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Rapid seismic vulnerability assessment by new regression-based demand and collapse models for steel moment frames

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Abstract. Predictive demand and collapse fragility functions are two essential components of the probabilistic seismic demand analysis that are commonly developed based on statistics with enormous, costly and time consuming data gathering. Although this approach might be justified for research purposes, it is not appealing for practical applications because of its computational cost. Thus, in this paper, Bayesian regression-based demand and collapse models are proposed to eliminate the need of time-consuming analyses. The demand model developed in the form of linear equation predicts overall maximum inter-story drift of the lowto mid-rise regular steel moment resisting frames (SMRFs), while the collapse model mathematically expressed by lognormal cumulative distribution function provides collapse occurrence probability for a given spectral acceleration at the fundamental period of the structure. Next, as an application, the proposed demand and collapse functions are implemented in a seismic fragility analysis to develop fragility and consequently seismic demand curves of three example buildings. The accuracy provided by utilization of the proposed models, with considering computation reduction, are compared with those directly obtained from Incremental Dynamic analysis, which is a computer-intensive procedure.

Keywords: probabilistic seismic demand analysis; demand model; collapse function; Bayesian regression; incremental dynamic analysis

1. Introduction

Probabilistic Seismic Demand Analysis (PSDA) is an approach for computing the probability of attaining or exceeding a specific seismic demand (d) for a given structure under a spectrum of possible earthquakes (Cornell 1996). This concept has been adopted, by Pacific Earthquake Engineering Research (PEER) center, as a "foundation on which structural performance assessment can be based" (http://peer.berkeley.edu/news/2000spring/index.html), and constitutes the basis of some performance-based seismic guidelines, such asFEMA 350 (2000) for steel moment resisting frame (SMRF) buildings. The PSDAis mathematically expressed by

$$P\left[D(IM,\Theta)\geq d\right]=\int P\left[D\left(IM,\Theta\right)\geq d\ |IM=im\right].\left|{}^{dH_{IM}(im)}\!\!\big/_{v}\right| \quad (1)$$

where, H_{IM} (im) is the seismic hazard function of the desired earthquake intensity (im) at the designated site, v represents the mean annual rate of the occurrence of the events of interest, e.g., events with intensity measure greater than a designated minimum value, $D(IM, \Theta)$ reflects seismic demand, and Θ is a vector of unknown parameters in the predictive demand model. In addition, the term $P\left[D\left(IM,\Theta\right)\geq d\ |IM=im\right]$ indicated seismic fragility function computed by

$$\left[1-\Phi\left(\frac{Ln(d)-\lambda_{Ln(D|im)}}{\sigma_{Ln(D|im)}}\right)\right]\times\left(1-P\left(\text{Collapse}\right)\right)+P\left(\text{Collapse}\right) \quad (2)$$

In this equation, $\lambda_{Ln(D|im)}$ and $\sigma_{Ln(D|im)}$ are the median and standard deviation of the seismic demand given im in the logarithmic space, Φ indicates standard normal distribution function, and P(Collapse) denotes the probability of collapse for a given im.

As mathematically expressed, the demand model and collapse fragility function are essential to execute a PSDA. These models are commonly developed based on statistics obtained from nonlinear response history analyses and/or experimental tests. For example, Ramamoorthy *et al*. (2006), based on a large number of nonlinear response history analyses, developed bilinear probabilistic demand models for a hypothetical two-story reinforced concrete (RC) frame building before and after retrofitting. The predictive models of the maximum inter-story drift are utilized in fragility analyses to evaluate the retrofitting impact on reducing the vulnerability of the building during earthquake events. This approach is also applied in some other studies to develop demand models for structural vulnerability assessment under earthquake loading (Adeli *et al*. 2011, Bai *et al*. 2011, Bayat and Daneshjoo 2015, Bayat *et al*. 2015a, b, Ghowsi and Sahoo 2015, Jalali *et al*. 2012,

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Le-Trung *et al*. 2010, O'Reilly and Sullivan 2016, Ruiz-García and Miranda 2010, Tang and Zhang 2011). Some other studies focus on developing demand models based on available deterministic models and experimental tests. This methodology originally presented by Gardoni *et al*. (2002) became the basis of some subsequent studies to develop capacity models for structural element. Some novel works employing this methodology to develop demand models are Choe *et al*. (2008), Sharma *et al*. (2014), Tabandeh and Gardoni (2014), Zhu *et al*. (2007). Although developing probabilistic demand model based on nonlinear response history analyses and/or experimental test results might be adopted for research purposes, it is not appealing for practical purposes because of its computational cost. Thus, developing ready-made demand and collapse fragility function eliminating the need of time-consuming analyses would be of great interest for practical purposes. To this end, Kia and Banazadeh (2016) developed the first-generation ofthe ready-made demand model of the low-to-mid- rise regular steel moment frames, and then, Based on the proposed model, closed-form fragility analyses is performed to demonstrate applicability of this approach.

This paper which is a continuation of our previous work presents next-generation generic demand model and new collapse fragility function of the low-to mid-rise regular steel moment resisting frames (SMRFs). The overall maximum inter-story drift which has been conventionally used as the vulnerability indicator for moment frames is selected as a target demand, and probabilistic model with thelinear formulation in the logarithmic space is developed to predict it. It is worth noting that in a parallel study developing predictive models for two other engineering demand parameters playing important roles in the next-generation PBEE, i.e., maximum story drift ratio and peak absolute floor acceleration, are ongoing. A generic collapse fragility function is also presented and formulated by a cumulative lognormal distribution function to predict structural collapse probability.

This paper is organized into six sections. Following this introduction, next section discusses about methodology implemented to achieve objectives of the paper. Then, a full discussion about generic moment frames is presented. Formulations of probabilistic demand model and collapse function are next presented. Finally, as an application of the proposed probabilistic models, seismic fragility and demand curves of some example buildings are developed, and compared against those obtained from the incremental dynamic analysis.

