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Dynamic response of adjacent structures connected by friction damper

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Abstract. Dynamic response of two adjacent single degree-of-freedom (SDOF) structures connected with friction damper under base excitation is investigated. The base excitation is modeled as a stationary white-noise random process. As the force-deformation behavior of friction damper is non linear, the dynamic response of connected structures is obtained using the equivalent linearization technique. It is observed that there exists an optimum value of the limiting frictional force of the damper for which the mean square displacement and the mean square absolute acceleration responses of the connected structures attains the minimum value. The close form expressions for the optimum value of damper frictional force and corresponding mean square responses of the coupled undamped structures are derived. These expressions can be used for initial optimal design of the friction damper for connected structures. A parametric study is also carried out to investigate the influence of system parameters such as frequency ratio and mass ratio on the response of the coupled structures. It has been observed that the frequency ratio has significant effect on the performance of the friction damper, whereas the effects of mass ratio are marginal. Finally, the verification of the derived close from expressions is made by correlating the response of connected structures under real earthquake excitations.

Keywords: adjacent structures; dynamic response; earthquake; friction damper; optimization; system parameters.

1. Introduction

The natural disturbances like strong wind and earthquakes produce excessive structural vibrations, which creates human discomfort and many times lead to catastrophic structural failure as well. In last decade, significant efforts have been given for design of engineering structures with various control strategies to increase their safety and reliability against strong earthquakes. Many energy dissipation devices and control systems have been developed to reduce the excessive structural vibrations due to natural disturbances. These control strategies are able to modify dynamically the response of structure in a desirable manner, thereby termed as protective systems for the new structures and the existing structures can be retrofitted or strengthened effectively to withstand future seismic activity. The control system and the structure do not behave as independent dynamic system but rather interact with each other. In addition, the interaction effects also occur between the excitation and structure (i.e. soil-structure interaction). The control system can be classified as

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active, passive, semi-active and hybrid system. Passive control system does not require any additional energy for its operation and control. Viscous damper, tuned mass dampers, tuned liquid dampers (dynamic absorbers), visco-elastic damper, metallic yielding dampers, friction dampers, shunted piezoceramics dampers, magnetic dampers and base isolators are other mechanism used for the passive vibration control (Housner *et al*. 1997). More complete details on the mechanics and working principles of these devices can be found in reference (Soong and Dargush 1997). The friction damper elements are one of the types of supplemental energy dissipation devices that have been introduced to enhance the seismic response of buildings. These types of devices dissipate input energy through work done by the frictional forces. The behavior of the devices are nearly unaffected by amplitude, frequency, temperature, or the number of applied loading cycles.

In modern city, many buildings are often built closely to each other because of limited availability of land and preference for centralized services. During an earthquake mutual pounding (severe loading condition during an earthquake) may also occur, which has been observed in the 1985 Maxico City earthquake (Bertero 1985), the 1989 Loma Prieta earthquake (Kasai and Maison 1992) and many others. Among the various structural control techniques, the coupling of two adjacent structures with suitable mechanisms when possible is a developing methodology for mitigation of the structural vibrations. This mechanism does not require additional space for installation of control devices and the free space available between adjacent structures can be effectively utilized for installation of control devices. This mechanism is working on the concept of exerting control forces upon one another to reduce the overall responses of the system and also prevent the mutual pounding between two adjacent structures. For adjacent buildings having their floors in alignment, they are suggested using hinged links to connect two neighboring floors, to prevent mutual pounding between adjacent structures during an earthquake (Westermo 1989). To avoid pounding damages during earthquakes the concept of linking adjacent fixed-base buildings has been introduced and verified analytically and experimentally by number of researchers (Filiatrault and Folz 1992, Constantinou and Symans 1993, Weidlinger 1996). The optimal values for the distribution of passive dampers have been found to interconnecting two adjacent structures of different heights (Luco and De Barros 1998). The dynamic characteristics of two SDOF systems coupled with a Viogt model-defined damper subjected to stationery white-noise excitation have been examined by means of statistical energy analysis techniques (Zhu and Iemura 2000). The method for analyzing the random seismic response of a structural system consisting of two adjacent buildings interconnected by non-linear hysteretic damping devices have been developed by Ni et al. (2001). The developed method is applicable to the structural system with an arbitrary number of stories and with connecting dampers at arbitrary stories. The combinations of the both displacement and velocity-dependent energy dissipating devices have been used to overcome the shortcomings in each type (Tsai *et al*. 2003). A stochastic optimal coupling-control method has been proposed for adjacent building structures with control devices under random seismic excitation (Ying *et al*. 2003). The experimental study of dynamic response of single-storey steel frames equipped with friction damper device to lateral harmonic excitation has also been carried out by Mualla *et al*. (2002). The dynamic behaviors of two SDOF structures connected with a friction damper under harmonic ground acceleration have been investigated (Bhaskararao and Jangid 2006a). The close-form expressions in terms of system parameters and excitation were derived for necessary conditions to initiate the stick-stick and slip-slip modes. The influence of important parameters structural damping ratios, frequency ratio, mass ratio and damper slip force on the maximum displacement of friction damper has been also investigated for its effective design in coupling the adjacent structures. The seismic response of adjacent multi-degree-of-freedom (MDOF) structures connected with friction damper having same slip force as well as different slip force has been also investigated (Bhaskararao and Jangid 2006b, 2006c). A parametric study has been also conducted to investigate the optimum slip force of the damper as well as the optimal placement of the dampers. The results of previous studies indicate that friction damper is effective in reducing structural responses. Bhaskararao and Jangid (2007) developed the close from expressions for optimum viscous damper for connecting adjacent SDOF structures under harmonic and stationary white-noise excitations. Using a stochastic equivalent linearization technique, an analytical expression has been developed for estimating optimum material damping values in resilient friction base isolation system for a rigid superstructure (Jangid and Banerji 1998). Stochastic response of bridges isolated by friction pendulum system has been obtained using the time dependent equivalent linearization technique due to non-linear force-deformation behavior of the friction pendulum system (Jangid 2008). These studies explain the use of time dependent equivalent linearization technique to obtain the seismic response of the bridges.

The present study is aimed to investigate the dynamic response of two adjacent SDOF structures connected with friction damper under base excitation modeled as stationary white-noise excitation. The specific objectives of the study are (i) to derive the dynamic response of adjacent SDOF structures connected by the friction damper (ii) to ascertain the optimum value of the limiting friction force of friction damper (iii) to derive the close-form expressions for the optimum value of damper slip force and corresponding responses of the structure, (iv) to examine the effect of important parameters such as frequency ratio and mass ratio on the optimum value of limiting friction force and corresponding response, and (v) to verify derived close from expressions by correlating the response of connected structures under real earthquake excitations.

2. Structural model

Consider two adjacent SDOF structures connected with a friction damper as shown in the Fig. 1. The two structures are assumed to be of the same height and are connected with friction damper at their floor level. The adjacent structures are idealized as two SDOF coupled system and referred as Structures 1 and 2. Both structures are assumed to be subjected to the same ground acceleration. The frictional force mobilized in the damper has typical Coulomb-friction characteristics. Depending upon the system parameters and excitation level, the connected structures may vibrate together without any slip in the friction damper (referred as non-slip mode) or vibrate independently if the frictional force in the damper exceeds the limiting value (i.e. vibration in the slip mode).

Let m_1, k_1, c_1 and m_2, k_2, c_2 be the mass, stiffness and damping coefficient of the Structures 1 and 2, respectively. Let $\omega_1 = \sqrt{k_1/m_1}$ and $\omega_2 = \sqrt{k_2/m_2}$ be the natural frequencies and $\xi_1 = c_1/2m_1\omega_1$ and $\xi_2 = c_2/2m_2\omega_2$ be the damping ratios of Structures 1 and 2, respectively.

