor, the robustness function $\hat{\alpha}$ is regarded as zero, which means that no variability due to the uncertainty of structural parameters can be allowed. In the case of $\hat{\alpha}_1(\mathbf{X}_1^c, f_c) > \hat{\alpha}_2(\mathbf{X}_2^c, f_c)$, a design more robust than $\mathbf{X}_2^c$ can be achieved by $\mathbf{X}_1^c$.

Fig. 2 illustrates the relationship between the robustness function and the allowable domain of structural design to satisfy the performance criterion f_c for 2-dimensional interval parameters. The robustness function $\hat{\alpha}$ is derived as the worst case of the objective function, i.e., the upper bound of the objective function $\bar{f}$ in $U(\mathbf{X}^c, \hat{\alpha})$. However, when the number of the combinations of uncertain

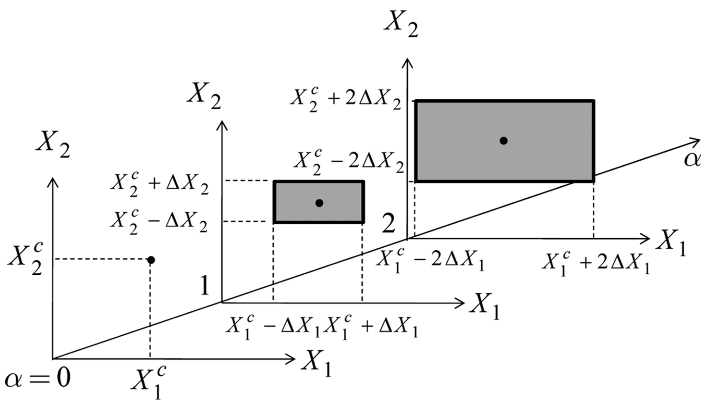


Fig. 1 Variation of uncertainty set of interval model

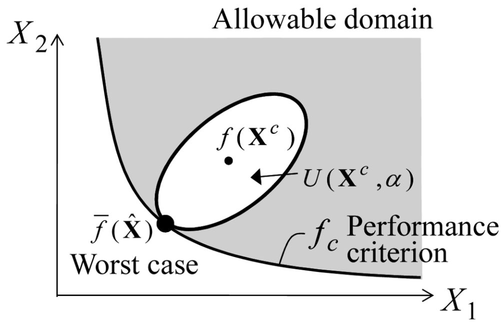


Fig. 2 Robustness function for performance criterion

parameters is huge, it may be a hard task to evaluate the worst case of the objective function reliably. For this reason, an efficient uncertainty analysis method is desired which can evaluate the upper bound of the objective function considering the uncertainty of the structural parameters accurately and reliably.

3. Efficient uncertainty analysis based on interval analysis

In this section, the URP (Updated Reference-Point) method proposed by Fujita and Takewaki (2011) originally for stochastic input is explained which can be used as an efficient uncertainty analysis to obtain the robustness function $\hat{\alpha}$. Since the URP method is based on the interval analysis using an approximation of first- and second-order Taylor series expansion (Fujita and Takewaki 2011), the formulation of Taylor series expansion in the interval analysis and the achievement of second-order Taylor series expansion proposed by Chen *et al*. (2009) are explained briefly.

3.1 Interval analysis using taylor series expansion

An approximate objective function f^* using first- and second-order Taylor series expansion can be expressed as

$$f^{*}(\mathbf{X})=f\left(\mathbf{X}^{c}\right)+\sum_{i=1}^{N} f_{,X_i}\left(X_i-X_i^{c}\right)+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} f_{,X_iX_j}\left(X_i-X_i^{c}\right)\left(X_j-X_j^{c}\right) \tag{4}$$

where $(\ )_{,Xi}$ and $(\ )_{,XiXj}$ denote first-order differentiation $\partial f(\mathbf{X})/\partial X_i|_{Xi=Xi^c}$ and second-order differentiation $\partial^2 f(\mathbf{X})/\partial X_i \partial X_j|_{Xi=Xi^c, Xj=Xj^c}$ of the objective function at the nominal value. Therefore, $f_{,X_i}$ and $f_{,X_iX_j}$ correspond to a gradient of f and to a component of the Hessian matrix of f for the nominal model, respectively.

In order to evaluate the upper bound of f^*, a basic theorem of "*inclusion monotonic*" in the interval analysis is often assumed. The theorem of "*inclusion monotonic*" is assumed in some of the previous studies on interval analysis, e.g. Chen *et al*. (2009). If the natural interval extension f^I of f is inclusion monotonic, the objective function f satisfies

$$\left\{f(\mathbf{X}): x_i \in X_i^I, i=1,2,\cdots,N\right\} \subseteq f\left(X_1^I, X_2^I, \cdots, X_N^I\right) \tag{5}$$

The right-hand side of Eq. (5) denotes the interval (range) of the function f determined by the end-point combinations. From Eq. (5), as long as the arguments of f are constrained between intervals (the lower and upper bounds a and b in $[a, b]$ are also called 'intervals'), the variation of f for any interval parameter value within the intervals should be included in the range of values for the intervals. Based on the theorem of "*inclusion monotonic*", we can derive the upper bound of f by iterative calculations with all end-point combinations, i.e., the upper and lower bounds of interval parameters. However, when the number N of interval parameters is extremely large, this primitive approach needs much computational time caused by a large combination number of interval parameters. Although the computational effort can be reduced by the approximation of Taylor series expansion in Eq. (4), the number of iterative calculations with all end-point combinations is the same with the interval analysis method based on the theorem of "*inclusion monotonic*", e.g. Dong and Shah (1987).