2. Methodology

In this paper, a generic demand model with the linear formulation in the logarithmic space is proposed to predict overall maximum inter-story drift of the low to mid- rise regular steel moment resisting frames (SMRFs). A number of 81 generic SMRFs are initially developed that represent various geometrical characteristics and different design choices and styles. The frames are then modeled in OpenSees (Mazzoni *et al*. 2006) software using lumped plasticity method.Details about generic frame parameters and the corresponding ranges of assigned values are presented in section "Generic Steel Moment Resisting Frame" along with nonlinear modeling concerns and methods. Afterwards, the frame models are subjected to incremental dynamic analysis (IDA) (Vamvatsikos and Cornell 2002). Within the IDA, algorithmic nonlinear dynamic analyses are performed that utilize increasingly scaled levels of intensity measure (*IM*) for a selected set of ground motion records. The structural responses are then recorded at each *IM* level and scaling continues until structural side-way collapse occurrence is detected. In this paper, the spectral acceleration at the fundamental period of buildings ($Sa(T_1)$) which is suitable for low to mid-rise SMRFs (Adeli *et al*. 2011b, Shome and Cornell 2000)was employed to represent earthquake intensity measure. The overall maximum inter-story drift ($\theta_{\max}$) is also considered as a demand of interest to evaluate the seismic performance of existing buildings. In the present study, a ground motion bin consisting of 82 far-field records is developed. The majority of the ground motion records have been proposed by FEMA P-695(2009) guideline and others are added by the authors utilizing the same selection rules as those used in FEMA P-695. A comprehensive structural data-base is finally established due to these extensive nonlinear dynamic analyses. The data-base is divided into two parts, collapse and non-collapse data. The generated collapse and non-collapse data are then respectively input to Bayesian regression analyses to develop regression-based collapse and demand models. Using the established regression-based models, the need of preparing structure-specific database to develop building demand model and collapse function will be eliminated. Finally, as an application of the proposed probabilistic models, demand curves of some example buildings are developed, and compared against those obtained through the computationally expensive procedure, i.e., IDA.

3. Bayesian regression analysis

A linear regression model consists of 4 parts, including: explanatory functions $h_i(x)$ formulated in terms of some physical measurable independent variables collected in vector "x", regression coefficients "θ_i", the term that reflects model error "ε" and a response variable "y" predicted by the model. It is supposed that ε is a normal random variable with standard deviation equals σ. The mathematical expression of the linear regression model is

$$y = \theta_1 h_1(x) + \theta_2 h_2(x) + \dots\dots\dots + \theta_k h_k(x) + \sigma\varepsilon \tag{3}$$

Even if explanatory functions in Eq. (3) have nonlinear formulation, it can be considered as a linear regression model because of the linear formulation respect to θ_i. In many cases a nonlinear problem can be treated by above linear explanatory function either in a logarithmic space or by discretizing into different linear parts. The main function of regression analysis is to calculate θ_i based on observed data. A classical regression analysis, based on developed finite-size sample population, provides a point estimation of the model parameters. This methodology puts in a specific type of uncertainty named statistical uncertainty to the problem. To address this type of uncertainty, when a regression model is developed, the Bayesian statistical inference was introduced. In Bayesian regression, the statistical uncertainty is explicitly addressed by treating θ_i and σ as random, rather than constant, variables whose probability distributions are determined using Bayesian updating rule

$$f\left(\theta_i,\sigma\right)=c.L\left(\theta_i,\sigma\right).p\left(\theta_i,\sigma\right) \tag{4}$$

where $p(\theta_i, \sigma)$ is the distribution of model parameters before gathering observation, $L(\theta_i, \sigma)$ reflects the probability of observing data corresponding to given θ_i values and is called likelihood function, c is a normalizing factor and $f(\theta_i, \sigma)$ denotes posterior (i.e., updated) distribution of (θ_i, σ) brought about by the new evidence of data.

By assuming a normally distributed error term, ε, and in the case of non-informative priors, Box and Tiao (2011) showed that the posterior distribution of model parameters, θ_i, and squared standard deviation, σ^2, are multivariate t and inverse chi-square distributions, respectively.

$$f(\theta)=\frac{\Gamma(\frac{1}{2}(\nu+k)).s^{-k}\sqrt{\left|H^T H\right|}}{\left[\Gamma(\frac{1}{2})\right]^k\Gamma(\frac{\nu}{2})(\sqrt{\nu})^k}\times\left[1+\frac{(\theta-\hat{\theta})^T H^T H(\theta-\hat{\theta})}{\nu s^2}\right]^{-\frac{\nu-k}{2}}$$

$$f\left(\sigma^2\right)=\upsilon s^2\chi_\upsilon^{-2} \tag{5}$$

$$\hat{\theta}=(H^T H)^{-1}H^T Y \quad ,\nu=n-k$$

$$s^2=\frac{1}{\nu}(Y-\hat{Y})^T(Y-\hat{Y}) \ , \hat{Y}=H\hat{\theta}$$

Where H is a n-by-k dimensional matrix which contains all n observations of explanatory functions and Y is the n-dimensional vector of response variable observations. Once the posterior distribution is known, mean vector M_θ and covariance matrix $\Sigma_{\theta\theta}$ can be readily computed.

4. Generic steel moment resisting frames

4.1 Index frame configurations

One of the main objectives of the present study is to develop generic demand models that can be directly applied in seismic vulnerability analysis of the low-to mid-rise regular SMRFs. It is important to develop astructural model that can balance between generality and accuracy. To this end, the concept of generic SMRF is adopted in this paper. This concept has been employed by various researchers to evaluate theseismic performance of the moment resisting frames (Ruiz-García and Miranda 2010, Chintanapakdee and Chopra 2003, Medina and Krawinkler 2004, Estevaand Ruiz 1989). In these studies, single-bay generic frames were used to simulate theresponse of multi-bay SMRFs. Although single-bay generic frames have been reported to provide reliable results, these frames are not able to realistically simulate the conditions of an interior joint (Zareian and Krawinkler 2006). Thus, in this study, a family of three-bay generic steel moment frames is used. To widen the generality of the obtained demand models, a number of properties of the developed generic frames have been selected to be varied. These properties, opted based on data provided in (Medinaand Krawinkler 2004, Zareian and Krawinkler 2006), include number of stories, N, fundamental period, T, pattern for beams' Strength/Stiffness Distribution, *SSD*, along building height, and the ratio of yield base shear to the building weight, *CY*. In this study, generic SMRFs with the number of stories N equal to 4, 6 and 8 are utilized to represent a range of low to mid-rise structures. For each number of stories, three fundamental periods equal to $0.1N$, $0.15N$ and $0.2N$ are considered to cover a realistic variation range of the SMRFs fundamental periods (Goel and Chopra1997). For each period, three base shear levels are considered for the generic frames. The *CY* ratios corresponding to each level differ for various structures. These ratios, however, are determined in accordance with similar scenarios for all frames. These scenarios establish two lower and upper bounds for the yield base shear, represented by "Low" and "High" *CY* levels, according to the limit values postulated by ASCE 7-10 (2010)for determining response modification factor, R, occupancy group, seismic design category, soil type and over-strength factor, Ω. A "Moderate" *CY* level is also defined which provides an intermediate level of yield base shear. The range of value considered for *CY* is presented in Table 1. In addition, three different patterns for beams' Strength/Stiffness Distribution, *SSD*, along building height are considered. These three patterns are: "Shear"," Uniform" and "Intermediate". The value of *SSD* is calculated by Eq. (6)

$$SSD=\frac{\sum_{i=2}^{N}\left(1+\frac{\left(\frac{I_i}{I_1}+\frac{M_i}{M_1}-2\frac{V_i}{V_1}\right)}{\left(1-\frac{V_i}{V_1}\right)}\right)}{N-1} \tag{6}$$

