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A remedy for a family of dissipative, non-iterative structure-dependent integration methods

Shuenn-Yih Chang* and Tsui-Huang Wu[a]

Department of Civil Engineering, National Taipei University of Technology, 1, Sec. 3, Jungshiau East Road, Taipei 106-08, Taiwan

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Abstract. A family of the structure-dependent methods seems very promising for time integration since it can simultaneously have desired numerical properties, such as unconditional stability, second-order accuracy, explicit formulation and numerical dissipation. However, an unusual overshoot, which is essentially different from that found by Goudreau and Taylor in the transient response, has been experienced in the steady-state response of a high frequency mode. The root cause of this unusual overshoot is analytically explored and then a remedy is successfully developed to eliminate it. As a result, an improved formulation of this family method can be achieved.

Keywords: overshooting, steady-state response, local truncation error, structure-dependent integration method

1. Introduction

The integration method is a very powerful technique for the dynamic analysis (Bayat *et al*. 2015, Fattah *et al*. 2015, Kaveh *et al*. 2015, Rezaiee-Pajand and Alamatian 2008, Rezaiee-Pajanda and Hashemian 2016, Rezaiee-Pajand and Karimi-Rad 2017, Rezaiee-Pajand *et al*. 2011, 2017, Romero *et al*. 2012, Su *et al*. 2014) and thus many integration methods have been developed for time integration. A family of structure-dependent integration methods with favorable numerical properties, such as unconditional stability, second-order accuracy, explicit formulation and numerical damping, has been successfully developed for structural dynamics (Chang 2014a). This family method, which is referred as Chang dissipative method (CDM) herein for brevity, can have controllable numerical dissipation and a zero damping ratio can also be achieved. It is promising for solving an inertia-type problem, where the total response is controlled by the low frequency modes while the high frequency modes contribute insignificantly (Belytschko and Hughes 1983), since it can integrate unconditional stability and explicit formulation simultaneously. A step size is not constrained by stability conditions due to unconditional stability. Meanwhile, no nonlinear iterations are involved due to the explicitness of each time step. Hence, an appropriate time step can be chosen to conduct the time integration without involving an iteration procedure and thus it is very computationally efficient for solving inertia-type problems. Notice that the favorable numerical dissipation can filter out the spurious participation of high frequency modes and enhance the stable computations.

A peculiar overshoot in the early transient response has been found by Goudreau and Taylor (1972) for the Wilson-method (Bathe and Wilson 1973), and the root cause of this overshoot has been analytically explored by Hilber and Hughes (1978). Thus, this property must be thoroughly evaluated in the development of a new integration method (Chang 2002, 2009, 2010, 2014b, 2015, 2016, Chang *et al*. 2015, Gao *et al*. 2012, Verma *et al*. 2015). The CDM has been analytically verified and numerically confirmed that it has no such overshoot. However, it exhibits a new type of overshoot in the steady-state response of a high frequency mode rather than in the early transient response. This overshooting has never been found in the conventional dissipative integration methods, such as HHT-α method (Hilber *et al*. 1977), WBZ-α method (Wood *et al*. 1981) and the generalized-α method (Chung and Hulbert 1993). Apparently, the new type of overshoot is different from that found by Goudreau and Taylor (1972) since it only occurs for nonzero dynamic loading while that found by Goudreau and Taylor is not related to dynamic loading. In this work, the overshoot is thoroughly studied and a remedy is proposed to overcome this overshoot.

2. Chang dissipative method

In structural dynamics or earthquake engineering, the equation of motion for a single degree of freedom system can be generally expressed as

$$m\ddot{u} + c\dot{u} + ku = f \tag{1}$$

where $\ddot{u}$, $\dot{u}$ and u correspond to the acceleration, velocity and displacement; m, c and k are the mass, viscous damping coefficient and stiffness, respectively; and f is an applied dynamic loading. The use of CDM to solve Eq. (1) can be generally expressed as

*Corresponding author, Professor
E-mail: changsy@ntut.edu.tw
[a]Graduate Student
E-mail: fd005@mail2000.com.tw

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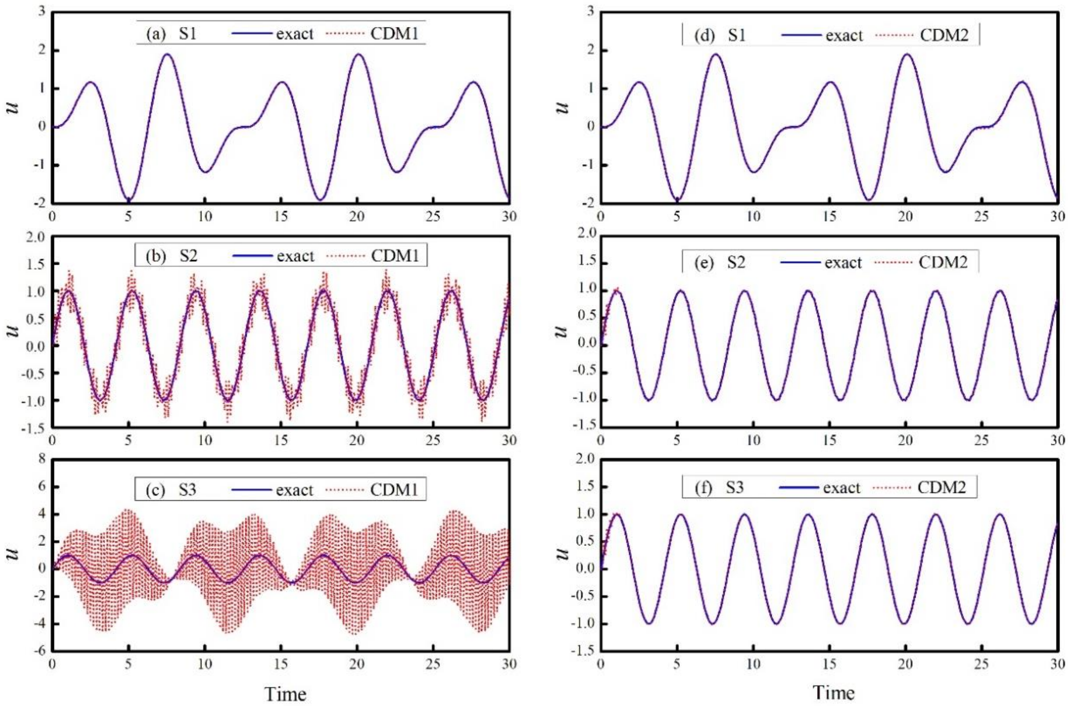