By using the approximation of Taylor series expansion, iterative response analyses such as time-history analysis for evaluating the objective function can be avoided. However, the computation of full elements of the Hessian matrix requires hard computational load when N is large, especially for numerical sensitivity analysis, i.e., the finite difference analysis using gradient vectors. For this reason, a simpler approach has been proposed by Chen *et al*. (2009) where the non-diagonal elements of the Hessian matrix are neglected.

An approximate objective function f^{**} using second-order Taylor series expansion with only diagonal elements can be rewritten from Eq. (6) as

$$f^{**}(\mathbf{X})=f(\mathbf{X}^{c})+\sum_{i=1}^{N}\{f_{,X_i}(X_i-X_i^{c})+\frac{1}{2}f_{,X_iX_i}(X_i-X_i^{c})^2\} \tag{6}$$

From Eq. (6), we can evaluate the increment of the objective function by using first- and second-order Taylor series expansion as the sum of the increments of the objective function in each one-dimensional domain. The perturbation $\Delta f_i(\mathbf{X})$ of the objective function by the variation of the i-th interval parameter X_i can be described as

$$\Delta f_i\left(X_1^{c},\cdots,X_i,\cdots,X_N^{c}\right)=f_{,X_i}\left(X_i-X_i^{c}\right)+\frac{1}{2}f_{,X_iX_i}\left(X_i-X_i^{c}\right)^2 \tag{7}$$

The validity of the neglect of cross terms has been demonstrated in the reference (Fujita and Takewaki 2011). In Eq. (7), the interval extension of the one-dimensional perturbation can be derived as

$$\begin{aligned}&\Delta f_i^I(X_1^c,\cdots,X_i,\cdots,X_N^c)\\&=\begin{bmatrix}\min\left[\Delta f_i(X_1^c,\cdots,\underline{X}_i,\cdots,X_N^c),\Delta f_i(X_1^c,\cdots,\overline{X}_i,\cdots,X_N^c)\right],\\ \max\left[\Delta f_i(X_1^c,\cdots,\underline{X}_i,\cdots,X_N^c),\Delta f_i(X_1^c,\cdots,\underline{X}_i,\cdots,X_N^c)\right]\end{bmatrix}\end{aligned}\tag{8}$$

From Eq. (8), the upper bound $\overline{\Delta f_i}$ of Eq. (7) can be derived by the comparison of Δf_i with structural parameter sets $\mathbf{X} = \{X_1^c,\cdots,\underline{X}_i,\cdots,X_N^c\}$ and $\mathbf{X} = \{X_1^c,\cdots,\overline{X}_i,\cdots,X_N^c\}$. Since Eq. (7) is a function of $\mathbf{X}$, it is natural to define an upper bound of that function. Finally, substituting $\underline{\Delta f_i}\,(i=1,\cdots,N)$ into Eq. (6), the interval extension of the approximate objective function f^{**} can be obtained as

$$f\left(\mathbf{X}^I\right)\approx\left[f\left(\mathbf{X}^c\right)+\left[\sum_{i=1}^{N}\underline{\Delta f_i}\left(X_i^I\right)\right],f\left(\mathbf{X}^c\right)+\left[\sum_{i=1}^{N}\overline{\Delta f_i}\left(X_i^I\right)\right]\right]\tag{9}$$

It is remarkable that the number of calculations in Eq. (9) is reduced to $2N$ from 2^N in Eq. (6). However, it should be mentioned that, because of the approximation by Taylor series expansion, the deterioration of accuracy can not be avoided when the level of uncertainties of interval parameters is large.

3.2 Proposed search algorithm for critical combination of interval parameters

The approximation using Taylor series expansion can reduce computational load dramatically in the interval analysis. However, we should take into account that the result by such approximation may include errors. Furthermore, although some of the interval analysis methods are based on "*inclusion monotonic*", it is not necessarily appropriate to assume the monotonic variation of the objective function for dynamic responses. When the objective function has a non-monotonic property in $U(\mathbf{X}^c, \alpha)$, the extreme value of the objective function may occur not on the bound of interval parameters but in an inner feasible domain of interval parameters. Even in such a case, we can achieve the robustness evaluation by reanalyzing the structural response via a reliable response analysis for the estimated critical combination of interval parameters (worst case). In this section, an efficient search algorithm for the critical combination of interval parameters is presented which makes the approximate objective function maximum by using first- and second-order Taylor series expansion.

Consider Eq. (7) again. When the perturbation $X_i - X_i^c$ of the structural parameter in Eq. (7) is denoted by ΔX_i, Eq. (7) can be transformed into

$$\Delta f_i\left(\Delta X_i\right)=\frac{1}{2}f_{,X_iX_i}\left(\Delta X_i+\frac{f_{,X_i}}{f_{,X_iX_i}}\right)^2-\frac{{f_{,X_i}}^2}{2f_{,X_iX_i}}\tag{10}$$

