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Vibration-mode-based story damage and global damage of reinforced concrete frames

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Abstract. An attempt is conducted to explore the relationship between the macroscopic global damage and the local damage of shear-type RC frames. A story damage index, which can be expressed as multi-variate functions of modal parameters, is deduced based on the tridiagonal matrix of the shear-type frame. The global damage model is also originated from structural modal parameters. Due to the connection of modal damage indexes, the relationship between the macroscopic global damage and the local story damage is reasonably established. In order to validate the derivation, a case study is carried out via an 8-story shear-type frame. The sensitivities of modal damage indexes to the location and severity of local story damages are studied. The evolution of the global damage is investigated as well. Results show that the global damage is sensitive to the degree of story damage, but it's not sensitive to its location. As the number of the damaged stories increases, more and more modes will be involved. Meanwhile, the global damage evolution curve changes from the concave shape to the S-type and then finally transforms into the convex shape. Through the proposed story damage, modal damage and global damage model, a multi-level damage assessment method is established.

Keywords: vibration mode; story damage; modal damage; global damage; frame

1. Introduction

A crucial step in the structural assessment is choosing an appropriate damage model to assess or describe the damage degrees of structures or components. Damages of reinforced concrete (RC) frames caused by earthquakes may be due to excessive deformations, or may be in the form of accumulated damage sustained under repeated load reversals, which usually appear as strength and stiffness degradation. Damage is a progressive process, which begins with the component deterioration and failure, gradually transforms to the story damage, and finally leads to the failure of the whole structure. Most of the present damage models are trapped in their own levels, e.g., component level, story level or structural level, etc. The evolution process of the migration and the mutual influence between different damage levels are often ignored. However, the local damage and the global damage are always closely related. Establishing the relationship between the local and the global damage can clearly reveal the migration and evolution law of the damage.

Theoretically, study of the damage relationship between the local and the global damage of frame structures can be conducted in two different ways. One is a weighted combination of the local damage, section damage or component damage, etc., to obtain the global damage. It's mostly similar to or an extension of the Park-Ang damage model (Park and Ang 1985). In the study of Heo and Kunnath (2013), the material level damage parameters, i.e., the concrete fiber damage index and the steel fiber damage index, were combined at the element, story and structural level by using weighting factors. Scotta *et al.* (2009) also used the combination of the concrete damage and steel damage to obtain the global damage, while the concrete damage index and the steel damage index are different to Heo and Kunnath's (2013). The displacement part of the Park-Ang damage index was modified by Kunnath *et al.* (1991) to correct the damage evaluation of the elastic phase. In the IDARC program (Reinhorn *et al.* 2009), the Park-Ang damage model was further modified by using the moment and rotation relationship of the cross section instead of the force and deformation relationship of the component. Benefit from the moment-rotation relationship Guo *et al.* (2016) extended the Park-Ang damage model to three dimensional cases. Based on a large number of beam and column experiments, the Park-Ang damage model is considered to be reasonable at the component level. But for further weighting to obtain the global damage, big deviations may occur because of the lacking of experimental data at the structural level and the amplification of errors for weighting several times. By using the weighted method, the local damage and global damage are associated with each other clearly. But this kind of global model is trapped with the weighted forms and the selection of the weighting coefficients. The determination of the weighting coefficients sometimes may be not physically based. There is no obvious way of determining

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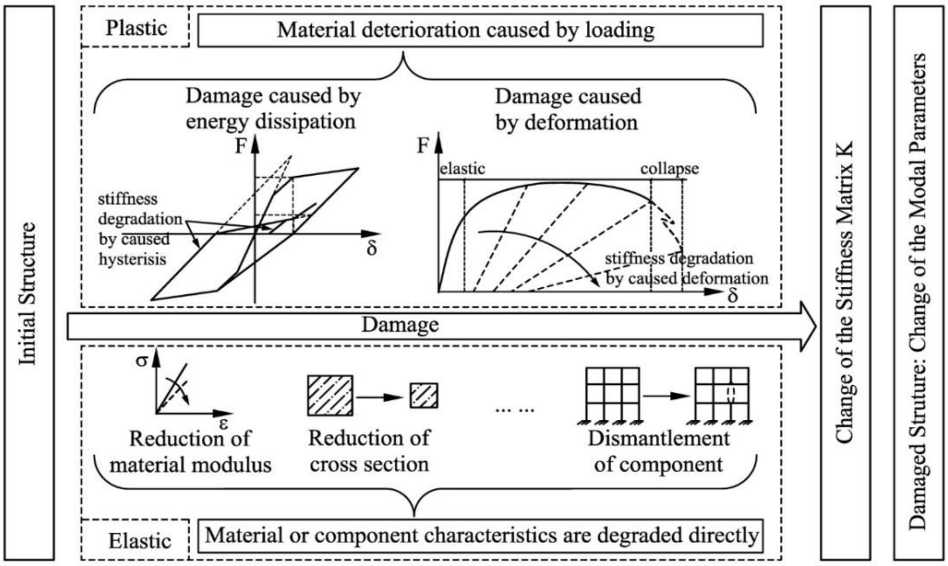


Fig. 1 Damage induced methods

the weighting that should be given to different structural elements or different levels of damage. And the global index can only be as reliable as the local values from which it is derived. In addition, the calculation of the weighted method is always tedious.