Fig. 1 Forced vibration response obtained from CDM for using Δt=0.1 sec

$$\begin{aligned} & m a_{i+1} + c v_{i+1} + (1+\alpha) k d_{i+1} - \alpha k d_i = (1+\alpha) f_{i+1} - \alpha f_i \\ & d_{i+1} = (1-\beta_1) d_{i-1} + \beta_1 d_i + \beta_2 (\Delta t) v_i + \beta_3 (\Delta t)^2 a_i \\ & v_{i+1} = v_i + (\Delta t)\left[(1-\gamma) a_i + \gamma a_{i+1}\right] \end{aligned} \tag{2}$$

where a_i, v_i, d_i and f_i are the nodal acceleration, velocity, displacement and external force at the i-th time step correspondingly; and the coefficients β_1 to β_3 are:

$$\beta_1 = 1 + \frac{\alpha\beta\Omega_0^2}{1+2\gamma\xi\Omega_0 + (1+\alpha)\beta\Omega_0^2}, \quad \beta_2 = \frac{1+2\gamma\xi\Omega_0}{1+2\gamma\xi\Omega_0 + (1+\alpha)\beta\Omega_0^2}$$

$$\beta_3 = \frac{\frac{1}{2} - 2\left(\beta - \frac{1}{2}\gamma\right)\xi\Omega_0}{1+2\gamma\xi\Omega_0 + (1+\alpha)\beta\Omega_0^2} \tag{3}$$

where $\Omega_0=\omega_0(\Delta t)$ and $\omega_0 = \sqrt{k_0/m}$ is the natural frequency. Notice that the initial stiffness k_0 is generally different from the stiffness k since k may vary for a nonlinear system. The symbol ξ represents a viscous damping ratio; and α, β and γ are the parameters to control numerical properties. To have favorable numerical properties, the following relations for α, β and γ are generally recommended

$$-\frac{1}{3} \le \alpha \le 0 \quad , \quad \beta = \frac{(1-\alpha)^2}{4} \quad \text{and} \quad \gamma = \frac{1}{2} - \alpha \tag{4}$$

Notice that the coefficients β_1 to β_3 are functions of the initial structural properties and the step size. Hence, CDM is a family of structure-dependent integration methods.

3. Overshooting phenomenon

In order to illustrate the unusual overshooting behavior that might occur in the steady-state response of a high frequency mode for CDM, a simple example is considered next by comparing the numerical solution obtained from CDM to the exact solution. For this purpose, an undamped, single degree of freedom system subject to a sine loading is solved. The equation of motion can be simply adjusted form Eq. (1) by taking c=0 and $f = k_0 \sin(\bar{\omega} t)$. Notice that a linear elastic system is assumed and thus k=k_0 in Eq. (1) is adopted in this example. Apparently, an exact solution of this example can be obtained from the fundamental theory of structural dynamics and is found to be

$$u(t) = \frac{1}{1-\theta^2}\sin(\bar{\omega} t) - \frac{\theta}{1-\theta^2}\sin(\omega_0 t) \tag{5}$$

where $\omega_0 = \sqrt{k_0/m}$ is the natural frequency of the system and $\theta = \bar{\omega}/\omega_0$ is defined. In the limiting case of $\omega_0 \to \infty$ (or $\theta \to 0$), the steady-state response R_{ste} and the transient response R_{tra} become

$$\begin{aligned} R_{ste} &= \lim_{\theta\to 0} \frac{1}{1-\theta^2}\sin(\bar{\omega} t) \approx \sin(\bar{\omega} t) \\ R_{tra} &= \lim_{\theta\to 0} \frac{\theta}{1-\theta^2}\sin(\omega_0 t) \approx 0 \end{aligned} \tag{6}$$

This implies that the solution $u(t)$ is dominated by the steady-state response for a high frequency mode while the response from the transient response is insignificant. Consequently, the solution is found to be $u(t) \approx \sin(\bar{\omega} t)$ for a high frequency mode. Notice that a large ω_0 or a small θ can be used to imply a high frequency mode and vice versa.

Three different systems can be obtained from this

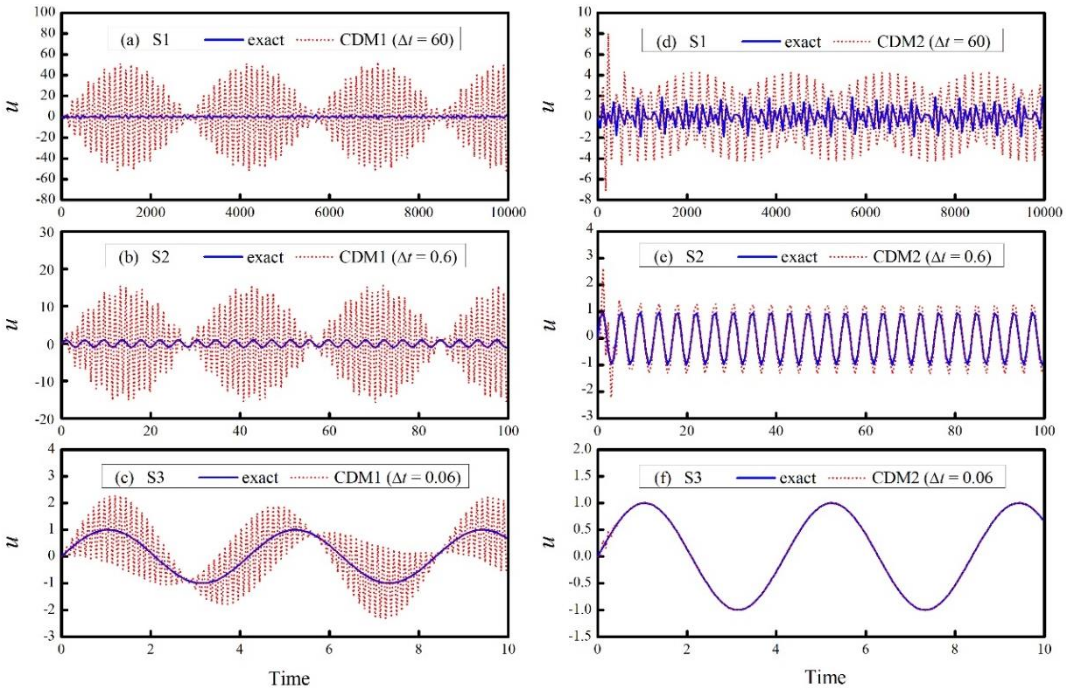

Fig. 2 Forced vibration response obtained from CDM for S1, S2 and S3

example by specifying the values of k_0=1, 10^4 and 10^6 in addition to m=1. The three systems are referred as S1, S2 and S3 for brevity. As a result, the natural frequencies corresponding to S1, S2 and S3 are found to be 1, 10^2 and 10^3 rad/sec. On the other hand, the driving frequency $\bar{\omega}=1.5$ rad/sec is assumed for each analysis. This leads to θ=1.5, 1.5×10^{-2} and 1.5×10^{-3} in correspondence to S1, S2 and S3. Apparently, the solutions for S2 and S3 are dominated by the steady-state response only while both the transient response and steady-state responses considerably contribute to the solution of S1. To demonstrate the unusual overshooting behavior that might experience for CDM, two members of CDM are used to calculate the responses. The member of α=0 is referred as CDM1 while that of α=−0.2 is referred as CDM2.