Let μ and β be the mass and frequency ratio of two structures, respectively, expressed as

$$\mu = \frac{m_1}{m_2} \tag{1}$$

$$\beta = \frac{\omega_2}{\omega_1} \tag{2}$$

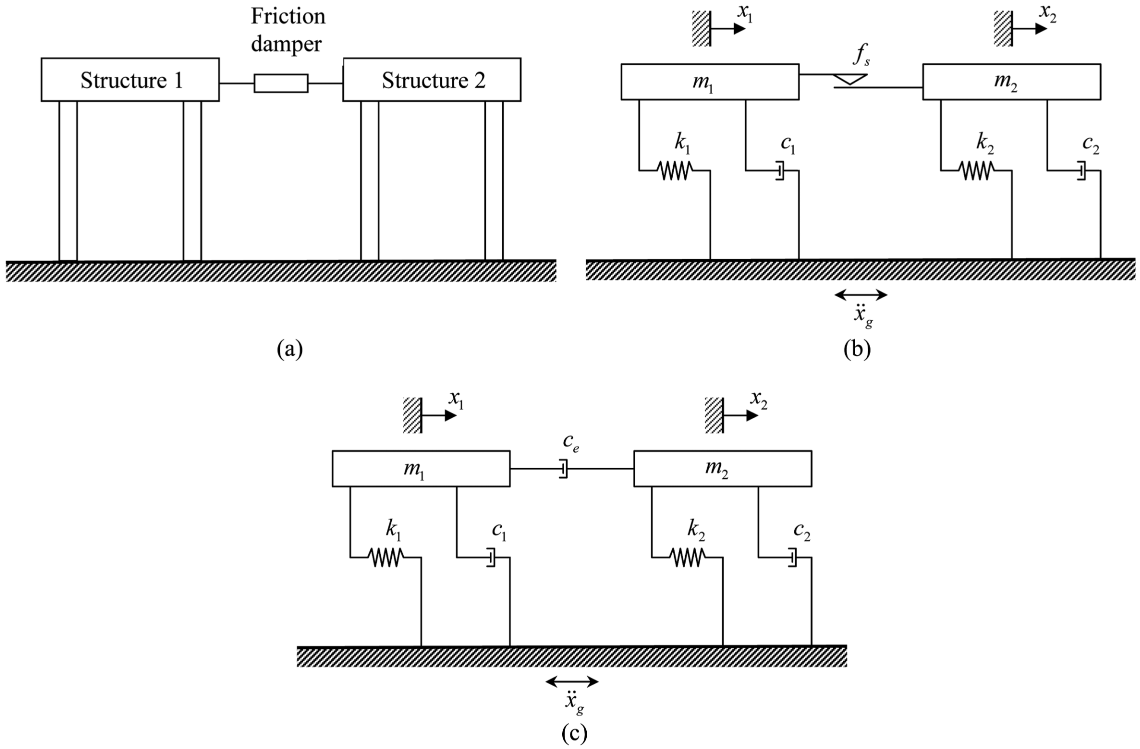

Fig. 1 Structural model of two SDOF adjacent structures connected with friction damper: (a) Adjacent structures connected with friction damper, (b) Mathematical model, (c) Equivalent linear model

Let f_s be the limiting friction force in the damper and it is also termed as slip force. The coupled system remains in the non-slip mode until the frictional force in the damper is less than the limiting frictional force. Whenever, the force in friction damper attains the limiting slip force, the condition for slippage will be initiated. Considering the coupled system subjected to ground motion, assuming the friction damper remains in slip mode, the equations of motion can be written as

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + f_s\,\mathrm{sgn}(\dot{x}_1 - \dot{x}_2) = -m_1\ddot{x}_g \tag{3}$$

$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 - f_s\,\mathrm{sgn}(\dot{x}_1 - \dot{x}_2) = -m_2\ddot{x}_g \tag{4}$$

where x_1 and x_2 are the displacement response relative to the ground of the Structures 1 and 2, respectively (as shown in the Fig. 1); $\ddot{x}_g$ is the ground acceleration; and sgn denotes the signum function.

The equations of motion of system connected with friction damper are non-linear and the corresponding equivalent linearized equations (Robert and Spanos 1990) in matrix form is given by

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_e & -c_e \\ -c_e & c_2 + c_e \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = -\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}\ddot{x}_g \tag{5}$$

where c_e is the equivalent constant.

The equivalent constant, c_e is obtained by minimizing the mean square error between linear and non-linear terms (Nigam 1983) and is expressed by

$$c_e = \sqrt{\frac{2}{\pi}} \frac{f_s}{\sigma_{\dot{x}_r}} \tag{6}$$

where $\dot{x}_r = (\dot{x}_1 - \dot{x}_2)$ is relative velocity of damper ends; and $\sigma_{\dot{x}_r}$ is the root mean square of the relative velocity, $\dot{x}_r$. It is to be noted that the non-linear phenomenon of the friction damper connecting the two adjacent structures still exists due to dependence of equivalent constant, c_e on the velocity response of the system, $\sigma_{\dot{x}_r}$.

The equivalent constant, c_e is expressed in the normalized form as

$$\xi_e = \frac{c_e}{2m_1\omega_1} \tag{7}$$

where ξ_e is the equivalent normalized damping constant.

From the Eq. (5), the absolute acceleration responses $\ddot{x}_{a_1}$ and $\ddot{x}_{a_2}$ of the Structures 1 and 2, respectively are expressed as

$$\ddot{x}_{a_1} = \ddot{x}_1 + \ddot{x}_g = -(c_1/m_1)\dot{x}_1 - (c_e/m_1)(\dot{x}_1 - \dot{x}_2) - (k_1/m_1)x_1 \tag{8}$$

$$\ddot{x}_{a_2} = \ddot{x}_2 + \ddot{x}_g = -(c_2/m_2)\dot{x}_2 + (c_e/m_2)(\dot{x}_1 - \dot{x}_2) - (k_2/m_2)x_2 \tag{9}$$

The structural control criteria depend on the nature of dynamic loads and the response quantities of interest. Minimizing the relative displacement or absolute acceleration of the system are always been considered as the control objective. In case of flexible structures, displacements are predominant that need to be mitigated. In case of stiff structures, accelerations which generate higher inertial force in structure are to be controlled.

3. Response to stationary white-noise excitation

Let the coupled system with equal structural damping ratio, $\xi_1 = \xi_2 = \xi$ be subjected to the base acceleration, $\ddot{x}_g$, modeled as Gaussian white-noise random process with constant power spectral density as S_0. The mean square displacement, $\sigma_{x_1}^2$ and $\sigma_{x_2}^2$ of Structures 1 and 2, respectively are expressed as (Nigam 1983)

$$\sigma_{x_1}^2 = \int_{-\infty}^{\infty} |x_1(i\omega)|^2 S_0 \ d\omega \tag{10}$$

$$\sigma_{x_2}^2 = \int_{-\infty}^{\infty} |x_2(i\omega)|^2 S_0 \ d\omega \tag{11}$$

where $x_1(i\omega)$ and $x_2(i\omega)$ are the harmonic transfer function for displacement responses, x_1 and x_2, respectively and are expressed by

$$x_1(i\omega) = \frac{\omega^2 - \omega_2^2 - i\omega(\Delta_d + \Delta_2)}{\begin{array}{c}\omega^4 - \omega^2(\omega_1^2 + \omega_2^2 + \Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1}) + \omega_1^2\omega_2^2 \\ -i\omega^3(\Delta_1 + \Delta_2 + \Delta_d) + i\omega(\omega_1^2\Delta_2 + \omega_2^2\Delta_1 + \omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1})\end{array}} \tag{12}$$

$$x_2(i\omega) = \frac{\omega^2 - \omega_1^2 - i\omega(\Delta_d + \Delta_1)}{\begin{array}{c}\omega^4 - \omega^2(\omega_1^2 + \omega_2^2 + \Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1}) + \omega_1^2\omega_2^2 \\ -i\omega^3(\Delta_1 + \Delta_2 + \Delta_d) + i\omega(\omega_1^2\Delta_2 + \omega_2^2\Delta_1 + \omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1})\end{array}} \tag{13}$$

$$\Delta_1 = c_1 / m_1;\ \Delta_2 = c_2 / m_2;\ \Delta_{d1} = c_e / m_1;\ \Delta_{d2} = c_e / m_2;\ \Delta_d = \Delta_{d1} + \Delta_{d2} \tag{14}$$

The mean square displacement responses are obtained by solving the integral of Eqs. (10) and (11) and using the technique given by Cremer and Heckl (1973). The solution of the integrals are given by the following close form expressions

$$\sigma_{x_1}^2 = \frac{2\pi S_0}{\omega_1^3}\left[\frac{\begin{array}{l}\beta(1+\beta)^2\xi((-1+\beta)^2 + 4\beta\xi^2) \\ +(1+\beta)((-1+\beta)^2(1+\beta)\mu + 4\beta(2+\beta+\beta^2+2(1+\beta)\mu)\xi^2)\xi_e \\ +4(1+\mu)(1+\mu+\beta(3+\beta^2+(3+\beta)\mu))\xi\xi_e^2 \\ +4(1+\mu)^3\xi_e^3\end{array}}{\begin{array}{l}4(\beta(1+\beta)^2\xi^2((-1+\beta)^2+4\beta\xi^2) + (1+\beta)^2(\mu+\beta)\xi((-1+\beta)^2+8\beta\xi^2)\xi_e \\ +((1+\beta^2)^2\mu + 4(\beta^4+\mu^2+3\beta^3(1+\mu)+3\beta\mu(1+\mu)+\beta^2(1+\mu(4+\mu)))\xi^2)\xi_e^2 \\ +4(1+\mu)(\mu+\beta)(\mu+\beta^2)\xi\xi_e^3)\end{array}}\right] \tag{15}$$

$$\sigma_{x_2}^2 = \frac{2\pi S_0}{\omega_2^3}\left[\frac{\begin{array}{l}\beta^2(4\beta(1+\beta)^2\xi^3 + \xi_e + 4(1+\beta)(\mu+\beta(2+\mu+2\beta(1+\mu)))\xi^2\xi_e \\ \quad +\beta^2\xi_e(-2+\beta^2+4(1+\mu)^3\xi_e^2) \\ \quad +\xi((-1+\beta^2)^2 + 4(1+\mu)(\mu+\beta(1+\beta(3+\beta)(1+\mu)))\xi_e^2))\end{array}}{\begin{array}{l}4(\beta(1+\beta)^2\xi^2((-1+\beta)^2+4\beta\xi^2) + (1+\beta)^2(\mu+\beta)\xi((-1+\beta)^2+8\beta\xi^2)\xi_e \\ +((1+\beta^2)^2\mu + 4(\beta^4+\mu^2+3\beta^3(1+\mu)+3\beta\mu(1+\mu)+\beta^2(1+\mu(4+\mu)))\xi^2)\xi_e^2 \\ +4(1+\mu)(\mu+\beta)(\mu+\beta^2)\xi\xi_e^3)\end{array}}\right] \tag{16}$$

