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Capacity spectrum method based on inelastic spectra for high viscous damped buildings

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Abstract. In the present study a capacity spectrum method based on constant ductility inelastic spectra to estimate the seismic performance of structures equipped with elastic viscous dampers is presented. As the definition of the structures' effective damping, due to the damping system, is necessary, an alternative method to specify the effective damping ratio ξeff is presented. Moreover, damping reduction factors (B) are introduced to generate high damping elastic demand spectra. Given the elastic spectra for damping ratio ξeff, the performance point of the structure can be obtained by relationships that relate the strength demand reduction factor (R) with the ductility demand factor (μ). As such expressions that link the above quantities, known as R – μ – T relationships, for different damping levels are presented. Moreover, corrective factors (Bv) for the pseudo-velocity spectra calculation are reported for different levels of damping and ductility in order to calculate with accuracy the values of the viscous dampers velocities. Finally, to evaluate the results of the proposed method, the whole process is applied to a four-storey reinforced concrete frame structure and to a six-storey steel structure, both equipped with elastic viscous dampers.

Keywords: simplified analysis method; capacity spectrum method; passive energy dissipation systems; high damping spectra; inelastic spectra; linear viscous damping

1. Introduction

The implementation of supplemental damping on the buildings is based on the concept of the designated energy dissipation. The energy does not dissipate primarily from the main structural members by their plastic behavior, but from the damping devices. These systems activate only due to horizontal loading and are easily replaced after potential damage (Karavasilis 2016, Withle *et al.* 2012, Seo *et al.* 2014, Symans *et al.* 2008, Constantinou *et al.* 1993, Whittaker *et al.* 1993). Passive energy dissipation systems such as viscous and viscoelastic dampers contribute to increasing the effective damping of the structure and hence to reducing its seismic demand. The application of passive energy dissipation systems as a method to enhance the seismic response of structures has been studied since early 1990. At the same time the first regulations for the usage of such systems were presented by Whittaker *et al.* 1993).

The control of the demanded displacement of the structure due to the seismic designed must be necessary, with or without considering additional damping systems. Therefore, provisions regarding the analysis methods of such buildings have been added to regulations like FEMA 368 and FEMA 273 for new and existing buildings accordingly. The Nonlinear Dynamic Procedure (NDP) seems to be the most accurate analysis method since all the contemporary regulations accept the nonlinear behavior of the structure, both for the seismic design and assessment. Despite the reliability of the NDP even though the uncertainties related with those (Whittaker *et al.* 2001), simplified static procedures like the Linear Static Procedure (LSP) (FEMA 273, FEMA 368) and Nonlinear Static Procedure (NSP) (FEMA 273) were introduced by the regulations.

From the simplified procedures the linear is appropriate for the seismic design and the nonlinear is more suitable for the seismic evaluation of an existing structure (Whittaker *et al.* 2003). Although the accuracy of the simplified methods has been examined extensively for both elastic (Whittaker *et al.* 2003, Sadek *et al.* 2000, Ramirez *et al.* 2001, 2003, Pavlou and Constantinou 2004 a, b) and inelastic methods (Tsopelas *et al.* 1997, Ramirez *et al.* 2002a, b), seismic assessment of structures which are expected to respond nonlinearly can be provided with more accuracy by the NSP as it takes into consideration the actual nonlinear response of the structural members (Tsopelas *et al.* 1997).

NSP introduced by FEMA 274 (Method 2) is based on the Capacity Spectrum Method (CSM) which was initially developed by Freeman (Freeman *et al.* 1975, Freeman 1978). In nearly most alternatives of NSP which were investigated throughout the years, the performance point of the structure is calculated by an repetitive process so as to equilibrates the demand with the capacity given initially the pushover curve (capacity) and the elastic demand spectra with damping ratio ξ=5%. An equivalent SDOF is considered throughout this operation with period T=T_{eq} and viscous damping $\xi=\xi_{eq}$. The equivalent damping ξ_{eq} is specified as the sum of the inherent damping (ξ_o), the additional damping due to the damping devices (ξ_D) and the

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hysteretic damping expressed in terms of equivalent viscous damping (ξ_{hyst}). Damping Reduction Factors (B) for certain values of the effective damping, are used to decrease the elastic spectra. The values of the B factor are provided in the literature (Sadek *et al.* 2000, Ramirez *et al.* 2002a, Palermo *et al.* 2013) and in the seismic codes.