Fig. 1 shows the numerical results obtained from CDM1 and CDM2 with Δt=0.1 sec. This time step is small enough to reliably integrate the transient response for S1 since $\Delta t/T_0=1/(20\pi)$, where $T_0=2\pi/\omega 0=2\pi$, while it leads to significant period distortion for both S2 and S3. On the other hand, this time step is also small enough to accurately integrate the steady-state response since $\Delta t/\bar{T}_0=3/40\pi$, where $\bar{T}_0=2\pi/\bar{\omega}_0=4\pi/3$ (Chang 2006). Either CDM1 or CDM2 give reliable results for S1 while an overshooting phenomenon is generally found in the results obtained from CDM1 and CDM2 for both S2 and S3. It is manifested from Fig. 1(a) to 1(c) that the overshooting phenomenon becomes more significant as the natural frequency increases; and there is almost no overshoot for a low frequency mode. Very similar phenomena are also found in Fig. 1(d) to 1(f). Since numerical damping can suppress the spurious growth of high frequency mode the results obtained from CDM1 show more significant overshooting behavior for S2 and S3 due to zero damping when compared to CDM2. As a summary, it is evident that CDM experiences an unusual overshooting behavior in the steady-state response of a high frequency mode.

The time steps of Δt=60, 0.6 and 0.06 sec are also used to solve the three systems S1, S2 and S3, respectively and each time step is too large to yield an accurate transient response of the corresponding system since the value of $\Delta t/T_0$ is found to be as large as 9.55 for each system. On the other hand, it is found that Δt=60 sec is too large to obtain a reliable steady-state response for S1 while Δt=0.6 sec may provide an acceptable steady-state response for S2. In addition, an accurate steady-state response can be achieved by using Δt=0.06 sec for S3. Numerical results obtained from CDM1 and CDM2 are shown in Fig. 2. In general, an overshoot behavior is also found in each plot of this figure. It seems that Fig. 2(a) shows a very significant overshoot while a less significant overshoot is found in Fig. 2(f). Again, the results obtained from CDM2 show less overshoot than for those obtained from CDM1 since it has numerical dissipation to suppress the high frequency response.

4. Cause of overshooting phenomenon

To explore the cause of the overshoot in the steady-state response of a high frequency mode, a local truncation error is determined from a forced vibration response rather than a free vibration response since the dynamic loading is closely related to the unusual overshoot. A local truncation error is defined as the error occurred in each time step by using the differential equation to replace the corresponding difference equation. Hence, the approximate difference equation of CDM can be derived from Eq. (2) by eliminating the velocities and accelerations and it is found to be

$$d_{i+2} - A_1 d_{i+1} + A_2 d_i - A_3 d_{i-1} + A_f = 0 \tag{7}$$

where

$$A_1 = 2 - \frac{1}{D}\left[2\xi\Omega_0 + \left(-\alpha\beta + \alpha\gamma + \tfrac{1}{2}\alpha + \gamma + \tfrac{1}{2}\right)\Omega_0^2\right]$$

$$A_2 = 1 - \frac{1}{D}\left[2\xi\Omega_0 + \left(-2\alpha\beta + 2\alpha\gamma + \gamma - \tfrac{1}{2}\right)\Omega_0^2\right]$$

$$A_3 = \frac{1}{D}\alpha\left(\beta - \gamma + \tfrac{1}{2}\right)\Omega_0^2$$

$$\begin{aligned} A_f = &-\frac{1}{BD}(1+\alpha)\left[\left(\gamma + \tfrac{1}{2}\right) + \left(-\beta + \gamma^2 + \tfrac{1}{2}\gamma\right)2\xi\Omega_0\right]\tfrac{1}{m}(\Delta t)^2 f_{i+1} \\ &+ \frac{1}{BD}\left[\left(2\alpha\gamma + \gamma - \tfrac{1}{2}\right) + \left(2\alpha\gamma^2 - 2\alpha\beta - \beta + \gamma^2 - \tfrac{1}{2}\gamma\right)2\xi\Omega_0\right]\tfrac{1}{m}(\Delta t)^2 f_i \\ &+ \frac{1}{BD}\alpha\left[\left(-\gamma + \tfrac{1}{2}\right) + \left(\beta - \gamma^2 + \tfrac{1}{2}\gamma\right)2\xi\Omega_0\right]\tfrac{1}{m}(\Delta t)^2 f_{i-1} \end{aligned} \tag{8}$$

where $B = 1 + 2\gamma\xi\Omega_0$ and $D = 1 + 2\gamma\xi\Omega_0 + (1+\alpha)\beta\Omega_0^2$ are further defined for brevity. Since a lot of algebraic manipulations are involved, it is very complicated to obtain these coefficients of the approximate displacement difference equation. As a result, after replacing Eq. (2) by Eq. (7), the local truncation error for CDM is found to be

$$E = \frac{1}{(\Delta t)^2}\left[u(t+2\Delta t) - A_1 u(t+\Delta t) + A_2 u(t) - A_3 u(t-\Delta t) + A_f\right] \tag{9}$$