Where I_i and M_i indicate, respectively, moment of inertia and plastic moment of i^{th} story beams, and V_i represents design story shear calculated at thei^{th} story. According to Eq. (6), the overall *SSD* of a structure is computed by, first, calculating the *SSD* index at each story level using interpolation technique and, then, averaging the story indexes over building height. The *SSD* parameter takes values between 1 and 3 bands which correspond, respectively, to "Shear" and "Uniform" distributions. A "Shear" distribution implies that moment of inertia and bending strength of beams are distributed in proportion to the story shears obtained from applying the design code lateral load pattern. This distribution leads to a straight line deformed shape under mentioned loading (Zareian and Krawinkler 2006). A "Uniform" distribution, on the other hand, suggests equal moments of inertia and bending strengths for all beams along the height. That is, the cross section assigned to the beams of the first story is also considered for the beams of other stories and provide an

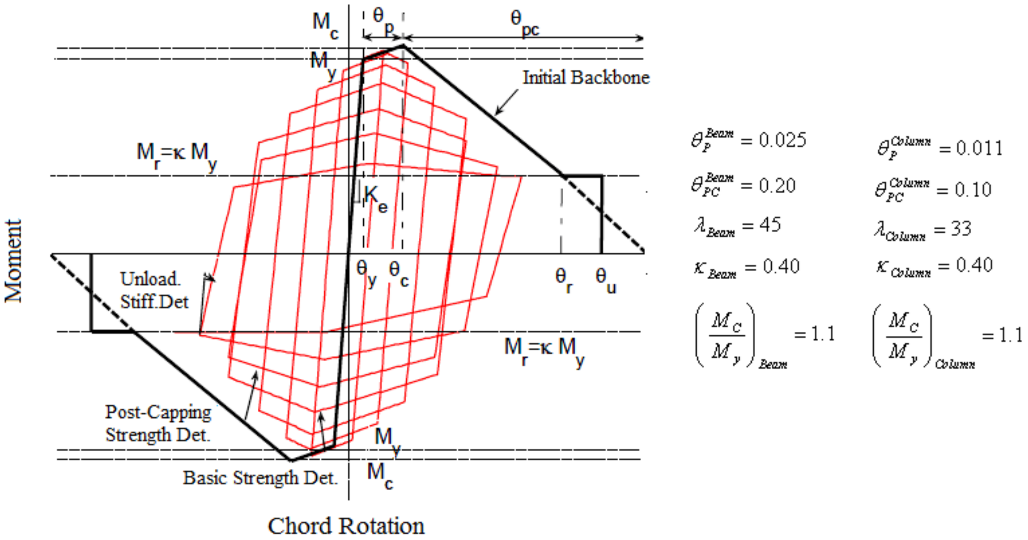

Fig. 1 Modified Ibarra-Medina-krawinkler deterioration model (Reprinted from OpenSees wiki)

upper bound for beams stiffness and strength distributions. This pattern represents those structural designs in which the designer decides to use a similar cross section for beams in several stories because of availability of structural material, the cost of using joints with different detailing, simplicity in design and construction, etc. For intermediate beam strength/stiffness assignments performed in practical SMRF design problems, $1<SSD<3$ values are logically expected. Columns moment of inertia is assumed to be equal to those of the story beams. However, this assumption, by considering the beam length is three times of column height, does not noticeably affect the performance of the studied low-to mid-rise SMRFs due to the minor contribution of columns in lateral deformation of the shear-dominated frames (Zareian and Krawinkler 2006). Moreover, columns strength is assigned to each story with therespect to the strong column–weak beam concept. Considering these parameter values, a total number of 81 generic frames are developed to be subjected to IDA.

5. Frames modeling

Concentrated plasticity concept is utilized to model nonlinear behavior of the generic SMRFs. For this purpose, elastic beam-column elements accompanied by nonlinear rotational springs at both ends are adopted to model members' nonlinearity in OpenSees software. The hysteretic behavior of the rotational springs is modeled using Bilin material with parameters set to the mean value of data obtained from 350 experimental tests conducted on steel beam-to-column connections (Lignos and Krawinkler 2010) (Fig. 1). Basic strength, post capping strength and unloading stiffness deterioration modes are considered in theformulation of this material according to Rahnama and Krawinkler (1993) deterioration rule. *P*-Delta effects have also been accounted for in the models, and a Rayleigh damping matrix (defined as a linear combination of the mass matrix and stiffness matrix) is computed using 2% of the critical damping applied at the first and vibration period corresponding to 90% of the modal mass of the system. To properly model the structure, stiffness proportional damping is applied only to the frame elements and not to the highly rigid truss elements that link the frame and leaning column, nor to the leaning column itself. In addition, zero-length elements have no contribution in the stiffness proportional damping. It is noted, using plastic hinges in combination with Rayleigh damping in a nonlinear dynamic analysis requires special considerations. Without proper modeling of structural elements, damping force become unrealistically large at plastic hinges location. Thus, in the present study, the simple methodology first proposed by Medina and Krawinkler (2004) and then enhanced by Zareianand Krawinkler (2006) is implemented to solve this deficiency.

6. Developing demand model

6.1 General form of model

According to the structural response observations provided by subjecting the generic frames to IDA, a linear model in the logarithmic space is employed to define demand model. This mathematical expression conforms to the perceptional of a structural performance curve (IDA curve). Also, expressing the model in logarithmic space ensures that the model satisfies the normality (implying that model error is normally distributed) and homoscedasticity (denoting that standard deviation of model error is constant) assumptions. Eq. (7) exhibits ageneral form of story-specific demand models developed in the present study.

$$Ln(D) = a + b\,Ln(Sa(T_1)) + u \tag{7}$$

In this equation, overall maximum inter-story drift, *D*, is correlated to first mode spectral acceleration, $Sa(T_1)$, which reflects seismic intensity in units of *g* and includes uncertainty inherent in the earthquake hazard; the *Ln* sign denotes natural logarithm, *u* is a term reflecting model error and is supposed to be a normal random variable with zero mean and unknown standard deviation, σ_D, and $\Theta=(a,b)$ is a vector of unknown normal random model parameters. Application of the demand models in the form of Eq. (7) requires the model parameters a, b, and σ_D to be known. On the other hand, providing structure-specific estimations for these statistical parameters requires collecting a large quantity of observations that is often time-consuming and impractical for design purposes. Therefore, developing regression models to predict mean and standard deviation of the model parameters a, b, and σ_D in terms of generic frame characteristics T, N, SSD, and CY would be of great interest. To develop each of these sub-models, Bayesian statistical inference is implemented to consider statistical uncertainty arising from theuse of finite-size sample population (see section "Bayesian Regression Analysis"). This paper assumes the normal marginal distribution for regression coefficients to reflect statistical uncertainty associated with the proposed regression equations.This assumption is supported by the fact that *t*-distribution, posterior distribution resulting from Bayesian regression, asymptotically approaches a normal distribution when the number of data is large.