This transformation is just a simple mathematical transformation. From Eq. (10), it can be seen that the increment of the objective function with respect to X_i is parabolic in the one-dimensional domain. By using Eq. (10), we can search the target position of the uncertain structural parameter X_1 which maximizes the objective function based on the second-order Taylor series expansion. For instance, when $f_{,X_iX_i}<0$, the target position $\hat{X}_i$ of the i-th interval parameter X_i which maximizes Eq. (10) can be derived explicitly as

$$\hat{X}_i = \begin{cases} X_i^c - f_{,X_i} / f_{,X_iX_i} & \left(\left| f_{,X_i} / f_{,X_iX_i} \right| \le \Delta X_i \right) \\ X_i^c + \Delta X_i & \left(-f_{,X_i} / f_{,X_iX_i} \ge 0, \left| f_{,X_i} / f_{,X_iX_i} \right| > \Delta X_i \right) \\ X_i^c - \Delta X_i & \left(-f_{,X_i} / f_{,X_iX_i} < 0, \left| f_{,X_i} / f_{,X_iX_i} \right| > \Delta X_i \right) \end{cases} \tag{11}$$

The first case of Eq. (11) indicates that the critical value occurs in the inner domain. On the other hand, the second and third cases mean that the critical value occurs at the boundaries.

The target positions of the other interval parameters can be obtained successively in a similar way. The feature of this proposed methodology is that we consider a possibility of occurrence of the extreme value in an inner range of interval parameters and that only the first- and second-order sensitivities of the objective function are needed.

Eq. (11) indicates that the target position of X_i can be derived by first- and second-order sensitivities $f_{,X_i}$, $f_{,X_iX_i}$ of the objective function with respect to X_i. For evaluating these sensitivities, we need to define a reference point. From the general point of view, it may be natural that first- and second-order sensitivities with diagonal elements only are calculated at the reference point of the nominal model. However, it is difficult to consider the influence of the interaction between the interval parameters in this approach. The authors have proposed the updated reference-point method (URP method) where the different computational procedure for the evaluation of first- and second-order sensitivities is applied. A detailed flow of the computation procedure of the URP method is explained below. The conceptual diagram of the URP method for 2-dimensional interval parameters is shown in Fig. 3.

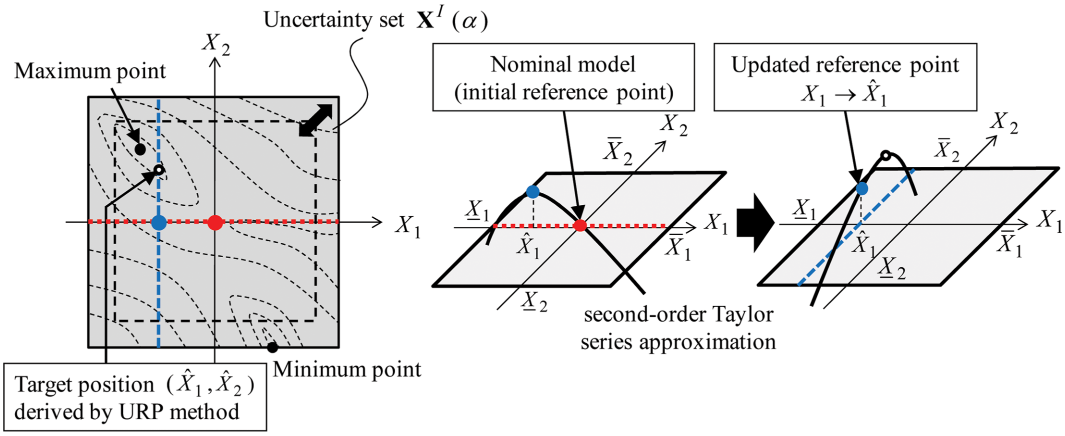


Fig. 3 Conceptual diagram of the URP method

Step 1 Calculate the first-order sensitivities $f_{,X_i}(i=1,\cdots,N)$ of the objective function for the nominal model.

Step 2 Sort the absolute values $|f_{,X_i}|(i=1,\cdots,N)$ of the first-order sensitivities in descending order to give the priority to the parameter with the largest sensitivity. Sort also the interval parameters as $\mathbf{X}_A=\{X_{A1},\cdots,X_{AN}\}$ corresponding to this.

Step 3 Calculate the second-order sensitivity of the objective function with respect to the interval parameter X_{Ak}. The second-order sensitivity $\partial^2 f/\partial X_{Ak}{}^2$ can be derived as a scalar value. When $k\geq 2$, i.e., the reference point of the objective function has been updated, the first-order sensitivity of the objective function with respect to the interval parameter X_{Ak} should be calculated again.

Step 4 Derive the target position $\hat{X}_{Ak}$ of the interval parameter where the approximate objective function $f^{**}(\mathbf{X})$ is maximized. The problem in this step can be stated as

$$\left.\begin{array}{l}\text{Find } \hat{X}_{Ak}\\ \text{so as to maximize or minimize } f^{**}(\mathbf{X})\\ \text{subject to } X_{Ak}\in(X_{Ak})^I, X_{Al}\text{:current value } (l\neq k)\end{array}\right\} \tag{12}$$

Step 5 Update the set of interval parameters from current one X_{Ak} to $\hat{X}_{Ak}$.

Step 6 Update the corresponding system structural matrices such as **C** and **K** at the new reference point.

Step 7 Update k to k+1. Repeat Step 2 through Step 6 until k becomes N.

4. Application to building structure with passive dampers

Any structural properties or responses, such as eigenvalue, static and dynamic responses, can be employed as the objective function in the proposed URP methods. From the view point of seismic structural design, the applicability of the URP method to building structures with passive dampers is investigated as an example where the objective function is defined as maximum interstorey drift for a set of recorded ground motions.