In this derivation, $u(t)$ and $f(t)$ are assumed to be continuously differentiable up to any required order and thus $u(t+2\Delta t)$, $u(t+\Delta t)$, $u(t-\Delta t)$, $f(t+\Delta t)$ and $f(t-\Delta t)$ can be expanded into the finite Taylor series at t. After substituting A_1, A_2, A_3 and A_f into (9), the local truncation error for CDM is found to be

$$\begin{aligned} E = &\frac{1}{D}\left(\alpha + \gamma - \tfrac{1}{2}\right)\left[\Omega_0\omega_0\dot{u}_i + \tfrac{1}{2}\Omega_0^2\ddot{u}_i + 2\xi\Omega_0(\Delta t)\dddot{u}_i - \tfrac{1}{m}(\Delta t)\dot{f}_i - \tfrac{1}{2m}(\Delta t)^2\ddot{f}_i\right] \\ &+ \frac{1}{D}\left(\gamma - \tfrac{1}{2}\right)2\xi\Omega_0\ddot{u}_i - \frac{1}{D}\left(\alpha\gamma + \tfrac{1}{2}\alpha + \beta - \tfrac{1}{6}\right)2\xi\Omega_0(\Delta t)\dddot{u}_i \\ &- \frac{1}{D}\left(\alpha\gamma - \tfrac{1}{2}\alpha + \beta - \tfrac{1}{12}\right)(\Delta t)^2\ddddot{u}_i + \frac{1}{BD}2\beta\xi\Omega_0\tfrac{1}{m}(\Delta t)\dot{f}_i + \frac{1}{D}\beta\tfrac{1}{m}(\Delta t)^2\ddot{f}_i \\ &+ \frac{1}{BD}2\left(\tfrac{1}{2} + \alpha\right)\beta\xi\Omega_0\tfrac{1}{m}(\Delta t)^2\ddot{f}_i \end{aligned} \tag{10}$$

In general, a first-order accuracy can be achieved for CDM. Whereas, a second-order accuracy can be further achieved for the satisfaction of $\gamma = \tfrac{1}{2} - \alpha$ in addition to zero damping.

For the case of $\gamma = \tfrac{1}{2} - \alpha$, the first term on the right-hand side of Eq. (10) will disappear. It seems that the last three error terms are the key issue to cause an overshooting phenomenon in the steady-state response of a high frequency mode for CDM since they can be alternatively expressed as

$$E_{last-3-terms} = \frac{1}{BDk_0(\Delta t)}2\beta\xi\Omega_0^3\dot{f}_i + \frac{1}{Dk_0}\beta\Omega_0^2\ddot{f}_i + \frac{1}{BDk_0}2\left(\tfrac{1}{2} + \alpha\right)\beta\xi\Omega_0^2\ddot{f}_i \tag{11}$$

Clearly, the first term is cubically proportional to Ω_0 while the rest two terms are quadratically proportional to Ω_0. This indicates that the steady-state response will be almost unaffected by a low frequency mode while it might be very significantly affected by a high frequency mode. This local truncation error seems to explain why an overshooting behavior occurs in the steady-state response of a high frequency mode while there is no such an overshoot for a low frequency mode.

4. An improved formulation

There is a great motive to propose a remedy to eliminate the adverse overshooting behavior in the steady-state response of a high frequency mode. The main concept to develop such a remedy originates from a slight modification of CDM so that the adverse error terms in the local truncation error can be removed. It seems feasible to propose a remedy by adding an extra loading term in the difference equation for displacement increment. As a result, the modified form of CDM can be expressed as

$$\begin{aligned} &ma_{i+1} + cv_{i+1} + (1+\alpha)kd_{i+1} - \alpha kd_i = (1+\alpha)f_{i+1} - \alpha f_i \\ &d_{i+1} = (1-\beta_1)d_{i-1} + \beta_1 d_i + \beta_2(\Delta t)v_i + \beta_3(\Delta t)^2 a_i + p_{i+1} \\ &v_{i+1} = v_i + (\Delta t)\left[(1-\gamma)a_i + \gamma a_{i+1}\right] \end{aligned} \tag{12}$$

where p_{i+1} is an extra loading term. For brevity, this modified form of CDM will be referred as MCDM. Based on this formulation, the local truncation error for MCDM can be also obtained by using the same procedure to derive that for CDM and is found to be

$$\begin{aligned} E = &\frac{1}{D}\left(\alpha + \gamma - \tfrac{1}{2}\right)\left[\Omega_0\omega_0\dot{u}_i + \tfrac{1}{2}\Omega_0^2\ddot{u}_i + 2\xi\Omega_0(\Delta t)\dddot{u}_i - \tfrac{1}{m}(\Delta t)\dot{f}_i - \tfrac{1}{2m}(\Delta t)^2\ddot{f}_i\right] \\ &+ \frac{1}{D}\left(\gamma - \tfrac{1}{2}\right)2\xi\Omega_0\ddot{u}_i - \frac{1}{D}\left(\alpha\gamma + \tfrac{1}{2}\alpha + \beta - \tfrac{1}{6}\right)2\xi\Omega_0(\Delta t)\dddot{u}_i \\ &- \frac{1}{D}\left(\alpha\gamma - \tfrac{1}{2}\alpha + \beta - \tfrac{1}{12}\right)(\Delta t)^2\ddddot{u}_i + \frac{1}{BD}2\beta\xi\Omega_0\tfrac{1}{m}(\Delta t)\dot{f}_i + \frac{1}{D}\beta\tfrac{1}{m}(\Delta t)^2\ddot{f}_i \\ &+ \frac{1}{BD}2\left(\tfrac{1}{2} + \alpha\right)\beta\xi\Omega_0\tfrac{1}{m}(\Delta t)^2\ddot{f}_i - p_{i+2} + \left(1 - \frac{1}{B}2\xi\Omega_0\right)p_{i+1} \end{aligned} \tag{13}$$

Apparently, this equation is almost the same as Eq. (10) except that the last two error terms are functions of the extra loading term that is intentionally introduced into the difference equation for displacement increment.

The extra loading term p_{i+1} must be appropriately determined so that the three adverse terms in Eq. (11) can be eliminated. As a result, p_{i+1} is found to be

$$p_{i+1} = \frac{1}{D}\beta\tfrac{1}{m}(\Delta t)^2\left(f_{i+1} - f_i\right) + \frac{1}{D}\alpha\beta\tfrac{1}{m}(\Delta t)^2\left(f_{i+1} - 2f_i + f_{i-1}\right) \tag{14}$$

Notice that the extra loading term is also structure dependent and the denominator is the same as that for β_1 to β_3. After determining the extra loading term, the local truncation error for CDM is found to be