In the initial modeling efforts, a variety of sub-model forms in which T, N, SSD, and CY were directly utilized as explanatory variables were examined. Failure of the tried forms to acceptably predict a, b, and σ_D parameters led us to replace each individual explanatory variable by an explanatory function, $h(x)$, in the form of Eq. (8), where the power terms m_1 to m_4 are picked from the vector {-3,-2,-1, 0, 1, 2, 3}. This change led to a large number of different sub-model forms which were successively assessed for selecting the one with lowest regression residual.

$$h(x) = T^{m_1} . N^{m_2} . SSD^{m_3} . CY^{m_4} \tag{8}$$

Although finding the best model form is an essential step in a model fitting program, it is not sufficient for demonstrating the reasonableness of the selected model. Therefore, theassessmentproceeded with plotting the model predictions against observed data, the model residuals versus the predicted values of the dependent variable and quantiles of the model residual against normal theoretical quantiles. The latter which is known as Q-Q plot depicts the residual values against the value of inverse Normal CDF at u/n point, with u being the number of the residual in ordered vector of residuals and n denoting the number of observations. In thecase of normally distributed residuals, the points align with a 45° line. Similarly, the degree to which plotted prediction data align with a 45° line implies the accuracy of the model. In addition, the proposed regression model is acceptable regarding the heteroscedasticity if the residuals fall within fairly horizontal lines on both sides of the zero axis. Further information about regression diagnosis techniques can be found in technical texts (for example Haldar and Mahadevan2000).

7. Overall maximum inter-story drift model

According to the above description, a large number of regression model forms are examined among which some exhibited inadequate prediction while some other suffered from heteroscedasticity, non-normality and/or non-linearity. In conclusion, according to the procedure described at the end of theprevious section, the models that best predict a, b, and σ_D are

$$a = \alpha_1 + \alpha_2 \frac{CY \times T}{N} + \alpha_3 T + \alpha_4 \frac{N^2 \times SSD \times CY^2}{T} + \alpha_5 CY + \sigma\varepsilon \tag{9}$$

$$b = \alpha_1 + \alpha_2 \frac{CY^2}{N^2 \times T^2} + \alpha_3 T + \alpha_4 SSD + \alpha_5 \frac{N}{T^2 \times CY} + \sigma\varepsilon \tag{10}$$

$$\sigma_D = \alpha_1 + \alpha_2 \frac{N^2 \times T^2}{SSD \times CY^2} + \alpha_3 T + \alpha_4 \frac{N^2 \times SSD}{T} + \alpha_5 \frac{1}{T^2 \times CY} + \sigma\varepsilon \tag{11}$$

In these relations, ε is a standard normal random variable, σ is thestandard deviation of the regression model

Table 1 The range of values for seismic yield base shear coefficient, *CY*

		N=4	
	T=0.4	*T*=0.6	*T*=0.8
C_y^{Lower}	0.30	0.25	0.20
$C_y^{Moderate}$	0.45	0.375	0.30
C_y^{Upper}	0.60	0.50	0.40
		N=6	
	T=0.6	*T*=0.9	*T*=1.2
C_y^{Lower}	0.25	0.2	0.15
$C_y^{Moderate}$	0.40	0.35	0.25
C_y^{Upper}	0.60	0.50	0.35
		N=8	
	T=0.8	*T*=1.2	*T*=1.6
C_y^{Lower}	0.20	0.15	0.10
$C_y^{Moderate}$	0.35	0.275	0.18
C_y^{Upper}	0.50	0.40	0.32

Table 2 Posterior statistics of the regression coefficients implemented in Eq. (9)-Eq. (11)

	a		*b*		σ_D	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
α_1	-4.35	0.046	1.01	0.021	0.447	0.0231
α_2	15.22	0.844	0.583	0.14	2.77e-5	2.12e-6
α_3	1.01	0.031	-0.124	0.014	-0.08	0.018
α_4	0.0067	0.001	0.039	0.004	2.42e-4	7.08e-5
α_5	-3.34	0.171	0.003	0.0002	0.01	0.0013
σ	0.067	0.0056	0.03	0.0025	0.032	0.0027

Table 3 Correlation matrix of the regression coefficients implemented in Eq. (9) ~ Eq. (11)

	Correlation matrix of the regression coefficients				
	α_1	α_2	α_3	α_4	α_5
α_1	1.00				
α_2	0.39	1.00			
α_3	-0.86	-0.58	1.00		
α_4	0.45	0.56	-0.38	1.00	
α_5	-0.72	-0.84	0.68	-0.77	1.00
	Correlation matrix of the regression coefficients				
	α_1	α_2	α_3	α_4	α_5
α_1	1.00				
α_2	-0.46	1.00			
α_3	-0.86	0.53	1.00		
α_4	-0.4	0	0	1.00	
α_5	-0.7	0.08	0.54	0	1.00
	Correlation matrix of the regression coefficients				
	α_1	α_2	α_3	α_4	α_5
α_1	1.00				
α_2	0.43	1.00			
α_3	-0.91	-0.60	1.00		
α_4	-0.3	0.10	0.01	1.00	
α_5	-0.84	-0.33	0.74	0.05	1.00

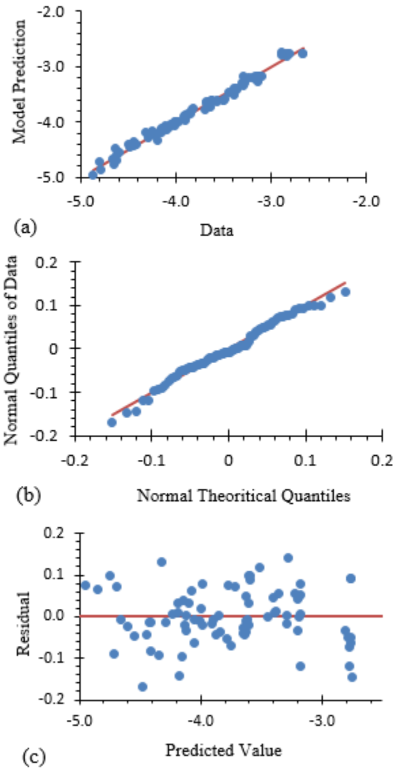

Fig. 2 Graphical diagnoses of the Eq. (9) (a) Prediction model plot; (b) Quantile-Quantile plot to assess the normality; (c) Residual plot to assess homoscedasticity

error and *T*, *N*, *CY*, and *SSD* represent SMRF characteristics previously introduced in section "Index Frame Configurations". Table 2 shows posterior statistics of Eqs. (9) to (11) parameters. The posterior correlation coefficients

