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Reconstruction of missing response data for identification of higher modes

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Abstract. The problem of reconstruction of complete building response from a limited number of response measurements is considered. The response at the intermediate degrees of freedom is reconstructed by using piecewise cubic Hermite polynomial interpolation in time domain. The piecewise cubic Hermite polynomial interpolation is preferred over the spline interpolation due to its trend preserving character. It has been shown that factorization of response data in variable separable form via singular value decomposition can be used to derive the complete set of normal modes of the structural system. The time domain principal components can be used to derive empirical transfer functions from which the natural frequencies of the structural system can be identified by peak-picking technique. A reduced-rank approximation for the system flexibility matrix can be readily constructed from the identified mass-orthonormal mode shapes and natural frequencies.

Keywords: inverse problem; modal identification; orthogonal decomposition; system identification; structural dynamics.

1. Introduction

With rapid developments in the sensor technology and easy availability of computing platforms for data processing, the system identification from the recorded system response data is increasingly being considered for rapid health monitoring of structural systems (Sohn *et al*. 2004, Brownjohn 2007). In principle, this is an inverse vibration problem wherein the system parameters are determined from the analysis of input-output data pairs. An overview of the process of system identification in structural engineering has been presented recently (Alvin *et al*. 2003). Some early attempts at structural system identification were based on estimating the coefficients of system parameter matrices (inertia, damping, and stiffness) so as to minimize a norm of the error between observed and analytically predicted response (Distefano and Pena-Pardo 1976, Hart and Yao 1977). The direct estimation of parameters is generally found to be sensitive to the noise contamination of the recorded input-output time histories. Moreover, non-uniqueness of the identified parameters has also been reported (Udwadia and Sharma 1978).

Some output-only modal identification methods have been proposed in recent years, such as, frequency domain decomposition (FDD) (Brincker *et al*. 2001) and its predecessor based on complex mode indication function (CMIF) (Shih *et al*. 1988) for operating in frequency domain.

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The singular value decomposition (SVD) of the cross-spectral density matrix of the measured vibration records is computed at each frequency and the frequency at which the computed singular value attains a local maximum corresponds to the natural frequency of the structural system and the associated left singular vector corresponds to the normal mode at that natural frequency. However, the full mode shape needs to be obtained by a suitable interpolation between the identified mode shape ordinates at the monitored degrees of freedom (DOF), which limits its applicability to the identification of lower modes. For identification of higher modes with numerous sign reversals, a more dense instrumentation to improve spatial resolution is often necessary. For operations in time-domain, the eigensystem realization algorithm (ERA) (Juang and Pappa 1985), or its equivalent the stochastic subspace identification (SSI) (Peeters and de Roeck 2001) and the natural excitation technique (NExT) (James III *et al*. 1993) have received considerable attention in recent years.

To use ERA, it is necessary to have an a priori information about the order of the state-space model to be identified from the impulse response data of the structural system arranged in the form of a block Hankel matrix (Juang and Pappa 1985, Juang 1987). There has been some progress in recent times on the selection of an appropriate model order by means of stabilization diagrams (Peeters *et al*. 2008, Zhang *et al*. 2005). The required impulse response data may be obtained from NExT as cross-correlation functions between a set of vibration records from different locations in the structural system. Although the basic assumptions in the formulation of NExT are too restrictive (stationary response to white noise excitation) it has been reported that useful estimates of impulse response functions can still be obtained when the vibration records are non-stationary and the excitation process is not a white noise. Yi and Yun studied the relative performance of various modal identification methods (Yi and Yun 2004). It is generally agreed that the identification of system parameters is quite sensitive to the signal contamination due to noise and the choice of an appropriate model order for identification, which is often difficult to specify a priori in the case of large-scale civil engineering systems. Further, a typical instrumentation programme involves deployment of vibration transducers only at a few carefully selected locations so as to maximize the information content of the response measurement for use in system identification studies (Udwadia 1994, Heredia-Zavoni and Esteva 1998, Datta *et al*. 2002, Limongelli 2003). Nevertheless, the limited response measurements entail some loss of information and the identification of reduced order models from these selective response data is often error prone. This uncertainty associated with the identified system parameters does not inspire much confidence in using those as diagnostic tools in structural health monitoring (Choudhury 2007). The low spatial resolution of vibration data may also lead to erroneous conclusions regard- ing consistency of the identified modal vectors (Allemang 2003) which can, in turn, adversely affect the estimation of some damage indicator functionals like modal strain energy (Shi *et al*. 2000, Jaishi and Ren 2007).

In this study, it is shown that the spatial resolution of the vibration signatures can be improved by using a suitable interpolation scheme to reconstruct the missing response data. Further, a new method for modal identification is proposed which is based on decomposition of the array of absolute acceleration response data into variable separable form wherein the variation with respect to space coordinates is separated from the time dependent part as in the case of normal mode analysis. The identification of the modal parameters allows us to construct a reduced-rank approximation of the system flexibility matrix, which is known to be a good damage indicator for structural health monitoring studies.

2. System configuration

A seven story symmetric reinforced concrete building with rigid floor diaphragm and with irregular stiffness and mass distribution is considered for the study. We assume that the building response measurements (absolute acceleration) are available at three floor levels, namely, 1, 4, and 7, which correspond to the optimal sensor locations for this building (Datta *et al*. 2002), in addition to the ground acceleration time history. A schematic plan of the building with sensor locations is shown in Fig. 1. The mass parameter is assumed as m = 3200 kg and the story stiffness parameter is considered as $k = 6.5 \times 10^6$ N/m and the undamped natural frequencies for this shear building model are found to be $\omega_i \in [12.92,32.65,49.74,71.13,85.31,93.09,104.10]$ rad/s. The equation of motion for base excitation of the example building is given by