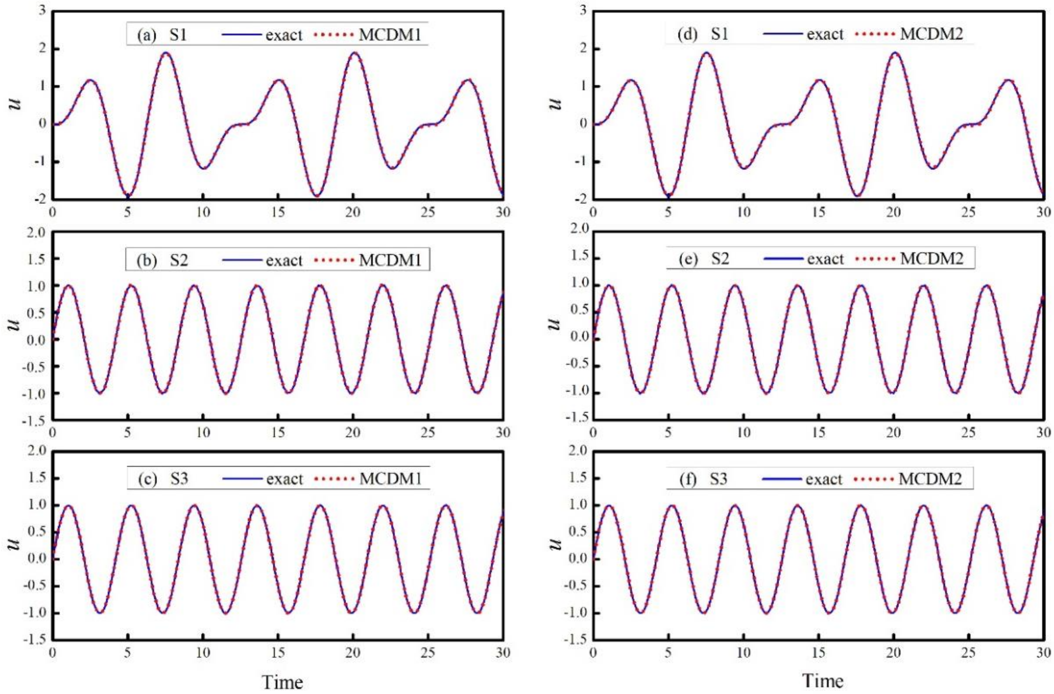

Fig. 3 Forced vibration response obtained from MCDM for using Δt=0.1 sec

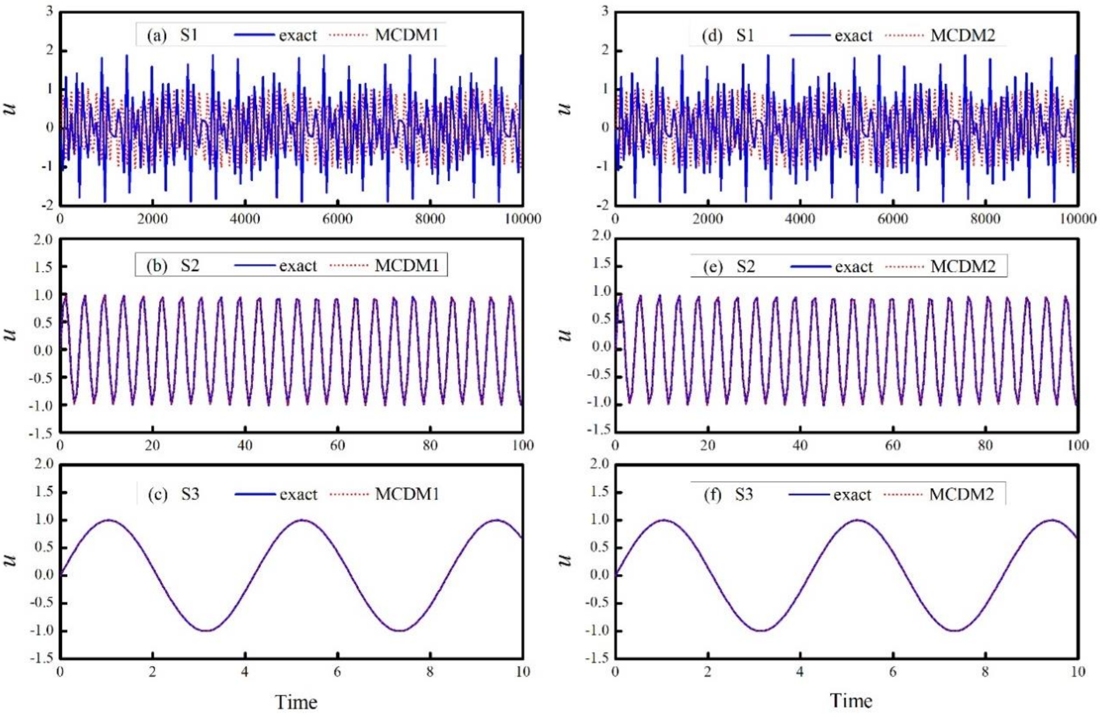

Fig. 4 Forced vibration response abtained from MCDM for S1, S2 and S3

$$
\begin{aligned}
E = & \frac{1}{D}\left(\alpha+\gamma-\tfrac{1}{2}\right)\left[\Omega_0\omega\dot{u}_i+\tfrac{1}{2}\Omega_0^2\ddot{u}_i+2\xi\Omega_0\left(\Delta t\right)\dddot{u}_i-\tfrac{1}{m}\left(f_{i+1}-f_i\right)\right] \\
& -\frac{1}{D}\left(\alpha\gamma+\tfrac{1}{2}\alpha+\beta-\tfrac{1}{6}\right)2\xi\Omega_0\left(\Delta t\right)\dddot{u}_i-\frac{1}{D}\left(\alpha\gamma-\tfrac{1}{2}\alpha+\beta-\tfrac{1}{12}\right)\left(\Delta t\right)^2\ddddot{u}_i \\
& +\frac{1}{D}\left(\gamma-\tfrac{1}{2}\right)2\xi\Omega_0\ddot{u}_i
\end{aligned}
\qquad (15)
$$

It is apparent that the three adverse error terms as shown in Eq. (11) are no longer in this local truncation error. In addition, the first term on the right-hand side of this equation becomes zero for the case of $\gamma=\frac{1}{2}-\alpha$. Whereas, the damping term is only linearly proportional to Ω_0 and the rest term is independent of Ω_0. This strongly indicates that the overshooting behavior in the steady-state response of a high frequency mode is eliminated by the extra loading term.

