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On the effect of the micromechanical models on the free vibration of rectangular FGM plate resting on elastic foundation

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Abstract. In this research work, free vibrations of simply supported functionally graded plate resting on a Winkler-Pasternak elastic foundation are investigated by a new shear deformation theory. The influence of alternative micromechanical models on the macroscopic behavior of a functionally graded plate based on shear-deformation plate theories is examined. Several micromechanical models are tested to obtain the effective material properties of a two-phase particle composite as a function of the volume fraction of particles which continuously varies through the thickness of a functionally graded plate. Present theory exactly satisfies stress boundary conditions on the top and the bottom of the plate. The energy functional of the system is obtained using Hamilton's principle. The closed form solutions are obtained by using Navier technique, and then fundamental frequencies are found by solving the results of eigenvalue problems. Finally, the numerical results are provided to reveal the effect of explicit micromechanical models on natural fundamental frequencies.

Keywords: FG plates; micromechanical models; Winkler-Pasternak elastic foundation; a new shear deformation theory; free vibration

1. Introduction

Functionally graded materials (FGMs) are the advanced materials in which thermal-mechanical properties are varied continuously as a function of position along certain dimension(s) of the structure to achieve a required function. The concept of FGMs was first proposed in 1984 by a group of material scientists in Japan (Koizumi 1993, 1997). Typically, FGMs are made from a mixture of metals and ceramics. Due to excellent characteristics of ceramics in heat and corrosive resistances combined with the toughness of metals, the interlaminar stresses at the interface would vanish (Meksi *et al.* 2018, Bellifa *et al.* 2017, Ait Yahia *et al.* 2015, Abualnour *et al.* 2018, Bousahla *et al.* 2014, Zidi *et al.* 2014).

Due to the increased relevance of the FGMs structural components in the design of aerospace and engineering structures, their vibration characteristics have attracted the attention of many scientists in recent years.

Using two-dimensional higher-order plate theory, Matsunaga (2008) developed a set of fundamental dynamic equations and presented analytical solutions for simply-supported rectangular plates, while Qian *et al.* (2004) utilized a meshless local Petrov-Galerkin method to solve the governing equations of higher-order shear and normal deformable plate theory and to elucidate the static deformation and vibration behaviors of square plates. Bouderba *et al.* (2016) studied the thermal buckling response of functionally graded sandwich plates with various boundary conditions by using a simple first-order shear deformation theory.

Vel and Batra (2004) proposed three-dimensional solutions for vibrations of simply supported rectangular plates. Reddy (2000) presented the formulation and finite element models for static and dynamic analysis of FG plates using the third-order shear deformation theory. Ferreira *et al.* (2006) analyzed the free vibration of FG plates based on the first and third-order shear deformation theories using the Mori-Tanaka homogenization method and the global collocation method with multiquadratic radial basis functions. Abrate (2006) studied the free vibration, static buckling, and static deformation of the FGM plates. It was shown that the natural frequencies, buckling loads and static deflections of functionally graded plates are always proportional to those of homogeneous isotropic plates. Dehghan and Baradaran (2011) solved the eigenvalue equations based on a mixed finite element (FE) and differential quadrature (DQ) method to obtain the natural frequency and buckling load parameters. Amal Alshorbagy *et al.* (2011) studied the free vibration analysis of FG beams is investigated using numerical finite element method.

Talha and Singh (2010) studied free vibration and static analysis of FGM plates using modified HSDT kinematics. The fundamental equations are obtained using variational principle by considering the stress free boundary conditions

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at the top and bottom faces of the plate.

Hadji *et al.* (2011) investigated the free vibration analysis of functionally graded material (FGM) sandwich rectangular plates. The theory presented is variationally consistent and strongly similar to the classical plate theory in many aspects. It does not require the shear correction factor, and gives rise to the transverse shear stress variation so that the transverse shear stresses vary parabolically across the thickness to satisfy free surface conditions for the shear stress. Benachour *et al.* (2011) developed a model for free vibration analysis of plates made of functionally graded materials with an arbitrary gradient. Closed form solutions are obtained by using Navier technique, and then fundamental frequencies are found by solving the results of eigenvalue problems. Attia *et al.* (2015) have studied the free vibration of temperature-dependent functionally graded (FG) plates.

Tounsi *et al.* (2013) presented a refined trigonometric shear deformable plate theory for thermoelastic bending of FGM sandwich plates. The same theory was used to study the mechanical behavior of FG plates (Bourada *et al.* 2012, 2012, Kettaf *et al.* 2013, Khalfi *et al.* 2014, Attia *et al.* 2015, Bouderba *et al.* 2013, Ait Amar Meziane *et al.* 2014, Draiche *et al.* 2014, Nedri *et al.* 2014, Sadoune *et al.* 2014). Bennoun *et al.* (2016) presented a new five variable refined plate theory for vibration analysis of functionally graded sandwich plates.

Hamidi *et al.* (2015) developed a sinusoidal plate theory for the thermomechanical bending analysis of functionally graded sandwich. This theory has five unknowns and incorporates the effect of stretching. Belabed *et al.* (2014) presented an efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates. Their theory accounts for both shear deformation and thickness stretching effects. Bessaim *et al*. (2013) have presented a new higher-order shear and normal deformation theory for the bending and free vibration analysis of sandwich plates with FG face sheets. Sekkal *et al.* (2017) presented an analytical solution for buckling and dynamic problems of functionally graded plates. They propose a new quasi-3D higher shear deformation theory. Zine *et al.* (2018) analyzed the bending and free vibration of multilayered plates and shells by utilizing a new higher order shear deformation theory (HSDT).

Recently, Tounsi *et al.* (2016) studied the buckling and vibration of functionally graded sandwich plate by using a new 3-unknowns non-polynomial plate theory. Houari *et al*. (2016) used the same theory to analyze the bending and free vibration analysis of functionally graded (FG) plates.

Because of the widespread applications of foundations in engineering, the interaction between engineering components and elastic media has been considered in many researches recently.

To describe the interactions of the plate and foundation as more appropriate as possible, scientists have proposed various kinds of foundation models.

The mechanical behavior of one-parameter elastic foundation was first discussed by Winkler (1867), whereas Pasternak (1954) considered a two-parameter model in analyzing elastic foundations. Kerr (1964) presented various models for elastic and viscoelastic foundations.

Based on third-order shear deformation theory, Baferani *et al*. (2011) presented an accurate solution for free vibration of functionally graded thick rectangular plates resting on elastic foundation.

Benyoucef *et al*. (2010) analyzed the bending response of FG thick plate resting on elastic foundation by using a new hyperbolic displacement model. Yaghoobi and Yaghoobi (2013) studied the buckling behavior of symmetric sandwich plates with FG face sheets resting on an elastic foundation using the first-order shear deformation plate theory (FSDPT) and subjected to mechanical, thermal and thermo-mechanical loads. Bouderba *et al*. (2013) used a refined plate theory to investigate the thermomechanical bending response of functionally graded plates resting on Winkler-Pasternak elastic foundations. Recently, Khalfi *et al*. (2013) studied the thermal buckling of solar functionally graded plate (SFGP) resting on two-parameter Pasternak's foundations using a refined and simple shear deformation theory. Also the same plate theory which accounted for a quadratic variation of the transverse shear strains across the thickness of functionally graded plates resting on elastic foundation was developed by Thai and Choi (2011). Yaghoobi and Fereidoon (2014) presented a simple refined nth-order shear deformation theory for mechanical and thermal buckling analysis of FG plates resting on elastic foundations.