An alternative CSM, for structures without additional damping, by using inelastic spectra, have been examined by Fajfar and Gašperšič (1996) and Fajfar (1999), as well as Chopra and Goel (1999a, b). This method differs from the classical CSM as the reduction of the capacity spectra is performed through relationships between the strength demand reduction factor of the structure R and the ductility μ. These types of relationships are referred to as R–μ–T and there are many proposed in the literature (Miranda and Bertero 1994, Vidic *et al.* 1994, Hidalgo and Arias 1990, Riddell and Newmark 1979). Except of Ridell and Newmark (1979) and Palermo *et al.* (2013) who proposed R–μ–T relationships for systems with higher values of viscous damping, the rest of the studies proposed R–μ–T relationships for damping ratio 5%, while Ramirez *et al.* (2002a) associates the elastic displacement with the expected inelastic, taking into account the additional damping not by using the R–μ–T relationships but through a coefficient C_1.

Considering that for stiff structural systems and in the case of high values of ductility, the inelastic spectra are more accurate than the elastic one (Fajfar and Gašperšič 1996, Fajfar 1999, Chopra and Goel 1999a, b), a CSM based on constant ductility inelastic spectra has been introduced, in order to evaluate the seismic response of structures equipped with elastic viscous dampers. In order to perform this method, the determination of the effective damping due to supplemental viscous damping devices is a basic requirement. As such, a simplified modal viscous damping is examined and the accuracy of this method is justified by comparing the results with those obtained by the FEMA relationships. Moreover, to determine the elastic demand of high damping systems, damping reduction factors (B) are presented, as well as R–μ–T relationships for systems with damping ratio up to 50%. Furthermore, pseudo-velocity correlation factors are introduced. Finally, the proposed CSM using high damping-inelastic spectra is performed into a four and six-storey RC and steel frame respectively.

2. CSM for structures equipped with viscous dampers based on inelastic spectra

In the present study a CSM based on constant ductility-yield point inelastic spectra has been adopted, in order to estimate the expected deformations of structures with supplemental viscous damping. According to the procedure, the performance point result by the relationship between the strength reduction factor R with the displacement ductility of the system μ. Therefore, the effective damping of the structure must be limited to the sum of the inherent damping (ξ_o) with the additional damping due to the damping devices (ξ_D), neglecting the hysteretic damping expressed in terms of equivalent viscous damping (ξ_{hyst}). The above sum may be cited as ξ_{eff}. Thus, simplified modal viscous damping is examined for the determination of the additional damping due to the damping devices (ξ_D). The modal viscous damping is calculated by transforming the equation of motion of a MDOF system with viscous dampers, to its eigenvectors without damping.

Given the elastic demand spectra usually with 5% damping, elastic demand spectra with ξ_{eff} damping ratio can be determined by applying damping reduction factors (B). Then, the inelastic demand spectrum for damping ratio ξ_{eff} can be constructed by the use of R–μ–T equations. Finally, the performance point can be estimated by inelastic spectra, in a yield point response spectra format as it developed by Aschheim and Black (2000).

Specifically, a modal analysis is initially required in order to specify the dynamic characteristics of the structure. Provided that the dynamic structural response could be described by the fundamental mode, the capacity spectrum method can be applied. As explained above, the determination of the effective damping is expressed by the following relationship

$$\xi_{eff} = \xi_o + \xi_D \tag{1}$$

The capacity of the structure is given by the pushover curve, which has to be converted to an equivalent bilinear capacity spectrum. To define the demand, the first step is to construct the elastic spectrum for damping equal to the effective damping of the structure ξ_{eff}. The modification of the elastic spectrum with 5% damping to the spectrum with damping ratio ξ_{eff} is performed with the use damping reduction factors (B) as follows (Ramirez *et al.* 2002a)

$$S_a(T,\xi_{eff}) = \frac{S_a(T,5\%)}{B(T,\xi_{eff})}, \quad S_d(T,\xi_{eff}) = \frac{S_d(T,5\%)}{B(T,\xi_{eff})} \tag{2}$$

where S_a (T,5%) and S_d (T,5%), is the demanded acceleration and displacement respectively for 5% damping ratio, S_a (T,ξ_{eff}) and S_d (T,ξ_{eff}) is the demanded acceleration and displacement respectively for damping ratio ξ_{eff} and B (T,ξ_{eff}) is the damping reduction factor for damping ratio ξ_{eff}.