The mean square acceleration, $\sigma_{a_1}^2$ and $\sigma_{a_2}^2$ of Structures 1 and 2, respectively are expressed by

$$\sigma_{a_1}^2 = \int_{-\infty}^{\infty} |\ddot{x}_{a_1}(i\omega)|^2 S_0\ d\omega \tag{17}$$

$$\sigma_{a_2}^2 = \int_{-\infty}^{\infty} |\ddot{x}_{a_2}(i\omega)|^2 S_0\ d\omega \tag{18}$$

where $\ddot{x}_{a_1}(i\omega)$ and $\ddot{x}_{a_2}(i\omega)$ are the harmonic transfer function for absolute acceleration responses, $\ddot{x}_{a_1}$ and $\ddot{x}_{a_2}$, respectively which are expressed by

$$\ddot{x}_{a_1}(i\omega)=\frac{\begin{array}{l}\omega_1^2\omega_2^2-\omega^2(\Delta_1\Delta_2+\Delta_1\Delta_{d2}+\Delta_2\Delta_{d1}+\omega_1^2)\\ \qquad -i\omega^3(\Delta_1)+i\omega(\omega_1^2\Delta_2+\omega_2^2\Delta_1+\omega_1^2\Delta_{d2}+\omega_2^2\Delta_{d1})\end{array}}{\begin{array}{l}\omega^4-\omega^2(\omega_1^2+\omega_2^2+\Delta_1\Delta_2+\Delta_1\Delta_{d2}+\Delta_2\Delta_{d1})+\omega_1^2\omega_2^2\\ \quad -i\omega^3(\Delta_1+\Delta_2+\Delta_d)+i\omega(\omega_1^2\Delta_2+\omega_2^2\Delta_1+\omega_1^2\Delta_{d2}+\omega_2^2\Delta_{d1})\end{array}} \tag{19}$$

$$\ddot{x}_{a_2}(i\omega)=\frac{\begin{array}{l}\omega_1^2\omega_2^2-\omega^2(\Delta_1\Delta_2+\Delta_1\Delta_{d2}+\Delta_2\Delta_{d1}+\omega_2^2)\\ \qquad -i\omega^3(\Delta_2)+i\omega(\omega_1^2\Delta_2+\omega_2^2\Delta_1+\omega_1^2\Delta_{d2}+\omega_2^2\Delta_{d1})\end{array}}{\begin{array}{l}\omega^4-\omega^2(\omega_1^2+\omega_2^2+\Delta_1\Delta_2+\Delta_1\Delta_{d2}+\Delta_2\Delta_{d1})+\omega_1^2\omega_2^2\\ \quad -i\omega^3(\Delta_1+\Delta_2+\Delta_d)+i\omega(\omega_1^2\Delta_2+\omega_2^2\Delta_1+\omega_1^2\Delta_{d2}+\omega_2^2\Delta_{d1})\end{array}} \tag{20}$$

The mean square acceleration responses are obtained by solving the integral of Eqs. (17) and (18) as per Cremer and Heckl (1973) and are given by

$$\sigma_{a_1}^2=2\pi S_0\omega_1\left[\frac{\begin{array}{l}\beta(1+\beta)^2\xi((-1+\beta)^2+4\beta\xi^2)(\xi+4\xi^3)\\ +(1+\beta)((1+\beta)\mu(1+4\xi^2)((-1+\beta)^2+8\beta\xi^2)\\ +4\beta^2\xi^2(3-\beta+2\beta^2+4(1+3\beta)\xi^2))\xi_e\\ +4\xi(3\beta^3+\beta^4+\beta^5+4\beta\mu+\beta^2\mu+3\beta^3\mu+2\beta^4\mu+\mu^2+3\beta\mu^2+\beta^2\mu^2\\ \qquad +4(\beta+\mu)(\mu+\beta(3\mu+\beta(2+3\beta+\mu)))\xi^2)\xi_e^2\\ +4(\beta^2+\mu)((1+\mu)(\beta^2+\mu)+4(\beta+\mu)^2\xi^2)\xi_e^3\end{array}}{\begin{array}{l}4(\beta(1+\beta)^2\xi^2((-1+\beta)^2+4\beta\xi^2)+(1+\beta)^2(\mu+\beta)\xi((-1+\beta)^2+8\beta\xi^2)\xi_e\\ \quad +((1+\beta^2)^2\mu+4(\beta^4+\mu^2+3\beta^3(1+\mu)+3\beta\mu(1+\mu)+\beta^2(1+\mu(4+\mu)))\xi^2)\xi_e^2\\ \quad +4(1+\mu)(\mu+\beta)(\mu+\beta^2)\xi\xi_e^3)\end{array}}\right] \tag{21}$$

$$\sigma_{a_2}^2=2\pi S_0\omega_1\left[\frac{\begin{array}{l}\beta^2(1+\beta)^2\xi(1+4\xi^2)((-1+\beta)^2+4\beta\xi^2)\\ +\beta(1+\beta)((-1+\beta)^2\beta(1+\beta)+4(2\mu+\beta(1+\beta+\beta^2+\beta^3-\mu+3\beta\mu))\xi^2\\ \qquad +16\beta(2\beta(1+\beta)+(3+\beta)\mu)\xi^4)\xi_e\\ +4\xi(\beta^3+3\beta^4+\beta^5+2\beta\mu+3\beta^2\mu+\beta^3\mu+4\beta^4\mu+\mu^2+\beta\mu^2+3\beta^2\mu^2\\ \qquad +4\beta(\mu+\beta)(3\mu+\beta(1+\beta(3+\beta)+2\mu))\xi^2)\xi_e^2\\ +4(\beta^2+\mu)((1+\mu)(\beta^2+\mu)+4(\beta+\mu)^2\xi^2)\xi_e^3\end{array}}{\begin{array}{l}4(\beta(1+\beta)^2\xi^2((-1+\beta)^2+4\beta\xi^2)+(1+\beta)^2(\mu+\beta)\xi((-1+\beta)^2+8\beta\xi^2)\xi_e\\ \quad +((1+\beta^2)^2\mu+4(\beta^4+\mu^2+3\beta^3(1+\mu)+3\beta\mu(1+\mu)+\beta^2(1+\mu(4+\mu)))\xi^2)\xi_e^2\\ \quad +4(1+\mu)(\mu+\beta)(\mu+\beta^2)\xi\xi_e^3)\end{array}}\right] \tag{22}$$

The variation of frequency response functions for displacement and absolute acceleration responses for an undamped and 5% structural damping are shown in the Figs. 2 and 3, respectively for different values of ξ_e. The two SDOF structures with their mass ratio $\mu = 1$ and frequency ratio, $\beta = 2$ are considered implying that the Structure 1 is relatively soft and Structure 2 is stiff. From the above Figs, it is interesting to note that with the increase in the ξ_e, the value of peak frequency response functions decrease and after reaching certain minimum value it increases with further increase of ξ_e. For low values of the ξ_e, the peak value of frequency response function occurs at the natural frequency of the respective structures and it shifts to the combined frequency of the structures for higher values of the ξ_e. This indicates that there is certain value of ξ_e for which the area of the frequency response function will become the minimum and yielding the minimum value of the mean square response. It is also observed that for connected undamped system all the curves passes through the fixed point (refer point *S*), which is not observed in-case of the damped connected structures.

The variations of mean square displacement and acceleration responses of the two structures against the equivalent normalized damping constant, ξ_e are shown in the Fig. 4 for different damping ratios of the connected structures (i.e. $\xi = 0$, 0.02 and 0.05). It is observed from the graph that as ξ_e increases, the mean square displacement and mean square acceleration responses decreases up to certain value and with further increase in ξ_e the mean square responses increase. There exists an optimum value of ξ_e to yield the minimum mean square responses under stationary white-noise random excitation. Thus, for a given structural system connected with a friction damper, there exists an optimum value of the frictional force for which the absolute accelerations and displacements of connected structure attain the minimum value.