The other way is a reverse deduction from the structural global damage to the local damage, which usually starts with the macroscopic global damage model. The macroscopic global damage model can reflect the damage state of the whole structure directly and can be applied conveniently with a concise calculation. But the macroscopic global damage models originated from structural global characteristics, such as drift (Ghobarah 2001, Wen and Kang 2001), modal parameters (Stubbs *et al.* 2010, Massumi and Moshtagh 2013, Loh *et al.* 2016), etc., usually are not sensitive to the local damages. Deducing from global damage to local damage is a one-to-many problem, which makes it almost impossible to be achieved. So another method may be considered to connect the macroscopic global damage with the story damage or the component damage. Based on the continuum damage mechanics, DiPasquale and Cakmak (1989) built a relationship between the global damage and the local damage. The parameter-based global damage indices are related to the local stiffness degradation through operations of averaging over the body volume. In the case of damage in RC structures, stiffness degradation can be related to micro-cracking of concrete, and yielding of reinforcement bars.

Based on DiPasquale and Cakmak's research (1989), the relationship between the story damage and the global damage is established in this paper. Starting with the tridiagonal stiffness matrix of the shear frame, and by introducing the story damage factor and the modal damage model, a multi-level damage assessment method is established. By this means, the deficiency of the weighted method as well as the insensitivity of the macroscopic global damage model to local damages are avoided. The macroscopic global damage model is obtained by comparing the changes of the structure modal parameters before and after the damage. Based on the modal parameters, the location of the damage and its damage degree are also obtained, that is the story damage can also be expressed in the form of modal parameters. The global damage and the local damage are therefore linked together. Through a case study, the sensitivity of different modal damages on the location and severity of story damages, as well as the evolution of the global damage, are investigated. The proposed method is supposed to be used to predict the damage of designed structures and to assess the damage of the real structures.

2. The mechanism of structural damage and the development of modal damage

Damage and failure of a structure are usually the results of the material deteriorations under external factors. Continuum damage mechanics and micro damage mechanics are the effective approaches to describe the process from physical evolution of the microdefects to the macroscopic performance degradation and finally to the failure of the component. However, for large civil engineering structures, the concepts and microscopic parameters of the continuum damage mechanics and the micro damage mechanics (e.g., tensor damage variable, micro-crack density or porosity, etc.) are difficult to be used to study the damage state of the whole structure. The evolution of those parameters should be reflected on some macroscopic physical quantities, which can be easily observed and measured, such as structural drift (Ghobarah 2001), vibration period (Massumi and Moshtagh 2013), observed displacement (Song *et al.* 2016), etc.

In the study of simulating the structural damage, two different ways, as shown in Fig. 1, are usually adopted. One is to induce damage in the structure by loading (wind, earthquake, etc.). It's a plastic method. The damage induced by loading can be subdivided into energy-dissipationcaused damage and deformation-caused damage. The structure under this circumstance has gone into the nonlinear stage, which is originated from the deterioration of material properties. The other is an elastic method, such as reducing the modulus of the material or the area of the cross section, weakening the connections, dismantling some components, etc. By this means, the parameters of the structure are changed directly, and the damaged structure is actually still elastic. The elastic method can characterize the structural damage on a macroscopic level. And to some extent it can reflect the structure damage state well with an unreal physical mechanism. Actually it obtains the equivalent elastic structure of the damaged one. Thus, no matter which method is used, the results are the same in most cases. Damage of the structure will lead to the change of the structural stiffness matrix, which in the case of measurable physical quantities, will be reflected in the change of the modal parameters. The change of the structural modal parameters can be used to assess the damage states of the structure.

Modal damage model uses the modal parameters before and after the damage to assess the structural damage state. The modal parameters of the initial structure and the equivalent elastic structure are easy to be acquired. But for the damaged inelastic structure, doubts will appear. Strictly, modal decomposition cannot be used when the structure is under the inelastic state. The modes are coupled with each other. However, when the damaged structure stays stable on the equilibrium position, the structure will reveal the properties of elasticity under micro-vibrations. The elastic properties will be different from the initial elastic structure. The damaged structure usually will encounter obvious residual deformation after the external load is removed. The unloading stiffness and the reloading stiffness will be less than the initial one, and they are related to the maximum deformation and the hysteresis dissipated energy in the process of loading. Meanwhile, the micro-vibration characteristics of the damaged structure are also decided by the degraded parameters. The changes of the structural modal parameters are closely related to the stiffness degradation. So, when the structure enters the nonlinear stage, the modal damage model will still be reasonable.

Up to now, the statement of 'modal damage' has been mentioned in many literatures. But most of them are used to detect or locate the damage. Only a few of them are used to assess the degree of the structural damage. The so-called modal damage indicator was first introduced by Koyluoglu *et al.* (1998) to assess the severity of the structural damage. In fact, the modal damage indicator is actually the softening index proposed by DiPasquale and Cakmak (1990). The softening index considers only the fundamental modal parameters. Later, Chinese scholars (Zhu *et al.* 2004, Wang 2008, He *et al.* 2014, 2017) also used the terminology, modal damage, in their papers. Zhu *et al.* (2004) extended the modal damage model to multi-modes. The global damage was obtained by combining the considered multi-modal damages. But the calculation process is limited by the modal pushover analysis method. An improvement was made by He *et al.* (2014, 2017) based on the Zhu *et al.* (2004) model and the softening index (DiPasquale and Cakmak 1989), in which the multi-modal damages were

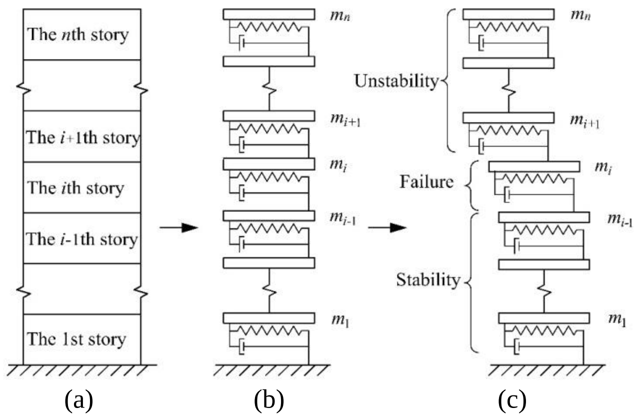


Fig. 2 Simplified model of shear-type frame

obtained through the modal decomposition rather than the pushover analysis. In the view of modal damage, the global damage of a multi-degree-of-freedom system can be represented as an aggregation of the damages in the vibrational modes under consideration.