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\mathbf{r}\ddot{u}_g \tag{1}$$

where, **M** is the diagonal lumped mass matrix; **C** is a damping matrix assumed to be characterized by a modal damping of 5% in each mode; **K** is the stiffness matrix; **r** is the vector of rigid body inuence coefficients, which in the case of shear buildings is a vector of ones; **u** is the vector of displacements relative to the base motion; and $\ddot{u}_g$ denotes the base acceleration. The system of equations in Eq. (1) is solved for relative displacement **u**, relative velocity $\dot{\mathbf{u}}$, and relative acceleration $\ddot{\mathbf{u}}$ by using the Newmark constant-average acceleration method. The absolute acceleration response at DOFs 1, 4 and 7 are considered to be those picked up by the accelerometers and are obtained by algebraic addition of the ground acceleration time history with

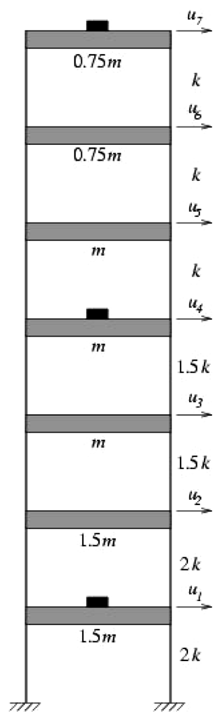


Fig. 1 Schematic diagram of the example building and sensor locations

the respective relative acceleration response time histories at each time instant. To simulate the effect of noise in transducer and/or recorder, 5% zero mean, white Gaussian noise is added to each of these three absolute acceleration response time histories and also to the ground acceleration time history (after the seismic response analysis).

3. Data reconstruction

Since only a few of the total DOFs are actually monitored the resolution of the spatial information is rather coarse which restricts the identification to lower modes only. As the structural response at a group of neighbouring DOFs is not expected to be too different, it is hoped that the response at these intermediate DOFs can be obtained from a suitable interpolation of the response measured at a few distributed locations. Let us assume that the absolute acceleration response is measured at 1, 4, and 7 floor levels of the example building and the response at floor levels 2, 3, 5, and 6 are to be estimated by interpolation. The problem can be addressed either in frequency domain, or in time domain. It is known that the information regarding temporal evolution of the waveform is contained in the Fourier phase spectrum (Shrikhande and Gupta 2001). Evidence of the wave passage efect, which manifests as the phase shifts, in the seismic response of buildings has been reported from the analysis of data from instrumented buildings (Todorovska *et al.* 2001a, 2001b). Thus it appears natural to approach the reconstruction of response data by interpolation in frequency domain. However, the phase information in the vibration signatures is very sensitive to the signal contamination by the noise of transducer and recorder and the results of reconstruction of missing data by Fourier synthesis of the interpolated Fourier amplitude and phase spectrum are not very encouraging. Therefore, we prefer to operate in the time domain for data reconstruction. The next important aspect of the data reconstruction problem is the decision regarding which interpolant to adopt. Limongelli used cubic spline interpolation at each time instant between the acceleration measured at a few selected floors to obtain the acceleration time history at other floor levels (Limongelli 2003). Since the cubic splines are based on the continuity of second order derivative of the interpolant variable at the knots, the interpolated function is too smooth and is not a good choice if the function is in some way discontinuous—a possibility when the structure undergoes nonlinear deformation in response to earthquake shaking. In such cases it is recommended to use piecewise cubic Hermite interpolation polynomial (PCHIP) which is also trend preserving (Fritsch and Carlson 1980).

Assuming a set of data points (x_k, y_k), $k = 1, 2, ..., n$, the PCHIP interpolation can be constructed by standard cubic Hermite interpolation (the same as used in the interpolation of the displacement field within an Euler-Bernoulli beam element ensuring continuity of zeroth and first derivatives) with the following pre-processing steps to estimate the function derivatives (or, tangents) at nodes

1. Compute the slopes of secant lines between successive points

$$\Delta_k = \frac{y_{k+1} - y_k}{x_{k+1} - x_k}$$

for $k = 1, 2, ..., n - 1$.

2. Initialize the tangent at every data point as the average of the secants

$$m_k = \frac{\Delta_{k-1} + \Delta_k}{2}$$

for k = 2, 3, ..., $n-1$; these may be updated in further steps. For the endpoints, use the one-sided diferences: $m_1 = \Delta_1$ and $m_n = \Delta_{n-1}$.

3. For k = 1, 2, ..., $n-1$, if $\Delta_k = 0$ (if two successive $y_k = y_{k+1}$ are equal), then set $m_k = m_{k+1} = 0$, as the spline connecting these points must be at to preserve monotonicity. Ignore steps 4 and 5 for those k.

4. Let $\alpha_k = m_k/\Delta_k$ and $\beta_k = m_{k+1}/\Delta_k$. If α and β are computed to be zero then the input points are not monotone. In such cases, piecewise monotone curves can still be generated by choosing $m_k = m_{k+1} = 0$, although global monotonicity is not possible.

5. To prevent overshoot and ensure monotonicity, the function

$$\phi(\alpha, \beta) = \alpha - \frac{(2\alpha + \beta - 3)^2}{3(\alpha + \beta - 2)}$$

must have a value greater than zero. One simple way to satisfy this constraint is to restrict the magnitude of vector (α_k, β_k) to a circle of radius 3. That is, if $\alpha^2 + \beta^2 > 9$ then set $m_k = \tau_k \alpha_k \Delta_k$ where $\tau_k = \frac{3}{\sqrt{\alpha_k^2 + \beta_k^2}}$.