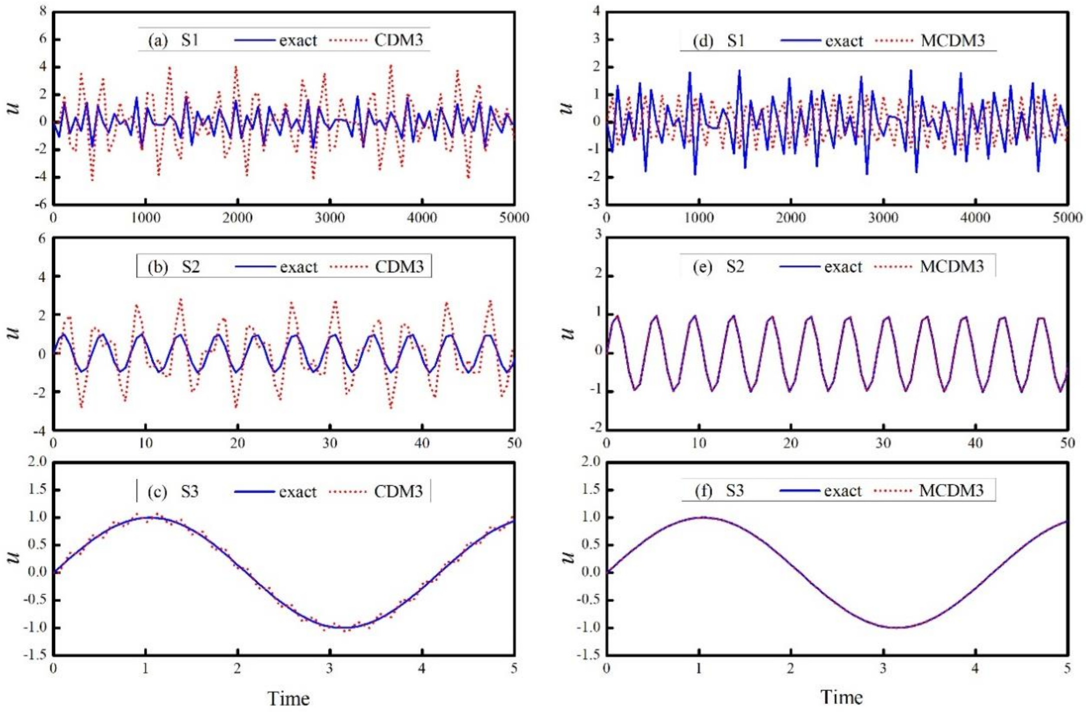

Fig. 5 Forced vibration response obtained from CDM3 and MCDM3 for S1, S2 and S3

4. Numerical verifications

In order to confirm the effectiveness of the proposed remedy, two numerical examples will be cautiously examined next by using MCDM. Notice that MCDM1 and MCDM2 are correspondent to CDM1 and CDM2. Notice that the only difference between CDM and MCDM in formulation is with or without the extra loading term. On the other hand, a structure-dependent integration method in the reference (Chang 2009) is also a member of CDM and thus MCDM but it is not in the subfamily of that defined in Eq. (4). In fact, it can be simply achieved by taking α=0 and β=γ=1/2. This member possesses no numerical dissipation and will be considered as CDM3 and MCDM3 in the subsequent numerical study.

4.1 Examplie 1

The illustrated example for a forced vibration response of a single degree of freedom system is solved again by using MCDM and the calculated solutions are plotted in Fig. 3. Apparently, the results obtained from MCDM1 and MCDM2 for using Δt=0.1 sec are overlapped together with the exact solutions in each plot of this figure. Consequently, it is verified to that the remedy can eliminate the adverse overshooting behavior in the steady-state response of a high frequency mode. On the other hand, the time step in correspondence to $\Delta t/T_0$=9.55 for each system is also used to compute the forced vibration response for using MCDM1 and MCDM2. The calculated results are shown in Fig. 4. In contrast to Fig. 2, the effectiveness of the proposed remedy is thoroughly confirmed since there is no overshoot in each plot of this figure although the value of $\Delta t/T_0$ is as large as 9.55, which is generally corresponding to a high frequency mode.

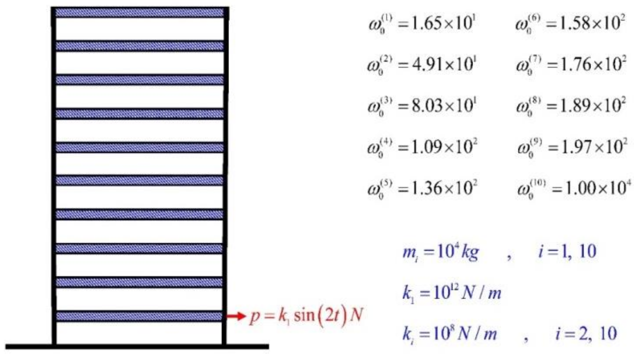

Fig. 6 A 10-story building and tis vibration properties

To illustrate that CDM3 will experience the unusual overshooting in the steady-state response of a high frequency mode and to confirm that MCDM3 can eliminate it, the three systems S1, S2 and S3 subject to the sine loading are also solved by CDM3 and MCDM3 with the time step in correspondence to $\Delta t/T_0$=9.55 for each system and the numerical results are plotted in Fig. 5. Fig. 5(a) to 5(c) attest to that an overshoot behavior is experienced if using CDM3 to calculate the numerical results. Whereas, no overshooting is found in Fig. 5(d) to 5(f) and thus it is confirmed that the proposed remedy can effectively eliminate the unusual overshooting.

4.2 Example 2

A 10-story shear-beam building is designed to show that the unusual overshooting behavior in the steady-state response of a high frequency mode might experience in a real structure in practical applications. The building and its

