# Improvement, analytical verification and application of RC frame beam-column joint models

Guoxi Fan<sup>\*1</sup>, Debin Wang<sup>2a</sup> and Jing Jia<sup>1a</sup>

<sup>1</sup>School of Engineering, Ocean University of China, No.238 Songling Road, Laoshan District, Qingdao City, China <sup>2</sup>School of Civil and Safety Engineering, Dalian Jiaotong University, No.794 the Yellow River Road, Shahekou District, Dalian City, China

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**Abstract.** Previous experimental researches indicate that reinforced concrete beam-column joints play an important role in the mechanical properties of moment resisting frame structures, so as to require proper design. In order to get better understanding of the beam-column joint performance, a rational model needs to be developed. Based on the former considerations, two typical models for calculating the shear carrying capacity of the beam-column joint including the inelastic reinforced concrete joint model and the softened strut-and-tie model are selected to be introduced and analyzed. After examining the applicability of two typical models mentioned earlier to interior beam-column joints, several adjustments are made to get better predicting of the test results. For the softened strut-and-tie model, four adjustments including modifications of the depth of the diagonal strut, the inclination angle of diagonal compression strut, the smeared stress of mild steel bars embedded in concrete, as well as the softening coefficient are made. While two adjustments for the inelastic reinforced concrete joint model including modifications of the confinement effect due to the column axial load and the correction coefficient for high concrete are made. It has been proved by test data that predicted results by the improved softened strut-and-tie model or the modified inelastic reinforced concrete joint model are consistent with the test data and conservative. Based on the test results, it is also not difficult to find that the improved beam-column joint model can be used to predict the joint carrying capacity and cracks development with sufficient accuracy.

**Keywords:** beam-column joint; carrying capacity; cracks development; axial compression ratio; softened strut-and-tie model; inelastic joint model

### 1. Introduction

For reinforced concrete frame structures subjected to inelastic response under earthquake loading, reinforced concrete beam-column joints are critical regions for developing frame action and ensuring that inertial loads are transferred through the frame to the foundation. So it can significantly influence the earthquake response of reinforced concrete frame structures. Also, reinforced concrete beam-column joints are one of the most complex, least studied, and important structural components of a frame structure. Post-earthquake reconnaissance efforts have attributed the collapse of many reinforced concrete frames to the failure of joints (Adibi et al. 2017). Several studies indicate that beam-column joints of RC frames are considered as critical regions (Rajagopal et al. 2014, Tsonos 2014, Costa et al. 2013, Asha and Sundararajan 2014). Therefore, the reinforced concrete beam-column joint plays an important role in the mechanical properties of the frame structure, and require proper design.

The usual earthquake resistant design philosophy of reinforced concrete frame structure involves three main requirements, which may be described as follows. Firstly, the ultimate carrying capacity of the beam-column joint should be greater than the flexural yielding force of the adjacent beam and column, and should not degrade before the beam reaches its required ductility. So it allows the beam to form plastic hinge adjacent to the beam-column joint. Secondly, the flexural carrying capacity ratio of the column to the beam at a joint is required to be greater than 1.0 to meet the requirements of ACI-ASCE 352 (ACI 352R-02 2002), which can ensure the formation of the beam plastic hinge rather than that of the column at large displacement levels. Thirdly, the anchorage of beam and column reinforcement in the joint should be sufficient to avoid running counter to ductile capacity of the joint. However, experimental studies indicate that they undergo large inelastic shear deformations even when the usual earthquake resistant design philosophy is followed. Therefore, a model needs to be developed to properly evaluate the performance of the beam-column joint.

At present, numerous theoretical models have been proposed to investigate the mechanical properties of beamcolumn joints. Park and Paulay (Paulay and Priestley 1992, Paulay and Park 1984, Paulay *et al.* 1978) pointed out that 2 kinds of shear resistant mechanisms including diagonal strut mechanism and truss mechanism existed in the joint core area. However, this model does not consider the constraint effect of stirrups on the concrete in the joint core area. Subsequently, the softened truss model was proposed which

<sup>\*</sup>Corresponding author, Lecturer

E-mail: fanguoxi-6688@163.com <sup>a</sup>Ph.D.

was one of five kinds of models belonging to the reinforced concrete unified theory, and satisfied equilibrium, compatibility and constitutive laws. Hsu (1988) proposed the rotating angle softened truss model, while Pang and Hsu (1996) proposed the fixed angle softened truss model. Ji et al. (2001) pointed out that the rotating angle softened truss model could not explain the existence of shear stress due to aggregate interlock and dowel action of reinforcement bars, and was no longer suitable for higher accuracy structure calculation using the constitutive relation of bare reinforcement bars. In regard to the fixed angle softened truss model, a problem was ignored that the direction of cracks was not only related with the external stress, but also related with the ratio of two perpendicular directions of reinforcement bars. In contrast, the softened strut-and-tie model is often termed as a rational model for determining the shear carrying capacity of beam-column joints (Hwang and Lee 1999, Hwang and Lee 2000) The softened strutand-tie model originates from the strut-and-tie concept and satisfies equilibrium, compatibility, and constitutive laws of cracked reinforced concrete, which has been proposed for determining the shear strength of beam-column joints.