In case that the elastic demand spectrum intersect the capacity spectrum to it elastic region, the structure is expected to response elastically. Otherwise, constant ductility-yield point inelastic spectra using the R–μ–T relations have to be constructed. The demanded ductility of the structural systems is estimated throughout the intersect point of the inelastic spectrum with the capacity spectrum at the yield point. The inelastic spectra are defined by the following relationships (Miranda and Bertero 1994, Chopra and Goel 1999 a, b)

$$S_{a,y}(T,\xi_{eff}) = \frac{S_{a,el}(T,\xi_{eff})}{R(\mu,\mathrm{T},\xi_{eff})}, \quad S_{d,y}(T,\xi_{eff}) = \frac{S_{d,el}(T,\xi_{eff})}{R(\mu,\mathrm{T},\xi_{eff})} \tag{3}$$

where $S_{a,el}$ (T,ξ_{eff}) is the demanded acceleration for elastic response and damping ratio ξ_{eff}, $S_{a,y}$ (T,ξ_{eff}) is the demanded acceleration at yielding and damping ratio ξ_{eff}, $S_{d,el}$ (T, ξ_{eff}) and $S_{d,y}$ (T, ξ_{eff}) the spectral displacements respectively and

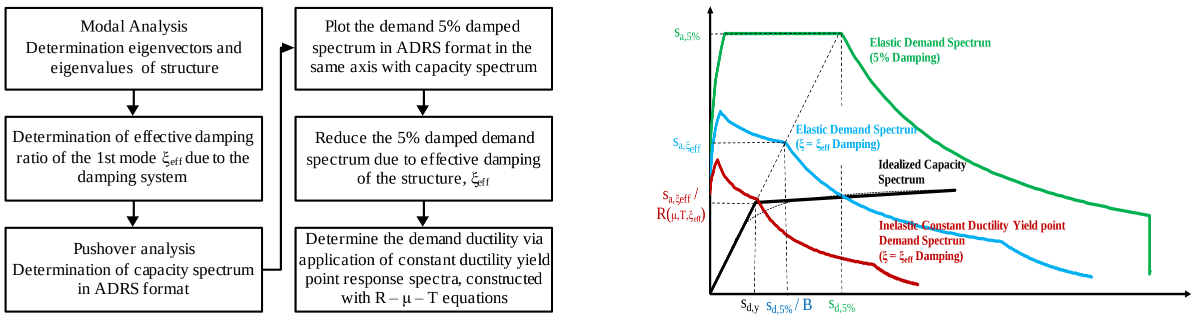


Fig. 1 Flowchart diagram of the calculation scheme

R $(\mu$, T,$\xi_{eff})$ is the strength reduction factor for displacement ductility demand μ and damping ratio ξ_{eff}.

The calculation scheme of the proposed CSM for buildings equipped with viscous dampers, as it is in detail described above, can be summarized in Fig. 1.

The calculation of the damping devices developing forces are of great importance. As the viscous dampers are velocity depended devices, the estimation of the dampers ends' relative velocity is essential. Given the spectral velocity S_v of the fundamental structural mode, the dampers' developing forces are given by the equation

$$F_{D,i} = \Gamma^* S_v(T)\varphi_{ij} C_i \cos\theta_i \tag{4}$$

where $F_{D,i}$ is the damping force of the device i, Γ* is the participation factor of the fundamental mode, θ_i is the angle to the horizontal of the device i, C_i is the damping coefficient of the device i and φ_{ij} is the 1st modal ordinate between the ends of the damper i.

However, the pseudo-velocity spectra (PS_v) are in common used, instead of the velocity spectra (S_v). Nevertheless, these values are not equivalent at the whole period range of the spectra. In that view, Sadek *et al.* (2000) examined 72 horizontal seismic excitations for damping ratios ξ=2–60%, concluding that spectral velocity may be considered equal to the spectral pseudo-velocity at the period region nearby the period value T=0.5 s. The same remark has been highlighted also by Hatzigeorgiou and Pneumatikos (2013). For that reason, a velocity corrective factor (B_v) is introduced, in order to estimate with increasing accuracy the spectral velocities, as presented in Eq. (5)

$$S_V = \frac{PS_V}{B_v} \tag{5}$$

In the following subsections of this section, all the parameters which are presented above, are thoroughly defined. Particularly, an alternative method of specifying the effective period of the structure (ξ_{eff}), as well as relationships to determine the damping reduction factors (B) are presented. Moreover, the introduced strength reduction factors (R–μ–T) and the adopted velocity correlation factors (B_v) for different levels of damping ratios and displacement ductility ratios are given.

2.1 Effective damping

2.1.1 Conventional assumptions of equivalent damping ratio

The methodology which is extensively used and adopted by FEMA 274 and 368 to calculate the effective viscous damping of a structural system with additional damping is presented below. The total amount of the viscous damping is estimated through the ratio of the dissipated energy per circle of motion W_{Diss} divided by the work of the restoring forces of an equivalent elastic oscillator W_R (FEMA 273, Ramirez *et al.* 2001, Chopra 2001)

$$\xi_{eff} = \frac{1}{4\pi}\frac{W_{Diss}}{W_R} \tag{6}$$

In the case of linear viscous dampers, the above relation can be written

$$\xi_D = \frac{T_{eff}\sum c_j \cos^2\theta_j \varphi_{rj}^2}{4\pi\sum m_i \varphi_i^2},\ \xi_D = \frac{\sqrt{\mu}T_{el}\sum c_j \cos^2\theta_j \varphi_{rj}^2}{4\pi\sum m_i \varphi_i^2} \tag{7}$$

where c_j is the damping coefficient of the j device, θ_j is the devices' angle to the horizontal, φ_{rj} is the relative modal ordinate, T_{eff} is the period corresponding to the lateral stiffness for the ductility ratio μ. From Eq. (7) it can be observed that the level of the additional damping depends on the effective period of the equivalent elastic oscillator. Relations based on Eq. (6) are also used when nonlinear damping is assumed (Diotallevi *et al.* 2012, Landi *et al.* 2014).