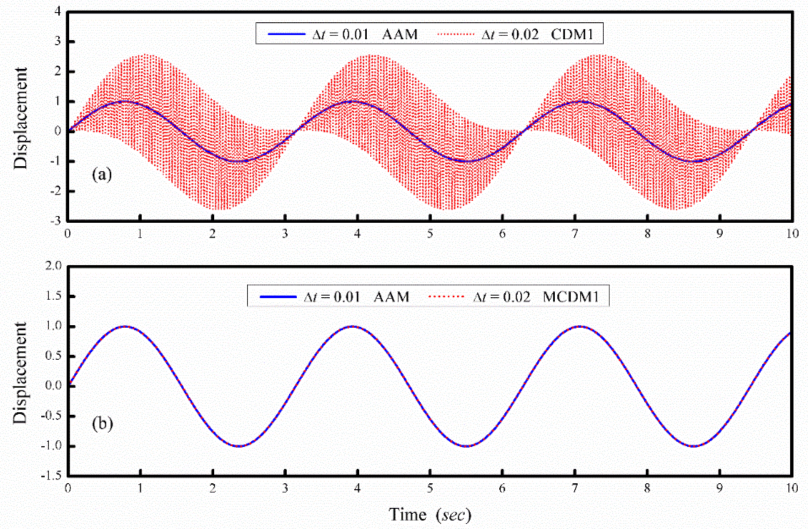

Fig. 7 Forced vibration response to 10-story building at bottom story

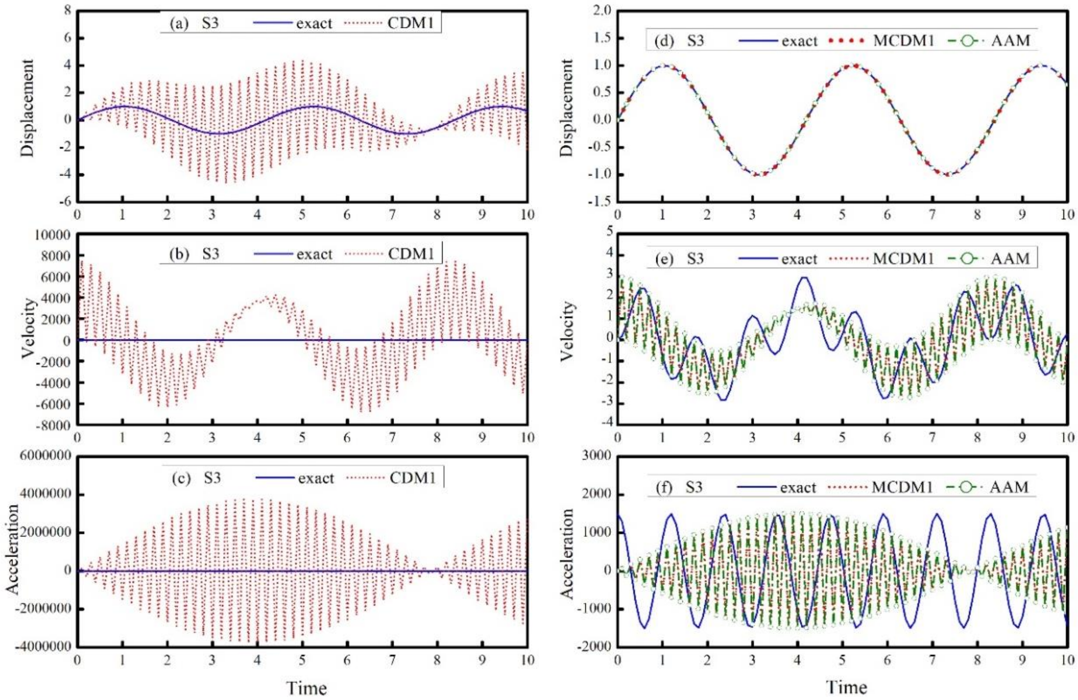

Fig. 8 Forced vibration response to S3 for using CDM1 and MCDM1 with Δt=0.1 sec

structural properties are shown in Fig. 6. Notice that only the bottom story of the building is subjected to a dynamic loading of k_1 sin(2t). It is found that the lowest natural frequency of the system is 16.52 rad/sec while the highest natural frequency is 10^4 rad/sec. The methods of CDM1 to CDM3 and their corresponding modified methods MCDM1 to MCDM3 with a time step of Δt=0.02 sec are applied to calculate the numerical results. The results obtained from CDM1 and MCDM1 are plotted in Fig. 7. For comparison, the solution obtained from the use of the constant average acceleration method (AAM) with Δt=0.01 sec is considered as a reference solution.

It is manifested from Fig. 7(a) that the response obtained from CDM1 for the bottom story of the 10-story building exhibits a very significant overshooting behavior. Whereas, this overshooting is eliminated by MCDM1 as shown in Fig. 7(b). Notice that the displacement response at the bottom story is dominated by the steady-state response since the period of this displacement response is exactly equal to π sec, which is the period of the applied sine loading. Very similar phenomena are also found for CDM2 and MCDM2 as well as CDM3 and MCDM3 and thus their results are not displayed herein repeatedly.

4.3 Example 3

It is analytically and numerical verified that an overshoot will occur in high frequency state-state responses for CDM and a remedy can be developed to overcome it. It is of interest to explore whether this overshoot also occurs

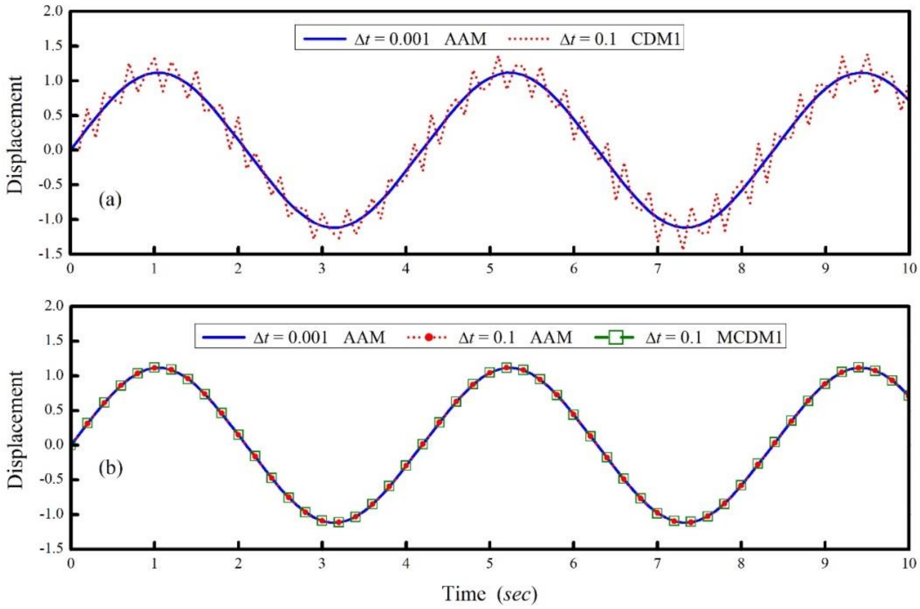

Fig. 9 Forced vibration response to S3 with stiffness softening

in velocity and acceleration and whether the remedy is also effective for eliminating velocity and acceleration. It seems that the results of S3 as shown in Fig. 1(c) and 3(c) can be also applied to reveal the overshoot behaviors in velocity and acceleration. As a result, the same problem of S3 is calculated by CDM1 and MCDM1 with Δt=0.1 sec and the results are plotted in Fig. 8. For comparison, the results obtained from the constant average acceleration method (AAM) with the same step size are also shown in this figure. Fig. 8(a) to 8(c) reveal that an overshoot in high frequency steady-state responses not only occur in displacement but also in velocity and acceleration. On the other hand, it is manifested from Fig. 8(d) to 8(f) that the remedy can eliminate the overshoot in high frequency steady-state responses not only in displacement but also in velocity and acceleration since the results obtained from CDM1 overlap with those obtained from AAM and exhibit no overshoot behaviors.