In addition, a general analytical model was proposed by Attaalla (2004). However, the wide beam effect and the carrying capacity reduction of the joint due to the presence of eccentricity are not included in this model. Lowes and Altoontash (2003) developed a joint model that provided a representation of the nonlinear mechanism of the joint behavior by developing the constitutive relationships of materials, geometric and design parameters and implementing a four-node 12 degree-of-freedom element. Shin and LaFave (2004), Kim and LaFave (2007) investigated the effects of some key parameters such as concrete compressive strength, joint reinforcement and axial load effect using the data from 26 beam-to-column connection tests. Afterwards, an analytical model was proposed to estimate the hysteretic joint shear stress versus strain behavior by employing modified compression field theory. Kim and LaFave (2008) used statistical methods to evaluate the effect of many key parameters on the joint behavior. Based on those models, an inelastic reinforced concrete joint model was proposed by Unal and Burak (2013). The developed model includes many parameters that take into account the effect of eccentricity, wide beams, transverse beams and presence of slab on the seismic behavior of the connection region, besides concrete compressive strength, effective joint width and joint transverse reinforcement ratio (Unal and Burak 2012).

Among those models, the softened strut-and-tie model can be used as a tool to clarity the roles of different parameters, besides the particular use in the strength prediction of discontinuity regions. And the inelastic reinforced concrete joint model proposed by Unal and Burak (Unal and Burak 2013, Unal and Burak 2012) can predict not only the joint shear strength versus strain relationship, but also the inelastic behavior of members. This paper aims to introduce and analyze the applicability of two typical models mentioned earlier to interior beamcolumn joints. Then several adjustments for two typical models are made to get better predicting of the test results.



Fig. 1 Mechanical model of the interior reinforced concrete joint

# 2. Definition of the joint horizontal shear carrying capacity

Before introducing the analytical model, the forces around and within a beam-column joint should be identified. The earthquake-induced forces acting on an interior beam-column joint are shown in Fig. 1. The corresponding joint horizontal shear carrying capacity  $V_{jh}$  can be calculated by

$$V_{jh} = T_{bs} + C_{bs} + C_{bc} - V_c \tag{1}$$

where  $V_c$  is the shear carrying capacity of the column;  $T'_{bs}$  is the tension force of beam top longitudinal reinforcement at the right side of the column;  $C_{bs}$  is the tension force of beam top longitudinal reinforcement at the left side of the column;  $C_{bc}$  is the compression force of concrete at the left side of the column, as shown in Fig. 1.

#### 3. Modification of the softened strut-and-tie model

The softened strut-and-tie model consists of the diagonal, horizontal, and vertical mechanisms as shown in Fig. 2. The diagonal mechanism is a single diagonal compression strut as shown in the shaded area of Fig. 2(a), i.e., the area which an arrow points to. It is assumed that the direction of the diagonal compression strut coincides with the direction of the principal compressive stress of the concrete. The horizontal mechanism is composed of one horizontal tie and two flat struts, while the proposed vertical mechanism includes one vertical tie and two steep struts. Horizontal stirrups in the joint constitute the horizontal tie, and flat struts are shown in the shaded area of Fig. 2(b). The vertical tie is made up of the intermediate column bars, and steep struts are shown in the shaded area of Fig. 2(c). The parameters in Fig. 2 are defined as follows, where  $\theta$  is the inclination angle of diagonal compression strut, while  $h_b$ and  $h_c^{"}$  are the distances between the extreme longitudinal reinforcement in the beams and columns, respectively. Bond deterioration along the beam and column reinforcement is assumed, and the principal stress is concentrated along the diagonal strut to cause shear failure.

On the basis of the softened strut-and-tie model derived



Fig. 2 Joint shear-resisting mechanisms

previously, Hwang and Lee (2002) proposed a simple predicting procedure, which dealt with the carrying capacity prediction of reinforced concrete discontinuity regions failing in diagonal compressions without getting lost in trivialities. The following assumptions are put forward, according to the proposed simple predicting procedure. (1) For joints where a beam hinge occurs at the face of the column, the spalling of the compression zone in the beam is frequently observed. Since the crushing of concrete produces a small compression zone in the beam, the neglect of  $a_b$  (the depth of the compression zone in the beam) in computing  $a_s$  (the depth of the diagonal strut) is assumed. (2) The column axial compression load always provides a beneficial effect on the joint shear carrying capacity. (3) The yield stress of the bare steel bars is adopted. (4) The value of softening coefficient  $\zeta$  is simplified.