To accurately model FGM, knowing the effective or bulk material properties as a function of individual material properties and geometry especially at micromechanics level is essential. In the last few years, different models have been proposed to estimate the effective properties of FGMs with respect to reinforcement volume fractions (Shen and Wang 2012, Jha *et al.* 2013).

Weng (2003) investigated the effective bulk moduli of two functionally graded composites by means of change of the dependent variable. Rahman and Chakraborty (2007) proposed a stochastic micromechanical model for predicting probabilistic characteristics of three phase FGMs. Pindera *et al.* (1995) used a computational micromechanical model, the generalized method of cells, to predict local stress in the fiber and matrix phases of FGMs. Fang *et al.* (2007), developed a micromechanics-based elastodynamic model to predict the dynamic behavior of two-phase functionally graded materials. Zuiker (1995) reviewed the micromechanical modeling of FGMs and concluded that the self-consistent method (SCM) provided good estimates, with minimal effort, and with no need for empirical fitting of parameters for the silicon carbide (SiC) - carbon (C) FGMs. Gasik (1998) studied the efficiency of the simplest micromechanical models to provide the most accurate estimates of FGM components with an arbitrary nonlinear three-dimensionally orientation of phases. Reiter and Dvorak (1997) used the transition function with Mori-Tanaka method (MTM) and SCM to predict the thermo-mechanical properties in C/SiC FGMs. Kar and Panda (2015) investigated the free vibration responses of shear deformable functionally graded single/doubly curved panels under uniform, linear and nonlinear temperature fields. The micromechanical material model of functionally graded material was computed using Voigt model in conjunction with the power-law distribution.

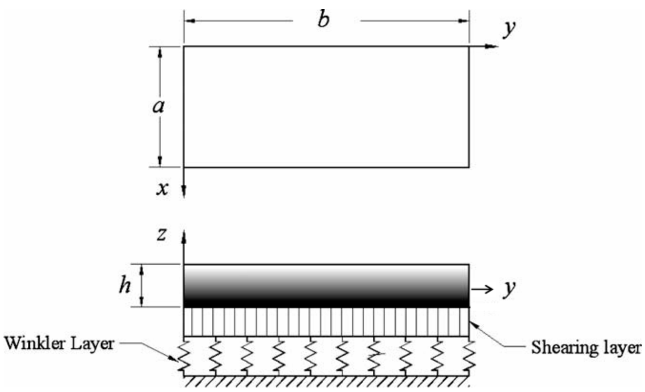


Fig. 1 FGM plate resting on elastic foundation

To assess the effect of the micromechanical models on vibrational response of a simply supported FG plates resting on an elastic foundation, a new shear deformable plate theory is presented. Different micromechanical models are examined to obtain the effective material properties of FGMs with power-law function distributions of volume fraction within the thickness of the plate. Using an analytical method, the governing equations are treated and the effects of Voigt, Reuss, LRVE, Tamura and Mori-Tanaka models on the natural fundamental frequencies of the FG plate resting on elastic foundation are investigated.

2. Effective properties of FGMs

Unlike traditional microstructures, in FGMs the material properties are spatially varying, which is not trivial for a micromechanics model Yu and Kidane 2014).

A number of micromechanics models have been proposed for the determination of effective properties of FGMs. In what follows, we present some micromechanical models to calculate the effective properties of the FG plate.

2.1 Voigt model

The Voigt model is relatively simple; this model is frequently used in most FGM analyses estimates Young's modulus E of FGMs as (Mishnaevsky Jr. 2007, Zimmerman 1994)

$$E(z) = E_i\, VF(z) + E_m(1 - VF(z)) \tag{1}$$

2.2 Reuss model

Reuss assumed the stress uniformity through the material and obtained the effective properties as (Mishnaevsky Jr. 2007, Zimmerman 1994)

$$E(z) = \frac{E_i\, E_m}{E_i(1 - VF(z)) + E_m VF(z)} \tag{2}$$

2.3 Tamura model

The Tamura model uses actually a linear rule of mixtures, introducing one empirical fitting parameter known as "stress-to-strain transfer" (Gasik 1995, Zuiker 1995)

$$q = \frac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \tag{3}$$

Estimate for q=0 correspond to Reuss rule and with q=±∞ to the Voigt rule, being invariant to the consideration of with phase is matrix and which is particulate. The effective Young's modulus is found as

$$E(z) = \frac{(1 - VF(z))\, E_m\left(q - E_i\right) + VF(z) E_i\left(q - E_m\right)}{(1 - VF(z))\left(q - E_i\right) + VF(z) E_i\left(q - E_m\right)} \tag{4}$$

2.4 Description by a representative volume element (LRVE)

The local representative volume element (LRVE) is based on a "mesoscopic" length scale which is much larger than the characteristic length scale of particles (inhomogeneities) but smaller than the characteristic length scale of a macroscopic specimen (Ju and Chen 1994). The LRVE is developed based on the assumption that the microstructure of the heterogeneous material is known. The input for the LRVE for the deterministic micromechanical framework is usually volume average or ensemble average of the descriptors of the microstructures.

Young's modulus is expressed as follows by the LRVE method (Akbarzadeh *et al*. 2015)

$$E(z) = E_m\left(1 + \frac{VF(z)}{FE - \sqrt[3]{VF(z)}}\right), \quad FE = \frac{1}{1 - \dfrac{E_m}{E_i}} \tag{5}$$

2.5 Mori-Tanaka model

According to Mori-Tanaka homogenization scheme, the effective Bulk Modulus (K) and the effective shear modulus (G) are given by (Belabed *et al*. 2014, Benveniste 1987, Mori and Tanaka 1973)

$$E(z) = E_m + \left(E_c - E_m\right)\left(\frac{V_C}{1 + \left(1 - V_C\right)\left(E_c / E_m - 1\right)\left(1 + \upsilon\right)/\left(3 - 3\upsilon\right)}\right) \tag{6a}$$

Where

$$V_C = \left(0,5 + \frac{z}{h}\right)^p \tag{6b}$$

In all models outlined above, E_i, V_i (i=c,m) are the Young's modulus and the volume fraction of the phase material respectively. The subscripts c and m refer to the ceramic and metal respectively. The volume fractions of the ceramic and metal phases are related by V_c+V_m=1, and V_c is expressed as

$$V_c = \left(0,5 + \frac{z}{h}\right)^p, \quad p \geq 0 \tag{7}$$

The effective mass density ρ is given by the rule mixtures as (Natarajan *et al.* 2011, Benachour *et al.* 2011, Bessaim *et al.* 2013, Yaghoobi and Torabi 2013, Tounsi *et al.* 2013, Ould Larbi *et al.* 2013, Hebali *et al.* 2014), regardless of the utilized micromechanical models

$$\rho = \rho_c V_c + \rho_m V_m \tag{8}$$

3. Kinematics and strains

In this study, further simplifying supposition are made to the conventional HSDT so that the number of unknowns is reduced. The displacement field of the conventional HSDT is given by

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + f(z)\varphi_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + f(z)\varphi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{9}$$

u_0, v_0, w_0, φ_x and φ_y are the five unknown displacement of the mid-plane of the plate. By considering that $\varphi_x = k_1\int \theta(x,y,t)\,dx$ and $\varphi_y = k_2\int \theta(x,y,t)\,dy$ (Bourada *et al.* 2016).