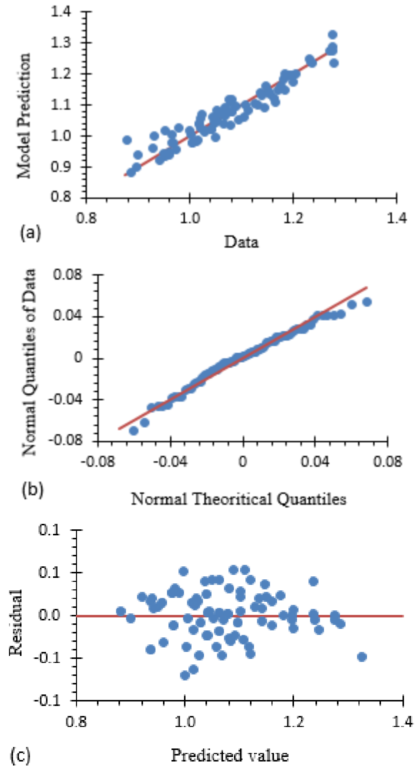

Fig. 3 Graphical diagnoses of the Eq. (10) (a) Prediction model plot; (b) Quantile-Quantile plot to assess the normality; (c) Residual plot to assess homoscedasticity

calculated between the regression coefficients are also presented in Table 3.

In addition, graphical diagnoses of the proposed regression models are performed in Fig. 2, for Eq. (9) regarding the descriptions given in section "General Form of Model". Fig. 2(a) exhibits the median model predictions versus observed data. The points are relatively close to the 45^o line (solid line) which is an indication that the model provides reasonable predictions. In Fig. 2(b), the residual quantiles of the model are plotted against normal theoretical quantiles. Again, the relative alignment of the data points to the 45^o solid line demonstrates the acceptable normality of the model. Also, in Fig. 2(c), the data points are shown to distribute fairly between two horizontal bands on both sides of the vertical axes, which demonstrates homoscedasticity. Similar curves were also obtained for two other equations, i.e., Eqs. (10) and (11), which are respectively demonstrated in Fig. 3 and Fig. 4.

In this paper, the potential dependency between demand model parameters, i.e., *a* and *b*, is also evaluated and regressed in Eq. (12).

$$\rho_{a,b} = Sin\left(\frac{\pi}{2} - \left[1.66 - 0.315\frac{T^2}{N \times CY} - 1.27T \times CY + 1.61CY\right]\right) \qquad (12)$$

8. Collapse fragility function formulation

Fragility function is referred to the conditional

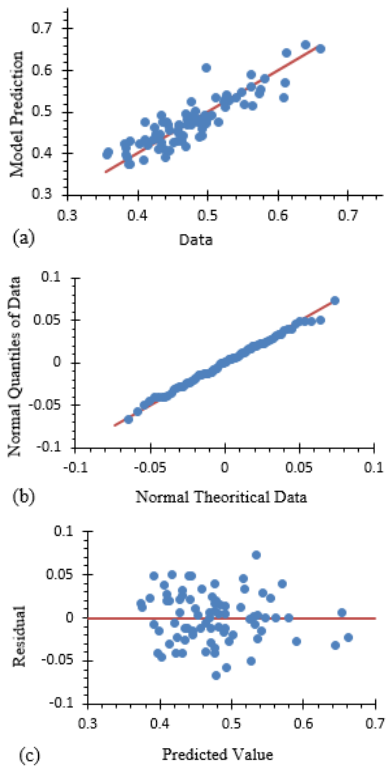

Fig. 4 Graphical diagnoses of the Eq. (11) (a) Prediction model plot; (b) Quantile-Quantile plot to assess the normality; (c) Residual plot to assess homoscedasticity

probability of occurring the collapse limit state given a ground motion with $IM=x$. The collapse fragility function is commonly formulated using conventional lognormal cumulative distribution function (CDF), a choice that has been supported by numerous research programs during the past decade in disparate fields (Lu 2008). It is described mathematically as follow

$$P\left(Collapse \mid IM = x\right) = F\left(x\right) = \int\left[\frac{1}{\sqrt{2\pi}\xi x}\exp\left(-\frac{1}{2}\left(\frac{Ln\left(x\right)-\lambda}{\xi}\right)^2\right)\right]dx \tag{13}$$

where λ and ξ are the two model parameters. These parameters are calculated from the information on the mean μ and standard deviation σ of the sample population by following equations (Rahnama and Krawinkler 1993)

$$\lambda = Ln\left(\mu\right) - \frac{1}{2}\xi^2 \qquad \xi^2 = Ln\left[1+\left(\frac{\sigma}{\mu}\right)^2\right] \tag{14}$$

Developing sample population requires performing time consuming IDA of the structural model under an appropriate set of ground motion records. To eliminate this difficulty, Bayesian regression equations, based on same procedure previously used in developing regression equations for parameters of the demand model, are extracted to estimate mean μ and standard deviation σ of collapse fragility function. Among a variety of

Table 4 Posterior Statistics of the regression coefficients implemented in Eq. (15) & Eq. (16)

	$\mu_{Collapse}$		$\sigma_{Collapse}$	
	Mean	Standard deviation	Mean	Standard deviation
α_1	1.862	0.039	1.683	0.054
α_2	-0.213	0.033	0.125	0.028
α_3	-1.420	0.032	-1.072	0.050
α_4	0.102	0.013	-0.112	0.017
α_5	1.233	0.140	-0.833	0.051
σ	0.074	0.006	0.112	0.009

Table 5 Correlation matrix of the regression coefficients implemented in Eq. (15) & Eq. (16)

	Correlation matrix of the regression coefficients				
	α_1	α_2	α_3	α_4	α_5
α_1	1.00				
α_2	-0.72	1.00			
α_3	0.09	-0.58	1.00		
α_4	-0.29	0.15	0.16	1.00	
α_5	-0.64	0.71	-0.68	-0.28	1.00
	Correlation matrix of the regression coefficients				
	α_1	α_2	α_3	α_4	α_5
α_1	1.00				
α_2	-0.32	1.00			
α_3	-0.58	0.71	1.00		
α_4	-0.37	-0.43	-0.31	1.00	
α_5	-0.24	-0.46	-0.43	0.20	1.00

mathematical expressions examined in this study, the model forms providing best prediction for $\mu_{Collapse}$ and $\sigma_{Collapse}$ are

$$Ln\left(\mu_{Collapse}\right) = \alpha_1 + \alpha_2\frac{1}{N^2 \times CY^2} + \alpha_3 T + \alpha_4\frac{N \times CY}{SSD} + \alpha_5 CY \times T + \sigma\varepsilon \tag{15}$$

$$Ln\left(\sigma_{Collapse}\right) = \alpha_1 + \alpha_2\frac{SSD}{T^2 \times CY \times N^2} + \alpha_3 T + \alpha_4 SSD + \alpha_5\frac{1}{N \times CY} + \sigma\varepsilon \tag{16}$$

The parameters used in these equations are those previously used in Eqs. (9) to (11). Posterior statistics of the regression coefficients and the correlations existing between them are also presented in Tables 4 and 5, respectively. In addition, graphical diagnoses of the regression Eqs. (15) and (16) are respectively performed in Fig. 5 and Fig. 6.