After the above pre-processing, evaluation of interpolated function is equivalent to cubic Hermite polynomial using the data x_k, y_k and m_k for k = 1, 2, ..., n. To evaluate at x, find the smallest value larger than x, x_{upper} and the largest value smaller than x, x_{lower} among x_k such that $x_{\text{lower}} \leq x \leq x_{\text{upper}}$. Calculate

$$h = x_{\text{upper}} - x_{\text{lower}} \text{ and } t = \frac{x - x_{\text{lower}}}{h}$$

to convert to standard unit interval form and then the interpolant is obtained as

$$f_{\text{interpolated}}(x) = y_{\text{lower}} h_{00}(t) + h m_{\text{lower}} h_{10}(t) + y_{\text{upper}} h_{01}(t) + h m_{\text{upper}} h_{11}(t) \tag{2}$$

where $h_{ij}(t)$ are the basis functions for the cubic Hermite polynomials and may be given as

$$\begin{aligned} h_{00}(t) &= 2t^3 - 3t^2 + 1 \qquad & h_{10}(t) &= t^3 - 2t^2 + t \\ h_{01}(t) &= -2t^3 + 3t^2 \qquad & h_{11}(t) &= t^3 - t^2 \end{aligned} \tag{3}$$

The PCHIP interpolation scheme is readily available as a standard part of various scientific computation tools, including MATLAB (Moler, 2004) and GNU Octave (Eaton 2002), for example.

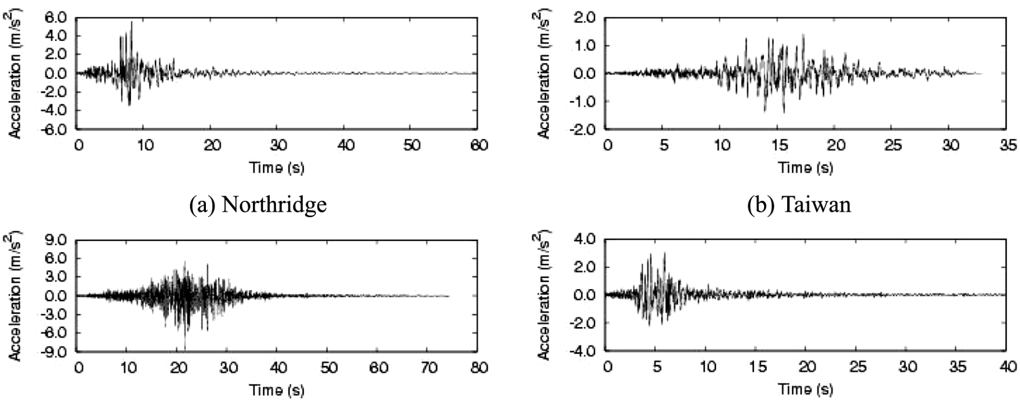
For the present problem, the height of floors constitute the independent variables (x_k) and the absolute accelerations at sensor locations constitute the dependent variables (or, function values y_k) at any specified time instant. The absolute acceleration at intermediate floors at the time instant under consideration may be then estimated by using the PCHIP interpolation scheme described above. The numerical experiments on the reconstruction of missing response data for the example building is considered for four earthquake ground motions recorded in different geographical locations, namely:

1. Northridge Earthquake ($M_s \sim 6.8$) of January 17, 1994, Component: 90, recorded at Castaic, California, Old Ridge Route, Station Latitude-Longitude: 34.2057, -118.5539, at the hypocentral distance of 44.4 km and with peak ground acceleration (PGA) of 5.57 m/s^2.
2. Taiwan Earthquake ($M_L \sim 7.3$) of November 14, 1986, Component: 0, recorded at Lotung, SMART-1 Array, Station: E03, Station Latitude-Longitude: 23.9918, 121.8332, at the hypocentral distance of 72.9 km and with 1.39 m/s^2 PGA.
3. El Salvador Earthquake ($M_s \sim 7.8$) of January 13, 2001, Component: 360, recorded at Santiago de Maria, Station Latitude-Longitude: 13.4860, -88.4710, at the hypocentral distance of 79.7 km and with 8.64 m/s^2 PGA, and
4. Uttarkashi Earthquake ($M_s \sim 7.0$) of October 19, 1991, Component: 75, recorded at Uttarkashi, Station Latitude-Longitude: 30.73, 78.45, at the hypocentral distance of 34 km and with 3.04 m/s^2 PGA.

Fig. 2 shows all excitation time histories used for computing the seismic response of the example building. These excitation time histories are representative of a diverse set of ground motion characteristics. The absolute acceleration response at the intermediate floors is first obtained by interpolation of response recorded at sensor locations as described earlier. The quality of the interpolated time history at k th DOF is assessed from the relative root mean square (r.m.s.) error (ε_k) defined as

$$\varepsilon_k = \left[\frac{\sum_{i=1}^{n}\{\ddot{u}_k^t(t_i) - \hat{\ddot{u}}_k^t(t_i)\}^2}{\sum_{i=1}^{n}\{\ddot{u}_k^t(t_i)\}^2}\right]^{1/2} \times 100\% \tag{4}$$

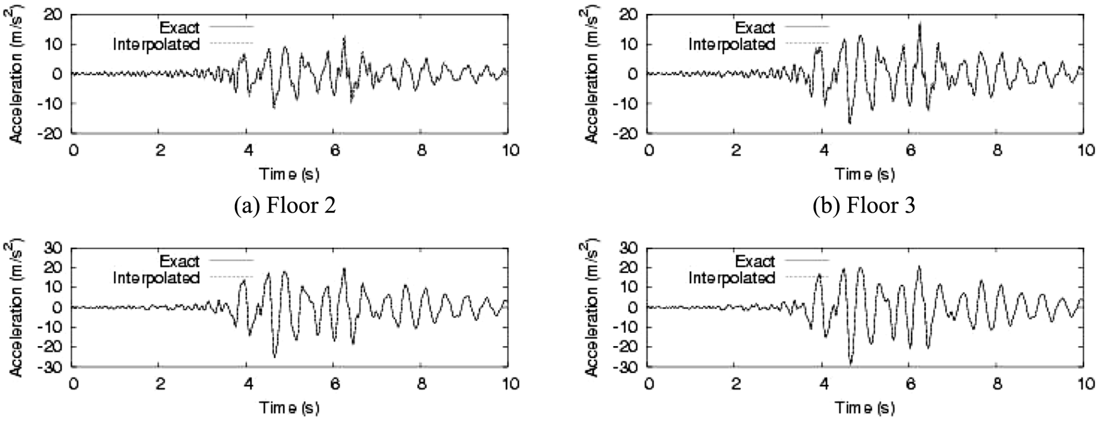
where, $\ddot{u}_k^t(t)$ denotes the true absolute acceleration response time history at k th DOF and $\hat{\ddot{u}}_k^t(t)$ represents the interpolated absolute acceleration time history at k th DOF. The reconstructed response time histories—for the case of excitation by the Uttarkashi earthquake motion—are compared with the true response time histories in Fig. 3. Only first 10 s data is shown in order to have a closer look at the quality of agreement between the two sets of time histories. The 10 s interval is chosen for that corresponds to the interval of most intense shaking. It can be seen that the