The displacement fields mentioned above can be written as follows

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,y,t)\,dx \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2 f(z)\int \theta(x,y,t)\,dy \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{10}$$

The integrals defined in the above equations shall be resolved by a Navier type method and the displacement fields can be rewritten as

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1 A' f(z)\frac{\partial \theta}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2 B' f(z)\frac{\partial \theta}{\partial y} \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{11}$$

Where

$$k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{12}$$

The coefficients A' and B' are expressed according to the Navier type solution and they are given by

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2} \tag{13a}$$

And

$$\alpha = \frac{m\pi}{a}, \quad \beta = \frac{n\pi}{b} \tag{13b}$$

The shape function *f*(*z*) is given as follows

$$f(z) = z - \frac{h\sinh\left(\frac{10z}{h}\right)}{10\cosh(5)} + \frac{h}{100} \tag{14a}$$

The transverse shear strain function is an even function which is the first derivation of the shape function ($g(z)=f'(z)$). Therefore, the shear shape function is presented in the present theory to satisfy zero stresses at top and bottom surfaces of the plate. The shear function is obtained as

$$f(z) = z - \frac{h\sinh\left(\frac{10z}{h}\right)}{10\cosh(5)} + \frac{h}{100} = z - \frac{h\dfrac{e^{\left(\frac{10z}{h}\right)} - e^{\left(\frac{-10z}{h}\right)}}{2}}{10\dfrac{e^{(5)}+e^{(-5)}}{2}} + \frac{h}{100} \tag{14b}$$

$$g(z) = 1 - \frac{e^{\left(\frac{10z}{h}\right)} + e^{\left(\frac{-10z}{h}\right)}}{e^{(5)}+e^{(-5)}} \quad \text{where} \quad g\left(\pm\frac{h}{2}\right) = 0 \tag{14c}$$

It can be seen that the displacement field in Eqs. (10) and (11) contains only four unknowns u_0, v_0, w_0 and θ.

The kinematic relations can be obtained as follows

$$\begin{aligned} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} &= \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \\ \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} &= g(z)\begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} \end{aligned} \tag{15}$$

Where

$$\begin{aligned} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} &= \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \\ \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} &= \begin{Bmatrix} k_1 A'\dfrac{\partial^2 \theta}{\partial x^2} \\ k_2 B'\dfrac{\partial^2 \theta}{\partial y^2} \\ (k_1 A' + k_2 B')\dfrac{\partial^2 \theta}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} = \begin{Bmatrix} k_2 B'\dfrac{\partial \theta}{\partial y} \\ k_1 A'\dfrac{\partial \theta}{\partial x} \end{Bmatrix} \end{aligned} \tag{16}$$

4. Constitutive relations

The linear constitutive relations are

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \frac{E(z)}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{(1-\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & \dfrac{(1-\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & \dfrac{(1-\nu)}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{17}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{yx}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{yx}) are the stress and strain components, respectively.

5. Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as

$$0=\int_0^T(\delta U+\delta U_F-\delta K)dt \tag{18}$$

Where δU is the variation of strain energy; δK is the variation of kinetic energy; and δU_F is the variation of strain energy of foundation.

The variation of strain energy of the plate stated as

$$\delta U=\int_A\int\begin{matrix}(\sigma_x\,\delta\varepsilon_x+\sigma_y\,\delta\varepsilon_y+\tau_{xy}\,\delta\gamma_{xy}\\+\tau_{xz}\,\delta\gamma_{xz}+\tau_{yz}\,\delta\gamma_{yz})dA\,dz\end{matrix} \tag{19}$$

Substituting Eqs. (15) and (17) into Eq.(19) and integrating through the thickness of the plate, Eq. (19) can be rewritten as

$$\delta U=\int_A\left\{\begin{matrix}N_x\dfrac{\partial\delta u_0}{\partial x}-M_x^b\dfrac{\partial^2\delta w_0}{\partial x^2}+k_1A'M_x^s\dfrac{\partial^2\delta\theta}{\partial x^2}\\+N_y\dfrac{\partial\delta v_0}{\partial y}-M_y^b\dfrac{\partial^2\delta w_0}{\partial y^2}+k_2B'M_y^s\dfrac{\partial^2\delta\theta}{\partial y^2}\\+N_{xy}(\dfrac{\partial\delta u_0}{\partial y}+\dfrac{\partial\delta v_0}{\partial x})-2M_{xy}^b\dfrac{\partial^2\delta w_0}{\partial x\partial y}\\+\left(k_1A'+k_2B'\right)M_{xy}^s\dfrac{\partial^2\delta\theta}{\partial x\partial y}+k_1A'S_{xz}^s\dfrac{\partial\delta\theta}{\partial x}\\+k_2B'S_{yz}^s\dfrac{\partial\delta\theta}{\partial y}\end{matrix}\right\}dA \tag{20}$$

The stress resultants N, M, P, Q and R are defined by

$$\begin{Bmatrix}N_x, & N_y, & N_{xy}\\M_x^b, & M_y^b, & M_{xy}^b\\M_x^s, & M_y^s, & M_{xy}^s\end{Bmatrix}=\int_{-h/2}^{h/2}\left(\sigma_x,\sigma_y,\tau_{xy}\right)\begin{Bmatrix}1\\z\\f(z)\end{Bmatrix}dz,$$
$$\left(S_{xz}^s,S_{yz}^s\right)=\int_{-h/2}^{h/2}\left(\tau_{xz},\tau_{yz}\right)g(z)dz. \tag{21}$$

Using Eq. (17) in Eq. (21), the stress resultants of the FG plate can be related to the total strains by

$$\begin{Bmatrix}N\\M^b\\M^s\end{Bmatrix}=\begin{bmatrix}A & B & B^s\\B & D & D^s\\B^s & D^s & H^s\end{bmatrix}\begin{Bmatrix}\varepsilon\\k^b\\k^s\end{Bmatrix},\ \ S=A^s\gamma \tag{22}$$