9. Numerical example

As an application of the proposed relations, seismic fragility analysis and PSDA are performed for 4, 5 and 7-story SMRF designed with respect to American Institute of Steel Construction (AISC) and ASCE 7-10 specifications. Contrary to the 4 and 7-story buildings, 5-story building is designed so that it cannot satisfy maximum allowable drift criterion of the ASCE 7-10. The building is rectangular in

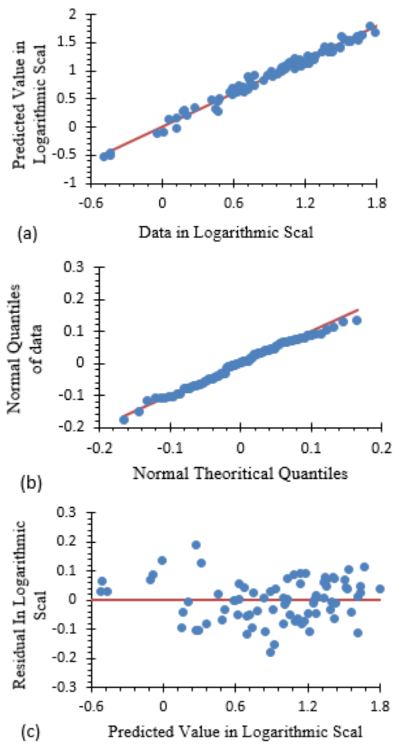

Fig. 5 Graphical diagnoses of the Eq. (15) (a) Prediction model plot; (b) Quantile-Quantile plot to assess the normality; (c) Residual plot to assess homoscedasticity

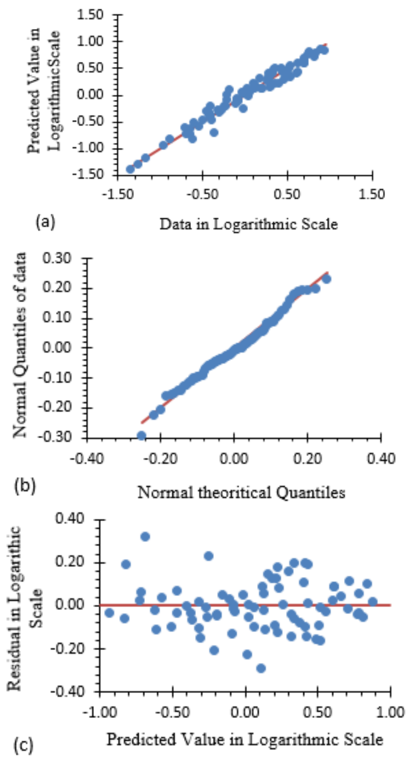

Fig. 6 Graphical diagnoses of the Eq. (16) (a) Prediction model plot; (b) Quantile-Quantile plot to assess the normality; (c) Residual plot to assess homoscedasticity

plan with a length of 22 meters and a width of 16 meters for 5 and 7-story buildings. Square plan with a length of 20

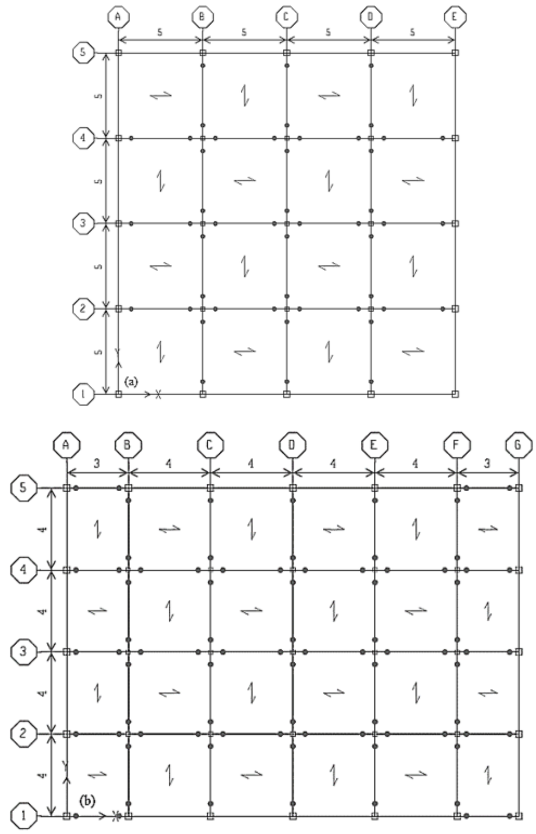

Fig. 7 (a) four-story building Plane (b) five and seven-storybuilding Plane

Table 6 Beams and columns geometries

STORY	4 Story-Building		5 Story-Building		7 Story-Building	
	Beam-Section	Column-Section	Beam-Section	Column-Section	Beam-Section	Column-Section
1	IPE 450	TUBE 400×400×12	IPE 450	TUBE 350×350×12	IPE 550	TUBE 400×400×15
2	IPE 450	TUBE 400×400×12	IPE 450	TUBE 350×350×12	IPE 550	TUBE 400×400×15
3	IPE 400	TUBE 350×350×12	IPE 450	TUBE 350×350×12	IPE 550	TUBE 400×400×15
4	IPE 400	TUBE 350×350×12	IPE 330	TUBE 300×300×12	IPE 500	TUBE 400×400×12
5	----	---	IPE 330	TUBE 300×300×12	IPE 500	TUBE 400×400×12
6	----	----	---	---	IPE 400	TUBE 350×350×12
7	----	----	---	---	IPE 400	TUBE 350×350×12

meters is considered forthe 4-story building. The first story is 2.8 meters high and the height of the remaining stories is 3.2 meters. Two perimeter steel moment frames in each direction along with composite steel deck floors are employed to carry lateral and gravity loads, respectively (Fig. 7). The model takes advantage of the building's regularity so that a two dimensional analytical model can be used to predict structural behavior under seismic loading in each direction. The effect of gravity load system during nonlinear dynamic analysis is also considered by introducing leaning columns. Rigid zones are used to define the joint regions and the inelastic behavior is concentrated

Table 7 The example buildings characteristics

Number of stories	*T*	SSD	CY
4	0.74	2.05	0.29
5	0.88	2.06	0.26
7	1.065	1.79	0.303

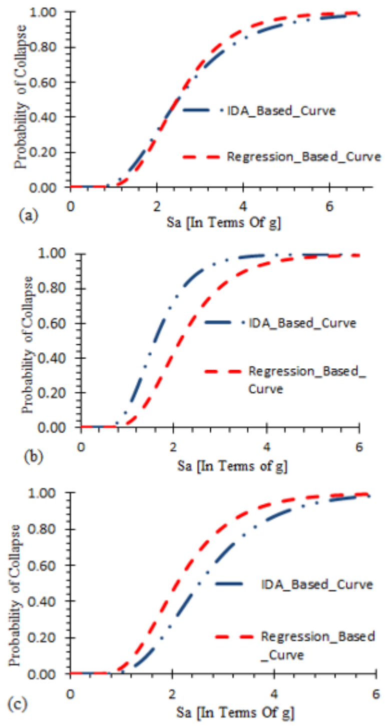

Fig. 8 Collapse curves of (a) four-story building, (b) five-storybuilding, (c) seven-story building

at the end of beam and column elements. Table 6 shows beams and columns geometries. In addition, the building characteristics used as input variables to the generic demand models are demonstrated in Table 7. It is worthwhile to mention that *CY* was directly obtained from pushover analysis. However, this quantity can be approximated by multiplying design base shear by the code-based over-strength factor.Although this approximation poses to some-extent inaccuracy to the problem, it dramatically simplifies a probabilistic decision-making analysis by completely eliminating the need of any structural analysis which would be useful in preliminary design.