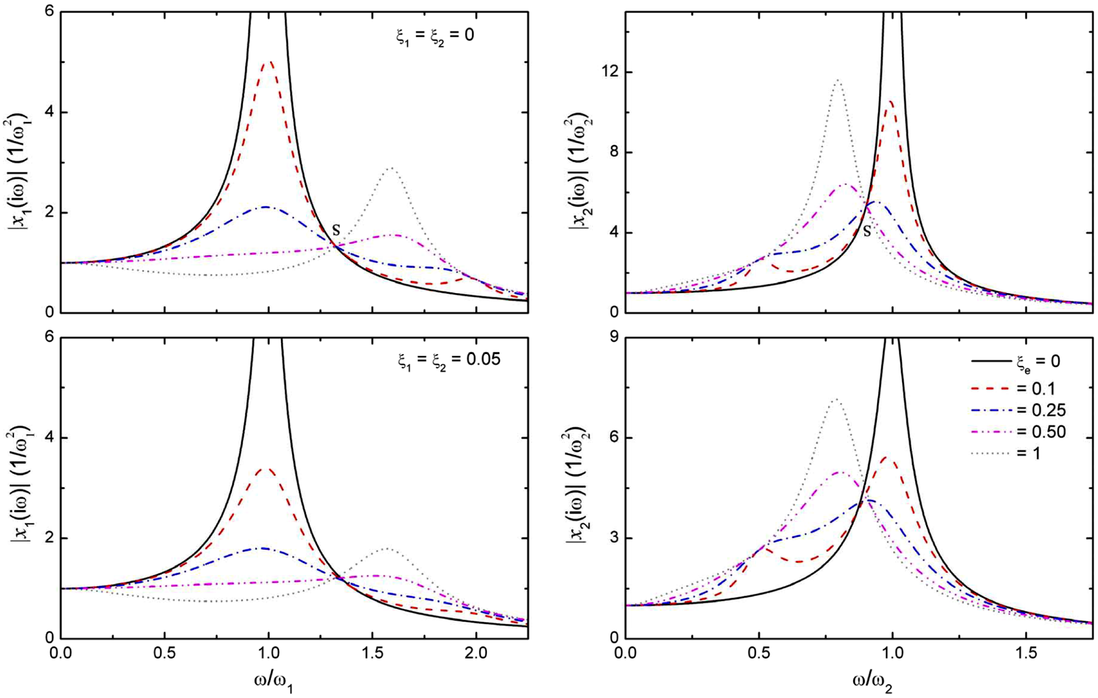

Fig. 2 Variation of frequency response functions of displacement responses against excitation frequency for different value of equivalent normalized constant ($\mu = 1$ and $\beta = 2$)

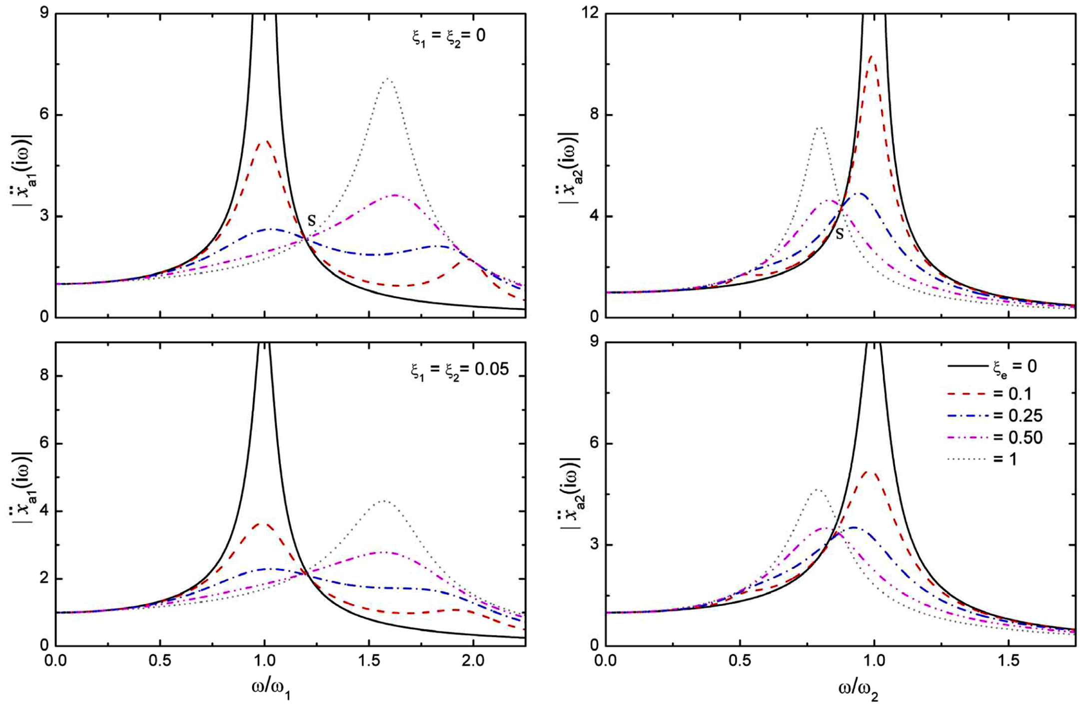

Fig. 3 Variation of frequency response functions of acceleration responses against excitation frequency for different value of equivalent normalized constant ($\mu = 1$ and $\beta = 2$)

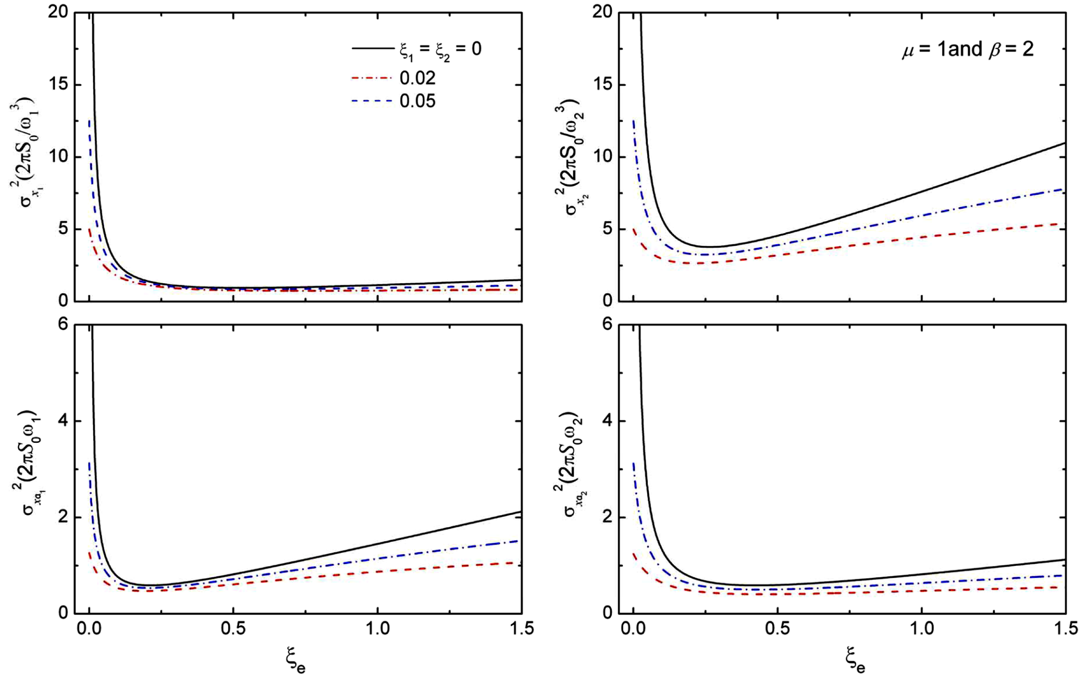

Fig. 4 Variation of mean square responses against equivalent normalized constant ξ_e of the friction damper ($\mu = 1$ and $\beta = 2$)

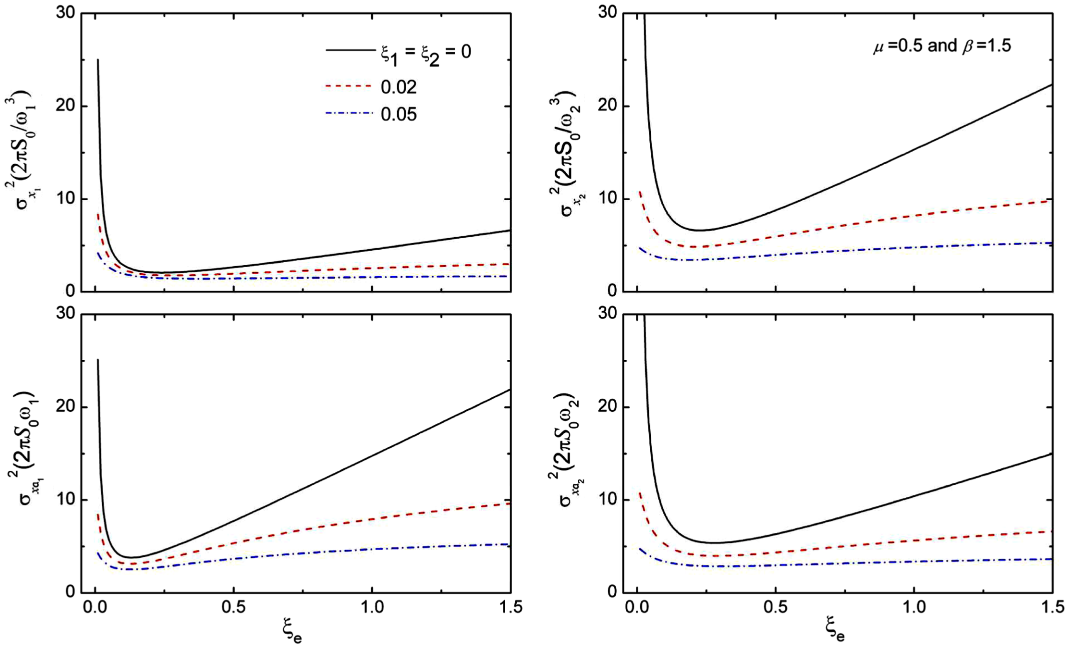

Fig. 5 Variation of mean square responses against equivalent normalized constant ξ_e of the friction damper ($\mu = 0.5$ and $\beta = 1.5$)