Where

$$N=\left\{N_x,N_y,N_{xy}\right\}^t,\ M^b=\left\{M_x^b,M_y^b,M_{xy}^b\right\}^t,\quad M^s=\left\{M_x^s,M_y^s,M_{xy}^s\right\}^t \tag{23a}$$

$$\varepsilon=\left\{\varepsilon_x^0,\varepsilon_y^0,\gamma_{xy}^0\right\}^t,\ k^b=\left\{k_x^b,k_y^b,k_{xy}^b\right\}^t,\quad k^s=\left\{k_x^s,k_y^s,k_{xy}^s\right\}^t \tag{23b}$$

$$A=\begin{bmatrix}A_{11} & A_{12} & 0\\A_{12} & A_{22} & 0\\0 & 0 & A_{66}\end{bmatrix},\ B=\begin{bmatrix}B_{11} & B_{12} & 0\\B_{12} & B_{22} & 0\\0 & 0 & B_{66}\end{bmatrix},\quad D=\begin{bmatrix}D_{11} & D_{12} & 0\\D_{12} & D_{22} & 0\\0 & 0 & D_{66}\end{bmatrix} \tag{23c}$$

$$B^s=\begin{bmatrix}B_{11}^s & B_{12}^s & 0\\B_{12}^s & B_{22}^s & 0\\0 & 0 & B_{66}^s\end{bmatrix},\ D^s=\begin{bmatrix}D_{11}^s & D_{12}^s & 0\\D_{12}^s & D_{22}^s & 0\\0 & 0 & D_{66}^s\end{bmatrix},\quad H^s=\begin{bmatrix}H_{11}^s & H_{12}^s & 0\\H_{12}^s & H_{22}^s & 0\\0 & 0 & H_{66}^s\end{bmatrix} \tag{23d}$$

$$S=\left\{S_{yz}^s,S_{xz}^s\right\}^t,\ \gamma=\left\{\gamma_{yz},\gamma_{xz}\right\}^t,\quad A^s=\begin{bmatrix}A_{44}^s & 0\\0 & A_{55}^s\end{bmatrix} \tag{23e}$$

Where A_{ij}, D_{ij} …etc., are the plate stiffness, defined by

$$\begin{Bmatrix}A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s\\A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s\\A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}Q_{11}\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)\begin{Bmatrix}1\\\nu\\\dfrac{1-\nu}{2}\end{Bmatrix}dz \tag{24a}$$

Where

$$Q_{11}=\frac{E(z)}{1-\nu^2}, \tag{24b}$$

And

$$\left(A_{22},B_{22},D_{22}\right)=\left(A_{11},B_{11},D_{11}\right)$$
$$\left(B_{22}^s,D_{22}^s,H_{22}^s\right)=\left(B_{11}^s,D_{11}^s,H_{11}^s\right) \tag{24c}$$

$$A_{44}^s=A_{55}^s=\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{E(z)}{2(1+\nu)}\left[g(z)\right]^2dz, \tag{24d}$$

The variation of kinetic energy is expressed as

$$\delta K=\int_V\left(\dot u\delta\,\dot u+\dot v\delta\,\dot v+\dot w_0\delta\,\dot w_0\right)\rho(z)dA\,d\,z \tag{25}$$

$$\delta K=\int_A \left\{\begin{aligned}&I_0[\dot u_0\delta\dot u_0+\dot v_0\delta\dot v_0+\dot w_0\delta\dot w_0]-I_1[\dot u_0\frac{\partial\delta\dot w_0}{\partial x}\\&+\frac{\partial\dot w_0}{\partial x}\delta\dot u_0+\dot v_0\frac{\partial\delta\dot w_0}{\partial y}+\frac{\partial\dot w_0}{\partial y}\delta\dot v_0]+J_1[k_1A'\dot u_0\frac{\partial\delta\dot\theta}{\partial x}\\&+k_1A'\frac{\partial\dot\theta}{\partial x}\delta\dot u_0+k_2B'\dot v_0\frac{\partial\delta\dot\theta}{\partial y}+k_2B'\frac{\partial\dot\theta}{\partial y}\delta\dot v_0]\\&+I_2[\frac{\partial\dot w_0}{\partial x}\frac{\partial\delta\dot w_0}{\partial x}+\frac{\partial\dot w_0}{\partial y}\frac{\partial\delta\dot w_0}{\partial y}]+K_2[(k_1A')^2\frac{\partial\dot\theta}{\partial x}\frac{\partial\delta\dot\theta}{\partial x}\\&+(k_2B')^2\frac{\partial\dot\theta}{\partial y}\frac{\partial\delta\dot\theta}{\partial y}]-J_2[k_1A'\frac{\partial\dot w_0}{\partial x}\frac{\partial\delta\dot\theta}{\partial x}\\&+k_1A'\frac{\partial\dot\theta}{\partial x}\frac{\partial\delta\dot w_0}{\partial x}+k_2B'\frac{\partial\dot w_0}{\partial y}\frac{\partial\delta\dot\theta}{\partial y}+k_2B'\frac{\partial\dot\theta}{\partial y}\frac{\partial\delta\dot w_0}{\partial y}]\end{aligned}\right\}dA \tag{26}$$

Where (·) dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density; and (I_0, I_1, J_1, I_2, J_2, K_2) are mass inertias defined as

$$\left(I_0,I_1,J_1,I_2,J_2,K_2\right)=\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(1,z,f,z^2,zf,f^2\right)\rho(z)\,dz \tag{27}$$

The variation of strain energy of foundation is expressed as

$$\delta U_F=\int_A f_e\delta w_0\,dA \tag{28}$$

Where f_e is the density of reaction force of foundation. For the Pasternak foundation model, it is given by

$$f_e=k_w w_0-k_p\nabla^2 w_0 \tag{29}$$

If the foundation is modeled as the linear Winkler foundation, the coefficient k_p in Eq. (29) is zero.

Substituting the expressions for δU; δU_F and δK from Eqs. (20), (28) and (26) into Eq. (18) and integrating by parts and collecting the coefficients of δu_0, δv_0, δw_0 and $\delta\theta$, the following equations of motion of the plate are obtained

$$\begin{aligned}&\delta u_0:\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\ddot u_0-I_1\frac{\partial\ddot w_0}{\partial x}+k_1A'J_1\frac{\partial\ddot\theta}{\partial x}\\&\delta v_0:\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\ddot v_0-I_1\frac{\partial\ddot w_0}{\partial y}+k_2B'J_1\frac{\partial\ddot\theta}{\partial y}\\&\delta w_0:\frac{\partial^2M_x^b}{\partial x^2}+2\frac{\partial^2M_{xy}^b}{\partial x\partial y}+\frac{\partial^2M_y^b}{\partial y^2}+k_w w_0-k_p\nabla^2w_0=I_0\ddot w_0\\&\qquad+I_1(\frac{\partial\ddot u_0}{\partial x}+\frac{\partial\ddot v_0}{\partial y})-I_2(\frac{\partial^2\ddot w_0}{\partial x^2}+\frac{\partial^2\ddot w_0}{\partial y^2})\\&\qquad+J_2(k_1A'\frac{\partial^2\ddot\theta}{\partial x^2}+k_2B'\frac{\partial^2\ddot\theta}{\partial y^2})\\&\delta\theta:-k_1A'\frac{\partial^2M_x^s}{\partial x^2}-(k_1A'+k_2B')\frac{\partial^2M_{xy}^s}{\partial x\partial y}-k_2B'\frac{\partial^2M_y^s}{\partial y^2}\\&+k_1A'\frac{\partial S_{xz}^s}{\partial x}+k_2B'\frac{\partial S_{yz}^s}{\partial y}=-J_1(\frac{\partial\ddot u_0}{\partial x}+\frac{\partial\ddot v_0}{\partial y})+J_2(\frac{\partial^2\ddot w_0}{\partial x^2}+\frac{\partial^2\ddot w_0}{\partial y^2})\\&\qquad-K_2\left[(k_1A')^2\frac{\partial^2\ddot\theta}{\partial x^2}+(k_2B')^2\frac{\partial^2\ddot\theta}{\partial y^2}\right]\end{aligned} \tag{30}$$