Global damage models, which are based on the modal parameters, can assess the structural damage state from a macroscopic point of view. It's relatively direct, and the structural modal information (modal shape, frequency, etc.) contains the local properties, which makes this kind of global damage model a broad prospect. Damages generally occur locally, but their effects exhibit in the changes of both local and global characteristics of the structure. The migration of the control mode relates to the localized development of the damages. By studying the relationship and the evolution of the story damage, modal damage and global damage, the inherent law of the structural damage performance, damage migration and evolution process will be further revealed.

3. Vibration-mode-based story damage model and global damage model

Damages of the structure change its vibration parameters. In the modal space, it's expressed as the stiffness degradation, flexibility increase, and the change of the vibration mode. By using these vibration parameters, the local damage of the structure can be located and assessed, while the global damage is obtained simultaneously. Before the derivation, some assumptions are made as follows: (1) the mass of the structure focuses on each floor, and the change of the mass before and after damage is neglected; (2) the beam and the slab of the frame are considered as rigid bodies; (3) the structural system is assumed to be a serial system, and any story failure will lead to the collapse of the whole structure. According to the above assumptions, the simplified model of the frame is as shown in Fig. 2. The third assumption is as shown in Fig. 2(c). The failure of a story means that a permanent unrecoverable failure mechanism is formed on the local scale. The whole structure is then divided into two parts by the failure story. The part, which still keeps in touch with the ground, can be treated according to the normal process. But another part above the failure story should be treated as an unstable mechanism. That means when any story of the structure is failure (the story damage index reaches or exceeds 1.0), the whole structure will enter a state of collapse (the global damage index is approaching 1.0).

3.1 Story damage model

The vibration equation of the shear frame structure can be written as follows

$$(K-\omega_j^2 M)\phi_j = 0 \tag{1}$$

Where, ω_j is the jth frequency of the structure, ϕ_j is the jth vibration mode of the structure. K and M are the stiffness matrix and mass matrix respectively. For the shear frame, the mass matrix M is a diagonal matrix, and the overall stiffness matrix can be obtained by the superposition of the stiffness matrix of every story. The expressions of K, M and ϕ_j can be expressed as follows

$$K = \sum_{i=1}^{n} K_i \tag{2}$$

Where

$$K_1 = \begin{bmatrix} k_1 & 0 & \cdots & 0 \\ 0 & 0 & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}$$

$$K_i = \begin{bmatrix} 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & 0 & & & & & & \cdot \\ \cdot & & \cdot & & & & & \cdot \\ \cdot & & & k_i & -k_i & & & \cdot \\ \cdot & & & -k_i & k_i & & & \cdot \\ \cdot & & & & & \cdot & & \cdot \\ \cdot & & & & & & 0 & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix} \begin{matrix} \\ \\ \\ i-1 \\ i \\ \\ \\ \\ \end{matrix} \quad (2 \le i \le n) \tag{3}$$

$$\qquad\qquad i-1 \quad i$$

$$M = \begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & m_n \end{bmatrix} \tag{4}$$

$$\phi_j = \begin{bmatrix} \varphi_{1j} & \cdots & \cdots & \varphi_{nj} \end{bmatrix}^T \tag{5}$$

Where, n is the number of stories, K_i is the element stiffness matrix of the ith story, k_i is the ith inter-story stiffness.

Substituting Eqs. (2)-(5) into Eq. (1) yields

$$k_i = \frac{\omega_j^2}{\Delta\varphi_{ij}} \sum_{n=i}^{n} m_n \varphi_{nj} \qquad j = 1, 2, ..., n \tag{6}$$

Where

$$\Delta\varphi_{ij} = \begin{cases} \varphi_{ij} - \varphi_{(i-1)j} & i = 2, 3, ..., n \\ \varphi_{ij} & i = 1 \end{cases} \tag{7}$$

After the structure is damaged, its stiffness matrix, frequencies and vibration modes will change (ignore the change of the structure mass matrix before and after the damage, and this is usually consistent with most of the actual situations). The vibration equation of the structure changes into

$$\left(K^d - \omega_j^{d2} M\right)\phi_j^d = 0 \tag{8}$$

$$K^d = K + \Delta K = K + \sum_{i=1}^{n} \alpha_i K_i \quad (-1 < \alpha_i < 0) \tag{9}$$

$$\phi_j^d = \phi_j + \Delta\phi_j = \phi_j + \sum_{r=1}^{n} \beta_{jr} \phi_r \tag{10}$$

$$\omega_j^d = \omega_j + \Delta\omega_j \tag{11}$$

Where, ΔK, $\Delta\phi_j$ and $\Delta\omega_j$ are the variation of the stiffness matrix, vibration mode and frequency respectively. α_i is the stiffness degradation factor. β_{jr} is the combination coefficient. Because the vibration modes of the structure are linearly independent, all model vectors will form a complete vector space. So, $\Delta\phi_j$ can be expressed as the linear combination of every mode.