2.1.2 Alternative derivation of effective damping

A simplified modal viscous damping which is calculated by transforming the equation of motion of a MDOF system with viscous dampers to its eigenvectors without damping is examined. The response of a damped MDOF oscillator can be computed by the superposition of equivalent SDOF oscillators' responses under harmonic vibration. In such case the eigen value problem is described by the expression

$$\left(\lambda^2\mathbf{m} + \lambda\mathbf{c} + \mathbf{k}\right)\boldsymbol{\phi} = 0 \tag{8}$$

The above constitutes a square eigen value problem resulting in complex eigen values and eigenvectors. Due to the fact that the solution of this problem requires eight times more calculations than without considering damping (Chopra 2001), an approximate method to estimate the response of MDOF oscillators has to be used. After transforming the equation of motion of a damped system with the eigenvectors of motion of the system without damping, the equation of motion in terms of modal coordinates is

$$\mathbf{M}^*\ddot{\mathbf{q}} + \mathbf{C}^*\dot{\mathbf{q}} + \mathbf{K}^*\mathbf{q} = -\mathbf{L}^*\ddot{u}_g(t) \tag{9}$$

where **M*** = $\mathbf{\Phi}^T$ **m Φ** is the generalized mass matrix, **C*** = $\mathbf{\Phi}^T$ **c Φ** is the generalized damping matrix, **K*** = $\mathbf{\Phi}^T$ **k Φ** is the generalized stiffness matrix and **L*** = $\mathbf{\Phi}^T$ **m δ** is the excitation vector.

To the equation above the generalized damping matrix is not necessarily diagonal resulting to n coupled SDOF oscillators (Chopra 2001). As the data out of the diagonal can be omitted, the equation of motion of the equivalent SDOF oscillator corresponding to the k mode of a MDOF system with linear viscous dampers and viscoelastic behavior of it structural members can be written

$$\mathrm{M}_k^*\ddot{\mathbf{q}} + C_k^*\dot{\mathbf{q}} + K_k^*\mathbf{q} = -\Gamma_k^*\mathrm{M}_k^*\ddot{u}_g(t) \tag{10}$$

where $\mathrm{M}_k^* = \varphi_{k,i}\cdot m_{ij}\cdot\varphi_{k,j}$, is the generalized mass of the k mode; $C_k^* = \varphi_{k,i}\cdot c_{ij}\cdot\varphi_{k,j}$, is the generalized damping of the k mode; $K_k^* = \varphi_{k,i}\cdot k_{ij}\cdot\varphi_{k,j}$, is the generalized stiffness of the k mode; $L_k^* = \varphi_{k,i}\cdot m_{ij}\cdot\delta_j$, is the excitation vector of the k mode.

Assuming that the response of the MDOF system can be described satisfactorily by the 1st mode, the effective damping of the equivalent SDOF is

$$\xi_{eff} = \xi_o + \frac{C_1^*}{2M_{1,eff}^* / \Gamma_1^{*2}\omega_1} \tag{11}$$

where ξ_o is the critical damping ratio of the structure without dampers, $C_1^* = \varphi_{1,i}\cdot c_{ij}\cdot\varphi_{1,j}$ is the generalized damping of the first mode, $\mathrm{M}_{1,\mathrm{eff}}^* = \left(\Gamma_1^*\right)^2\cdot\mathrm{M}_1^*$ is the effective modal mass of the first mode, $\Gamma_1^* = L_1^*/\mathrm{M}_1^*$ is the participation factor of the first mode and ω_1 is the natural vibration frequency of the first mode.

The data c_{ij} of the matrix $\mathbf{C_D}$, describes the damping force at the i degree of freedom where a unit of velocity is enforced to the degree of freedom j. The damping force is given by the expression

$$\mathbf{F_D} = c_D\cdot\left|\dot{\mathbf{u}}_{//}\right|^a\cdot\mathrm{sgn}(\dot{\mathbf{u}}_{//}) \tag{12}$$

where c_D is the damping coefficient of the damper, α is the exponential coefficient with values ranging from 0.1 to 1, $\dot{\mathbf{u}}_{//}$ is the relative translational velocity between the ends of the damper and sgn is the signum function that provides the correct sign for the damping force.