10. Collapse fragility curve

IDA of the three example buildings in the longitudinal direction is performed using OpenSees software. Collapse fragility curves are then directly developed based on collapse data obtained from IDA for the selected building, and compare with those developed in a few minutes based on parameters computed by Eqs. (15) and (16) (Fig. 7). Table 8 presents collapse model parameters calculated based on IDA and proposed equations. In Fig. 8, the curve developed according to Eq. (15) and Eq. (16) are shown in red dash line, while the IDA-based curves are illustrated using blue dashed-dotted-dotted lines.

Table 8 Collapse curve parameters

Number of story	Estimated by IDA		Mean value of Eqs. (13) and (14)	
	Mean	Standard Deviation	Mean	Standard Deviation
4	2.76	1.34	2.65	1.04
5	1.71	0.66	2.3	0.92
7	2.75	1.17	2.27	0.96

Table 9 Percentage errors at *Median* & *Median+Std* values of the collapse

Collapse Probability		4-Story Building	5-Story Building	7-Story Building
Median-Std	Sa-Exact	1.57	1.10	1.68
	Sa-predicted	1.69	1.46	1.39
	Percentage-error	**8%**	**33%**	**-17%**
Median	Sa-Exact	2.48	1.60	2.28
	Sa-predicted	2.46	2.13	1.89
	Percentage-error	**-0.8%**	**33%**	**-17%**
Median+Std	Sa-Exact	3.92	2.3	3.8
	Sa-predicted	3.60	3.13	3.13
	Percentage-error	**-8%**	**36%**	**-17%**

According to the presented graphs, the regression equation-based curves are able to capture both the variation trend and the numerical value of IDA-based curves. To numerically examine theaccuracy of the regression equation-based curves, percentage error (the difference between Approximate and Exact values, as a percentage of the Exact value) at the median and median ±Std values of the collapse are computed and presented in Table 9. According to the comparison results, it is concluded that the proposed regression equations provide areasonable approximation of the exact solution for 4 and 7-story buildings, i.e., buildings satisfying seismic design criteria.This level of accuracy of the proposed regression equations would be more appealing, if computational efforts required to developed IDA-based curve are considered. However, for 5-story building, collapse curve developed based on the proposed regression equations only providea rough estimate of the exact solution. In short, Eqs. (15) and (16) can be reasonably applied to rapidly estimate collapse capacity of the buildings satisfying seismic design criteria. However, they are not enough accurate to be implemented for buildings can't pass seismic design code criteria.

11. Seismic fragility analysis

In the following, Seismic fragility analyses are carried out for three example buildings using models developed both based on proposed equations and direct IDA. To this end, based on non-collapse observations obtained from IDA, overall maximum drift demand model in the form of Eq. (7) is first developed for three example buildings, and

Table 10 Demand model parameters computed using IDA

Number of Story	a	B	σ
4	-3.518	0.858	0.408
5	-3.255	0.954	0.355
7	-3.498	0.885	0.427

Table 11 Demand model parameters predicted by generic models (Eq. (9)-Eq. (11))

Number of Story	a	B	σ
4	-3.735	1.075	0.463
5	-3.612	1.053	0.444
7	-3.539	1.007	0.420

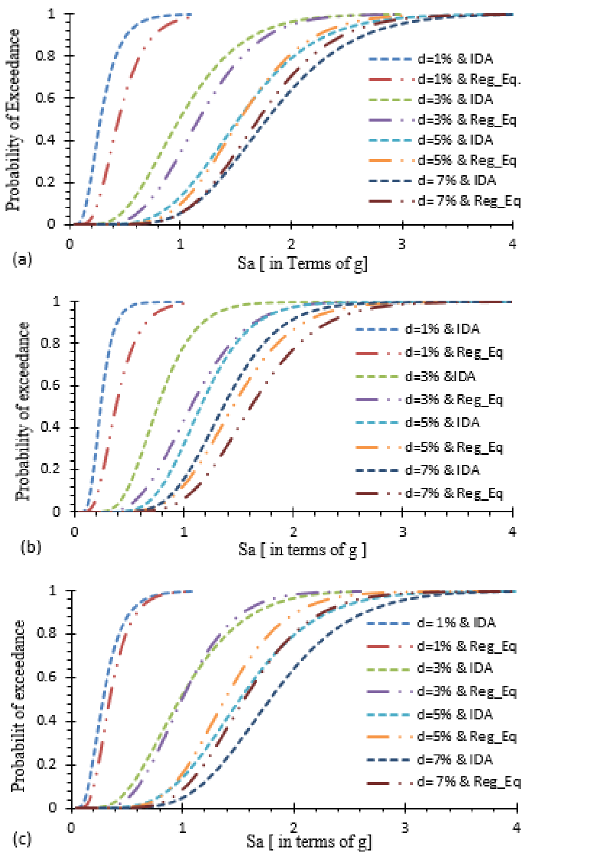

Fig. 9 Fragility curves (a) four-story building, (b) five-storybuilding, (c) seven-story building

then convolved with IDA-based collapse function in the context of Eq. (2) to obtain fragility curve. Fragility curves are, also, extracted using demand model and collapse function developed based on the proposed regression equations. The demand model parameters computed based on IDA and proposed relations, i.e., Eqs. (9), (10) and (11), are respectively presented in Tables 10 and 11. Fragility analysis is performed for thresholds d equal to 1%, 3%, 5%, 7%, and results are presented in the form of fragility curves (Fig. 9). These values are quite an arbitrary choice, but according to FEMA 350 limitation on collapse and immediate occupancy limit-states, it can be expected that selected thresholds reflect light to severe damage states and named SD_1, SD_2, SD_3 and SD_4 respectively.