If the story damage is expressed as the story stiffness degradation, the ith story damage index is as follows

$$d_i = -\alpha_i = 1 - \frac{k_i^d}{k_i} = 1 - \frac{\omega_j^{d2}}{\omega_j^2} \frac{\Delta\varphi_{ij}}{\Delta\varphi_{ij}^d} \frac{\sum_{n=i}^{n} m_n \varphi_{nj}^d}{\sum_{n=i}^{n} m_n \varphi_{nj}} \tag{12}$$

It should be noted that as the mass of the structure before and after the damage has been neglected, the stiffness degradation is the main direct cause of the change of structural modal parameters. This story damage index is based uniquely on the story stiffness degradation. It cannot account for the strength degradation.

3.2 Modal curvature for damage location

The modal curvature of the system is usually used to locate the structural damage (Yoon *et al.* 2009). When only a few of the stories are damaged, it will be convenient to use the modal curvature to locate the damage positions, rather than to calculate Eq. (12) story by story to obtain all the local damage indexes. It can greatly improve the computational efficiency. The expression of the modal curvature is as follows

$$\kappa_t = \frac{\varphi_{t+1} - 2\varphi_t + \varphi_{t-1}}{h^2} \tag{13}$$

Where, φ_t is the t-th element of the mode vector. h is the height of the story.

3.3 Global damage model and the correlation with story damage

As has been pointed out earlier, if the weighted method is adopted, deviation of the final global damage index will occur because of the selection of the weighting coefficient and the multiple weighting. In the reference (He *et al.* 2017), the multi-mode global damage model is used to assess the structural damage. It's defined as the loss ratio of potential energy stored in structures before and after the damage, with the combination rule based on the assumption of in-series independencies among modal damages. The simplified damage model is as follows

$$D_j = 1 - \frac{T_j^2}{T_j^{d2}} \tag{14}$$

$$D = \sqrt{1 - \prod_{j=1}^{n}(1 - D_j^2)} \tag{15}$$

Where, D_j is the jth modal damage index, D is the global damage index, T_j and T_j^d are the jth period of the initial structure and the damaged structure, respectively. n is the number of the considered modes.

Substituting Eqs. (10) and (12) into Eq. (14) yields

$$D_j = 1 - \frac{T_j^2}{T_j^{d2}} = 1 - \frac{\omega_j^{d2}}{\omega_j} = -\frac{1}{1+2\beta_{jj}} \frac{\sum_{r=1}^{n} \alpha_r k_r \delta\varphi_{jr}^2}{\sum_{r=1}^{n} k_r \delta\varphi_{jr}^2} \tag{16}$$

Where

$$\delta\varphi_{jr} = \begin{cases} \varphi_{j,r} - \varphi_{j,r-1} & r = 2,3,\ldots,n \\ \varphi_{j,r} & r = 1 \end{cases} \tag{17}$$

The vibration modes before and after damage is modified by normalizing the modal mass to unity. It gives

$$\phi_r^T M \phi_r = 1 \tag{18}$$

$$(\phi_r + \Delta\phi_r)^T M (\phi_r + \Delta\phi_r) = 1 \tag{19}$$

Substituting Eqs. (10) and (18) into Eq. (19), and omitting the high-order term yields

$$\beta_{jj} = 0 \tag{20}$$

Substituting Eqs. (12), (20) into Eq. (16), the modal damage becomes

$$D_j = 1 - \frac{T_j^2}{T_j^{d2}} = \frac{\sum_{r=1}^{n} d_r k_r \delta\varphi_{jr}^2}{\sum_{r=1}^{n} k_r \delta\varphi_{jr}^2} \tag{21}$$

The relationship between the modal damage and the story damage is established by Eq. (21). The modal damage is obtained by using the macroscopic global parameters. The story damage is obtained by considering the local stiffness degradation of the structure. Those two damage models are linked together through the modal parameters. It can be seen from Eq. (21) that by using the inter-story stiffness, the mode shape vector of the initial structure and the story damage index, the modal damage of the structure can be obtained. For a certain structure, story damages are the only variates. The structural modal damage is the function of story damages. And when the damage values of every story are the same, the modal damage will be equal to the story damages. Substitute Eq. (21) into Eq. (15), the global damage index can be gained. Through the modal damage index, the relationship between the global damage and the local damage is established. It also should be noted that the global damage model cannot account for the strength degradation.

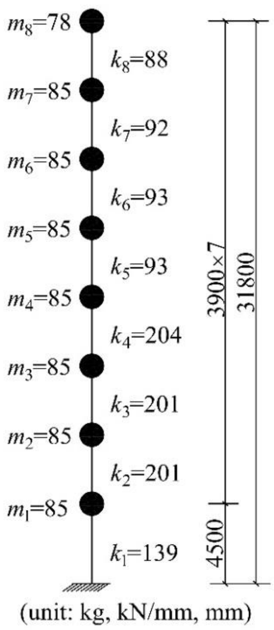


Fig. 3 Model information of the simplified shear-type frame

4. Validation

The story damage model, the damage location formula, the modal damage and the global damage model are verified via a simplified shear plane frame. Damages in the structure are induced by weakening the stiffness of the stories directly. The procedure is as shown in Fig. 1.