By substituting Eq. (22) into Eq. (30), the equations of motion can be expressed in terms of displacements (u_0, v_0, w_0 and θ) as

$$\begin{aligned}&A_{11}\frac{\partial^2u_0}{\partial x^2}+A_{66}\frac{\partial^2u_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2v_0}{\partial x\partial y}-B_{11}\frac{\partial^3w_0}{\partial x^3}\\&-(B_{12}+2B_{66})\frac{\partial^3w_0}{\partial x\partial y^2}+k_1A'B_{11}^s\frac{\partial^3\theta}{\partial x^3}+k_2B'B_{12}^s\frac{\partial^3\theta}{\partial x\partial y^2}\\&+\left(k_1A'+k_2B'\right)B_{66}^s\frac{\partial^3\theta}{\partial x\partial y^2}=I_0\ddot u_0-I_1\frac{\partial\ddot w_0}{\partial x}+k_1A'J_1\frac{\partial\ddot\theta}{\partial x}\\[6pt]&A_{22}\frac{\partial^2u_0}{\partial y^2}+A_{66}\frac{\partial^2v_0}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2u_0}{\partial x\partial y}-B_{22}\frac{\partial^3w_0}{\partial y^3}\\&-(B_{12}+2B_{66})\frac{\partial^3w_0}{\partial x^2\partial y}+k_1A'B_{12}^s\frac{\partial^3\theta}{\partial x^2\partial y}+k_2B'B_{22}^s\frac{\partial^3\theta}{\partial y^3}\\&+\left(k_1A'+k_2B'\right)B_{66}^s\frac{\partial^3\theta}{\partial x^2\partial y}=I_0\ddot v_0-I_1\frac{\partial\ddot w_0}{\partial y}+k_2B'J_1\frac{\partial\ddot\theta}{\partial y}\\[6pt]&B_{11}\frac{\partial^3u_0}{\partial x^3}+(B_{12}+2B_{66})\left(\frac{\partial^3u_0}{\partial x\partial y^2}+\frac{\partial^3v_0}{\partial x^2\partial y}\right)+B_{22}\frac{\partial^3v_0}{\partial y^3}\\&-D_{11}\frac{\partial^4w_0}{\partial x^4}-2(D_{12}+2D_{66})\frac{\partial^4w_0}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4w_0}{\partial y^4}+k_1A'D_{11}^s\frac{\partial^4\theta}{\partial x^4}\\&+\left(k_1A'+k_2B'\right)(D_{12}^s+2D_{66}^s)\frac{\partial^4\theta}{\partial x^2\partial y^2}+k_2B'D_{22}^s\frac{\partial^4\theta}{\partial y^4}+k_w w_0\\&-k_p\nabla^2w_0=I_0\ddot w_0+I_1(\frac{\partial\ddot u_0}{\partial x}+\frac{\partial\ddot v_0}{\partial y})-I_2(\frac{\partial^2\ddot w_0}{\partial x^2}+\frac{\partial^2\ddot w_0}{\partial y^2})\\&+J_2(k_1A'\frac{\partial^2\ddot\theta}{\partial x^2}+k_2B'\frac{\partial^2\ddot\theta}{\partial y^2})\\[6pt]&-k_1A'B_{11}^s\frac{\partial^3u_0}{\partial x^3}-(k_2B'B_{12}^s+\left(k_1A'+k_2B'\right)B_{66}^s)\frac{\partial^3u_0}{\partial x\partial y^2}\\&-(k_1A'B_{12}^s+\left(k_1A'+k_2B'\right)B_{66}^s)\frac{\partial^3v_0}{\partial x^2\partial y}-k_2B'B_{22}^s\frac{\partial^3v_0}{\partial y^3}\\&+k_1A'D_{11}^s\frac{\partial^4w_0}{\partial x^4}+\left(k_1A'+k_2B'\right)(D_{12}^s+2D_{66}^s)\frac{\partial^4w_0}{\partial x^2\partial y^2}\\&+k_2B'D_{22}^s\frac{\partial^4w_0}{\partial y^4}-(k_1A')^2H_{11}^s\frac{\partial^4\theta}{\partial x^4}-[2k_1A'k_2B'H_{12}^s\\&+\left(k_1A'+k_2B'\right)^2H_{66}^s]\frac{\partial^4\theta}{\partial x^2\partial y^2}-(k_2B')^2H_{22}^s\frac{\partial^4\theta}{\partial y^4}\\&+(k_1A')^2A_{55}^s\frac{\partial^2\theta}{\partial x^2}+(k_2B')^2A_{44}^s\frac{\partial^2\theta}{\partial y^2}=-J_1(\frac{\partial\ddot u_0}{\partial x}+\frac{\partial\ddot v_0}{\partial y})\\&+J_2(\frac{\partial^2\ddot w_0}{\partial x^2}+\frac{\partial^2\ddot w_0}{\partial y^2})-K_2\left[(k_1A')^2\frac{\partial^2\ddot\theta}{\partial x^2}+(k_2B')^2\frac{\partial^2\ddot\theta}{\partial y^2}\right]\end{aligned} \tag{31}$$

6. Navier solution for simply supported rectangular plates

Rectangular plates are generally classified in accordance with the type of support used. We are here concerned with the exact solution of Eq. (31) for a simply supported FG plate. The following displacement functions are chosen to satisfy the boundary conditions of plate and are selected as Fourier series

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}e^{i\omega t}\cos\alpha x\sin\beta y \\ V_{mn}e^{i\omega t}\sin\alpha x\cos\beta y \\ W_{mn}e^{i\omega t}\sin\alpha x\sin\beta y \\ \theta_{mn}e^{i\omega t}\sin\alpha x\sin\beta y \end{Bmatrix} \tag{32}$$

Where $i=\sqrt{-1}$, $\alpha=\pi/a$, $\beta=\pi/b$. ω is the natural frequency. U_{mn}, V_{mn}, W_{mn} and θ_{mn} are arbitrary parameters to be determined. Substituting Eq. (32) into Eqs. (31), the following eigenvalue equation is obtained

$$\left(\left[k\right]-\omega^2\left[M\right]\right)\left(\Delta\right)=\left(0\right) \tag{33}$$

Where

$$\left[K\right]=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} \tag{34a}$$

$$\left[M\right]=\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{12} & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix} \tag{34b}$$