According to reference (Zhao 2005), the height of concrete compression zone is related to the curvature ductility coefficient of beam, and the curvature ductility coefficient  $\mu_{\varphi}$  can be calculated by

$$\mu_{\varphi} = 1 + \frac{(\mu_{\Delta} - 1)}{3\frac{l_{p}}{l}(1 - 0.5\frac{l_{p}}{l})}$$
(2)

where  $l_p$  is the length of beam plastic hinge, and its value can be computed according to reference (Zhao 2005);  $\mu_{\Delta}$  is the displacement ductility coefficient. The displacement ductility coefficient can be taken as 4.0-6.0, which is in close agreement with the test results (Fan *et al.* 2014). Also, the curvature ductility coefficient  $\mu_{\varphi}$  can be calculated by

$$\mu_{\varphi} = \frac{1}{0.035 + 0.65\xi} \tag{3}$$

Calculated by the above formulations, the relative height of concrete compression zone of beam section can be obtained, which is approximate 0.2. And the column axial load provides a beneficial effect on the joint shear carrying capacity because it increases the depth of the strut as shown in Eq. (4), which is not consistent with the actual test results.

$$a_c = (0.25 + 0.85 \frac{N}{A_g f_c}) h_c \tag{4}$$

According to previous research by the authors (Fan et al. 2014), the softened strut-and-tie model is adjusted as

follows.

(1) When the neglect of  $a_b$  in computing  $a_s$  is made, the softened strut-and-tie model underestimates the joint shear carrying capacity for the cases with lower axial loads (Fan *et al.* 2014). Therefore, the depth of the compression zone in the beam can't be ignored in computing the depth of the diagonal strut. And the relative height of concrete compression zone of beam section can be obtained by the ductile design control criterion of reinforced concrete structures, which is approximate 0.2. So  $a_b$  in computing  $a_s$  can be taken as  $0.2h_0$  (effective height of beam section).

(2) The column axial compression load always provides a beneficial effect on the joint shear carrying capacity because it increases the depth of the strut, which is not consistent with the actual test results. Research shows that the high column axial compression load accelerate the deterioration of the joint shear resisting mechanism (Fan et al. 2014). The inclination angle  $\theta$  of diagonal compression strut is assumed to be oriented between the extreme longitudinal reinforcement in the columns, but this assumption is violated by the high column axial compression load, and this results in a steeper  $\theta$ . Besides, the horizontal shear capacity of the joint decreases with the increase of the inclination angle  $\theta$  of diagonal compression strut. Therefore, in order to consider the effect of column axial compression load more reasonably, the actual inclination angle  $\theta$  of diagonal compression strut is adjusted as follows. It can be calculated by Eq. (5) for the range of axial compression ratio less than 0.1. Otherwise, it is defined as Eq. (6).

$$\theta = \tan^{-1}(\frac{h_b}{h_c}) \tag{5}$$

$$\theta = \tan^{-1}\left(\frac{h_b^*}{h_c - \frac{2a_c}{3}}\right) \tag{6}$$

(3) The smeared stress of mild steel bars embedded in concrete  $f_{sy}$  is lower than the yield stress of the bare steel bars  $f_y$  (Belarbi and Hsu 1994). It is expressed as

$$f_{sy} = f_y (0.93 - 2B) \tag{7}$$

$$B = \frac{(f_{cr} / f_y)^{1.5}}{\rho} = \frac{(0.31\sqrt{f_c'} / f_y)^{1.5}}{\rho}$$
(8)

where  $\rho$  is the reinforcement steel ratio (limited to a minimum of 0.25%). The smeared stress of mild steel bars embedded in concrete  $f_{sy}$  is adopted to make the calculation more accurate.

(4) The average value of  $\zeta$  estimated by the simplified method is lower than that computed by the general method (Hwang and Lee 2002). So the softening coefficient should be computed by Eq. (9) based on the comparison of iterative results and simplified results.

$$\zeta = \frac{3.65}{\sqrt{f_c}} \le 0.52 \tag{9}$$

### 4. Calculation model of shear strength

The concrete of the joint core area is in the state of three dimensional compression considering the confinement effects of transverse reinforcement, where compressive stresses are taken as negative,  $\sigma_1 \ge \sigma_2 \ge \sigma_3$  (algebraic value). As mentioned earlier, the principal stress is concentrated along the diagonal strut to cause shear failure. Therefore, when final failure occurs as a result of diagonal compression failure, the diagonal compression strut will be crushed after the failure of the horizontal tie, and the principal compressive stress  $\sigma_1$  supplied by transverse reinforcement is 0, which is parallel to the joint horizontal shear carrying capacity direction. When a limiting tensile stress of transverse reinforcement is reached, the principal compressive stress  $\sigma_2$  supplied by the effective layers of transverse reinforcement is  $-0.09 f_c$  for the specimens with a relative large amount of transverse reinforcements in reference (Fan et al. 2014), which is perpendicular to the joint horizontal shear carrying capacity direction. The value of  $\sigma_2$  is calculated by Eq. (10).

$$\sigma_2 = \frac{F_h}{A_{scor}} \tag{10}$$

where  $F_h$  is the ultimate carrying capacity of the horizontal tie,  $A_{scor}$  is the area constrained by the joint transverse reinforcement.