4.1 Model information

The parameters of the simplified 8-story shear plane frame is as shown in Fig. 3. m_i is the lumped mass of the ith story, and k_i is the ith inter-story stiffness. The vibration periods of the structure is as shown in Table 1. T_i represents the initial structural period. T_d is the period of the damaged structure. When the degradation of the inter-story stiffness appears, the damage of the structure begins. The degradation of the inter-story stiffness will lead to the elongation of the structural periods. For example, when the third story encounters a 40% stiffness degradation, the periods of the structural are as shown in Table 1 (T_d). All periods of the structure are elongated.

4.2 Verification of the proposed method

The stiffness of the third story is weakened to obtain different damage levels. The damage location is then calculated by Eq. (13). The results are as shown in Fig. 4. As can be seen from the figure, when the third story is weakened, the modal curvature indexes of the second story and the third story fluctuate obviously comparing to the initial state, while the indexes of other stories are remained

Table 1 Vibration periods before and after the damage

Period	T_i (s)	T_d (s)
1st period	0.807	0.843
2nd period	0.312	0.315
3rd period	0.188	0.189
4th period	0.136	0.145
5th period	0.116	0.120
6th period	0.101	0.102
7th period	0.084	0.084
8th period	0.069	0.075

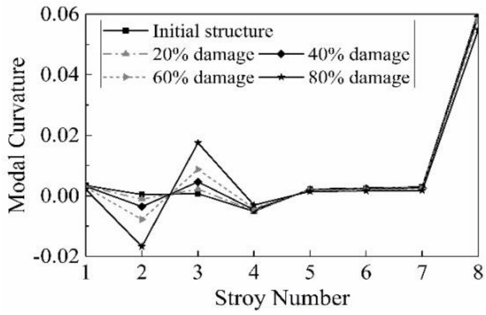


Fig. 4 Modal curvature for damage location

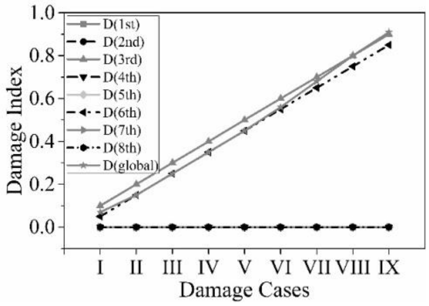
Table 2 Calculated values of d_2 and d_3 under different damage conditions

Formula	d_2 (Eq. (12), i=2)	d_3 (Eq. (12), i=3)
20% damage	$6.00\times10^{-15}\approx0$	0.2
40% damage	$-1.11\times10^{-14}\approx0$	0.4
60% damage	$8.88\times10^{-16}\approx0$	0.6
80% damage	$9.66\times10^{-15}\approx0$	0.8

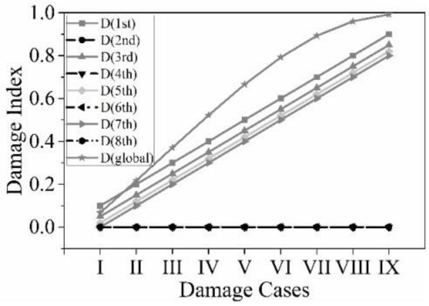
almost the same. It means that the damage of the structure may only occur in the second or third story. It has narrowed the scope of damage to a very small range, and it is conformed to the real situation.

According to Fig. 4, the changes of the modal curvature indexes are almost zero except for the second and the third story. It indicates that damage occurs only in those two stories. Let the damage index of the second story and the third story to be d_2 and d_3, respectively. Substitute d_2 and d_3 into Eq. (12) to calculate the damage indexes. The results are as shown in Table 2. It can be seen that the damage indexes of the second story are approximately equal to zero, and the damage indexes of the third story are completely consistent with the weakened cases. The damage location formula and the story damage model are feasible.

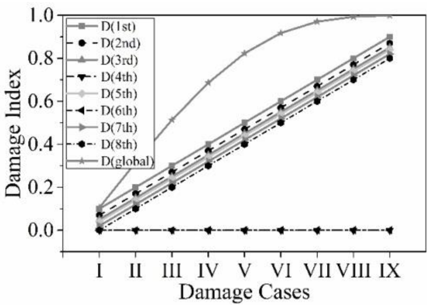
Substituting the story damage indexes into Eq. (21) and Eq. (15), the modal damage indexes and the global damage indexes of the frame then can be obtained. What is shown in Table 2 is the result of the case that only the third story is weakened. When different stories are weakened, the story damage indexes and the global damage indexes are as shown in Fig. 5 (I-IX in the figure represent different damage cases, D(1st)-D(8th) represent the 1st story damage to the 8th story damage, D(global) represent the global damage). What can be seen from Fig. 5 is that the formulas in the section 3 are still valid under different damage cases (adjacent-story damages, multi-story damages, etc.).



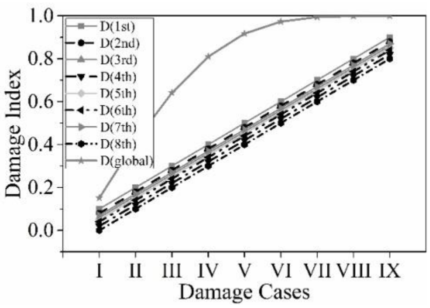
(a) Damage indexes with two stories weakened