Further, it is interesting to note from the Fig. 4 that the optimum ξ_e for connected damped system lies almost very close to that of connected undamped system and a slight variation in the optimum ξ_e does not have much affect on the minimum value of the mean square responses. It is also observed that the difference in resulting response of damped system considering (i) actual optimum ξ_e and (ii) ξ_e that corresponding to the undamped system is quite negligible. This brings out an important conclusion that the optimum ξ_e for connected undamped system can also be used for connected damped system. Similar effects structural damping on optimum ξ_e are depicted in Fig. 5 where the corresponding response are plotted for parameters mass ratio, $\mu = 0.5$ and frequency ratio, $\beta = 1.5$.

The optimum ξ_e for different system parameters and corresponding mean square responses are given in the Tables 1 and 2. From these tables, it is concluded that the optimum frictional force of the damper increases with the increase of the frequency ratio and decreases with the increase in the mass ratio. Further, the corresponding response at the optimum frictional force decreases with the increase of the frequency ratio for all system parameters.

4. Optimum ξ_e and responses for undamped connected system

4.1 Minimization of the mean square displacement response

Let us consider the friction damper connected adjacent structures with structural damping ratio, $\xi_1 = \xi_2 = 0$. From the Eq. (5), the frequency response function of the velocity responses of the two structures are given by

Table 1 Effect of damping in structure on optimum equivalent normalized constant for minimum value of mean square displacements

μ	Damping ratio	Optimum equivalent normalized constant, ξ_e								Mean square displacement							
		β = 1.25		1.5		1.75		2.0		β = 1.25		1.5		1.75		2.0	
		S_1*	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2
0.5	0.00	0.108	0.122	0.241	0.227	0.397	0.321	0.577	0.408	4.619	10.206	2.079	6.614	1.260	5.456	0.866	4.899
	0.02	0.126	0.114	0.276	0.210	0.458	0.297	0.672	0.378	3.529 (3.668)	6.480 (6.487)	1.768 (1.846)	4.869 (4.877)	1.108 (1.162)	4.228 (4.236)	0.773 (0.812)	3.894 (3.901)
	0.05	0.187	0.100	0.373	0.184	0.613	0.261	0.910	0.332	2.536 (2.807)	4.060 (4.073)	1.426 (1.601)	3.434 (3.458)	0.927 (1.055)	3.126 (3.153)	0.655 (0.755)	2.950 (2.978)
1.0	0.00	0.099	0.080	0.221	0.147	0.365	0.208	0.530	0.265	5.028	7.857	2.263	5.092	1.371	4.200	0.943	3.772
	0.02	0.113	0.073	0.247	0.135	0.407	0.191	0.595	0.244	3.842 (3.856)#	5.684 (5.692)	1.949 (1.958)	4.136 (4.149)	1.228 (1.235)	3.545 (3.556)	0.859 (0.863)	3.245 (3.257)
	0.05	0.158	0.063	0.313	0.117	0.508	0.166	0.742	0.211	2.726 (2.789)	3.857 (3.879)	1.581 (1.619)	3.158 (3.196)	1.044 (1.074)	2.830 (2.868)	0.746 (0.769)	2.649 (2.690)
1.5	0.00	0.087	0.057	0.194	0.105	0.320	0.149	0.465	0.190	5.741	7.330	2.582	4.748	1.565	3.913	1.076	3.154
	0.02	0.098	0.052	0.214	0.096	0.352	0.136	0.513	0.173	4.225 (4.232)	5.450 (5.478)	2.191 (2.200)	3.942 (3.964)	1.392 (1.397)	3.373 (3.384)	0.978 (0.983)	3.084 (3.093)
	0.05	0.133	0.045	0.264	0.082	0.427	0.117	0.619	0.173	2.914 (2.961)	3.789 (3.824)	1.749 (1.787)	3.073 (3.128)	1.177 (1.200)	2.747 (2.791)	0.850 (0.871)	2.567 (2.610)
2.0	0.00	0.077	0.043	0.170	0.080	0.281	0.113	0.408	0.144	6.538	7.240	2.939	4.677	1.781	3.86	1.225	3.466
	0.02	0.085	0.039	0.186	0.073	0.307	0.103	0.447	0.131	4.606 (4.607)	5.370 (5.370)	2.442 (2.448)	3.891 (3.901)	1.566 (1.571)	3.33 (3.338)	1.064 (1.110)	3.049 (3.054)
	0.05	0.114	0.034	0.227	0.062	0.366	0.088	0.529	0.112	3.081 (3.119)	3.760 (3.765)	1.911 (1.941)	3.044 (3.086)	1.308 (1.330)	2.722 (2.758)	0.955 (0.972)	2.547 (2.583)

#The value within the parenthesis indicates the response when ξ_e corresponding to zero damping ratio is considered.
*S_1 = Structure 1, S_2 = Structure 2.

Table 2 Effect of damping in structures on optimum equivalent normalized constant for minimum value of mean square accelerations

μ	Damping ratio	Optimum equivalent normalized constant, ξ_e								Mean square acceleration							
		β = 1.25		1.5		1.75		2.0		β = 1.25		1.5		1.75		2.0	
		S_1*	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2
0.5	0.00	0.078	0.139	0.131	0.278	0.167	0.414	0.192	0.544	6.351	8.982	3.811	5.389	2.992	4.231	2.598	3.674
	0.02	0.078	0.143	0.128	0.285	0.162	0.422	0.187	0.554	4.728 (4.727)	5.394 (5.395)	3.142 (3.142)	3.846 (3.847)	2.543 (2.544)	3.206 (3.206)	2.235 (2.236)	2.868 (2.868)
	0.05	0.078	0.141	0.125	0.294	0.158	0.436	0.182	0.572	3.411 (3.411)	3.150 (3.150)	2.536 (2.538)	2.607 (2.608)	2.135 (2.198)	2.305 (2.306)	1.912 (1.914)	2.126 (2.127)
1.0	0.00	0.078	0.097	0.136	0.204	0.179	0.314	0.212	0.424	6.445	6.445	3.679	3.678	2.786	2.786	2.357	2.357
	0.02	0.076	0.100	0.132	0.209	0.173	0.321	0.205	0.433	4.893 (4.893)#	4.733 (4.733)	3.150 (3.154)	3.039 (3.041)	2.484 (2.485)	2.395 (2.396)	2.142 (2.142)	2.066 (2.067)
	0.05	0.075	0.102	0.126	0.218	0.165	0.335	0.194	0.450	3.499 (3.499)	3.298 (3.307)	2.578 (2.584)	2.394 (2.102)	2.140 (2.144)	1.977 (1.981)	1.892 (1.896)	1.745 (1.748)
1.5	0.00	0.071	0.073	0.129	0.158	0.175	0.250	0.211	0.345	7.030	5.743	3.873	3.163	2.857	2.332	2.367	1.933
	0.02	0.070	0.075	0.125	0.163	0.169	0.257	0.204	0.353	5.168 (5.168)	4.384 (4.390)	3.290 (3.290)	2.701 (2.701)	2.548 (2.552)	2.068 (2.068)	0.164 (2.165)	1.745 (1.745)
	0.05	0.070	0.078	0.120	0.171	0.161	0.269	0.193	0.368	3.600 (3.600)	3.149 (3.170)	2.659 (2.662)	2.199 (2.204)	2.189 (2.196)	1.764 (1.767)	1.919 (1.922)	1.524 (1.525)
2.0	0.00	0.064	0.057	0.120	0.127	0.166	0.206	0.204	0.289	7.777	5.492	4.164	2.945	3.007	2.126	2.450	1.732
	0.02	0.063	0.059	0.117	0.131	0.161	0.212	0.197	0.296	5.474 (5.474)	4.228 (4.228)	3.47 (3.470)	2.546 (2.546)	2.657 (2.660)	1.910 (1.910)	2.230 (2.230)	1.585 (1.586)
	0.05	0.063	0.062	0.112	0.139	0.154	0.222	0.187	0.310	3.701 (3.701)	3.075 (3.085)	2.752 (2.754)	2.101 (2.105)	2.257 (2.262)	1.654 (1.656)	1.965 (1.967)	1.407 (1.409)

#The value within the parenthesis indicates the response when ξ_e corresponding to zero damping ratio is considered.
*S_1 = Structure 1, S_2 = Structure 2.