According to the provisions of the appendix C of GB50010-2010 (GB50010-2010 2010), the improvement coefficient of the concrete compression strength under triaxial compression state can be approximated by linear interpolation as 1.09. The mean ratio of the measured joint horizontal shear carrying capacity to the joint horizontal shear carrying capacity calculated by the softened strut-and-tie model is 1.115 (Fan *et al.* 2014), which is close to the improvement coefficient of the concrete compression strength. As a result, the following assumptions may be made to derive the calculation model of shear strength.

(a) It is roughly assumed that the transverse reinforcements within the center half of the joint core are considered fully effective when computing the tensile force supplied by transverse reinforcements, and the other joint core transverse reinforcements are included at a rate of 50%.

(b) The concrete of the joint core area is in the state of three dimensional compression considering the confinement



Fig. 3 Mechanical model for the shear strength of concrete

effects of transverse reinforcement. When final failure occurs as a result of diagonal compression failure, the principal compressive stress  $\sigma_1$  is 0, and the principal compressive stress  $\sigma_2$  is assumed to be zero because its value is much smaller than the compressive strength of concrete.

During reversed cyclic loading, the concrete of the core area of the joint combination is in the state of composite shear compression, and the failure criterion is complex. At any point in the joint core area, the relationship between normal stress and shear stress satisfies Mohr's circle theory. According to previous research by the authors (Fan and Song 2014), the calculation model of shear strength can be obtained, as shown in Fig. 3, which can be expressed as

$$\tau = \begin{cases} (0.25 + 0.75n) \cdot f_c & n \le 0.2 \\ \sqrt{n - n^2} \cdot f_c & n > 0.2 \end{cases}$$
(11)

where *n* represents the axial compression ratio. According to the calculation model and Fig. 3, n=0.5 is the critical value. The increase of axial compression ratio plays a beneficial effect on the shear carrying capacity of the joint with axial compression ratio less than or equal to 0.5. In contrast, above the critical value, the shear carrying capacity of the joint decreases as axial compression ratio increases. Also, the axial compression ratio has a significant effect on the shear carrying capacity of the joint for its value less than or equal to 0.2.

# 5. Modification of the inelastic reinforced concrete joint model

The parameters which are believed to be influential on the seismic behavior of joints are collected in a database. By using statistical correlation methods, the most effective parameters are determined whereas the ones that have a slight effect on the shear behavior are neglected. Consequently, an equation to predict the maximum joint shear strength of the reinforced concrete beam-column joint subjected to earthquake loading is generated (Unal and Burak 2012). Final parameters in the equation are defined in terms of ratios and powers of some of the key individual parameters to accurately represent their effect on the capacity and obtain the minimum average error and the highest correlation with the experimental values. The resulting equation to define the maximum joint shear strength  $v_{j,u}$  is given below (Unal and Burak 2013).

$$\upsilon_{j,u} = JT \bullet (f_c^{'} \bullet f_y)^{1/6} \bullet \rho_{joint} \bullet EE \bullet CI \bullet NE \bullet WB \bullet SI$$
(12)

where *JT* is a parameter that takes into account the effect of the confinement provided by the surrounding beams.  $f_c$  is the concrete compressive strength.  $f_y$  is the yield strength of reinforcing bars.  $\rho_{joint}$  is a parameter that depends on the volumetric joint transverse reinforcement ratio for one layer of confinement reinforcement. *EE* shows the reduction in joint strength due to the presence of eccentricity. *CI* is the column index based on column aspect ratio that is used to account for the reduction of effective joint area in rectangular columns. *WB* gives the reduction in strength when wide beams are present in the loading direction. *SI* defines the confinement of the connection region due to the presence of a floor system. *NE* defines the confinement effect due to axial load as shown in Eq. (13).

$$NE = 1 + \frac{N}{A_{e} \bullet f_{c}} = 1 + n \tag{13}$$

Fig. 4 shows the influence trend of the axial compression ratio on the joint shear carrying capacity. It can be seen from Fig. 4 that according to the inelastic reinforced concrete joint model, the column axial compression load always provides a beneficial effect on the joint shear carrying capacity because it increases the confinement effect NE as shown in Eq. (13). This phenomenon occurs due to that this model does not consider the increasing compressive stress within the joint induced by the column axial compression load. Research shows that the high column axial compression load accelerate the deterioration of the joint shear resisting mechanism (Fan et al. 2014). However, Kitayama et al. (1987) conclude that a column axial compression stress less than  $0.5f_c$  does not affect joint strength, high axial compression stress accelerates strength loss in the diagonal compression strut that forms in the joint core area. Many other studies (Durrani and Wight 1985, Fujii and Morita 1991, Alaee and Li 2017) show that an increase in the column axial compression load induces an increase in the joint shear carrying capacity, however, the influential extent is different from each other. Alaee et al. (2015) conclude that the presence of column axial loading is only beneficial up to a certain level and the joint shear will reduce in higher axial loading levels. Further increase in the axial load results in the joint maximum shear reduction, similar to the observed trend in the models with a floor slab. The preceding discussion clearly shows that an axial load  $0.2f_c A_g$  will cause an optimal enhancement in strength of the models, with or without a floor slab (Alaee et al. 2015). It can be seen from that the difference between these conclusions is due to the difference in the axial compression ratio. And the consistent conclusion is that high axial compression load is unfavorable to the carrying capacity of the joint. Therefore, the effect of axial compression load should be considered in accordance with the actual level of axial compression ratio. The existing model should be adjusted according to the above analysis.