4.1 Structural model with passive dampers and selection of uncertain parameters

Consider an N-storey planar shear building model, as shown in Fig. 4, with viscous dampers and their supporting members. Let M_i, k_{fi}, c_{fi}, c_{di} and k_{bi} $(i=1,\cdots,N)$ denote the floor mass, the storey stiffness of the frame, the structural damping coefficient, the damping coefficient of the passive damper and the supporting member stiffness of the damper in the i-th storey, respectively. It has been shown that the supporting member stiffness of dampers plays an important role in the optimal distribution of dampers (Fujita *et al*. 2010a). The frame stiffness distribution in the nominal model is shown in Fig. 5 ($N=20$). The properties of structural parameter of the nominal model are shown in Table 1.

The equations of motion of the building with viscous dampers subjected to the horizontal ground motion can be expressed in time domain by

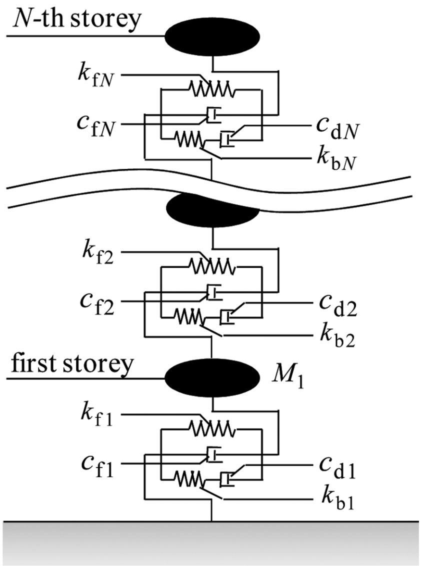


Fig. 4 Structural model with passive damper

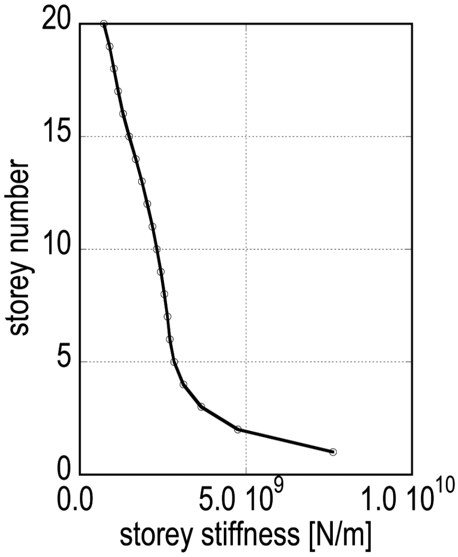


Fig. 5 Storey stiffness distribution

Table I Structural parameters of main frame

	20-storey building
Floor mass [kg]	$1024{\times}10^3$
Total damper capacity [Ns/mm]	$6.000{\times}10^8$
Supporting member stiffness [N/mm]	Ratio 1.0 to frame storey stiffness
Structural damping ratio (stiffness-proportional damping)	0.02
Fundamental natural circular frequency* with damper [rad/s]	3.927

*Complex eigenvalue analysis without dampers

$$\mathbf{M}\ddot{\mathbf{u}}(t)+(\mathbf{C}+\mathbf{C}_D)\dot{\mathbf{u}}(t)+(\mathbf{K}+\mathbf{K}_b)\mathbf{u}(t)=-\mathbf{M}\mathbf{r}\ddot{u}_g(t) \tag{13}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{C}_D$, $\mathbf{K}$ and $\mathbf{K}_b$ are the system mass, structural damping, damper damping, structural stiffness and supporting member stiffness matrices, respectively. Furthermore $\mathbf{r}=\{1,\cdots,1\}^T$ is the influence coefficient vector. By assuming that a viscous damper connected in series with its supporting member is treated as a detailed model in which a small lumped mass is allocated between the components of the dashpot and the spring, the components of the $\mathbf{K}$, $\mathbf{K}_b$ and $\mathbf{C}_D$ can be given by a linear combination of structural parameters k_{fi}, c_{di}, k_{di} $(i=1,\cdots,N)$. The Newmark-β method ($\beta=1/4$) has been employed to evaluate the maximum interstorey drift.

The structural parameters $\mathbf{c}_d=\{c_{d_i}\},\mathbf{k}_b=\{k_{b_i}\}$ and $\mathbf{k}_f=\{k_{f_i}\}$ are dealt with as interval parameters. The interval parameters of these uncertain structural parameters are described with uncertain parameter α by

$$\begin{aligned}\mathbf{c}_d^I &= \left[\mathbf{c}_d^c - \alpha\Delta\mathbf{c}_d, \mathbf{c}_d^c + \alpha\Delta\mathbf{c}_d\right] \\ \mathbf{k}_b^I &= \left[\mathbf{k}_b^c - \alpha\Delta\mathbf{k}_b, \mathbf{k}_b^c + \alpha\Delta\mathbf{k}_b\right] \\ \mathbf{k}_f^I &= \left[\mathbf{k}_f^c - \alpha\Delta\mathbf{k}_f, \mathbf{k}_f^c + \alpha\Delta\mathbf{k}_f\right]\end{aligned} \tag{14a,b,c}$$

The uncertainties of interval parameters $\mathbf{X}^I = \{\mathbf{c}_d^I, \mathbf{k}_b^I, \mathbf{k}_f^I\}$ are given by $\boldsymbol{\varepsilon} = \{\varepsilon_i\}$ $(i = 1,\cdots,3N)$ defined by

$$\varepsilon_i = \begin{cases} \Delta c_{d_i} / c_{d_i}^c & = 0.3 \ (i = 1,\cdots,N) \\ \Delta k_{b_{i-N}} / k_{b_{i-N}}^c & = 0.3 \ (i = N+1,\cdots,2N) \\ \Delta k_{f_{i-2N}} / k_{f_{i-2N}}^c & = 0.1 \ (i = 2N+1,\cdots,3N) \end{cases} \tag{15}$$

Eq. (15) assumes that the degrees of uncertainties of interval parameters is constant for all storeys on the same structural properties in the following numerical examples.