And

$$\begin{aligned}
a_{11} &= A_{11}\alpha^2 + A_{66}\beta \\
a_{12} &= (A_{11}\alpha^2 + A_{66})\alpha\beta \\
a_{13} &= -B_{11}\alpha^3 - (B_{12}+2B_{66})\alpha\beta^2 \\
a_{14} &= [k_2B'C_{12} + (k_1A' + k_2B')C_{66}]\alpha\beta^2 + k_1A'C_{11}\alpha^3 \\
\alpha_{22} &= A_{22}\beta^2 + A_{66}\alpha \\
a_{23} &= -B_{22}\beta^3 - (B_{12}+2B_{66})\alpha^2\beta \\
a_{24} &= [k_1A'C_{12} + (k_1A' + k_2B')C_{66}]\alpha^2\beta + k_2B'C_{22}\beta^3 \\
\alpha_{33} &= D_{11}\alpha^4 + D_{22}\beta^4 + 2(D_{12}+2D_{66})\alpha^2\beta + k_w \\
&\quad + k_p(\alpha^2+\beta) \\
a_{34} &= -k_1A'E_{11}\alpha^4 - (k_1A' + k_2B')(E_{12}+2E_{66})\alpha^2\beta \\
&\quad - k_2B'E_{22}\beta^4 \\
a_{44} &= [2k_1A'k_2B'G_{12} + (k_1A' + k_2B')^2G_{66}]\alpha^2\beta \\
&\quad + (k_1A')^2G_{11}\alpha^4 + k_2B')^2G_{22}\beta^4 + (k_1A')^2G_{55}\alpha \\
&\quad + (k_2B')^2F_{44}\beta \\
m_{11} &= m_{22} = I_0, m_{12} = 0, m_{13} = -I_1\alpha, \\
m_{14} &= k_1A'J_1\alpha, m_{23} = -I_1\beta, \\
m_{24} &= k_2B'J_1\beta, m_{33} = I_0 + I_2(\alpha^2+\beta), \\
m_{34} &= -J_2(k_1A'\alpha^2 + k_2B\beta^2), \\
m_{44} &= K_2[(k_1A')^2\alpha^2 + (k_2B')^2\beta^2],
\end{aligned} \tag{34c}$$

7. Results and discussion

In this study, the effect of micromechanical models on the free vibration analysis of FG plates on elastic foundation by a new shear deformation theory is suggested for investigation. Navier solutions for free vibration analysis of FG plates are presented by solving the eigenvalue equations. The Poisson's ratio is fixed at ν=0.3. Comparisons are made with available solutions in literature. In order to verify the accuracy of the present analysis, some numerical examples are solved. The material properties used in the present study are:

- Metal (Aluminium, Al): $E_M=70\times10^9$ N/m^2; ν=0.3; ρ_M=2702 kg/m^3.
- Ceramic (Alumina, Al_2O_3): $E_C=380\times10^9$ N/m^2; ν=0.3; ρ_C=3800 kg/m^3.

In all examples, the foundation parameters are presented in the non-dimensional form of $K_w=k_wa^4/D$ and $K_p=k_pa^2/D$, where $D=Eh^3/12(1-\nu^2)$ is a reference bending rigidity of the plate.

For simplicity, the following nondimensional fundamental frequency parameter is used in the numerical examples $\Omega=\omega a^3\sqrt{\rho_c h/D_c}$.

7.1 Comparison between different micromechanical models

The estimations of Young's modulus using the aforementioned five micromechanical models are compared in Fig. 2. The estimated results are depicted as a function of volume fraction of inclusions (ceramic). As shown in Fig. 2, Voigt and Reuss approximations plot upper and lower bounds for estimation of Young's modulus. While Reuss estimation for Young's modulus could be 48% lower than the Voigt estimation for the volume fraction of V_F= 0,5.

The estimates made by LRVE are within Tamura (q=−100 GPa). It is worth noting that the estimates by Tamura greatly depend on the value of q.

In addition, the estimation of Young's modulus given by the Mori-Tanaka model is slightly higher than that given by LRVE and Tamura.

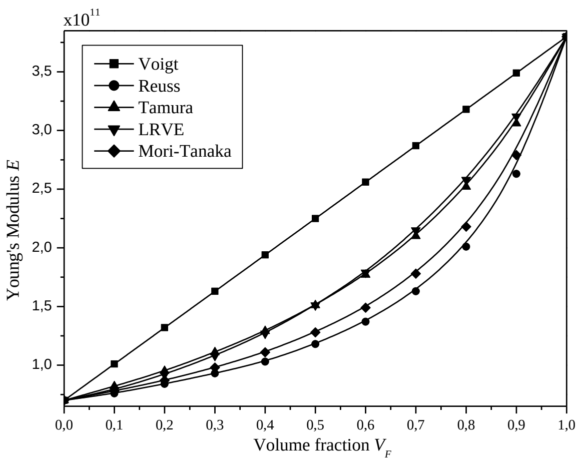


Fig. 2 Effective Young's modulus as function of volume fraction of ceramic for several micromechanical models

Table 1 Comparison of the first six natural frequencies of square AL/AL_2O_3 FG plate $\Omega = \omega h \sqrt{\rho_c / E_c}$

a/h	Mode n°	Mode (α,β,i)	Theory	Power law index "*p*"				
				p=0	*p*=1	*p*=4	*p*=10	*p*=∞ [a]
2	1	101	Matsunaga (2008)	0,5572	0,4375	0,3579	0,3313	0,2829
			Ait Atmane *et al.* (2010)	0,5524	0,4324	0,3554	0,3289	0,2812
			Present	0,5546	0,4338	0,3592	0,3323	0,2823
	2	111	Matsunaga (2008)	0,9400	0,7477	0,5997	0,5460	0,4773
			Ait Atmane *et al.* (2010)	0,9300	0,7725	0,6244	0,5573	0,4730
			Present	0,9337	0,7383	0,6008	0,5478	0,4753
	3	102	Matsunaga (2008)	0,9742	0,8005	0,6325	0,5664	0,4946
			Ait Atmane *et al.* (2010)	0,9742	0,8013	0,6356	0,5668	0,4958
			Present	0,9740	0,8092	0,6539	0,5671	0,4958
	4	112	Matsunaga (2008)	1,3777	1,1166	0,8731	0,7885	0,6995
			Ait Atmane *et al.* (2010)	1,3777	1,1209	0,8751	0,7895	0,7012
			Present	1,3777	1,1461	0,9260	0,8155	0,7012
	5	201	Matsunaga (2008)	1,5090	1,2163	0,9591	0,8588	0,7661
			Ait Atmane *et al.* (2010)	1,4907	1,1933	0,9466	0,8526	0,7587
			Present	1,4959	1,1935	0,9558	0,8600	0,7614
	6	103	Matsunaga (2008)	1,6078	1,3091	1,0008	0,9050	0,8163
			Ait Atmane *et al.* (2010)	1,6466	1,3391	1,0440	0,9426	0,8381
			Present	1,6466	1,3410	1,0507	0,9443	0,8381
5	1	101	Matsunaga (2008)	0,1120	0,08614	0,07356	0,06999	0,05686
			Ait Atmane *et al.* (2010)	0,1120	0,0860	0,07346	0,06984	0,05689
			Present	0,1119	0,0860	0,07371	0,07007	0,05697
	2	111	Matsunaga (2008)	0,2121	0,1640	0,1383	0,1306	0,1077
			Ait Atmane *et al.* (2010)	0,2113	0,1740	0,1520	0,1369	0,1075
			Present	0,2117	0,1634	0,1387	0,1308	0,1077
	3	102	Matsunaga (2008)	0,3874	0,3020	0,2502	0,2300	0,1967
			Ait Atmane *et al.* (2010)	0,3850	0,3000	0,2500	0,2300	0,1958
			Present	0,3861	0,3002	0,2512	0,2344	0,1965
	4	112	Matsunaga (2008)	0,3897	0,3236	0,2607	0,2337	0,1979
			Ait Atmane *et al.* (2010)	0,3897	0,3236	0,2606	0,2324	0,1983
			Present	0,3896	0,3240	0,2618	0,2306	0,1983
	5	201	Matsunaga (2008)	0,4658	0,3644	0,3000	0,2790	0,2365
			Ait Atmane *et al.* (2010)	0,4622	0,3674	0,3037	0,2794	0,2353
			Present	0,4639	0,3620	0,3014	0,2799	0,2361
	6	103	Matsunaga (2008)	0,5511	0,4567	0,3668	0,3243	0,2798
			Ait Atmane *et al.* (2010)	0,5511	0,4568	0,3668	0,3243	0,2804
			Present	0,5511	0,4584	0,3704	0,3262	0,2805
10	1	101	Matsunaga (2008)	0,02936	0,02246	0,01942	0,01861	0,01491
			Ait Atmane *et al.* (2010)	0,02934	0,02244	0,01941	0,01859	0,01493
			Present	0,02935	0,02244	0,01943	0,01861	0,01494
	2	111	Matsunaga (2008)	0,05777	0,04427	0,03811	0,03642	0,02933
			Ait Atmane *et al.* (2010)	0,05770	0,04718	0,04210	0,03832	0,02936
			Present	0,05773	0,04422	0,03815	0,03644	0,02938
	3	102	Matsunaga (2008)	0,1120	0,08614	0,07356	0,06999	0,05686
			Ait Atmane *et al.* (2010)	0,1120	0,08600	0,07346	0,06984	0,05689
			Present	0,1119	0,08597	0,07371	0,07007	0,05696
	4	112	Matsunaga (2008)	0,1381	0,1063	0,09045	0,08588	0,07012
			Ait Atmane *et al.* (2010)	0,1380	0,1092	0,09455	0,08764	0,07006
			Present	0,1379	0,1060	0,09067	0,08600	0,07017
	5	201	Matsunaga (2008)	0,1948	0,1620	0,1308	0,1153	0,09890
			Ait Atmane *et al.* (2010)	0,1948	0,1620	0,1308	0,1152	0,09916
			Present	0,1948	0,1621	0,1309	0,1153	0,09917
	6	103	Matsunaga (2008)	0,2121	0,1640	0,1383	0,1306	0,1077
			Ait Atmane *et al.* (2010)	0,2113	0,1740	0,1520	0,1368	0,1075
			Present	0,2117	0,1634	0,1387	0,1308	0,1077