(a) Northridge (b) Taiwan

(c) El Salvador (d) Uttarkashi

Fig. 2 Ground acceleration time histories used for computing seismic response data two sets of time histories are in good agreement. The quantitative measure of the error in estimation—as calculated by Eq. (4)—is tabulated in Table 1 for data reconstruction by using the data from 3 sensors located at 1st, 4th and 7th floors. The quality of approximation is comparable for the four test excitation cases considered with the global average error of interpolation is close to 10% (with relative r.m.s. error at individual floor levels ranging from 3-20%) for PCHIP interpolation. A 20% r.m.s. error in signal reconstruction at a DOF is similar to acquiring a vibration signature by using a data acquisition system with 14 dB signal to noise ratio (SNR)—bordering on the accept- able quality of signal for further processing which can be subsequently improved by using some denoising techniques (Chan and Tse 2009). However, no such denoising processing is adopted in this study. Similar error estimates are obtained for the case of interpolation by using cubic splines. However, we recommend the use of PCHIP interpolation owing to its trend preserving character. The extra smoothness of cubic splines due to enforced continuity of second derivative can potentially introduce unnatural kinks in the interpolated function. The PCHIP interpolation scheme to reconstruct the missing response data has been found to work well even in the cases of moment resistant frames sustaining damage during earthquake shaking (Patil 2009, Shiradhonkar 2009). It will be shown in the following that even with this moderate error in approximation due to small number of vibration recordings being used in data reconstruction, it is possible to extract stable mode shape information from the response data.



(a) Floor 2 (b) Floor 3

(c) Floor 5 (d) Floor 6

Fig. 3 Comparison of the analytical and interpolated time histories of absolute acceleration floor responses for the first 10 s of Uttarkashi earthquake motion

Table 1 Relative root mean square error of the interpolated time histories

DOF	Northridge motion	Taiwan motion	El Salvador motion	Uttarkashi motion
2	13.89%	15.64%	20.61%	20.74%
3	6.24%	7.77%	11.16%	10.01%
5	2.90%	4.06%	6.24%	4.78%
6	4.33%	4.80%	8.18%	5.02%
Average	6.84%	8.07%	11.55%	10.42%

4. Identification of modal parameters

The reconstruction of response time histories at intermediate DOFs provides the much needed stability constraints to the system identification procedures thereby improving the quality of estimated parameters. A major problem in the use of system identification procedures for damage estimation is that the damage is essentially a local phenomenon and is better correlated with higher modes of vibrations, which are sensitive to the localized effects. For this reason the changes in curvatures of mode shapes, or the modal strain energy functional are tracked as an indicator of the damage. Most of the system identification procedures, however, provide information about lower modes only which are not substantially affected by the localized damage in a small region. Further, there have been instances where the participation of higher modes was identified as the primary reason for damage in multi-storey buildings (Villaverde 1991). Therefore, it would be immensely useful to estimate as complete information about normal modes of the structural system as possible.

Let us consider the complete acceleration response data (recorded and interpolated) of an N -DOF system is arranged in the form

$$\mathbf{A} = [\mathbf{a}_1(t_i), \mathbf{a}_2(t_i), ..., \mathbf{a}_N(t_i)]^T \tag{5}$$

where, $\mathbf{a}_k(t_i) = [a_k(t_1), a_k(t_2), ..., a_k(t_n)]$ is the absolute acceleration time history of the k th DOF, and $\mathbf{A} \in \mathbb{R}^{N\times n}$. We look for the possibility of factorising this response data in the variable separable form such that the dependence with respect to space coordinates is separated from the time dependent component, i.e. $\mathbf{A}(x,t) = \mathbf{G}(x)\mathbf{F}(t)$. It is possible to decompose the space-time data set represented by the absolute acceleration response by using singular value decomposition (Golub and Loan 1996) as

$$\mathbf{A} = \underbrace{\mathbf{U}}_{N\times N}\underbrace{\Sigma\mathbf{V}^T}_{N\times n} \tag{6}$$

In this decomposition the columns of orthogonal matrix $\mathbf{U}(\in \mathbb{R}^{N\times N})$ —known as the left singular vectors of $\mathbf{A}$—represent the spatial distribution of various principal time history components represented by the columns of orthogonal matrix $\mathbf{V}(\in \mathbb{R}^{n\times n})$, also known as the right singular vectors of $\mathbf{A}$. The elements of $\Sigma(\in \mathbb{R}^{N\times n})$ are all 0's except for elements with identical row, column indices, i.e. σ_{ii}, $\forall i = 1, 2, ..., \min(N, n)$, containing non-negative scaling factors—known as singular values—that indicate the relative importance of the principal time series components in constituting the response at a particular DOF. It must be mentioned here that a similar SVD-based orthogonal decomposition has been presented earlier using covariance matrix (Feeny and Kappagantu 1998) and using power spectral density matrix (Han and Feeny 2003) of the response The proportionality of the proper orthogonal modes obtained by SVD-based measurements of displacement response data to the undamped normal modes of a linear system with mass distribution proportional to identity matrix has also been established (Kerschen and Golinval 2002). In practice, however, it is easier to measure absolute acceleration response, particularly, in the case of forced vibrations. In the following, we describe a one-to-one correspondence between the factors of orthogonal decomposition of the absolute acceleration data and the various terms in the mode-superposition of dynamic response of linear structural systems subjected to base excitations, such as earthquake ground motions.