4.2 Recorded ground motions

El Centro NS (1940), Taft EW (1952) and Hachinohe NS (1968) are used as representative recorded ground motions, whose maximum ground velocities are normalized as 50 cm/s. These earthquake ground motions are often used for structural design (Level 2 of large earthquake ground

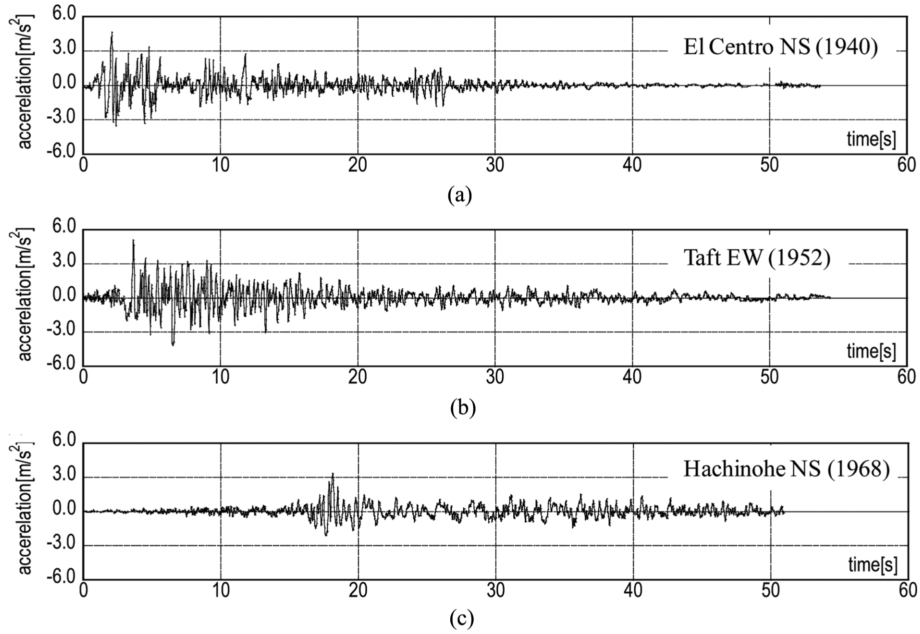


Fig. 6 Normalized recorded ground motions: (a) El Centro NS (1940), (b) Taft EW (1952) and (c) Hachinohe NS (1968)

motion) of high-rise and base-isolated buildings in Japan. Figs. 6(a)-(c) show the normalized recorded ground motions.

4.3 Robustness function for various damper distributions

In this section, the robustness functions defined by Eq. (3) for various damper distributions are evaluated by the URP method. Fig. 7 shows the comparison of maximum interstorey drifts of the building structure without dampers for three recorded ground motions. It could be observed from Fig. 7 that the interstorey drift is maximized at upper storeys higher than 15-th floor. For enhancing the seismic performance of the building structure, let us consider three different damper distributions: (1) uniform distribution, (2) added only from 10-th to 20-th storeys and (3) optimum distribution. In these various damper distributions, the total quantity of damping coefficients of the viscous dampers is given by a constant value as shown in Table 1. Fig. 8 shows the comparison of the maximum interstorey drifts with three different damper distribution shown in Fig. 9. The optimum damper distribution has been derived by the optimization algorithm developed by Fujita *et al*. (2010b) to minimize the maximum amplitude of interstorey drift transfer function.

Figs. 10(a)-(c) show the robustness functions with respect to the constraint on the maximum interstorey drift for the three representative recorded ground motions. These figures can be obtained by using the method in Section 3 and evaluating the maximum interstorey drift for various values of $\hat{\alpha}$. The maximum interstorey drift of the nominal model without considering uncertainty of the structural parameters can be seen in Figs. 10(a)-(c) at the uncertainty parameter $\hat{\alpha} = 0$. By comparing these nominal values of the objective function for various damper distributions, it is found that the most preferable response reduction can be obtained by the damper distribution from 11-th to 20-th storey in El Centro NS (1940) and Hachinohe NS (1968). On the other hand, the most preferable response reduction can be obtained by the optimal damper distribution in Taft EW (1952). Since the optimal damper distribution aims to suppress the maximum amplitude of interstorey-drift transfer function at the fundamental natural circular frequency, the response reduction can be achieved dramatically for the excitation, such as Taft EW (1952), whose predominant frequency is resonant to the fundamental natural circular frequency of the objective building structure. From Fig. 10(b), if the performance criterion with respect to the maximum interstorey drift is given by 0.03 m, the robustness function $\hat{\alpha}$ is nearly 0.8 for the optimal damper