[a]Fully metallic plate (*p*=∞), $\Omega_M = \omega h \sqrt{\rho_M / E_M}$.

7.2 Comparison studies

In this section, various numerical examples are described and discussed for verifying the accuracy of the new present shear deformation plate theory in predicting the free vibration behavior of simply supported FG plates. For the verification purpose, the results obtained by the present theory are compared with other theories existing in the literature, such as the 2D higher order theory of Matsunaga (2008), the high order shear deformation theory of Ait Atmane *et al.* (2010) and the results obtained by Akhavan *et al.* (2009).

In Table 1, the first six natural frequency parameters of the FG plate for different values of the power law p are compared with those of Matsunaga (2008) and Ait Atmane *et al.* (2010). "α" and "β" denote the wave numbers and "i" the mode order number which corresponds to displacement distributions in thickness direction. A good agreement is observed by the comparisons of the fundamental frequency parameter obtained by the present new shear deformation theory (with only four unknown functions) with other theories.

As a second example and in order to validate the present method in the case of plates resting on elastic foundation, the results for the fundamental natural frequency parameter of isotropic thick plate with three different values of thickness-to-length ratios, three different values of Winkler elastic coefficient and three values of shearing layer coefficient are presented in Table 2. It can be seen that the results are in close agreement.

It should be noted that for the values shown in Tables 1 and 2, the results of the present method are obtained for the Voigt model.

7.3 Parametric studies

The impact of the micromechanical models on the estimated fundamental frequency of FG plates is studied in this section.

In Fig. 3, the variations of non-dimensional fundamental frequencies of FG plate with the power law index p are given for different micromechanical models. Tree configurations are studied: plate without foundation, plate on Winkler foundation and plate resting on Pasternak foundation. It is seen from the figures that the increase of the power law index p produces a reduction of fundamental frequency values.

In addition, whatever the type of foundation used, the model of Voigt gives higher results compared to other micro-mechanical models. While the other models provide substantially similar results. The presence of Pasternak foundation is increasing the fundamental frequency and this whatever the value of the power law index or the model. The presence of the Winkler foundation has little influence on the values of the fundamental frequency.

Relative Percentage difference of fundamental frequency between micromechanical models versus power law index p is shown in Fig. 4.

The discrepancy between the estimated fundamental frequency of FGMs by the Voigt, Reuss and other micromechanical models depends considerably on the power law index p.

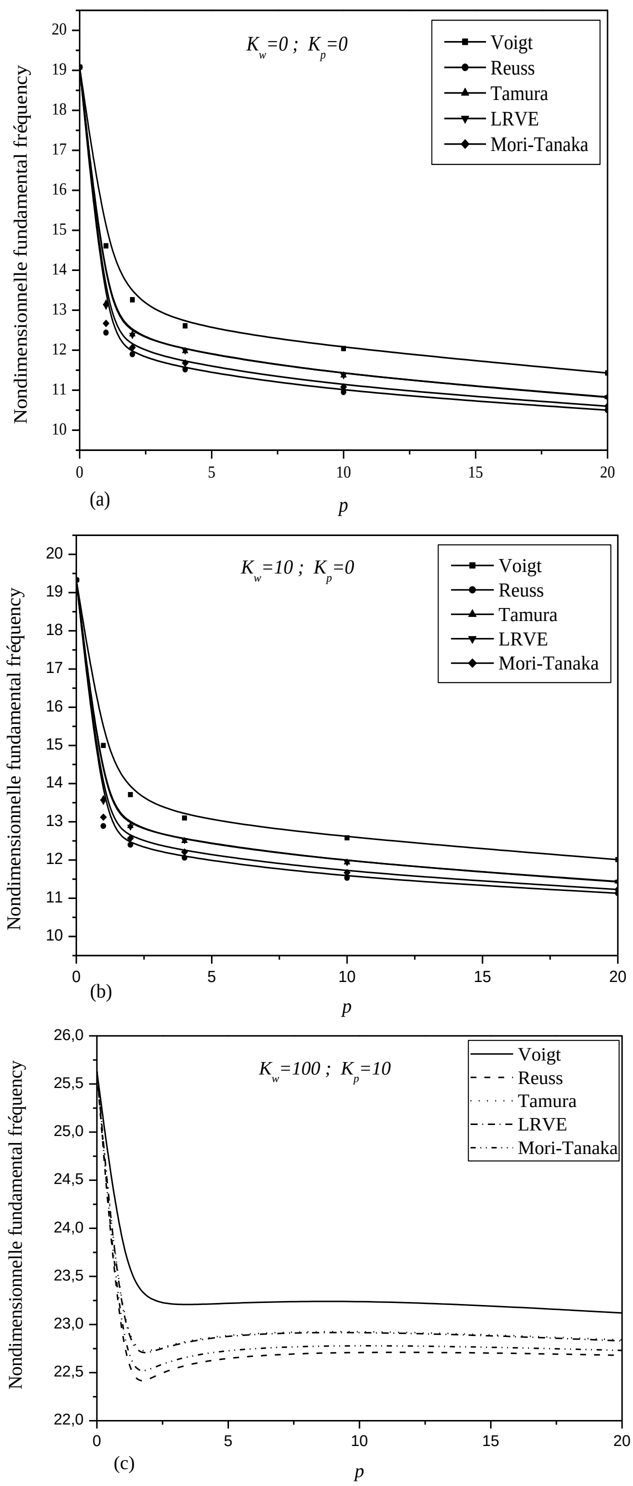


Fig. 3 The effect of the power law index p on nondimensional fundamental frequency of FG square plates with different configurations of elastic foundation a/h=10. (a) FG plate without elastic foundation, (b) FG plate resting on Winkler foundation, (c) FG plate resting on Pasternak foundation