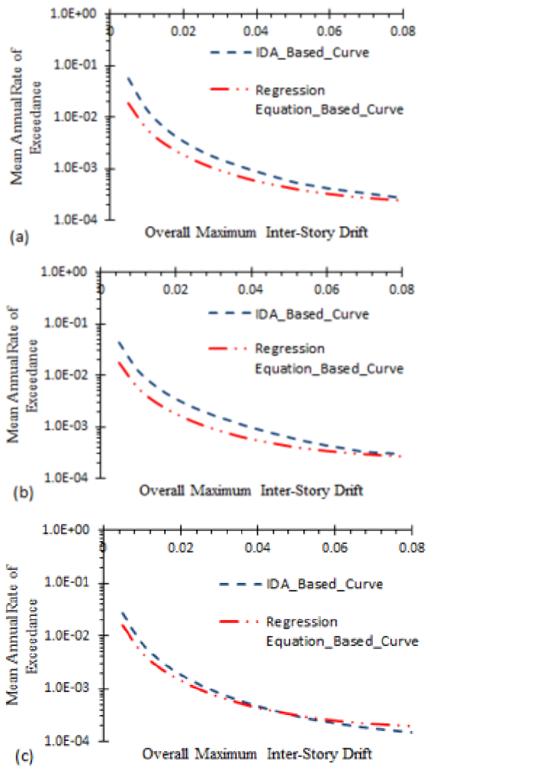

Fig. 10 Demand curve of (a) four-story building, (b) five-story building, (c) Seven- story building

As shown graphically, the curves developed by employing proposed regression equations appropriately follow the curves which are based on building specific demand model and collapse function. To numerically examine theaccuracy of the regression equation-based curves, percentage errors at median and median $\pm Std$ are computed and presented in Table 12. According to the values presented in this table and considering the regression-based fragility curves are developed with much less computational efforts, it is concluded‹for 4 and 7-story buildings‹the proposed regression equations provide a reasonable approximation of the exact solution for damage state with moderate to high consequences i.e., SD_2, SD_3 and SD_4. Although the accuracy level is reduced to some extent for 5-story building, regression-based fragility curves provide an acceptable approximation of the exact solutions for threshold values equal or greater than 5%. In addition, for d=1%, in all example buildings, the accuracy of the regression-based demand model falls significantly. However, this inaccuracy is not important for decision making due to less consequence this damage-state poses to the problem. In general, approximation provided by the developed regression-based demand model and collapse fragility function would be reliable for damage states particularly important for seismic mitigation decisions.

Having provided fragility curves, seismic demand curves can be readily developed for a designated site using Eq. (1). To numerically demonstrate this applicability, the above fragility curves are combined with seismic hazard curves of a site located at 52.665° east and 36.525° north and regressed by

Table 12 Percentage errors at *Median* & *Median+Std* values

	Probability of Exceedance		*d*=1%	*d*=3%	*d*=5%	*d*=7%
4-Story Building	*Median-Std*	Sa-Exact	0.175	0.63	1.05	1.27
		Sa-predicted	0.29	0.80	1.12	1.24
		Percentage-error	**66%**	**27%**	**7.0%**	**-2.0%**
	Media	Sa-Exact	0.29	1.00	1.52	1.78
		Sa-predicted	0.445	1.17	1.54	1.70
		Percentage-error	**53%**	**17%**	**1.0%**	**-4.0%**
	Median+Std	Sa-Exact	0.46	1.50	2.13	2.45
		Sa-predicted	0.69	1.64	2.07	2.28
		Percentage-error	**50%**	**9.0%**	**3.0%**	**-7.0%**
5-Story Building	*Median-Std*	Sa-Exact	0.17	0.53	0.85	1.01
		Sa-predicted	0.25	0.71	1.05	1.19
		Percentage-error	**47%**	**34%**	**24%**	**18%**
	Media	Sa-Exact	0.25	0.73	1.16	1.36
		Sa-predicted	0.38	1.06	1.45	1.61
		Percentage-error	**52%**	**45%**	**25%**	**18%**
	Median+Std	Sa-Exact	0.35	1.07	1.54	1.8
		Sa-predicted	0.59	1.50	1.93	2.14
		Percentage-error	**69%**	**40%**	**25%**	**19%**
7-Story Building	*Median-Std*	Sa-Exact	0.16	0.62	1.05	1.3
		Sa-predicted	0.24	0.69	1.01	1.15
		Percentage-error	**50%**	**11%**	**-4.0%**	**-12%**
	Media	Sa-Exact	0.30	0.99	1.53	1.8
		Sa-predicted	0.36	1.01	1.39	1.57
		Percentage-error	**20%**	**2%**	**-9.0%**	**-13%**
	Median+Std	Sa-Exact	0.48	1.50	2.12	2.72
		Sa-predicted	0.54	1.42	1.87	2.44
		Percentage-error	**13%**	**-5%**	**-12%**	**-10%**

$$\lambda_{T=0.74}(Sa) = 0.0011(Sa)^{-1.74}. \quad (17)$$

$$\lambda_{T=0.88}(Sa) = 0.0009(Sa)^{-1.702} \quad (18)$$

$$\lambda_{T=1.065}(Sa) = 0.0006(Sa)^{-1.715} \quad (19)$$

Where $\lambda(Sa)$ indicates the mean annual frequency and, indirectly represent the annual probability of exceedance of the *Sa*. Fig. 10 graphically illustrates differences between demand curves developed based on regression equations and those obtained from IDA. The results demonstrate the curves developed with much less computational efforts appropriately follow IDA-based curves, especially at the tail of the distribution where the probability is low but the consequence is high. It is worth noting, this part of the curve is particularly important for seismic mitigation decisions.

12. Conclusions

Seismic demand analysis is a probabilistic decision-making analysis which is widely implemented in research to evaluate the vulnerability of a building under earthquake loading. However, performing a probabilistic seismic demand analysis requires ingredients commonly developed using nonlinear response history analyses and/or experimental test results. Although, this methodology can be suitable for research purposes, it couldn't be appealing for practical purposes because of its computational costs. Thus, the focus of the present study is on the developing ready-made drift demand model and collapse fragility function, two essential ingredients of a probabilistic decision-making analysis, to eliminate the need of time-consuming data gathering procedure, when a probabilistic decision-making analysis is performed for a regular low to mid-rise steel moment resisting frames. The applicability of the proposed regression equations are investigated by doing seismic fragility analysis and probabilistic seismic demand analysis for three example buildings and results are compared with those obtained directly from IDA. The results indicate, although the proposed regression-based demand model and collapse fragility function could not be able to provide exact results, they are enough accurate to be used for rapid seismic vulnerability assessment, especially for buildings satisfying seismic design code criteria or the probability of high consequences events is sought. Note, this level of accuracy is achieved with low computational cost in comparison with the conventional method which is based on time-consuming building specific nonlinear response history analyses. In fact, the authors believe theavailability of such explicit regression-based models along with traditional design regulations helps extend a technical basis which is not only minimizing casualties by preventing structural collapse, but also makes it possible to approximate lyevaluate seismic vulnerability of a building that would be useful for decision making.

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