Moreover, Mitra *et al* (Mitra and Samui 2012, Mitra *et al.* 2011) utilized the binomial logistic regression model to



Fig. 4 Confinement effect of the axial compression ratio



Fig. 5 Relative importance of the design variables

determine with sufficient accuracy the failure patterns of interior beam-column joints, and established a database consisting of 110 two-dimensional interior beam-column joints to verify the correctness of the computed regression parameters. The magnitude of a regression parameter multiplied by the mean of its corresponding design variables indicates the relative importance of the design variables, as shown in Fig. 5. The conclusion is that the joint shear strength at yielding of the longitudinal reinforcement bars in the beam (*TYLD*) is the most influential parameter. And the axial compression stress (*PFC*) has a certain degree of influence on the carrying capacity of the joint.

In order to better evaluate the applicability of the inelastic reinforced concrete joint model, error between predicted and experimental shear carrying capacity are given as shown in Eq. (14). 27 specimens are selected to establish a database, and the calculation results are shown in Table 1 (Durrani and Wight 1985, Fujii and Morita 1991, Gentry and Wight 1994, Lee and Ko 2007, Raffaelle and Wight 1995, Teng and Zhou 2003). When the axial compression ratio larger than 0.2, the predicted results are even higher than the experimental results, such as specimens in reference (Fujii and Morita 1991). The study shows that if the inelastic reinforced concrete joint model is directly used to calculate the shear carrying capacity of the beam-column joint at higher axial compression ratio, it is unsafe due to overestimate the shear carrying capacity of the joint.

Research team	Specimen	$f_c^{'}$ (MPa)	$f_y$ (MPa)	$b_c$ (mm)	$h_c$ (mm)	$b_b$ (mm)	$h_b$ (mm)	n	$V_{exp}$ (kN)	$V_{pre}$ (kN)	Error (%)
Durrani	X1	34.3	352	362	362	279	419	0.054	840.0	732.5	-12.8
and Wight	X2	33.6	352	362	362	279	419	0.056	853.6	823.7	-3.50
(1985)	X3	31.0	352	362	362	279	419	0.053	628.7	719.5	14.44
	A1	40.2	297	220	220	160	250	0.076	412.0	268.8	-34.76
	A2	40.2	297	220	220	160	250	0.076	379.6	268.8	-29.19
	A3	40.2	297	220	220	160	250	0.227	412.0	306.5	-25.61
Fujii and Morite	A4	40.2	297	220	220	160	250	0.227	420.8	398.4	-5.32
(1991)	B1	30.0	297	220	220	160	250	0.068	246.2	203.1	-17.51
(1))1)	B2	30.0	297	220	220	160	250	0.068	213.9	203.1	-5.05
	B3	30.0	297	220	220	160	250	0.236	272.7	235.0	-13.82
	B4	30.0	297	220	220	160	250	0.236	287.4	305.5	6.30
Gentry and	WB1	27.6	441	356	356	864	305	0.026	616.1	668.0	8.42
Wight (1994)	WB2	27.6	441	356	356	762	305	0.026	643.1	635.7	-1.15
	SO	32.6	471	400	600	300	450	0.089	828.0	931.7	12.52
T 177	S50	34.2	471	400	600	300	450	0.085	789.0	879.7	11.50
Lee and Ko $(2007)$	W0	28.9	471	600	400	300	450	0.101	775.0	912.4	17.73
(2007)	W75	30.4	471	600	400	300	450	0.096	780.0	862.3	10.55
	W150	29.1	471	600	400	300	450	0.100	710.0	730.9	2.94
	1	28.6	441	356	356	254	381	0.025	650.8	626.1	-3.80
Raffaelle and	2	26.8	441	356	356	178	381	0.026	420.6	449.2	6.80
Wight (1995)	3	37.7	441	356	356	191	381	0.019	469.7	502.2	6.92
	4	19.3	441	356	356	191	559	0.036	412.4	457.3	10.89
	<b>S</b> 1	33.0	440	400	300	200	400	0.111	775.8	566.4	-26.99
Teng and Zhou (2003)	S2	34.0	440	400	300	200	400	0.108	772.2	534.7	-30.76
	<b>S</b> 3	35.0	440	400	300	200	400	0.105	742.5	452.4	-39.07
	S5	39.0	440	400	200	200	400	0.110	452.4	377.3	-16.60
	<b>S</b> 6	38.0	440	400	200	200	400	0.113	439.2	328.4	-25.23

Table 1 Database of selected beam-column joints

$$Error = \frac{V_{pre} - V_{exp}}{V_{exp}}$$
(14)