Assuming an oscillator with n DOF where each mass m_i is connected with the previous mass with a viscous damper that has damping coefficient c_i and angle θ_i to the horizontal, the damping matrix $\mathbf{C_D}$ can be computed as follows:

By moving each time a DOF with a unit of velocity $\dot{\mathbf{u}}_\mathbf{i} = \overline{1}$ whereas all the others DOF remain restrained, the dampers velocity in parallel to its direction is

$$\dot{\mathbf{u}}_{//,\mathbf{i}} = \dot{\mathbf{u}}_\mathbf{i}\cdot\cos\theta_i = \cos\theta_i \tag{13}$$

and the corresponding force

$$\mathbf{F_{d,i}} = c_i\cdot\cos^\alpha(\theta_i)\cdot\left|\dot{\mathbf{u}}_\mathbf{i}^a\right|\cdot\mathrm{sgn}(\dot{\mathbf{u}}) = c_i\cdot\cos^\alpha(\theta_i) \tag{14}$$

while the horizontal projection of the damper force is

$$\mathbf{F_{dx,i}} = c_i\cdot\cos^{\alpha+1}(\theta_i)\cdot\left|\dot{\mathbf{u}}_\mathbf{i}^a\right|\cdot\mathrm{sgn}(\dot{\mathbf{u}}) = c_i\cdot\cos^{\alpha+1}(\theta_i) \tag{15}$$

From the equilibrium of all the forces applied to each mass mi due to the dampers, the contents of the damping matrix $\mathbf{C_D}$ are given by the relationship

$$\text{For i = j} \quad C_{i,j} = c_i\cdot\cos^{a+1}(\theta_i) + c_{i+1}\cdot\cos^{a+1}(\theta_{i+1}) \tag{16a}$$

$$\text{For j = i+1 or i-1} \quad C_{i,j} = -c_j\cdot\cos^{a+1}(\theta_j) \tag{16b}$$

$$\text{For all the other cases} \quad C_{i,j} = 0 \tag{16c}$$

The above relationships, known by the literature, are presented for the sake of completeness. The effective damping of Eq (7), which is calculated by Eq. (6), is equivalent with the proposed effective damping of Eq. (11), although they are computed based on different assumptions.

2.2 High damping spectra

2.2.1 Elastic high damping spectra

For the application of the CSM, the spectra of the examined excitation must be defined. The most accurate way to define an elastic spectra is to integrate the differential equation of motion throughout the time. Nevertheless, reduction factors are usually used to reduce the elastic spectra that correspond to 5% damping ratio in order to take into consideration either the additional damping influence or the inelastic response. In the case of high damping elastic spectra the reduction has been performed by using damping reduction factors, defined as follows (Ramirez *et al.* 2002a)

$$B = \frac{S_d(T,5\%)}{S_d(T,\xi)} \tag{17}$$

where S_d (T,5%), is the demanded displacement for 5% damping ratio and S_d (T,ξ) is the demanded displacement for damping ratio ξ.

Several expressions of the reduction factor B are already presented in the literature and regulations (FEMA274, FEMA 368, EC-8, Sadek *et al.* 2000, Ramirez *et al.* 2002a, Palermo *et al.* 2013). Most of them are specified by bilinear or trilinear models. The common feature of all the proposed models is that the reduction factor remains constant beyond the period values that correspond to the constant acceleration area of the response spectra.

Analyses were performed using a set of 20 ground motions (Table 1) which have been scaled to the EC-8 response spectra for soil type C. It can be observed that the reduction did not remain constant above the area of the

Table 1 Earthquake events used in this study

Date	Earthquake	M_s	Station	Component
1941	Northern Calif-01	6.40	Ferndale City Hall	315
1951	Imperial Valley-03	5.60	ElCentro Array #9	000
1952	KernCounty	7.36	Taft Lincoln School	021
1961	Hollister-01	5.60	Hollister City Hall	271
1966	Parkfield	6.19	Cholame – Shandon Array #12	050
1967	Northern Calif-05	5.60	Ferndale City Hall	314
1968	BorregoMtn	6.63	ElCentro Array #9	180
1971	SanFernando	6.61	Castaic – Old Ridge Route	291
1973	PointMugu	5.65	Port Hueneme	270
1976	Friuli Italy-01	6.50	Barcis	000
1978	SantaBarbara	5.92	Cachuma Dam Toe	250
1978	TabasIran	7.35	Dayhook	L1
1979	Imperial Valley-06	6.53	Brawley Airport	225
1980	Livermore-01	5.80	APEEL 3E Hayward CSUH	146
1980	Mammoth Lakes-01	6.06	Long Valley Dam (Upr L Abut)	090
1980	VictoriaMexico	6.33	Cerro Prieto	315
1981	Taiwan SMART1(5)	5.90	SMART1 O07	EW
1981	Westmorland	5.90	Parachute TestSite	225
1984	MorganHill	6.19	San Juan Bautista_ 24 Polk St	213
1986	Mt. Lewis	5.60	Halls Valley	090