For linear systems, the absolute acceleration response may be expressed by rearranging Eq. (1) as

(assuming mass-orthonormal modes)

$$\begin{aligned} \mathbf{A} &= \ddot{\mathbf{u}} + \mathbf{r}\ddot{u}_g = -\mathbf{M}^{-1}\left[\mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u}\right] \\ &= \underbrace{-\mathbf{\Phi}}_{N\times N}\overbrace{\left[\lfloor 2\zeta_j\omega_j \rfloor\dot{\mathbf{q}} + \lfloor \omega_j^2 \rfloor\mathbf{q}\right]}^{N\times n} \end{aligned} \tag{7}$$

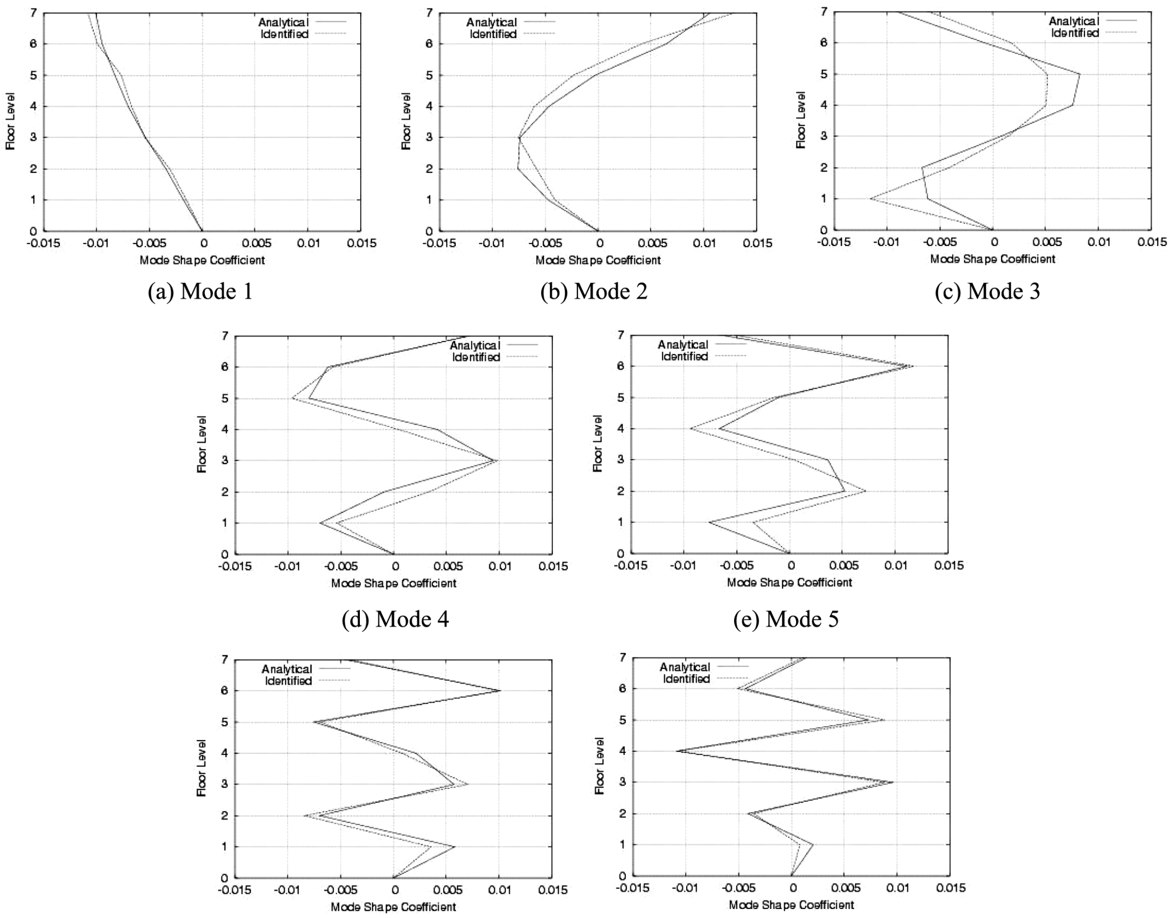
where, $\mathbf{\Phi}$ represents the mass-orthonormal (i.e. $\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi} = \mathbf{I}$) modal matrix, ζ_j and ω_j respectively denote the modal damping and natural frequency in the j th mode of vibration, $\lfloor 2\zeta_j\omega_j \rfloor$(= $\mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi}$) and $\lfloor \omega_j^2 \rfloor$(= $\mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}$) are diagonal matrices and $\mathbf{q}_j$ is the vector of modal coordinates. Comparing Eqs. (7) and (6), it may be inferred that the mass-orthonormal modal matrix $\mathbf{\Phi}$ is proportional to the left singular matrix $\mathbf{U}$ of the (complete) absolute acceleration response matrix $\mathbf{A}$. Since $\mathbf{U}$ is an orthogonal matrix of same dimensions as the modal matrix $\mathbf{\Phi}$ of the structural system, it should be possible to extract the mode shape information from $\mathbf{U}$. Thus for a positive definite mass matrix, which is usually available with a reasonable accuracy from analytical models for the structural system, and considering mass-orthonormalized mode shapes, we have

$$\begin{aligned} &\mathbf{U}^T\mathbf{U} = \mathbf{I} = \mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi} = \mathbf{\Phi}^T\mathbf{L}\mathbf{L}^T\mathbf{\Phi} \\ &\text{or, } \mathbf{\Phi} = \mathbf{L}^{-T}\mathbf{U} \end{aligned} \tag{8}$$