In addition, very limited guidance is provided by the ACI Code for design of structures with high-strength materials (ACI 318R-05 2005). However, high-strength materials (concrete with compressive strength in excess of 59 MPa, and reinforcement bars with yield strength in excess of 500 MPa) have been used in recent construction project. Joints designed using high-strength materials may result in joints for which seismic behavior is determined by joint failure and not beam yielding (Attaalla and Agbabian 2004, Sanada and Maruta 2004). If the inelastic reinforced concrete joint model is directly used to calculate the shear carrying capacity of beam-column joints which are designed using high-strength materials, it is unreasonable due to the different performance of high-strength materials. Based on the strut and truss model, Chen and Wang (2012) proposed a modified model, in which a correction coefficient for high concrete joint was considered. Results of the modified model are close and conservative to experimental results. Besides, the seismic performance of HSC beam-column joints with high-yield-strength steel reinforcements was evaluated through both an experimental and analytical approach in recent studies (Li and Leong 2014, Alaee and Li 2017). Increased bond-slip of reinforcing bars, decreased hysteretic energy dissipation, and lower joint shear strength are the key issues in designing beamcolumn joints using high-strength steel reinforcements. Research results show that using longitudinal beam reinforcements of a higher grade causes a slight decrease in bond strength in the joint region, and using high-strength steel reinforcements results in a smaller energy dissipation capacity. On the basis of research results, Alaee and Li (2017) proposed a modified friction bond strength model to further investigate the effect of using high-strength steel reinforcing bars. After that, it is proved that current code provisions could be used for the design of high-strength members (using high-strength steel reinforcements). Some limitations on beam-bar diameter to column-depth ratio should be suggested (Alaee and Li 2017). Therefore, the influence of high-strength steel reinforcements can be reflected by adjusting the beam-bar diameter to columndepth ratio.

Based on the former considerations, adjustments for the inelastic reinforced concrete joint model are made as follows.

(1) As mentioned above, according to the inelastic reinforced concrete joint model, the column axial compression load always provides a beneficial effect on the joint shear carrying capacity because it increases the confinement effect NE as shown in Eq. (13), which is not

consistent with the actual test results. According to the results of theoretical analysis and experimental research, it is known that the if the inelastic reinforced concrete joint model is directly used to calculate the shear carrying capacity of the beam-column joint at higher axial compression ratio, it is unsafe due to overestimate the shear carrying capacity of the joint. Besides, according to the calculation model (Fig. 3) and previous studies (Alaee *et al.* 2015), it is known that n=0.2 is the inflection point. Therefore, the confinement effect due to the column axial load may be shown as follows.

$$NE = \begin{cases} 1+n & n \le 0.2\\ 1.11+0.89n-2.22n^2 & n > 0.2 \end{cases}$$
(15)

(2) As mentioned above, if the inelastic reinforced concrete joint model is directly used to calculate the shear carrying capacity of beam-column joints which are designed using high-strength materials, it is unreasonable due to the different performance of high-strength materials. Based on the modified model proposed by Chen and Wang (2012), a correction coefficient  $\zeta_h$  for high concrete can be obtained. And the influence of high-strength steel reinforcements can be reflected by adjusting the beam-bar diameter to column-depth ratio (Alaee and Li 2017).

$$\xi_h = \frac{1}{0.023 f_c^{(0.94)}} \tag{16}$$

After aforementioned adjustments made, the joint shear carrying capacity is recalculated. Fig. 6 shows that satisfactory results are obtained for the comparison of experimental and predicted joint shear carrying capacity. After previously mentioned adjustments made, the average ratio of carrying capacity of beam-column joints computed by the inelastic reinforced concrete joint model and experimental results is 0.949. Considering the reduction factor of the softened strut-and-tie model, it can be known that both the modified inelastic reinforced concrete joint model can be used to predict the joint carrying capacity with sufficient accuracy.



Fig. 6 Comparison of experimental and predicted joint shear carrying capacity

# 6. Application of improved beam-column joint models

## 6.1 Prediction of the carrying capacity of beamcolumn joints

As mentioned earlier, Hwang and Lee (2000) selected 56 specimens to examine the applicability of the softened strut-and-tie model to interior beam-column joints. By comparing the experimental results of beam-column joints and the calculation results of the softened strut-and-tie model at home and abroad, it is found that the following problems exist in the softened strut-and-tie model. (1) Test results show that, because the neglect of  $a_b$  (the height of concrete compression zone of beam section) in computing  $a_s$  (the height of effective cross section of diagonal compression strut) is assumed, the proposed model underestimates the joint shear carrying capacity for the cases with lower axial loads. (2) The average value of the reduction factor of concrete strength estimated by the simplified method is lower than that computed by the general method (Hwang and Lee 2002). (3) The column axial load plays a beneficial role in the shear carrying capacity of beam-column joints in a certain range, because it increases the height of effective cross section of diagonal compression strut. But on the contrary, the column axial load provides an adverse effect on the joint shear carrying capacity due to the early crushed of the concrete in the core area of the joint when the column axial load is too large. The softened strut-and-tie model does not consider the adverse effect of the larger column axial load. (4) The smeared stress of mild steel bars embedded in concrete  $f_{sy}$  is lower than the yield stress of the bare steel bars  $f_{y}$  (Belarbi and Hsu 1994). However, the stress-strain relationship of the bare steel bars is adopted in the softened strut-and-tie model. In view of the above problems, four adjustments for the softened strut-and-tie model including modifications of the depth of the diagonal strut, the inclination angle  $\theta$  of diagonal compression strut, the smeared stress of mild steel bars embedded in concrete, as well as the softening coefficient are made.