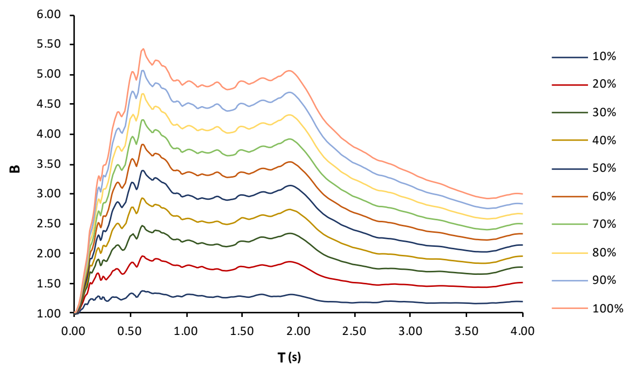


Fig. 2 Mean reduction of the elastic spectra due to damping.

Table 2 Values of Eq. (22) factors a, b and c

	ξ									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
a	1.46	1.92	2.34	2.82	3.40	4.16	5.26	7.09	10.71	80.40
b	-0.15	-0.20	-0.24	-0.27	-0.30	-0.33	-0.36	-0.40	-0.44	-0.55
c	-2.56	-1.75	-1.45	-1.28	-1.15	-1.04	-0.93	-0.83	-0.74	-0.59

constant spectral acceleration but it tends to descend (Fig. 2). Thus, in the present study the construction of a single continuum expression for the damping reduction factors that could take into account their reducing tendency at the higher values of periods is attempted.

The maximum acceleration of an oscillation with damping ratio ξ, larger than 5%, is smaller, due to the larger damping of the system. To specify the reduced seismic demand, it can be assumed that the variation of the

Table 3 Values of equations (24) factors a, b and c

	ξ		
	0.05	0.10	≥ 0.20
a	0.31	0.25	0.24
b	-0.97	-0.47	-0.13
c	1.00	0.95	0.94

maximum potential energy between the systems with damping ratio 5% ($E_{P,0.05}$) and ξ ($E_{P,\xi}$) would be equal with the energy that is dissipated due to an increase of the damping by $\Delta\xi$ ($E_{D,\Delta\xi}$) (Eq. (18)).

$$E_{P,0.05} = E_{P,\xi} + E_{D,\Delta\xi} \tag{18}$$

Therefore, the value of the dissipated energy due to the increase of the damping by $\Delta\xi$ must be defined. Considering a forced vibration by an harmonic external force $P(t) = P_o \cdot \sin(\omega t)$, the dissipated energy under one cycle of loading due to viscous damping $\xi^* = \Delta\xi$ is (Chopra 2001)

$$E_D = \int f_D du = \int_0^{2\pi/\omega} c^* \dot{u}^2 dt = \pi c^* \omega u_o^2 = 2\pi\xi^* \frac{\omega}{\omega_n} k u_o^2 \tag{19}$$

Combining the Eqs. (17)-(19) results to

$$B = \frac{Sa_{0.05}}{Sa_\xi} = \sqrt{1 + 4\pi\Delta\xi \frac{T_n}{T}} \tag{20}$$

where T is the period of the harmonic external force P(t) and T_n the natural period of the oscillator.

As the term T is difficult to specify mainly due to the uncertainties of the ground motions and the non-harmonic shape of a natural ground motion, a set of 20 ground motions were used to conclude to a function of the general form f (T, ξ) in order to describe the ratio T_n/T. By calibrating the Eq. (20) to the analytical results, the reduction factor of the spectra are given by the following relationships

$$B = \frac{Sa_{0.05}}{Sa_\xi} = \sqrt{1 + 4\pi(\xi - 0.05) f(T,\xi)} \tag{21}$$

$$f(T,\xi) = \mathrm{a}\left(\mathrm{e}^{b\frac{T}{T_o}} - \mathrm{e}^{c\frac{T}{T_o}} \right) \tag{22}$$

where a, b and c are based on the damping levels and they are listed in the Table 2 and T_o is the period that corresponds to the beginning of the constant spectral velocities area.