where, $\mathbf{M} = \mathbf{L}\mathbf{L}^T$ denotes Cholesky decomposition in terms of a lower triangular component ($\mathbf{L}$) and an upper triangular component ($\mathbf{L}^T$). The Eq. (8) provides the desired relation for structural mode shapes in terms of the left singular vectors of the response data. It is important to note that this formulation allows extraction of all modes—lower as well as higher—from the available response data and can be very helpful in system identification, model updating, and damage detection of structural systems. The mode shapes of the example building have been identified by using the aforementioned procedure from the response data (measured and interpolated) for the El Salvador excitation case with 3 measured and 4 interpolated time histories are shown in Fig. 4. The results for El Salvador case are shown here as those corresponding to the worst case in terms of interpolation errors. However, the mode shapes identified from other response data set are similar to the ones shown here indicating that the mode shape information is not very sensitive to the errors in the response data sets. It may be seen that the quality of the estimated mode shapes is very good even for higher modes of vibration—a definite advantage for their use in damage detection with the help of changes in mode shape curvature, or the modal strain energy functional. A measure to ascertain the quality of identified mode shapes is the modal assurance criterion (MAC) defined as a scalar constant relating the degree of consistency (linearity) between one modal vector and another reference modal vector as (Allemang 2003)

$$MAC = \frac{\left|\phi_{id}^H\phi\right|^2}{(\phi_{id}^H\phi_{id})(\phi^H\phi)} \tag{9}$$

where, ϕ represents the reference (analytical, in the present case) modal vector, ϕ_{id} denotes the identified vector, and the superscript $(\bullet)^H$ denotes the Hermitian (conjugate transpose) operator. A very good agreement of the estimated mode shapes with the analytical mode shapes were obtained as indicated by the MAC values as shown in Table 2. The temporal information encapsulated in the



(a) Mode 1 (b) Mode 2 (c) Mode 3

(d) Mode 4 (e) Mode 5

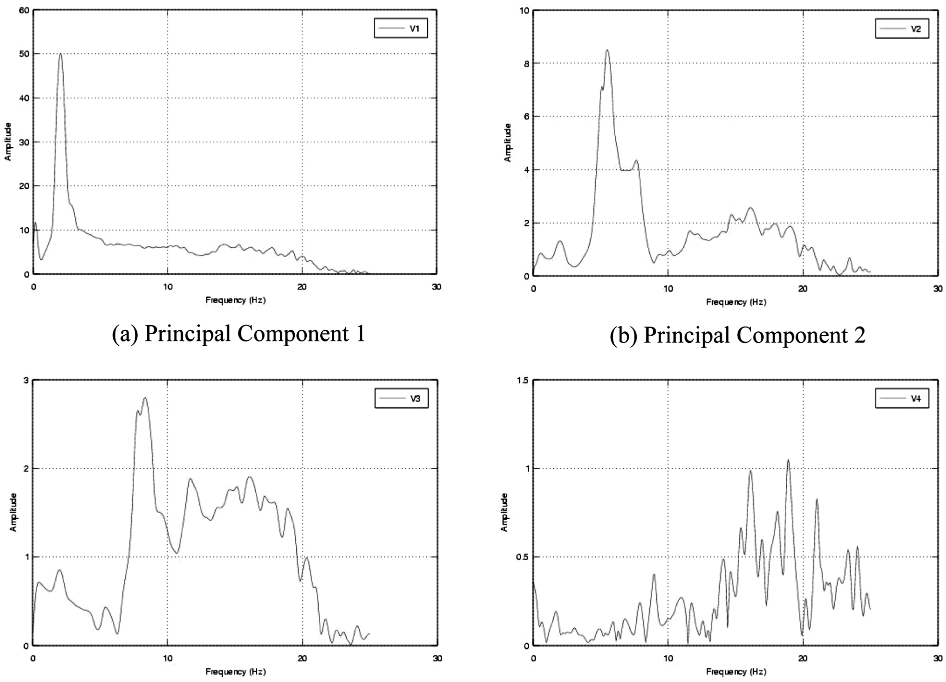
(f) Mode 6 (g) Mode 7

Fig. 4 Comparison of the analytical and estimated mode shapes

Table 2 Modal assurance criterion (MAC) values for the mode shapes identified from different data sets

Mode #	Northridge motion	Taiwan motion	El Salvador motion	Uttarkashi motion
1	0.9939	0.9939	0.9961	0.9953
2	0.9522	0.9514	0.9389	0.9361
3	0.7490	0.7469	0.7684	0.7125
4	0.8776	0.8848	0.8849	0.9040
5	0.8851	0.8872	0.8730	0.8759
6	0.9525	0.9503	0.9634	0.9149
7	0.9825	0.9825	0.9825	0.9825

product of the singular values and the right singular vector ($\mathbf{\Sigma V}^T$) represents the principal components of the absolute acceleration response in time domain and can be used to estimate natural frequencies by peak picking technique for empirical transfer function estimates (ETFE) (Ljung 1985). Considering



(a) Principal Component 1 (b) Principal Component 2

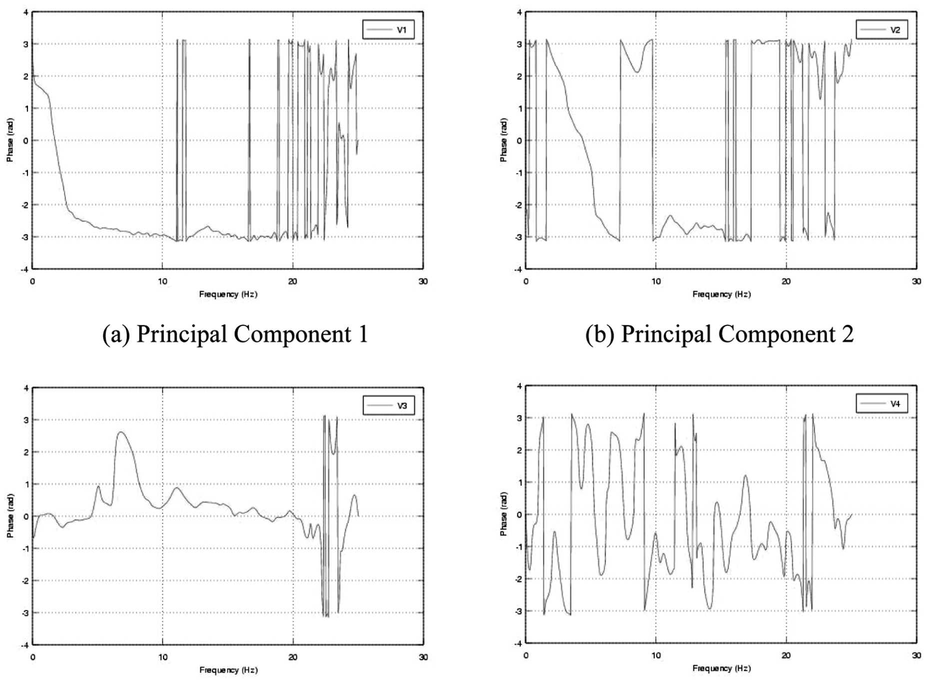
(c) Principal Component 3 (d) Principal Component 4