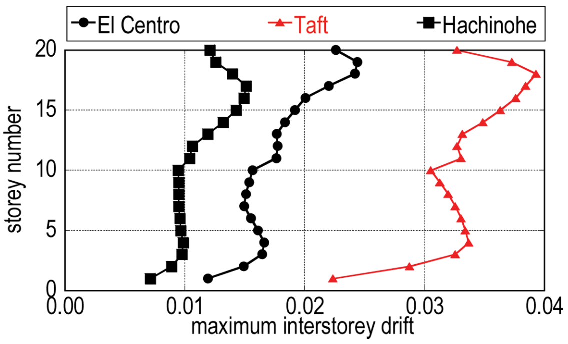


Fig. 7 Maximum interstorey drift without dampers

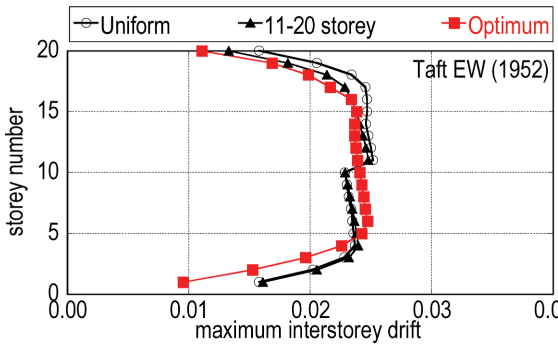


Fig. 8 Comparison of maximum interstorey drift with various damper distributions

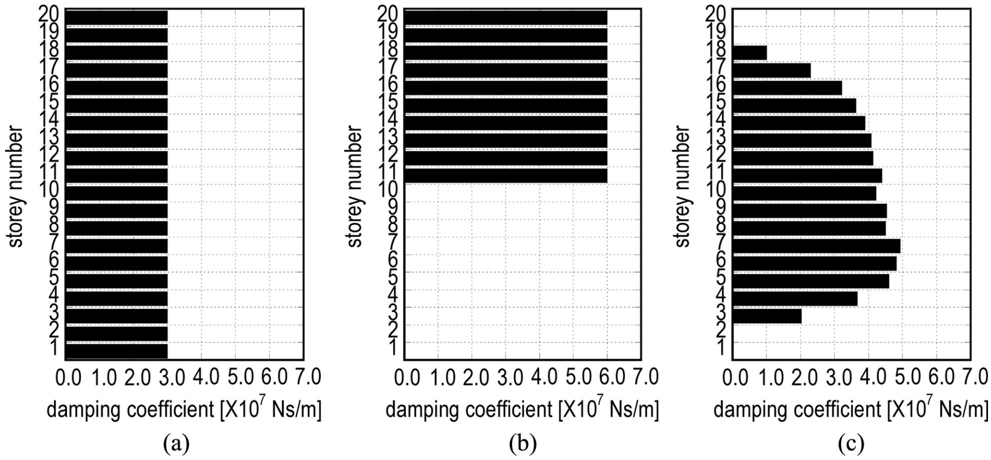


Fig. 9 Various damper distributions: (a) Uniform, (b) 11-20 storeys and (c) Optimum

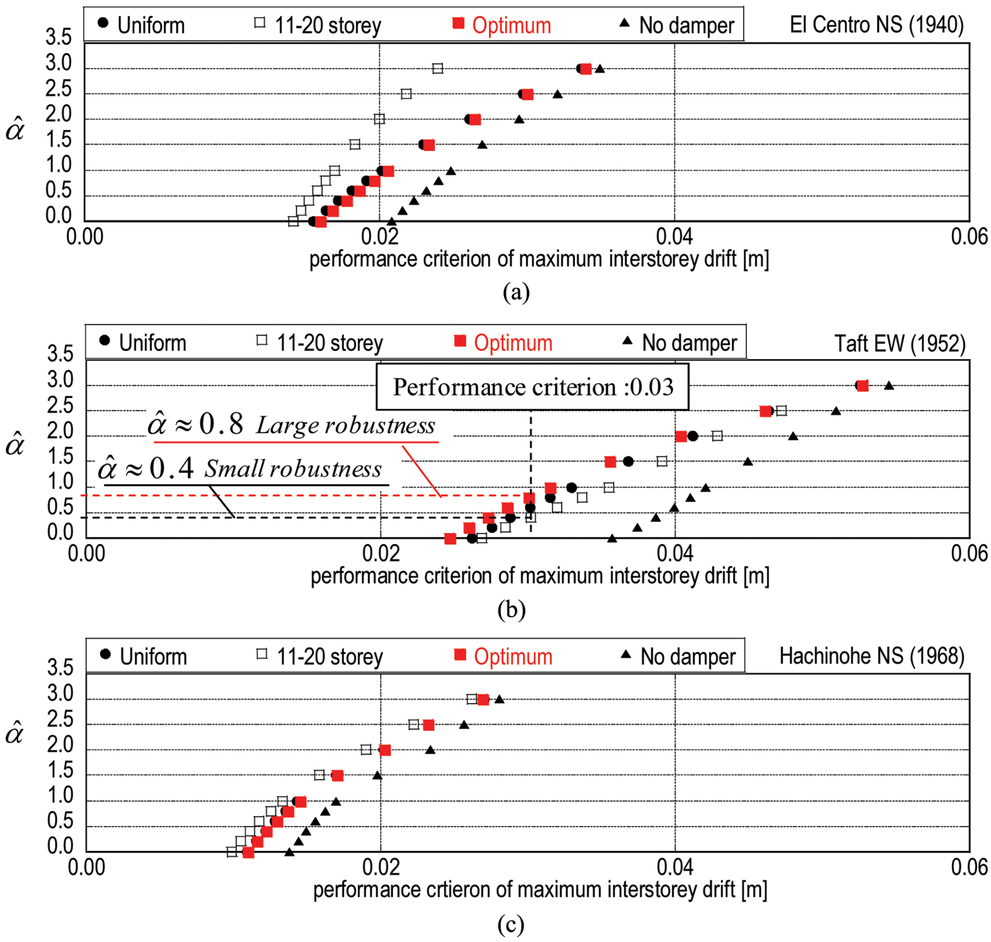
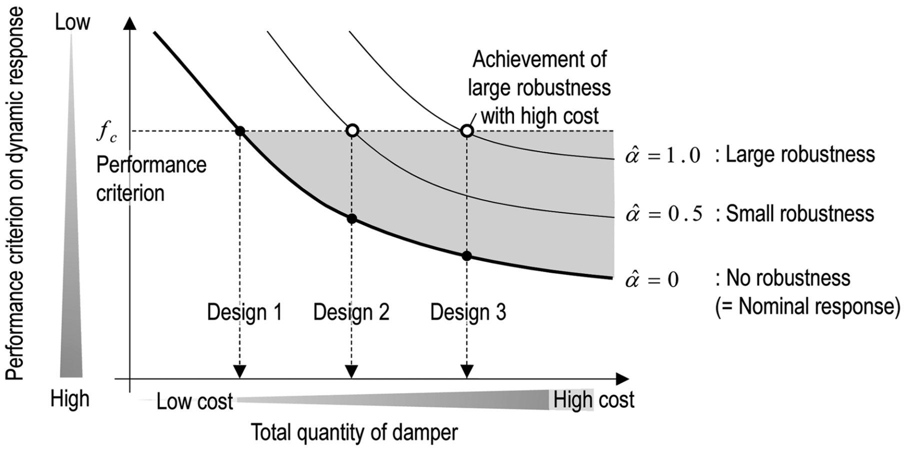