It is also of importance to show that the analytical results obtained from the studies of linear elastic systems are also applicable to nonlinear systems. For this purpose, the stiffness of S3 is slightly modified into the following form

$$k = k_0\left[1 - \frac{1}{10}\sqrt{|u|}\right] \tag{16}$$

Clearly, the stiffness will become softening after the system deforms. Numerical solutions obtained from CDM1, MCDM1 and AAM with Δt=0.1 sec are plotted in Fig. 9. The solution obtained from AAM with Δt=0.001 sec is considered as a reference solution for comparison. Fig. 9(a) reveals that CDM1 still experiences an overshoot in high frequency steady-state response for the nonlinear system. On the other hand, it is revealed by Fig. 9(b) that the proposed remedy can also remove an unusual overshoot in high frequency steady-state responses for nonlinear systems.

5. Conclusions

The previously published family method seems very useful for time integration since it can have desired numerical properties, such as unconditional stability, second-order accuracy, explicit formulation and controllable numerical dissipation. However, it might lead to an unusual overshoot in the steady-state response of a high frequency mode. This unusual overshoot can be detected by the local truncation error constructed from a forced vibration response rather than a free vibration response. In addition, it can be applied to propose an improved formulation of this family method. In fact, it can be achieved by adjusting the displacement difference equation with an additional load-dependent term. Consequently, it is analytically and numerically verified that the improved formulation of the previously published family method can effectively eliminate the unusual overshoot in the steady-state response of a high frequency mode.

References

Bathe, K.J. and Wilson, E.L. (1973), “Stability and accuracy analysis of direct integration methods”, *Earthq. Eng. Struct. Dyn.*, **1**, 283-291.

Bayat, M., Bayat, M. and Pakar, I. (2015), “Analytical study of nonlinear vibration of oscillators with damping”, *Earthq. Struct.*, **9**(1), 221-232.

Belytschko, T. and Hughes, T.J.R. (1983), *Computational Methods for Transient Analysis*, Elsevier Science Publishers B.V., North-Holland.

Chang, S.Y. (2002), “Explicit pseudodynamic algorithm with unconditional stability”, *J. Eng. Mech.*, ASCE, **128**(9), 935-947.

Chang, S.Y. (2006), “Accurate representation of external force in time history analysis”, *J. Eng. Mech.*, ASCE, **132**(1), 34-45.

Chang, S.Y. (2009), “An explicit method with improved stability property”, *Int. J. Numer. Meth. Eng.*, **77**(8), 1100-1120.

Chang, S.Y. (2010), “A new family of explicit method for linear

structural dynamics", *Comput. Struct.*, **88**(11-12), 755-772.
Chang, S.Y. (2014a), "A family of non-iterative integration methods with desired numerical dissipation", *Int. J. Numer. Meth. Eng.*, **100**(1), 62-86.
Chang, S.Y. (2014b), "Numerical dissipation for explicit, unconditionally stable time integration methods", *Earthq. Struct.*, **7**(2), 157-176.
Chang, S.Y. (2015), "Dissipative, non-iterative integration algorithms with unconditional stability for mildly nonlinear structural dynamics", *Nonlin. Dyn.*, **79**(2), 1625-1649.
Chang, S.Y. (2016), "A virtual parameter to improve stability properties for an integration method", *Earthq. Struct.*, **11**(2), 297-313.
Chang, S.Y., Wu, T.H. and Tran, N.C. (2015), "A family of dissipative structure-dependent integration methods", *Struct. Eng. Mech.*, **55**(4), 815-837.
Chung, J. and Hulbert, G.M. (1993), "A time integration algorithm for structural dynamics with improved numerical dissipation: the generalized-α method", *J. Appl. Mech.*, **60**(6), 371-375.
Fattah, M.Y., Hamoo, M.J. and Dawood, S.H. (2015), "Dynamic response of a lined tunnel with transmitting boundaries", *Earthq. Struct.*, **8**(1), 275-304.
Gao, Q., Wu, F., Zhang, H.W., Zhong, W.X., Howson, W.P. and Williams, F.W. (2012), "A fast precise integration method for structural dynamics problems", *Struct. Eng. Mech.*, **43**(1), 1-13.
Goudreau, G.L. and Taylor, R.L. (1972), "Evaluation of numerical integration methods in elasto- dynamics", *Comput. Meth. Appl. Mech. Eng.*, **2**, 69-97.
Hilber, H.M. and Hughes, T.J.R. (1978), "Collocation, dissipation, and 'overshoot' for time integration schemes in structural dynamics", *Earthq. Eng. Struct. Dyn.*, **6**, 99-118.
Hilber, H.M., Hughes, T.J.R. and Taylor, R.L. (1977), "Improved numerical dissipation for time integration algorithms in structural dynamics", *Earthq. Eng. Struct. Dyn.*, **5**, 283-292.
Kaveh, A., Aghakouchak, A.A. and Zakian, P. (2015), "Reduced record method for efficient time history dynamic analysis and optimal design", *Earthq. Struct.*, **8**(3), 639-663.
Rezaiee-Pajand, M. and Alamatian, J. (2008), "Implicit higher-order accuracy method for numerical integration in dynamic analysis", *J. Struct. Eng.*, **134** (6), 973–985.
Rezaiee-Pajand, M. and Karimi-Rad, M. (2017), "A family of second-order fully explicit time integration schemes", *Comput. Appl. Math.*, 1-24.
Rezaiee-Pajand, M., Sarafrazi, S.R. and Hashemian, M. (2011), "Improving stability domains of the implicit higher order accuracy method", *Int. J. Numer. Meth. Eng.*, **88** (9), 880-896.
Rezaiee-Pajanda, M. and Hashemian, M. (2016), "Time integration method based on discrete transfer function", *Int. J. Struct. Stab. Dyn.*, **16**(5), 1550009.
Rezaiee-Pajanda, M., Hashemian, M. and Bohlulyb, A. (2017), "A novel time integration formulation for nonlinear dynamic analysis", *Aerosp. Sci. Technol.*, **69**, 625-635.
Romero, A., Galvin, P. and Dominguez, J. (2012) "A time domain analysis of train induced vibrations", *Earthq. Struct.*, **3**(3), 297-313.
Su, C., Huang, H., Ma, H. and Xu, R. (2014), "Efficient MCS for random vibration of hysteretic systems by an explicit iteration approach", *Earthq. Struct.*, **7**(2), 119-139.
Verma, M., Rajasankar, J. and Iyer, N.R. (2015), "Numerical assessment of step-by-step integration methods in the paradigm of real-time hybrid testing", *Earthq. Struct.*, **8**(6), 1325-1348.
Wood, W.L., Bossak, M. and Zienkiewicz, O.C. (1981), "An alpha modification of Newmark's method", *Int. J. Numer. Meth. Eng.*, **15**, 1562-1566.

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