(b) Damage indexes with four stories weakened



(c) Damage indexes with six stories weakened



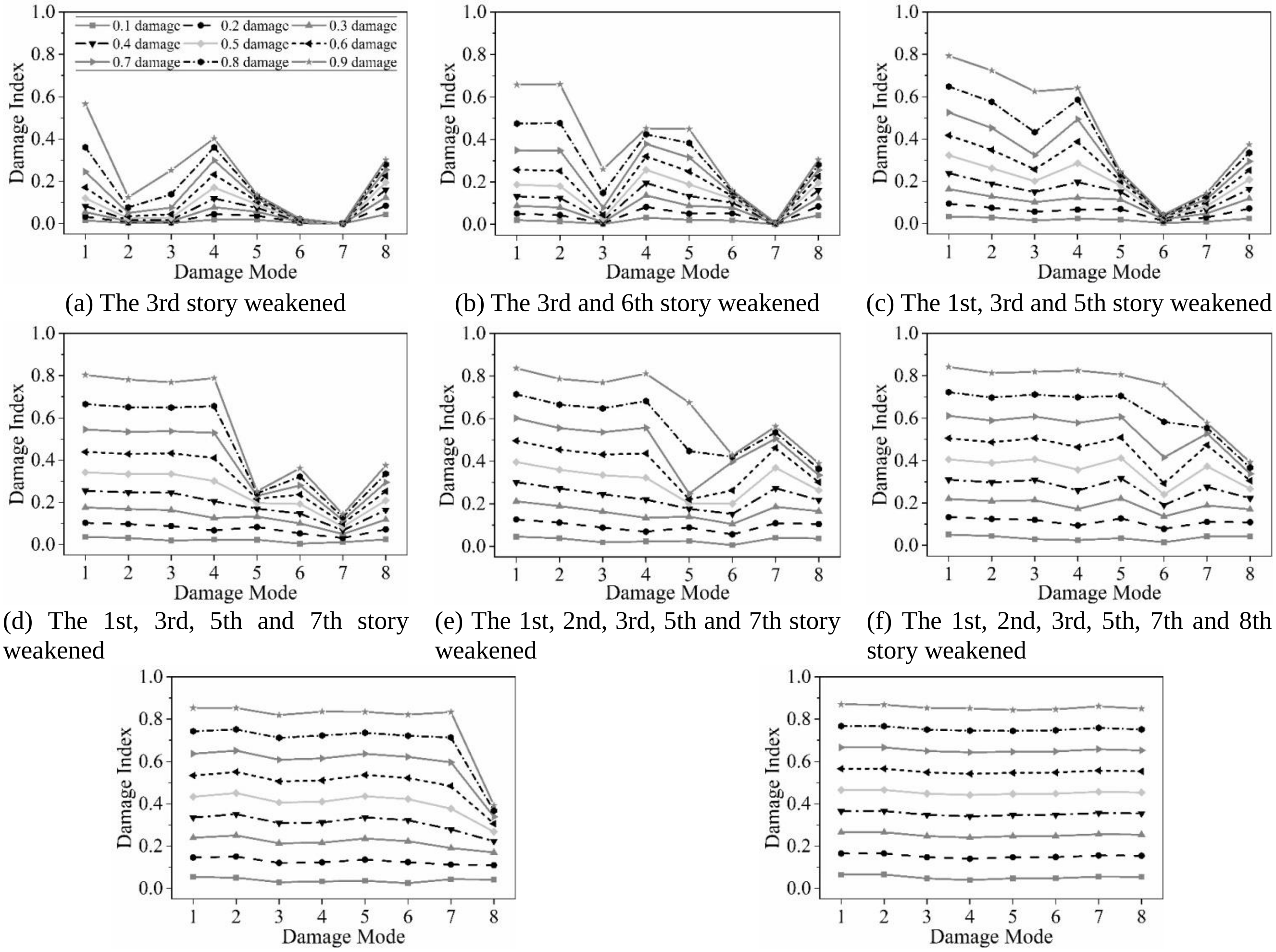
(d) Damage indexes with all stories weakened

Fig. 5 Story damage and global damage under different weakened cases

5. Story damage, modal damage and global damage

5.1 Influence of the number of damaged stories

When different story damages occur, the modal damage evolution curves and global damage evolution curves are as shown in Fig. 6 and Fig. 7. The meanings of the lines in Fig. 6(b) to Fig. 6(h) are the same as shown in Fig. 6(a) ('0.1 damage' to '0.9 damage' means the damage degree of the weakened stories is from 0.1 to 0.9). The label 'global-n' in Fig. 7 means that the number of the damaged stories in the structure is 'n'.



(a) The 3rd story weakened (b) The 3rd and 6th story weakened (c) The 1st, 3rd and 5th story weakened

(d) The 1st, 3rd, 5th and 7th story weakened (e) The 1st, 2nd, 3rd, 5th and 7th story weakened (f) The 1st, 2nd, 3rd, 5th, 7th and 8th story weakened

(g) The 1st, 2nd, 3rd, 5th, 6th, 7th and 8th story weakened (h) All stories weakened

Fig. 6 Modal damages under different weakened cases

As shown in Fig. 6, when the structural damages are slight, the differences between different modal damages are not obvious. But as damages increase, parts of the modes will be significant, some of the modal damages will control the structural damage. The sensitivity of the modal damage to the story damage gradually becomes distinct. And when the number of the damaged stories becomes bigger and bigger, more high-order modal damages are activated. More and more high-order modal damages are involved in the contribution of the global damage. As shown in Fig. 6(a), when the third story of the structure is damaged, only the 1st modal damage, the 4th modal damage and the 8th modal damage are sensitive to the structural damage. The control modes are the 1st mode and the 4th mode. But as the number of the damaged stories increases, more modes will contribute to the global damage. As shown in Fig. 6(h), when all stories of the structure are damaged, the eight modal damage indexes are almost the same. All modes take the same proportion in the contribution of the global damage. The increase of the modal damage indicates that the damage of the structure becomes more and more serious. The uneven fluctuation of the modal damages means the structure encounters an uneven damage state. Because the local failure will cause the failure of the whole structure, the structural damage resisting ability will not give full play if the uneven damage state occurs.