In order to verify the suitability of the above adjustments, 19 specimens are selected according to the test results of references (Fan et al. 2014, Durrani and Wight 1985, Fujii and Morita 1991, Fenwick and Irvine 1997, Birss 1978, Beckingsale 1980, Kitayama et al. 1991). For different specimens, the range of the compressive strength of concrete is 26.2-42.9MPa, while the range of the axial compression ratio is 0-0.44. For some specimens, the column axial load is taken as zero to simplify the tests and to create a severe loading condition for the joint. Besides, specimens with different loading speeds (0.4mm/s, 4mm/s or 40mm/s) are selected to verify the suitability of the improved softened strut-and-tie model, according to the test results of reference (Fan et al. 2014). Also, the geometry and size of the cross section and the reinforcement ratio are different from each other. Thus, the specimens selected encompass a wide range of material properties, reinforcement detailing, loading speeds and cross section geometry. After previously mentioned adjustments made,



Fig. 7 Prediction of cracks development

the average ratio of the carrying capacity of beam-column joints computed by the softened strut-and-tie model and experimental results is 0.8847, which is very close to the reduction factor. The above results show that the improved strut-and-tie model has better adaptability.

For the inelastic reinforced concrete joint model, it does not consider the increasing compressive stress within the joint induced by the column axial load so that the column axial load always provides a beneficial effect on the joint shear carrying capacity. Another problem is that the inelastic reinforced concrete joint model does not consider the different performance of high-strength materials. Thus adjustments about the confinement effect due to the column axial load, and correction coefficient for high concrete are made. Also, test results of specimens with different design parameters are selected to verify the suitability of the modified inelastic reinforced concrete joint model. The result shows that the average ratio of carrying capacity of beam-column joints computed by the inelastic reinforced concrete joint model and experimental results is 0.949. Based on the above results, it is not difficult to find that both the modified inelastic reinforced concrete joint model and the improved softened strut-and-tie model can be used to predict the joint carrying capacity with sufficient accuracy.

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### 6.2 Prediction of cracks development of beamcolumn joints

In order to make use of the improved beam-column joint model to predict the cracks development, a calculation program can be employed, and the calculation flow chart is shown in Fig. 7. The prediction process of cracks development can be determined under the premise of known material properties, reinforcement detailing, axial compression ratio and cross section geometry. The inclination angle  $\theta$  of diagonal compression strut is calculated according to Eqs. (5) or (6). Based on the value of  $\theta$  and the geometric relation in Fig. 2, the inclination angle  $\alpha$  of flat strut and the ratio of tensile force provided by the horizontal tie to the joint horizontal shear force  $\gamma_h$ can be obtained according to Eqs. (17) and (18), respectively.

$$\tan \alpha = \frac{1}{2} \tan \theta \tag{17}$$

$$\gamma_h = \frac{2\tan\theta - 1}{3} \tag{18}$$

Under the action of horizontal shear  $V_{jh}$ , the compression force of the flat strut  $D_1$ , the tensile force of the horizontal tie  $F_h$  and the compression force of the diagonal strut D can be calculated according to Eqs. (19), (20) and (21), respectively.

$$D_1 = \frac{F_h}{\cos \alpha} \tag{19}$$

$$F_h = \gamma_h \times V_{jh} \tag{20}$$

$$D = \frac{(1 - \gamma_h) \times V_{jh}}{\cos\theta} \tag{21}$$

The effective area of the diagonal strut  $A_{str}$  is defined as

$$A_{str} = a_s \times b_s \tag{22}$$

where  $b_s$  is the width of the effective cross section of diagonal strut, which is equal to the effective width of the beam-column joint.  $a_s$  is the depth of the effective cross section of diagonal strut, which can be determined as

$$a_s = \sqrt{a_b^2 + a_c^2} \tag{23}$$

Considering the value of  $a_b$ , Eqs. (4), (22) and (23) are used simultaneously to calculate the effective cross sectional area of the diagonal strut  $A_{str}$ . After that, the ultimate carrying capacity of the diagonal strut  $D_c$  and the ultimate carrying capacity of the horizontal tie  $F_{hs}$  can be obtained by material strength and geometry dimension of the cross section.