Fig. 5 Empirical transfer function (amplitude) estimates for the right singular vectors and the ground motion

only the first few principal components which correspond to the non-zero singular values reduces the effect of noise contamination and the estimated transfer function between the principal component and the base excitation time history allows us to estimate (damped) natural frequencies by peak picking technique. The estimated transfer functions for the El Salvador data set are shown in Figs. 5 and 6. Actually, a peak in the transfer function amplitude plot corres- ponds to a natural frequency if and only if it is associated with a sharp phase shift in the transfer function phase plots shown in Fig. 6. In the present case, the transfer function estimates derived from the first three principal components clearly provide estimates for the first four natural frequencies as: 2.06, 5.54, 7.68, and 11.68 Hz. Since the participation of higher modes in the seismic response of shear buildings is negligible, those are not easily identified from the transfer function. The modal damping may also be estimated by using the half-power bandwidth method for the estimated transfer functions. It must be mentioned here that the peak-picking method of modal identification is not very reliable in case of closely spaced modes. In such cases, more robust identification methods such as eigen realization algorithm, or sub-space identification should be used in conjunction with stabilization diagrams to ascertain the correct model order (Peeters *et al.* 2008, Zhang *et al.* 2005). However, the ETFE estimates are still valuable for the purpose of cross-validation.

Once the mass-orthonormal mode shapes and the associated natural frequencies are identified the system flexibility matrix can be easily constructed by using rank-1 update relationship as

$$F = \sum_{j=1}^{N_m} \frac{1}{\omega_j^2} \phi_j \phi_j^T \tag{10}$$



(a) Principal Component 1 (b) Principal Component 2

(c) Principal Component 3 (d) Principal Component 4

Fig. 6 Empirical transfer function (phase) estimates for the right singular vectors and the ground motion

where, $N_m(\le N)$denotes the number of natural frequencies (ω_j) identified from the ETFE plots and ϕ_j indicates the identified j th mass-orthonormalized mode shape. Each term in the summation of Eq. (10) represents a rank-1 update to the system flexibility matrix. This reduced rank approximation to the system flexibility matrix has been found to be within 3% of the true system flexibility matrix in terms of Frobenius norm for all example cases considered in this study where it is possible to identify up to four natural frequencies from the ETFE plots. The close agreement between the reduced-rank approximation and the true system flexibility matrix is due to negligible relative contribution of the higher modes to the system flexibility matrix due to the scaling factor $1/\omega_j^2$.

The modal identification and reduced rank approximation for the system flexibility matrix as above can also be performed over different time-windows of the same data set to detect the changes in mode shape curvatures and the system flexibility matrix which would be helpful in structural health monitoring (Patil 2009). Alternatively, the estimated modal parameters from different time windows can be used to define constraints in finite element model updating procedures (Shiradhonkar 2009).

5. Conclusions

A new method for reconstructing the building response data from limited measurements has been proposed. The piecewise cubic Hermite polynomial interpolation (PCHIP) is preferred for interpolation in time domain due to its trend preserving character. The quality of interpolated data is found to be bordering on the acceptable quality of signals for system identification studies.

An alternate formulation of SVD-based decomposition of absolute acceleration response data for extracting the mode shape information has also been proposed. It is shown that all mode shapes of the structural system can be identified with a few response measurements with the response at other DOFs being obtained through the interpolation procedure pro- posed earlier. A good agreement has been observed between the analytical and estimated mode shapes for lower modes which are global trend indicators in the response as well as the higher modes which capture the finer localized detail in the system response. The natural frequencies can be identified from the plots of empirical transfer function estimates for right singular vectors and the ground acceleration. The estimated natural frequencies and mass- orthonormalized mode shapes can be combined to construct a reduced-rank approximation for the system flexibility matrix to a good accuracy. The identified mode shapes and the approximation for the flexibility matrix can be obtained for different time windows of the same data set for detect any changes indicative of structural damage during an earthquake.

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References

Allemang, R. (2003), “The modal assurance criterion: twenty years of use and abuse”, *Sound Vib.*, **37**(8),14-23.

Alvin, K., Robertson, A., Reich, G. and Park, K. (2003), “Structural system identification: from reality to models”, *Comput. Struct.*, **81**(12), 1149-1176.

Brincker, R., Zhang, L. and Andersen, P. (2001), “Modal identification of output only systems using frequency domain decomposition”, *Smart Mater. Struct.*, **10**(3), 441-445.

Brownjohn, J. (2007), “Structural health monitoring of civil infrastructure”, *Philos. T. R. Soc. A*, **365**, 589-622, doi:10.1098/rsta.2006.1925.

Chan, J. and Tse, P. (2009), “A novel, fast, reliable data transmission algorithm for wireless machine health monitoring”, *IEEE T. Reliab.*, **58**(2), 295-304.

Choudhury, K. (2007), “On the reliability of a class of system identification techniques”, M.Tech. dissertation, Department of Earthquake Engineering, Indian Institute of Technology Roorkee, Roorkee-247667, India.

Datta, A., Shrikhande, M. and Paul, D. (2002), “On the optimal location of sensors in multi-storeyed buildings”, *J. Earthq. Eng.*, **6**(1),17-30.