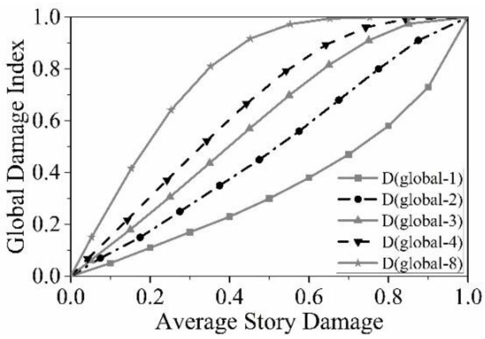
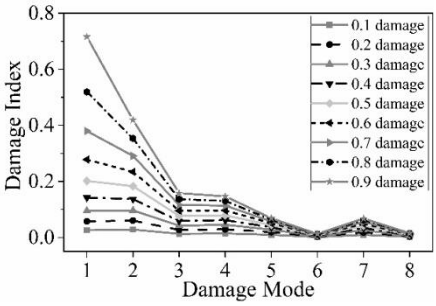
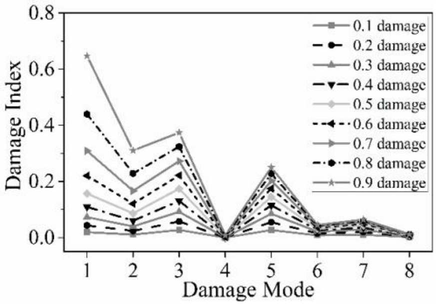


Fig. 7 Global damage evolution curves under different damage cases

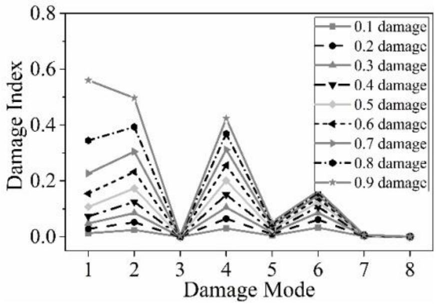
The global damage curves are as shown in Fig. 7. As the number of the damaged stories increases, the global damage evolution curve changes from the concave shape to the S-type and then finally transforms to the convex shape. In the real earthquake event, the structure failure or collapse is usually caused by the failure or collapse of several stories. It makes the damage evolution curves of most of the real structures close to the S-type curve.



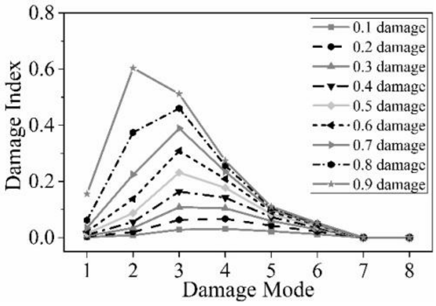
(a) The 1st story



(b) The 5th story



(c) The 6th story



(d) The 8th story

Fig. 8 Modal damage under different damage locations

5.2 Influence of the location of damaged story

When the location of the damaged story varies, the modal damage evolution curves are as shown in Fig. 8. The fluctuation of the modal damage in Fig. 8 indicates that different modal damages have different sensitivities to the location of the damaged story. When damage occurs only in the first story, the first and the second mode are the most sensitive modes. When damage occurs in the fifth story, the most sensitive modes change to the first, the second, the third and the fifth mode. The first, the second and the fourth mode are sensitive to the sixth story damage, and the second, the third and the fourth mode are sensitive to the

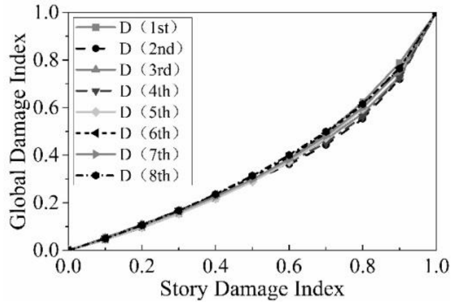


Fig. 9 Global damage evolution curves under different damage locations

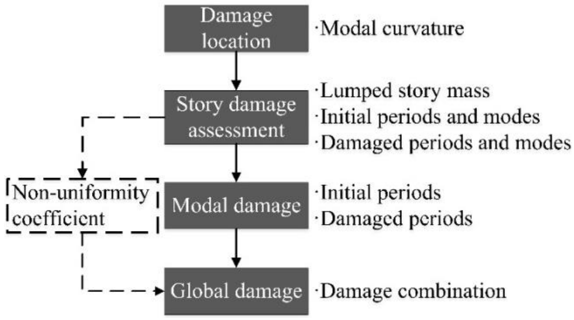


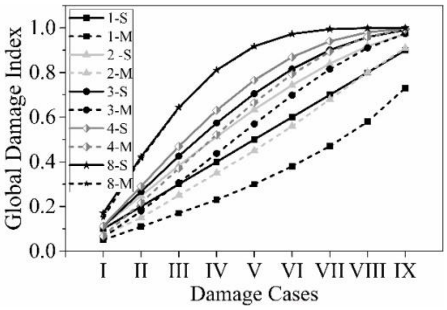
Fig. 10 The multi-level damage assessment process

eighth story damage. Roughly, the sensitivity of the fundamental mode decreases as the location of the damaged story rises. Simultaneously, higher order modal damages fluctuate heavily. That's because as the location of damaged story rises, higher modes are easy to be excited, and the changes of the higher order periods are severe than the fundamental period. As a result, the effect of each modal damage index should be reasonably considered in the process of calculating the global damage index. Inappropriate neglecting will affect the accuracy of the structural damage evaluation.