Fig. 3 displayed comparatively the damping reducing factors from Eq. (20) and the mean reduction calculated from the 20 ground motions by time history analysis. A special characteristic of the proposed continuum nonlinear expression is that it can be applied to the whole range of the spectra periods, while it describes the descending reducing

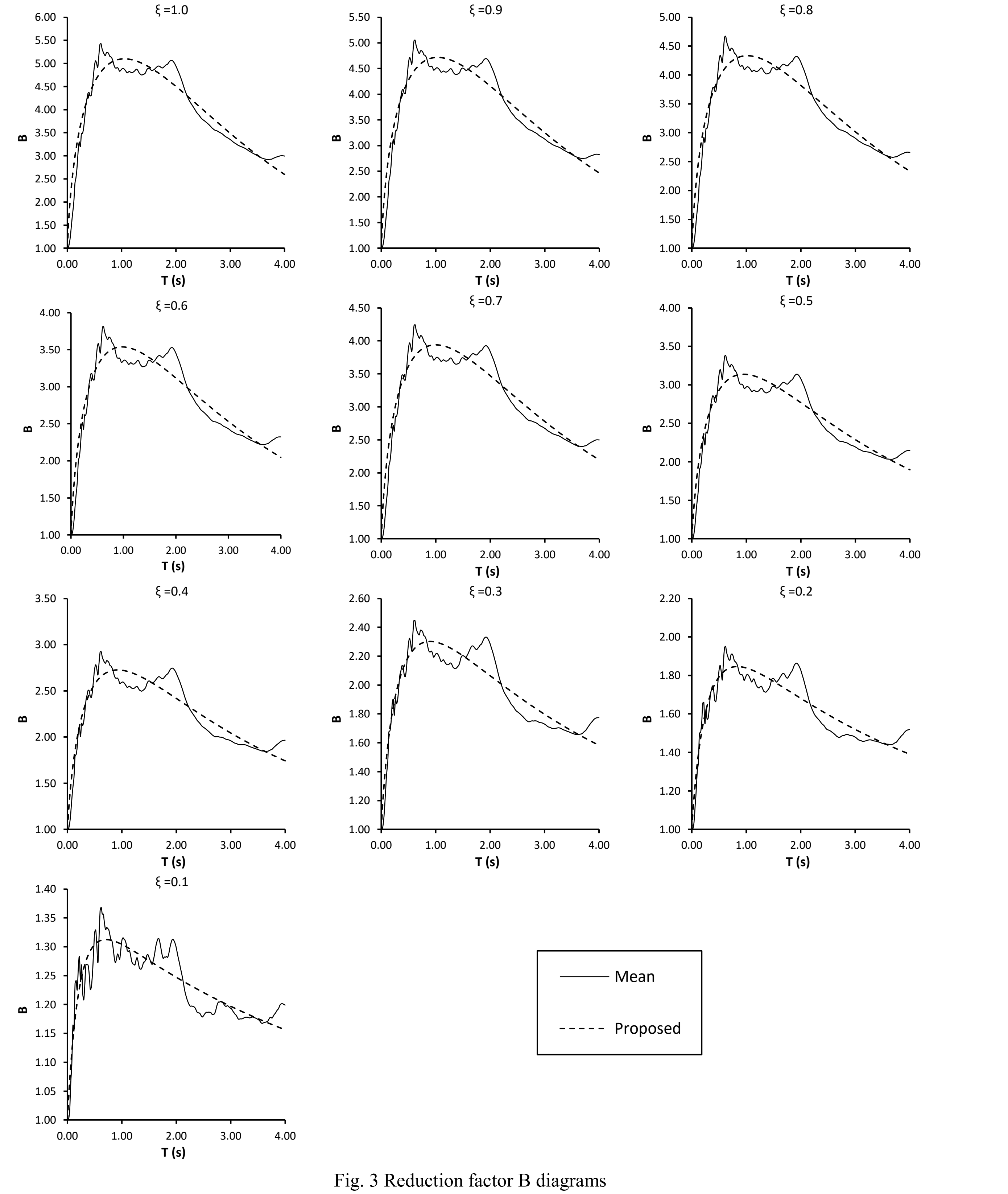


Fig. 3 Reduction factor B diagrams

factor beyond the constant spectral acceleration area.

Mentioned above, the reduction factor B values results by scaled, over the EC-8 spectrum, ground motion records. Despite the fact that the scaling may affect the results, the B factor (Eq. (21)) can be used for excitations compatible with the EC-8 spectra, due to the fact that is defined based on that. Moreover, B reduction factor could have a generalized application as it is parameterized over the period value T_o, which is determined by any code spectra.