Distefano, N. and Pena-Pardo, B. (1976), “System identification of frames under seismic loads”, *J. Eng. Mech. - ASCE*, **102**(EM2), 313-330.

Eaton, J.W. (2002), *GNU Octave manual*, Network Theory Limited, United Kingdom, http://www.octave.org.

Feeny, B. and Kappagantu, R. (1998), “On the physical interpretation of proper orthogonal modes in vibrations”. *J. Sound Vib.*, **211**(4), 607-616.

Fritsch, F. and Carlson, R. (1980), “Monotone piecewise cubic interpolation”, *SIAM J Numer. Anal.*, **17**(2), 238-246.

Golub, G. and Loan, C.V. (1996), *Matrix computations*, John Hopkins University Press, 3rd edition.

Han, S. and Feeny, B. (2003), “Application of proper orthogonal decomposition to structural vibration analysis”, *Mech. Syst. Signal Pr.*, **17**(5), 989-1001.

Hart, G. and Yao, J. (1977), "System identification in structural dynamics", *J. Eng. Mech. - ASCE*, **103**(EM6), 1089-1104.
Heredia-Zavoni, E. and Esteva, L. (1998), "Optimal instrumentation of uncertain structural systems subject to earthquake ground motions", *Earthq. Eng. Struct. Dyn*., **27**(4), 343-362.
Jaishi, B. and Ren, W.X. (2007), "Finite element model updating based on eigenvalue and strain energy residuals using multiobjective optimisation tech- nique", *Mech. Syst. Signal Pr*., **21**(5), 2295-2317.
James III, G., Carrie, T. and Laufer, J. (1993), "The natural excitation technique (NExT) for modal parameter extraction from operating wind turbines", Technical Report SAND92-1666, Sandia National Laboratories, Albuquerque, New Mexico, U.S.A. http://infoserve.library.sandia.gov/sand_doc/1992/921666.pdf.
Juang, J.N. (1987), "Mathematical correlation of modal parameter identification methods via system realization theory", *Int. J. Anal. Exper. Modal Anal*., **2**, 1-18.
Juang, J.N. and Pappa, R. (1985), "An eigensystem realization algorithm for modal parameter identification and model reduction", *J. Guid. Control Dynam*., **8**(5), 620-627.
Kerschen, G. and Golinval, J. (2002), "Physical interpretation of proper orthogonal modes using the singular value decomposition", *J. Sound Vib*., **249**(5), 849-865.
Limongelli, M. (2003), "Optimal location of sensors for reconstruction of seismic responses through spline function interpolation", *Earthq. Eng. Struct. Dyn*., **32**(7), 1055-1074.
Ljung, L. (1985), "On the estimation of transfer functions", *Automatica*, **21**(6), 677-696.
Moler, C. (2004), *Numerical computing with MATLAB*, Society for Industrial Applications of Mathematics (SIAM), Philadelphia, http://www.mathworks.com/moler/chapters.html.
Patil, A.D. (2009), "Structural health monitoring via flexibility update", M.Tech. dissertation, Department of Earthquake Engineering, Indian Institute of Technology Roorkee, Roorkee-247667, India.
Peeters, B. and de Roeck, G. (2001), "Stochastic system identification for operational modal analysis: a review", *J. Dyn. Syst. Measurement Control*, **123**(4), 659-667.
Peeters, B., Lau, J., Lanslots, J. and Van der Auweraer, H. (2008), "Automatic modal analysis — myth or reality?", *Sound Vib*., (March):17-21, http://www.sandvmag.com/downloads/0803peet.pdf.
Shi, Z., Law, S. and Zhang, L. (2000), "Structural damage detection from modal strain energy change", *J. Eng. Mech. - ASCE*, **126**(12), 1216-1223.
Shih, C., Tsuei, Y., Allemang, R. and Brown, D. (1988), "Complex mode indication function and its application to spatial domain parameter estimation", *Mech. Syst. Signal Pr*., **2**(4), 367-377.
Shiradhonkar, S.R. (2009), "Structural health monitoring via finite element model updating", M.Tech. dissertation, Department of Earthquake Engineering, Indian Institute of Technology Roorkee, Roorkee-247667, India.
Shrikhande, M. and Gupta, V. (2001), "On the characterization of the phase spectrum for strong motion synthesis", *J. Earthq. Eng.*, **5**(4), 465-482.
Sohn, H., Farrar, C., Hemez, F., Shunk, D., Stinemates, S., Nadler, B. and Czarnecki, J. (2004), "A review of structural health monitoring literature from 1996-2001", Technical Report LA-13976-MS, Los Alamos National Laboratory, U.S.A.
Todorovska, M., Ivanović, S. and Trifunac, M. (2001a), "Wave propagation in a seven-story reinforced concrete building: I. Theoretical models", *Soil Dyn. Earthq. Eng*., **21**(3), 211-223.
Todorovska, M., Ivanović, S. and Trifunac, M. (2001b), "Wave propagation in a seven-story reinforced concrete building: II. Observed wavenumbers", *Soil Dyn. Earthq. Eng*., **21**(3), 225-236.
Udwadia, F. (1994), "Methodology for optimum sensor locations for parameter identification in dynamic systems", *J. Eng. Mech. - ASCE*, **120**(2), 368-390.
Udwadia, F. and Sharma, D. (1978), "Some uniqueness results related to building structural identification", *SIAM J. Appl. Math*., **34**, 104-118.
Villaverde, R. (1991), "Explanation for the numerous upper floor collpses during the 1985 Mexico City earthquake", *Earthq. Eng. Struct. Dyn*., **20**(3), 223-241.
Yi, J.H. and Yun, C.B. (2004), "Comparative study on modal identification methods using output-only information", *Struct. Eng. Mech*., **17**(3-4), 445-466.
Zhang, Y., Zhang, Z., Hu, X. and Hua, H. (2005), "Modal parameter identification using response data only", *J. Sound Vib*., **282**(1-2), 367-380.

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