$$\dot{x}_1(i\omega) = \frac{[\omega^2 - \omega_2^2 - i\omega(\Delta_d)]i\omega}{\omega^4 + (i\omega)^3(\Delta_d) + (i\omega)^2(\omega_1^2 + \omega_2^2) + i\omega(\omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1}) + \omega_1^2\omega_2^2} \tag{23}$$

$$\dot{x}_2(i\omega) = \frac{[\omega^2 - \omega_1^2 - i\omega(\Delta_d)]i\omega}{\omega^4 + (i\omega)^3(\Delta_d) + (i\omega)^2(\omega_1^2 + \omega_2^2) + i\omega(\omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1}) + \omega_1^2\omega_2^2} \tag{24}$$

The frequency response function of the relative velocity at damper ends is given by

$$\dot{x}_r(i\omega) = \dot{x}_1(i\omega) - \dot{x}_2(i\omega) \tag{25}$$

Substituting the Eqs. (23) and (24) into Eq. (25), the frequency response function for the relative velocity at the two ends of damper is given by

$$\dot{x}_r(i\omega) = \frac{[\omega_1^2 - \omega_2^2]i\omega}{\omega^4 + (i\omega)^3(\Delta_d) + (i\omega)^2(\omega_1^2 + \omega_2^2) + i\omega(\omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1}) + \omega_1^2\omega_2^2} \tag{26}$$

The mean square relative velocity at damper ends, $\sigma_{\dot{x}_r}^2$ for an undamped coupled system subjected to stationary white-noise excitation will be

$$\sigma_{\dot{x}_r}^2 = \int_{-\infty}^{\infty} |\dot{x}_r(i\omega)|^2 S_0 \; d\omega \tag{27}$$

Substituting for the $\dot{x}_r(i\omega)$ from the Eqs. (26) into (27) and solving the integral will lead to

$$\sigma_{\dot{x}_r}^2 = \frac{\pi S_0(1+\mu)}{2\omega_1 \mu \xi_e} \tag{28}$$

From the Eqs. (6), (7) and (28), the normalized equivalent damping constant and slip force of friction damper can be expressed as

$$\xi_e = \frac{f_s^2}{m_1^2 \pi^2 \omega_1 S_0}\left(\frac{\mu}{1+\mu}\right) \tag{29}$$

$$f_s = \sqrt{\xi_e m_1^2 \pi^2 \omega_1 S_0 \left(\frac{1+\mu}{\mu}\right)} \tag{30}$$

For an undamped system ($\xi = 0$), the mean square displacement responses can be obtained from Eqs. (15) and (16) and are given by

$$\sigma_{x_1}^2 = \frac{2\pi S_0}{\omega_1^3}\left[\frac{1}{4\xi_e} + \frac{(1+\mu)^3 \xi_e}{(-1+\beta^2)^2 \mu}\right] \tag{31}$$

$$\sigma_{x_2}^2 = \frac{2\pi S_0}{\omega_2^3}\left[\frac{\beta}{4\mu\xi_e} + \frac{\beta^3(1+\mu)^3 \xi_e}{(-1+\beta^2)^2 \mu}\right] \tag{32}$$

The optimizing condition $\frac{d\sigma_{x_1}^2}{d\xi_e} = 0$ will result in the optimum ξ_e for the Structure 1 and after simplification the expression for the optimum equivalent damping is given by

$$\xi_e^{opt}\Big|_{x_1} = \frac{(-1+\beta^2)}{2(1+\mu)}\sqrt{\frac{\mu}{(1+\mu)}} \tag{33}$$

and the corresponding optimum slip force in friction damper can be given as

$$f_s^{opt}\Big|_{x_1} = m_1\pi\sqrt{\frac{S_0\omega_1(\beta^2-1)}{2\sqrt{\mu(1+\mu)}}} \tag{34}$$

Similarly $\frac{d\sigma_{x_2}^2}{d\xi_e} = 0$ will result in the optimum ξ_e for the Structure 2 as expressed by

$$\xi_e^{opt}\Big|_{x_2} = \frac{(-1+\beta^2)}{2\beta(1+\mu)}\frac{1}{\sqrt{(1+\mu)}} \tag{35}$$

and the corresponding optimum slip force in friction damper can be given as

$$f_s^{opt}\Big|_{x_2} = m_1\pi\sqrt{\frac{S_0\omega_2(-1+\beta^2)}{2\mu\beta^2\sqrt{(1+\mu)}}} \tag{36}$$

The close-form expression for the corresponding mean square displacement responses of two structures can be calculated by substituting the optimum ξ_e in the expressions of the mean square displacement responses which are given by

$$\sigma_{x_1}^2\Big|_{opt} = \frac{2\pi S_0}{\omega_1^3}\left[\frac{\sqrt{(1+\mu)^3}}{(-1+\beta^2)\sqrt{\mu}}\right] \tag{37}$$

$$\sigma_{x_2}^2\Big|_{opt} = \frac{2\pi S_0}{\omega_2^3}\left[\frac{\beta^2\sqrt{(1+\mu)^3}}{(-1+\beta^2)\sqrt{\mu}}\right] \tag{38}$$

The effect of frequency ratio of structures on f_s^{opt} and corresponding mean square displacement responses are shown in the Fig. 6 for different mass ratios of structures. It is observed from the figures that the increase in frequency ratio increases the optimum frictional force, f_s^{opt}. This is due to the reason that higher frequency ratio increase the relative displacement between the connected masses and thus, it leads friction damper to be more effective for energy dissipation, as a result, reduction of more displacement response. Further, the increase in mass ratio reduces the f_s^{opt} of friction damper. It is also observed from figure that the increase in frequency ratio decreases the mean square displacement of the two structures. Thus, it is concluded that the friction damper performs more effectively for displacement response control and the higher reduction in the mean square displacement responses can be achieved if the frequency of the connected structures are well separated. However, the increase in mass ratio increases the mean square displacement response of Structure 1, whereas it decreases the mean square displacement response of Structure 2.

It is to be noted that in order to use the Eqs. (33) to (38), the frequency ratio, β shall always be more than 1. This condition can always be achieved by keeping the Structure 1 as flexible among the two structures. In case the right side structure (i.e. Structure 2) of the connected system is flexible then the two connected system shall be rotated by 180 degree so that the left structure become flexible and right as stiff and calculate the system parameters accordingly.

4.2 Minimization of the mean square acceleration response

Considering an undamped system ($\xi = 0$), from the Eqs. (21) and (22), the mean square acceleration responses can be simplified to

$$\sigma_{a_1}^2 = 2\pi S_0 \omega_1 \left[\frac{1}{4\xi_e} + \frac{(1+\mu)(\beta^2+\mu)^2 \xi_e}{(-1+\beta^2)^2 \mu} \right] \tag{39}$$

$$\sigma_{a_2}^2 = 2\pi S_0 \omega_2 \left[\frac{\beta}{4\mu\xi_e} + \frac{(1+\mu)(\beta^2+\mu)^2 \xi_e^2}{(-1+\beta^2)^2 \mu\xi_e} \right] \tag{40}$$

The optimizing condition $\dfrac{d\sigma_{a_1}^2}{d\xi_e} = 0$ will result in the optimum ξ_e for acceleration response of Structure 1 and after simplification it will reduce to

$$\xi_e^{opt}\Big|_{\ddot{x}_{a_1}} = \frac{(-1+\beta^2)}{2(\beta^2+\mu)} \sqrt{\frac{\mu}{(1+\mu)}} \tag{41}$$

and the corresponding optimum slip force in friction damper is given as

$$f_s^{opt}\Big|_{\ddot{x}_{a_1}} = m_1 \pi \sqrt{\frac{S_0 \omega_1 (-1+\beta^2)}{2(\beta^2+\mu)} \sqrt{\frac{1+\mu}{\mu}}} \tag{42}$$

Similarly $\dfrac{d\sigma_{a_2}^2}{d\xi_e} = 0$ will result in the optimum ξ_e for the acceleration response of Structure 2 expressed by

$$\xi_e^{opt}\Big|_{\ddot{x}_{a_2}} = \frac{\beta(-1+\beta^2)}{2(\beta^2+\mu)} \frac{1}{\sqrt{(1+\mu)}} \tag{43}$$

and the corresponding optimum slip force in friction damper is given as

$$f_s^{opt}\Big|_{\ddot{x}_{a_2}} = m_1 \pi \sqrt{\frac{S_0 \omega_2 (-1+\beta^2)\sqrt{(1+\mu)}}{2\mu(\beta^2+\mu)}} \tag{44}$$

The mean square acceleration response at optimum ξ_e value are given by the following expressions

$$\sigma^2_{a_1}\Big|_{opt} = 2\pi S_0\omega_1\left[\frac{(\beta^2+\mu)\sqrt{(1+\mu)}}{(-1+\beta^2)\sqrt{\mu}}\right] \tag{45}$$

$$\sigma^2_{a_2}\Big|_{opt} = 2\pi S_0\omega_2\left[\frac{(\beta^2+\mu)\sqrt{(1+\mu)}}{(-1+\beta^2)\sqrt{\mu}}\right] \tag{46}$$

The effect of frequency ratio of the connected structures on f_s^{opt} and corresponding mean square acceleration response are shown in Fig. 7 for different mass ratios of the structures. It is seen from these figures that the f_s^{opt} increases with increase of the frequency ratio as observed in the Fig. 6. Further, the increase in mass ratio reduces the f_s^{opt} of friction damper for acceleration responses. It is also observed from figure that the increase in frequency ratio decreases the mean square acceleration responses corresponding to respective f_s^{opt} of the two structures. Thus, it is concluded that the friction damper performs more effectively for acceleration response control and the higher reduction in the mean square acceleration responses can be achieved if the frequencies are well separated. However, the increase in mass ratio increases the mean square acceleration response of Structure 1, whereas it decreases the mean square acceleration response of Structure 2.