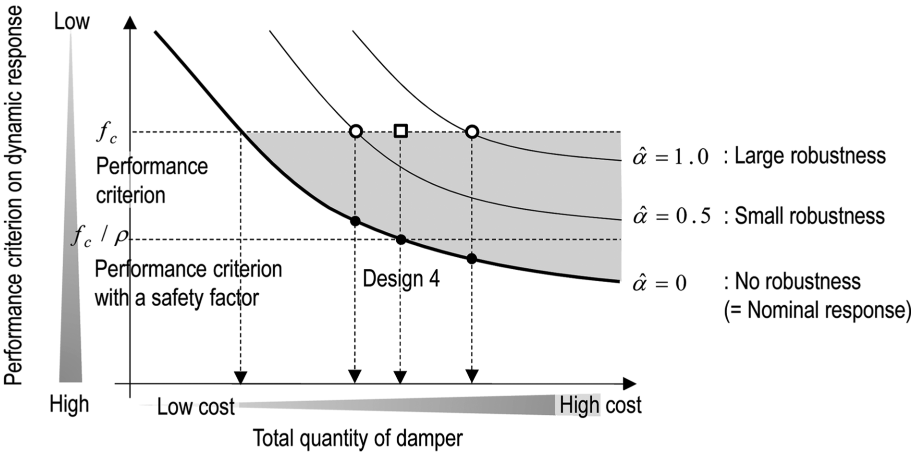
Fig. 10 Comparison of robustness functions for various damper distributions: (a) El Centro NS (1940), (b) Taft EW (1952) and (c) Hachinohe NS (1968)

distribution while nearly 0.4 for the 11th-20th storey distribution. From this comparison, it can be concluded that a large robustness can be obtained by the optimal damper distribution.

From the view point of seismic structural design, the total quantity of added dampers may be a principal design parameter. In the general design procedure, the total quantity of dampers is determined based on the condition whether the dynamic responses satisfy the constraints with a certain safety factor ρ. However, it is often ambiguous whether the values of these safety factors are appropriate or not. On the other hand, such total damper quantity can be derived in a more logical manner by using the 'robustness function' for the structural uncertainty. Fig. 11(a) shows the conceptual diagram of the re-design approach for determining the total damper quantity for a robust building structure, which can be derived by varying the robustness function with respect to the total damper quantity. The thick curve can be drawn by evaluating the dynamic response for various total quantities of dampers (nominal parameters). In Fig. 11(a), when the performance criterion f_c and the

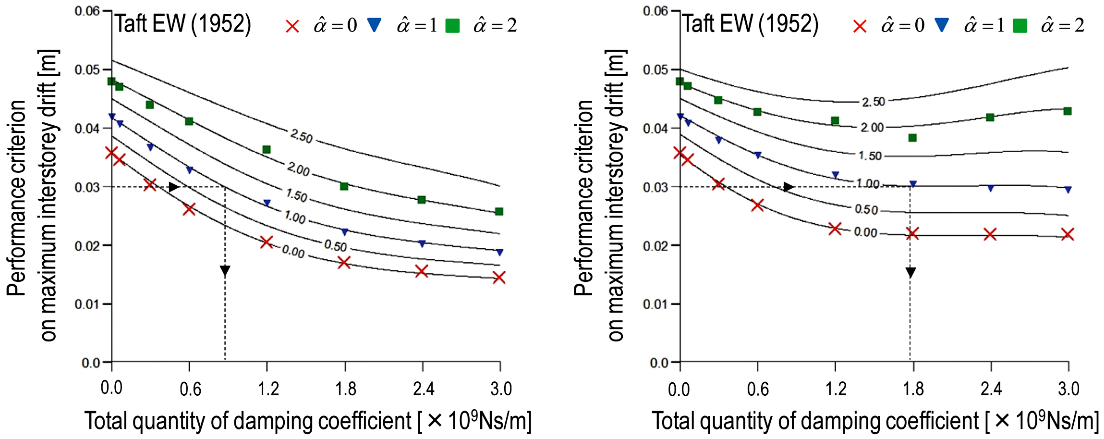


(a)