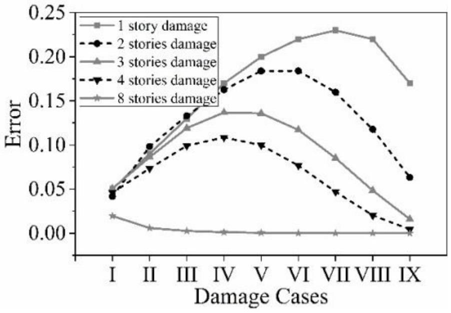
The global damage evolution curves under different damage locations are as shown in Fig. 9. The label, 'D (1st)' to 'D (8th)' in the figure means that damage occurs only in the first to the eighth story, respectively. It can be seen from the figure that the global damage index conforms the definition of structural damage. The initial value of damage is zero (undamaged), and the value of the failure state is 1.0. In addition, the global damage evolution curves are almost the same when only one story is damaged. It means that the global damage is sensitive to the damage degree of the story, but it's not sensitive to the location of the damage. It's the character that a global damage model should have. Small fluctuation and change of location of local damages shouldn't cause significant disturbance of the global damage. That's to say, global damage model should have good robustness. Only by this means can global damage model accurately grasp the damage state from the structure level.

6. Discussion

According to the deduction of the section 3 and the



(a) Global damage curves



(b) Relative error

Fig. 11 Modal-damage-based global damage and story-damage-based global damage

analysis of the section 5, a damage assessment process can be drawn. As shown in Fig. 10, by using the modal parameters of the structure before and after the damage, a set of simple, reliable multi-level structure damage assessment method is established.

Two methods are suggested in Fig. 10 to obtain the global damage index. One is by combining the modal damages, and the other one is directly by combining the story damages. A non-uniformity coefficient may be proposed to assess the distribution of the structural damage, i.e., the degree of the uneven damage, and to decide which method will be used to obtain the global damage. From the assumptions, the story damage and the modal damage have the similar properties: each story damage can be regarded as a series of independent system. The story damages are independent to each other, and when one of them reaches the limit value the whole structure reaches the failure state as well. So, it can be deducted that the global damage can be obtained through the combination of story damages by using the combination method of the modal damage. The result is shown in Fig. 11. It can be seen from the figure that as the number of damaged stories increases, the differences between the story-damage-combined global damage indexes and the modal-damage-combined global damage indexes become smaller and smaller. If most of the stories have the similar damage degree, the modal damage can be replaced by the story damage, and the assessment of the global damage will be acceptable. But if the damage in the structure is extremely uneven, the error of the assessment by using the story-damage-combined global damage index will be unacceptable. That's why the non-uniformity coefficient should be proposed to assess the distribution and severity of the story damage, and to decide which method should be used to obtain the global damage of the structure.

The proposed damage assessment method is based on the assumptions that the structure will form a sidesway collapse under extreme conditions. The stiffness degradation is the primary factor of the proposed damage model, which makes the proposed method cannot be applied to structures with no obvious stiffness degradation. And it cannot account for the strength degradation of the structure as well. And for those structures which may form a vertical collapse, the proposed method needs to be further verified. The proposed method is supposed to be used in both the numerical prediction of damage in designed structures and the experimental assessment of damage in real structures. When it is applied to designed structures, the method can provide a prediction of the damage state of the structure under various conditions. And the modal parameters are obtained by performing a modal analysis on the numerical model before and after the damage. When it is applied to real structures, the modal parameters are obtained from field tests, health monitoring systems or from the numerical analyses based on the real-time properties of the real structures. Because of the limited test technologies, noises in the monitoring signal and imprecision of the numerical model, the assessment of real structures may not be that accurate.

7. Conclusions

This study attempts to explore the damage mechanism of the reinforced concrete frame and the relationship between the story damage and the global damage. A failure mode of the shear-type frame, which can associate the story failure with the global failure, is introduced. Starting with the tridiagonal stiffness matrix of the shear frame, and by introducing the stiffness degradation story damage index and the modal damage model, a multi-level damage assessment method is established. Combined with the case study, the damage formulas are verified. And the conclusion can be drawn as follows,

The modal damage of the structure is the function of the story damage. The correlation between the story damage, and the global damage is reasonably established through the modal damage index. Different modal damages are sensitive to different location of story damages. In general, as the location of the damaged story rises, the sensitivity of the fundamental modal damage decreases, while the higher order modal damages fluctuate heavily. The global damage is only sensitive to the damage degree of the story, but it's not sensitive to the location of the local damage. The global damage evolution curves will be almost the same if only the location of the damage changes. As the number of the damaged stories increases, more and more modes will be involved. Simultaneously, the global damage evolution curve changes from the concave shape to the S-type and finally transforms to the convex shape.

Based on the modal damage model, the correlation between the story damage and the global damage of the shear-type frame is studied. However, the example structure applied in this study is too much simplified and the other patterns of failure modes aren't considered. In addition the plane frame, the extensive applications of the proposed method to the spatial structures with bidirectional vibrations, wall structures, mixed structures or structures with planar and vertical irregularities are also supposed to be improved in the further research. Because the modal damage model is not restricted to two dimensional or three dimensional structures, those extensive applications are possible to be achieved.

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