As shown previously, it is roughly assumed that the transverse reinforcements within the center half of the joint core are considered fully effective when computing the tensile force supplied by transverse reinforcements, and the other transverse reinforcements of the joint core are included at a rate of 50%. The starting point of the above treatment is to consider the inhomogeneity of stress

distribution in transverse reinforcements of the joint core. With the aforementioned calculation results, the following decisions can be made. If the contrast result is  $D_c \ge F_{hs}$ , the cracking carrying capacity of the diagonal compression strut  $D_{cr}$  and the yielding carrying capacity of the horizontal tie  $F_{hy}$  should be determined by Eqs. (24)-(26).

$$f_{cr} = 0.31\sqrt{f_c} \tag{24}$$

$$D_{cr} = f_{cr} \times A_{str} \tag{25}$$

$$F_{hy} = 2 \times (1 + 0.5 \times n) \times f_{yv} \times A_s \tag{26}$$

where  $f_{cr}$  is the cracking strength of concrete.  $f_c$  is the compressive strength of concrete cylinder. n represents the number of transverse reinforcements in the joint core area, in addition to the transverse reinforcements within the center half of the joint core.  $f_{yy}$  is the yield stress of transverse reinforcement.  $A_s$  is the sectional area of transverse reinforcement. Then, the sequence order of the generation of cracks in the diagonal compression strut and the yield of transverse reinforcements can be determined. If the contrast result is  $D_c < F_{hs}$ , this means that there are much more transverse reinforcements in the joint core area. According to the ratio of the yield strength to the ultimate strength of transverse reinforcements and the ratio of the cracking strength to the compressive strength of concrete cylinder, the following conclusion can be draw that cracks occur in the diagonal compression strut before the yielding of transverse reinforcements. Under the action of horizontal shear  $V_{jh}$ , the position of the first crack can be estimated by the carrying capacity of the diagonal compression strut and the flat strut.

Take the specimen of reference (Fan et al. 2015) for example, the calculation results are shown in Table 2. By comparison, it can be found that the ultimate carrying capacity of the diagonal strut  $D_c$  is much greater than that of the horizontal tie  $F_{hs}$ . In addition, the horizontal tie bears the greater horizontal shear force before yielding, so that the horizontal tie yielding before the diagonal strut failure. Since the shear resistant mechanism in the softened strutand-tie model is statically indeterminate, the diagonal strut can continue to bear forces after the horizontal tie yielding. According to the calculation results, it is not difficult to find that cracks occur in the area of the strut before transverse reinforcement yielding. Under the action of horizontal shear  $V_{ih}$ , the pressure beared by the flat strut is greater than that beared by the diagonal strut. However, the softened strutand-tie model provides that the cracking carrying capacity of the flat strut is smaller than that of the diagonal strut due to the smaller effective cross sectional area. Therefore, the first crack occurs in the flat strut region. Test results show when the first crack occurs, the strain of transverse

Table 2 Calculation results of improved softened strut-andtie model

-	θ (°)	α (°)	$\gamma_h$	$D_1$	$F_h$	D	Dc (kN)	$F_{hs}$ (kN)	D <sub>cr</sub> (kN)	$F_{hy}$ (kN)
	55.9	36.4	0.65	0.81 $V_{ih}$	$0.65V_{jh}$	$0.62V_{jh}$	985.99	196.12	69.92	169.36



Fig. 8 Test results of cracks development

reinforcement within the center half of the joint core is  $660.5 \times 10^{-6}$ , which is less than the yielding strain of transverse reinforcement ( $1433.3 \times 10^{-6}$ ). And the first crack occurs in the flat strut region, as shown in Fig. 8. This is also confirmed the adaptability of the improved strut-and-tie model in predicting cracks development, on the other hand.

### 7. Conclusions

The main objective of this paper is to introduce and analyze the applicability of two typical models mentioned earlier to interior beam-column joints. For this purpose, the calculation model about shear strength of concrete is derived under shear compression state to investigate the effect of the axial compression ratio. Based on the analytical results present in this paper, the following conclusions can be drawn:

• Four adjustments for the softened strut-and-tie model including modifications of the depth of the diagonal strut, the inclination angle  $\theta$  of diagonal compression strut, the smeared stress of mild steel bars embedded in concrete, as well as the softening coefficient are made. It has been proved by test data that predicted results by the improved softened strut-and-tie model are consistent with the test data and conservative. The average carrying capacity ratio of predicted results by the improved model and experimental results is 0.8847 without considering the reduction factor.

• Two adjustments for the inelastic reinforced concrete joint model including modifications of the confinement effect due to the column axial load and the correction coefficient for high concrete are made. After previously mentioned adjustments made, the average ratio of carrying capacity of beam-column joints computed by the inelastic reinforced concrete joint model and experimental results is 0.949.

• Based on the test results, it is not difficult to find that both the modified inelastic reinforced concrete joint model and the improved softened strut-and-tie model can be used to predict the joint carrying capacity with sufficient accuracy. The proposed prediction method can be used to predict cracks development, and can also be used to determine the sequence order of the generation of cracks in the diagonal compression strut and the yield of transverse reinforcements, and the position of the first crack.

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