(b)

Fig. 11 Redesign of total quantity of passive dampers for robust structures: (a) total quantity of dampers for no robustness, small robustness and large robustness for a given performance criterion and (b) total quantity of dampers for the robustness function representation and the safety factor representation



(a) (b)

Fig. 12 Variation of robustness functions with respect to varied total quantity of damping coefficient for Taft EW (1952): (a) Uniform distribution and (b) 11-20 storey distribution

robustness function $\hat{\alpha}$ are given (0, 0.5 and 1.0), an appropriate total damper quantity can be found (Design 1, 2, 3). If the value of $\hat{\alpha}$ is assumed to be large, the total damper quantity will increase and a large robustness can be achieved. Fig. 11(b) illustrates the relation of total quantities of passive dampers for the robustness function representation and the safety factor representation (Design 4). It can be observed that the robustness function of Design 4 with a safety factor ρ is between 0.5 and 1.0. A clear understanding may be possible of the meaning of the safety factor in terms of the structural robustness.

Figs. 12(a) and (b) show a comparison of variations of the robustness functions with respect to a varied total quantity of damping coefficient for Taft EW (1952) between (a) uniform distribution and (b) 11-20 storey distribution. The markers depicted in Figs. 12(a) and (b) have been determined by the robustness function evaluated by the URP method. The contour plot has been constructed from these robustness functions by assuming that the variation of the robustness function can be obtained by the third-order polynomial approximation. As explained in Fig. 11, the total damper quantity can be obtained by just satisfying the given performance criterion under the uncertainty of the structural parameters. For instance, when the performance criterion f_c on the maximum interstorey drift is assumed to be 0.03 as shown in Fig. 12, the total damper quantity for the uniform damper distribution is less than 0.8×10^9 Ns/m on $\hat{\alpha}=1.0$. On the other hand, the total damper quantity for the 11-20th storey distribution is about 1.8×10^9 Ns/m, which is more than twice that for the uniform distribution. In Fig. 12(b), no response reduction can be observed for the large damper quantity. This is because the location where the interstorey drift is maximized may be switched to a lower storey by the addition of dampers to upper storeys (11-20 storeys) and further introduction of dampers to upper storeys is ineffective.

It is noted that addition of damping to a given system enhances the robustness of the system response with respect to the variability of the input. As for the variabilities of both the input and system properties, the uncertainty level of the input possesses a trade-off relation with the uncertainty level of the system properties under a constant robustness requirement (Takewaki and Ben-Haim 2005, 2008).

5. Conclusions

A robustness function for the constraint on dynamic response of building structures with passive dampers subjected to ground motions has been defined based on the info-gap decision theory. For evaluating the reliable robustness, an efficient uncertainty analysis methodology for the robustness evaluation of a damped structure has been proposed which is aimed at finding the upper bound of dynamic response under uncertainties of structural parameters.

In the proposed uncertainty analysis method, a model of uncertain parameters has been defined by using an interval model. Although the basic theorem of "*inclusion monotonic*" is assumed in some of the interval analyses proposed by many researchers, the critical combination of interval parameters in a feasible domain, not only on the bounds but also in an inner domain of interval parameters, has been derived explicitly in the proposed method. By evaluating the extreme value of the objective function via the approximation of second-order Taylor series expansion, the upper bound of the objective function can be obtained straightforwardly for the predicted structural parameter set. To evaluate the upper bound of the objective function more accurately within a reasonable task, the URP (Updated Reference-Point) method has been proposed where the reference point to calculate first- and second-order sensitivities has been updated according to the variation of uncertain structural parameters.

Numerical examples using the robustness function has been presented for a 20-storey planar shear building including passive viscous dampers with supporting members by applying the proposed URP method. A detailed comparison of the robustness of the structures where the additional dampers have been distributed (1) uniformly, (2) from 11-th to 20-th storey and (3) optimally has been conducted for representative recorded ground motions. The optimum damper distribution has been derived by the optimization algorithm developed by the present authors to minimize the maximum amplitude of interstorey-drift transfer function. By comparing the robustness functions among the various damper distributions, the large robustness can be obtained in the design of optimal damper distribution especially for the excitation whose predominant frequency is resonant to the fundamental natural frequency of the building structure.

An approximate contour plot of the robustness function with respect to a varied total damper quantity has been constructed by using the proposed method. By comparing the contour plots of the robustness function for various damper distributions, a total damper quantity can be derived which satisfies the performance criterion under the uncertainty of the structural parameters. It has also been shown that it is possible to clarify the relation of total quantities of passive dampers for the robustness function representation and the safety factor representation. It is expected that this leads to a more robust damper design.

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