In order to establish the better performance of the friction damper for higher frequency ratio, the phase angle of the displacement response function of the Structures 1 and 2 is plotted in Fig. 8 for $\beta = 1$, 1.25, 1.5 and 2. As expected, the difference in the phase angle of the displacements of two

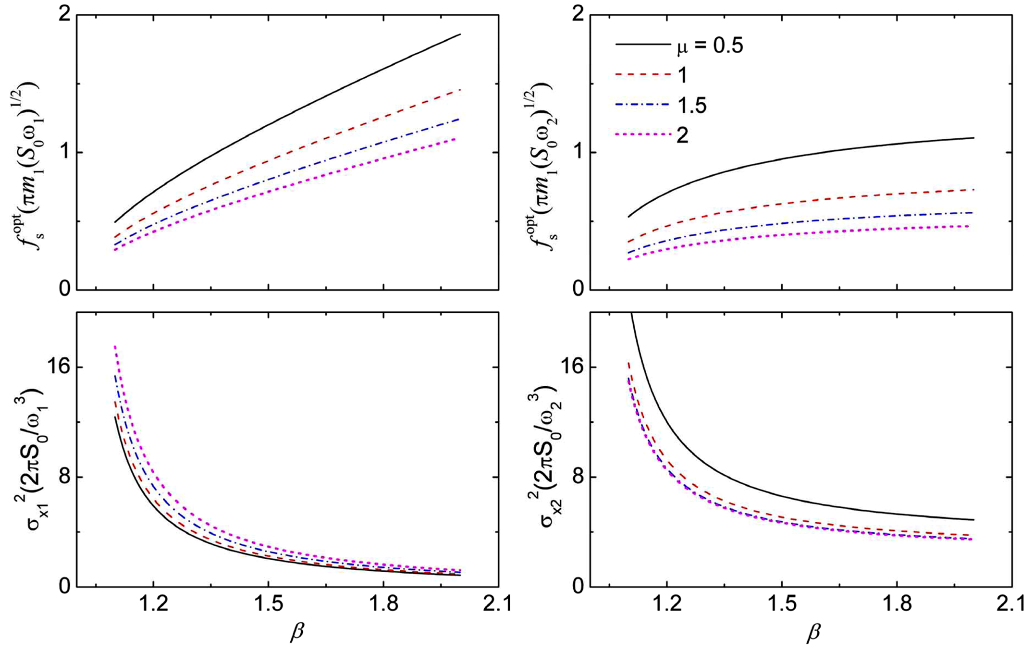

Fig. 6 Effects of frequency ratio on the optimum frictional force and corresponding mean square displacement responses

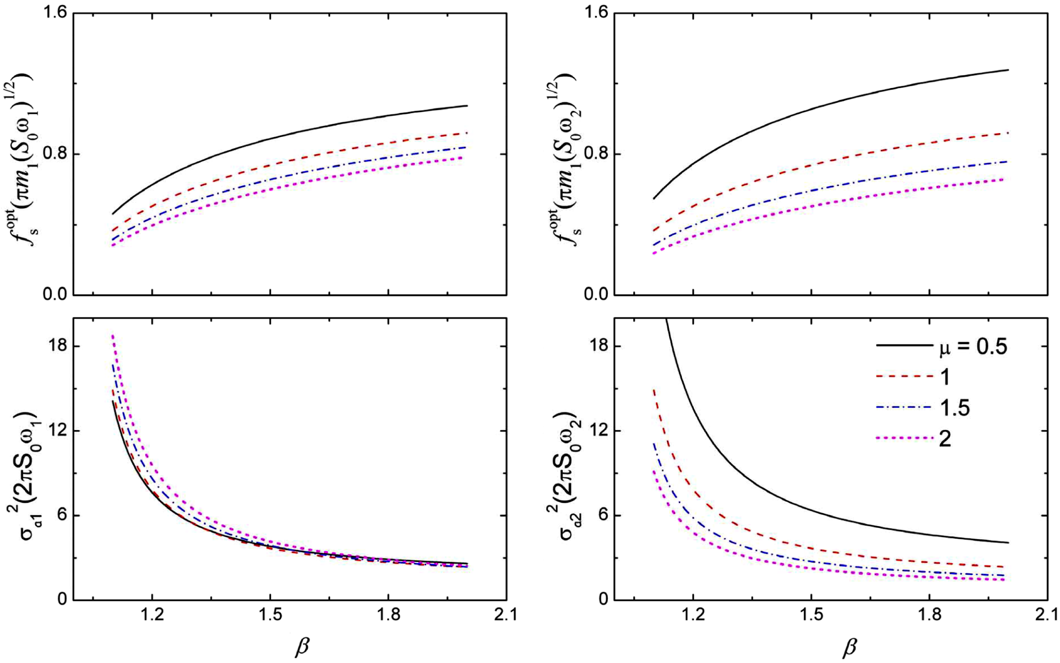

Fig. 7 Effects of frequency ratio on the optimum frictional force and corresponding mean square acceleration responses

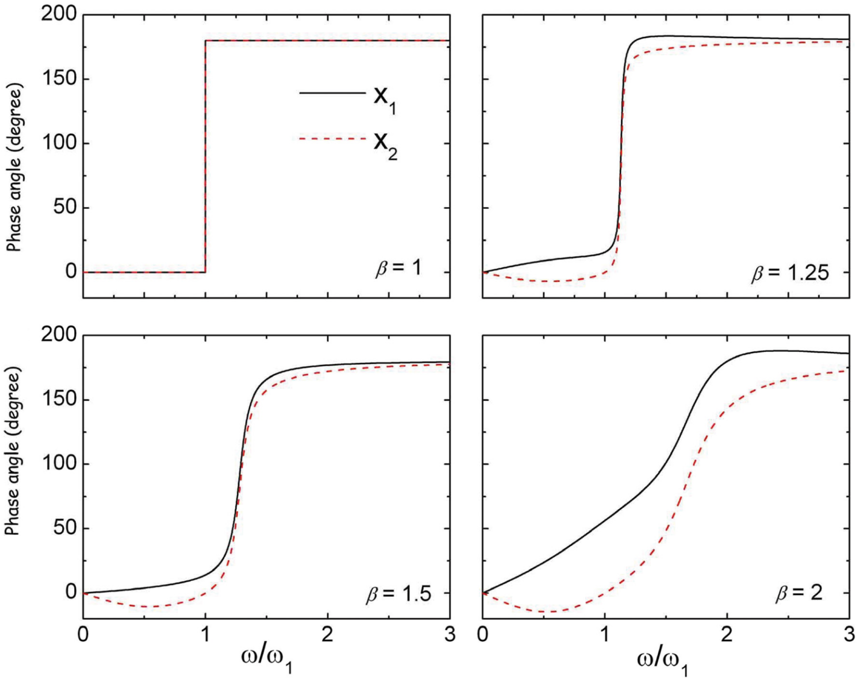

Fig. 8 Effects of frequency ratio on the phase angle of the displacement response of the connected structures ($\xi = 0$, $\xi_e = 0.5$ and $\mu = 1$)

structures increases with the increase of the frequency ratio. This will cause larger displacement in the friction damper and thus more dissipation of energy and subsequent response reduction.