The discrepancy between the Voigt model and other micro-mechanical models for the estimated values of the fundamental frequency reaches a maximum of 12%

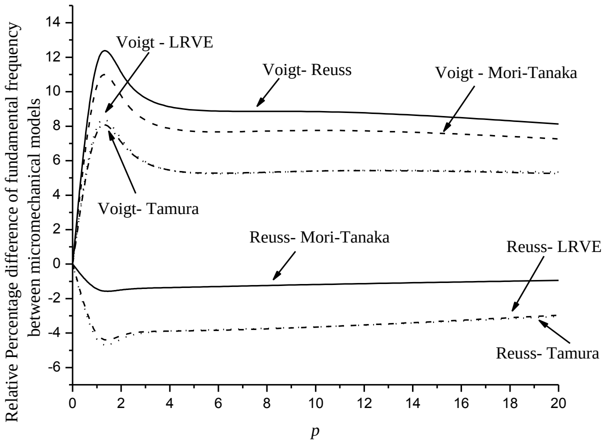
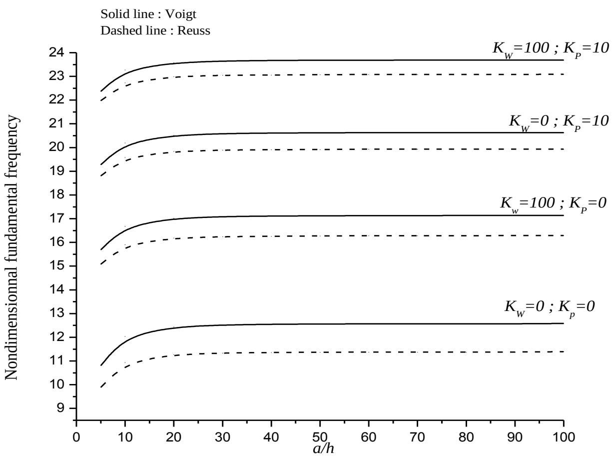
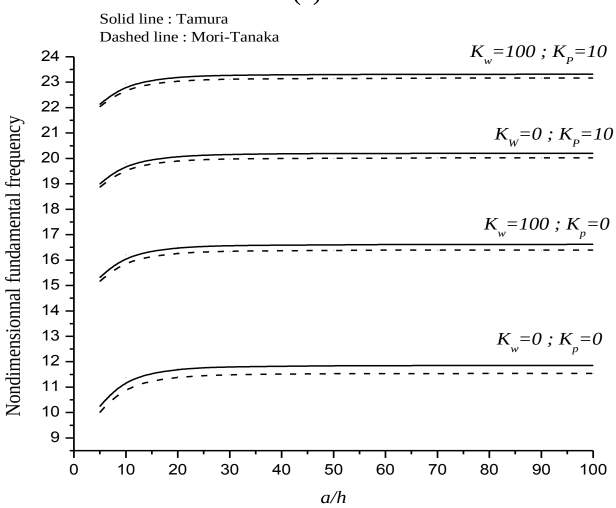


Fig. 4 Relative percentage difference of fundamental frequency between micromechanical models versus power law index p, a/h=10, a=b



(a)



(b)

Fig. 5 The effect of elastic foundation on nondimensionnal fundamental frequency of square FG plates for a range of the side-to-thickness ratio a/h. (a) comparison between Voigt-Reuss, (b) comparison between Tamura- Mori-Tanaka

between Voigt and Reuss and it is 11% between Voigt and Mori-Tanaka. While between Voigt and other models

Table 2 Comparison of the fundamental frequency parameter of isotropic square plate (α=β=1) $\Omega_c = \omega\, a^2 \sqrt{\rho_c / E_c}$

Thickness to length ratio	K_w, K_p	Theory		
		Akhavan *et al.* (2009)	Ait Atmane *et al.* (2010)	Present
h/a=0,001	0, 0	19,7391	19,7392	19,7391
	100, 10	26,2112	26,2112	26,2112
	1000, 100	57,9961	57,9962	57,9961
h/a=0,1	0, 0	19,0840	19,0658	19,0786
	100, 10	25,6368	25,6236	25,6328
	1000, 100	57,3969	57,3923	57,3955
h/a=0,2	0, 0	17,5055	17,4531	17,4899
	100, 10	24,3074	24,2728	24,2971
	1000, 100	56,0359	56,0311	56,0345

namely LRVE and Tamura it does not exceed 8%.

The second comparison shown in this figure is the discrepancy between the values of the fundamental frequency between the Reuss model and other micromechanical models. The difference is insignificant between Reuss and Mori-Tanaka and it reached a maximum of 5% between Reuss and other models. Therefore, the necessity of the proper micromechanical modeling of FGMs is evident to accurately estimate the fundamental frequency.

Fig. 5 shows the non-dimensional fundamental natural frequency versus side-to-thickness ratio for simply supported square FG plate. As it can be seen, the fundamental frequency increases for small values of the ratio and then maintain a constant shape. In addition, Winkler foundation parameter affects very little the value of the fundamental frequency as compared to that of Pasternak parameter. From the figures (a) and (b), we can see that there is a discrepancy between the results provided by Voigt and Reuss, while the results of Tamura and Mori-Tanaka are substantially similar.

This confirms what was already mentioned above that a proper micromechanical modeling of FGM should be defined.

8. Conclusions

A new higher order shear deformation theory was proposed to analyse dynamic behaviour of functionally graded plates resting on Winkler-Pasternak elastic foundations. Different micromechanical models were used to determine the effective properties of such plates.The equilibrium equations and associated boundary conditions of the plate are obtained using Hamilton’s principle. The Navier method is used for the analytical solutions of the FG plate with simply supported boundary conditions. The results obtained using this new theory, are found to be in excellent agreement with previous studies.

Furthermore, the influences of plate parameters such as power law index, aspect ratio, elastic foundation parameters and the effect of micromechanical models on the natural frequencies of FG rectangular plate have been comprehensively investigated.

From these results and comparisons between different micromechanical models, it has been found significant differences between some models. This proves the need for a proper micromechanical modelling of FGMs to accurately estimate the fundamental frequency.

Finally, we can say that the proposed model has great significance for design engineers dealing with plates resting on elastic foundation in obtaining the desired frequency parameters for their application.

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