5. Response to determinsitic earthquake excitation

In this section, the influence of frictional force of damper on response of two adjacent SDOF structures (refer Fig. 1) under deterministic earthquake excitation is investigated to correlate with the trends of stochastic response analysis studied earlier. Two real recorded earthquake ground motions namely the east-west component of Northridge, 1994 recorded at Sylmar station and north-south component of El-Centro, 1940 are considered. The time history of these selected earthquake motions are shown in the Fig. 9. The peak ground acceleration is 0.6 g and 0.32 g for Northridge and El-Centro earthquake motions, respectively. These earthquake ground motions are applied to a model of connected structures with $\xi = 0$, $\mu = 1$ and $\beta = 2$ (i.e. $\omega_1 = 2\pi$ rad/sec and $\omega_2 = 4\pi$ rad/sec). The response of the system is obtained by solving the coupled Eqs. (3) and (4) in the time domain using Newmarks's step-by-step method. The damper frictional force was considered to be non-linear and the damper remains in the non-sliding phase till the frictional force is less than the limiting frictional force. The damper enters into the sliding phase as soon as the force becomes equal to the limiting value. The transition from sliding to non-sliding phase takes place whenever the relative velocity, $\dot{x}_r$ changes the sign. The variation of the peak absolute acceleration and displacement of the two structures are plotted against the limiting frictional force of damper in Figs. 10 and 11 for Northridge and El-Centro motions, respectively. The responses are shown for three scaling factor (SF) i.e. 1, 1.5 and 2 of the earthquake motions to study the influence of the intensity of earthquake

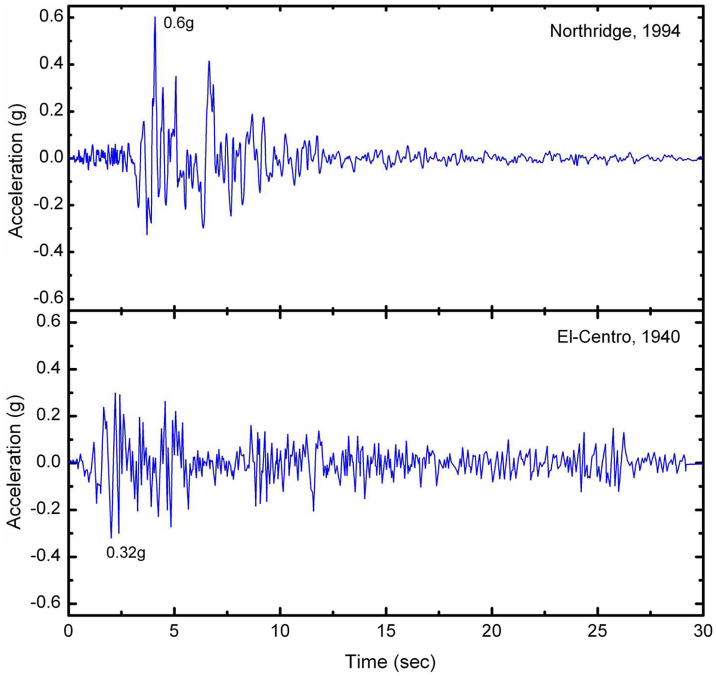

Fig. 9 Acceleration time history of Northridge 1994 and El-Centro 1940 earthquake motions

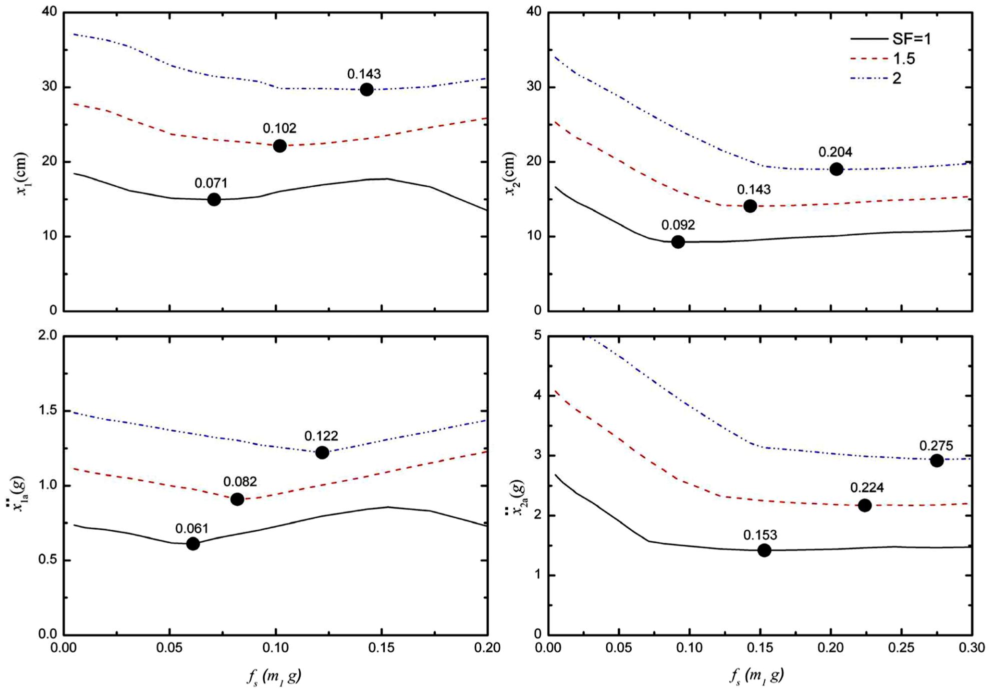

Fig. 10 Influence of the frictional force of the damper on the peak response of connected structures under Northridge, 1994 earthquake motion ($\xi = 0$, $\mu = 1$ and $\beta = 2$)

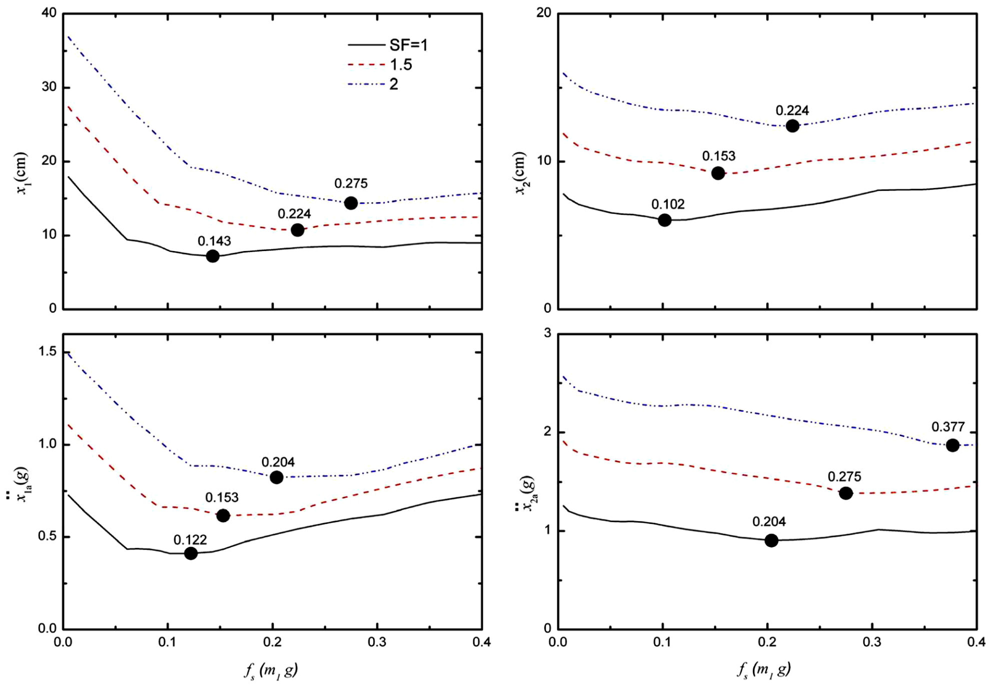

Fig. 11 Influence of the frictional force of the damper on the peak response of connected structures under El-Centro, 1940 earthquake motion ($\xi = 0$, $\mu = 1$ and $\beta = 2$)

excitation. The Figs. 10 and 11 indicate that there exists an optimum value of the damper frictional force for which the absolute accelerations and displacements of the two structures attain the minimum value. These trends of the results are similar to that obtained for the stochastic response. It is also observed from the Figs. 10 and 11 that the optimum damper frictional force linearly increases with the increase of the earthquake intensity (compare the optimum results for different values of SF marked with solid circle) confirming the relations derived in the Eqs. (34), (36), (42) and (44). In addition, the peak absolute accelerations and displacements of the two structures at optimum value of the damper frictional force also increases linearly with the increase of the earthquake intensity and validating the derived Eqs. (37), (38), (45) and (46). Thus, there is a good correlation of the influence of damper frictional force on the response of connected structures under deterministic time history analysis with the corresponding results obtained using statistical linearization approach under stochastic excitation.

6. Conclusions

The dynamic response of two adjacent SDOF structures connected with friction damper subjected to earthquake excitation is investigated. The method of equivalent linearization is used to obtain the dynamic response of the coupled system under stochastic excitation. Close-form expressions for optimum slip force of friction damper and corresponding mean square responses are derived. The effect of system parameters such as frequency ratio and mass ratio on the optimum slip force and corresponding mean square responses is investigated. From the trends of the results of present study, the following conclusions are drawn:

1. For a given structural system connected with friction damper, there exists an optimum friction frictional force for which the absolute accelerations and displacements of connected structures attains the minimum value.
2. The optimum frictional force of the damper increases with the increase of the frequency ratio and decreases with the increase in the mass ratio. The corresponding responses at optimum frictional force decreases with the increase of the frequency ratio.
3. The optimum equivalent normalized constant for connected damped system lies almost very close to that of connected undamped system. The optimum equivalent normalized constant for connected undamped system can be used for connected damped system.
4. The derived close from expressions for the optimum frictional force of the damper and corresponding responses can be effectively used for preliminary optimum design of structural system connected with the friction damper.
5. There is a good correlation of the influence of frictional force of the damper on the response of damper connected structural system building under deterministic time history analysis with the corresponding results obtained using statistical linearization approach under stochastic excitation.

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