The values of damping reduction factor B corresponding

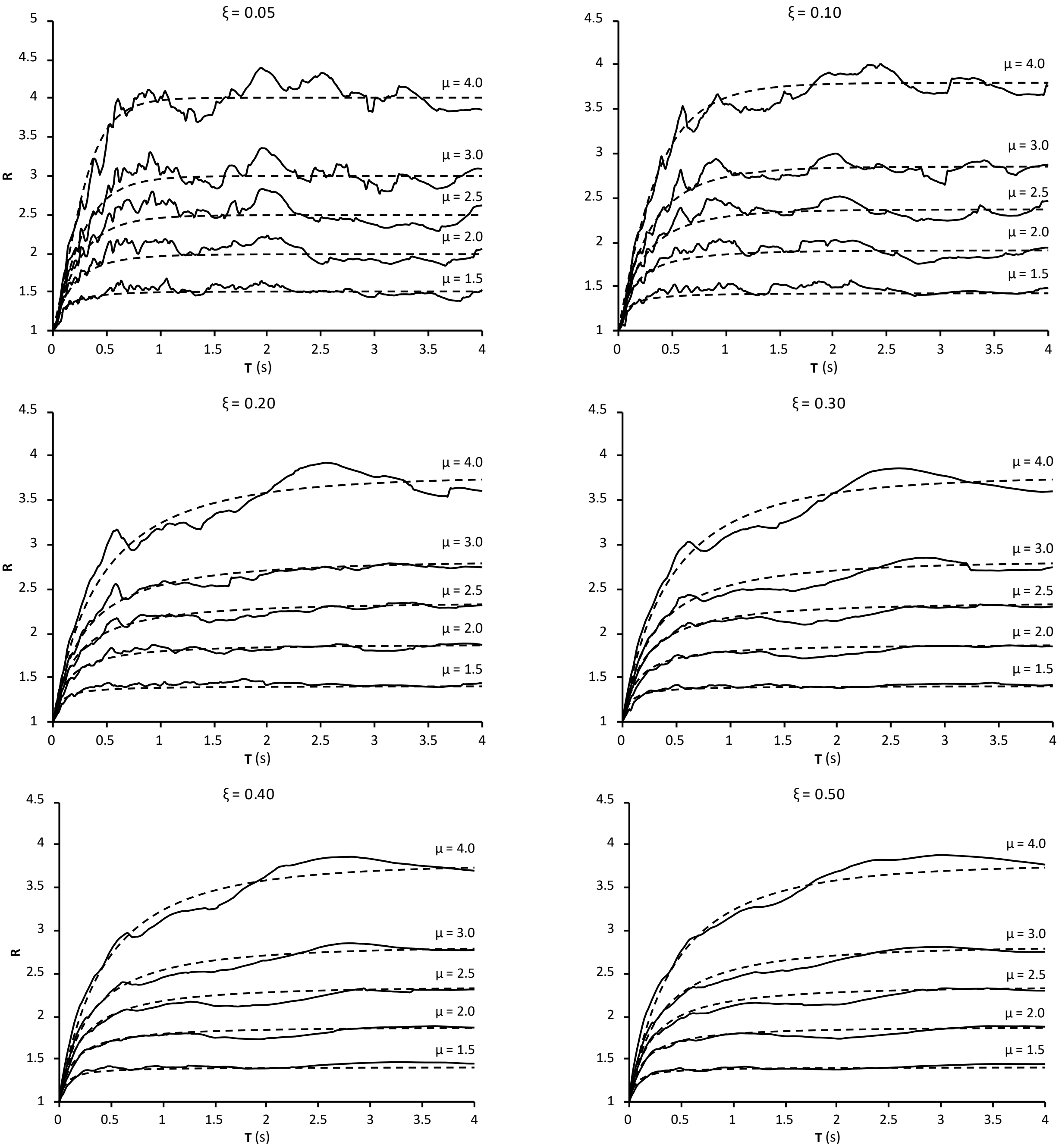


Fig. 4 Strength reduction factors R, by the analysis (solid line) and by the proposed relationships (dash line)

for long period structures are similar with those presented by Pavlou and Constantinou (2004 a, b) for near fault ground motions. It seems that these values are affected by the selection of the ground motions. However, that is not a considerable issue as long period structures has to be assessed using dynamic time history analyses due to the participation of higher order modes on the seismic response. Notwithstanding the B values corresponding to high period structures are not essential, the Eq. (21) can be used for the initial design and estimation of the contribution of the implemented dampers.

2.2.2 Constant ductility inelastic high damping spectra

In the case of structures that respond in the inelastic range, the application of the CSM requires the inelastic spectra of the examined excitation. As in the case of elastic response spectra, the most accurate way to define the inelastic spectra is to integrate the differential equation of motion throughout the time. Nevertheless, in practice, strength reduction factors (R) are usually used to reduce the elastic spectra. The strength reduction factor defined as the ratio of elastic demand strength (F_{el}) to inelastic demand strength (F_y) as follows (Miranda and Bertero 1994, Chopra and Goel 1999 a, b)

$$R = \frac{F_{el}}{F_y} = \frac{S_{a,el}}{S_{a,y}} \tag{23}$$

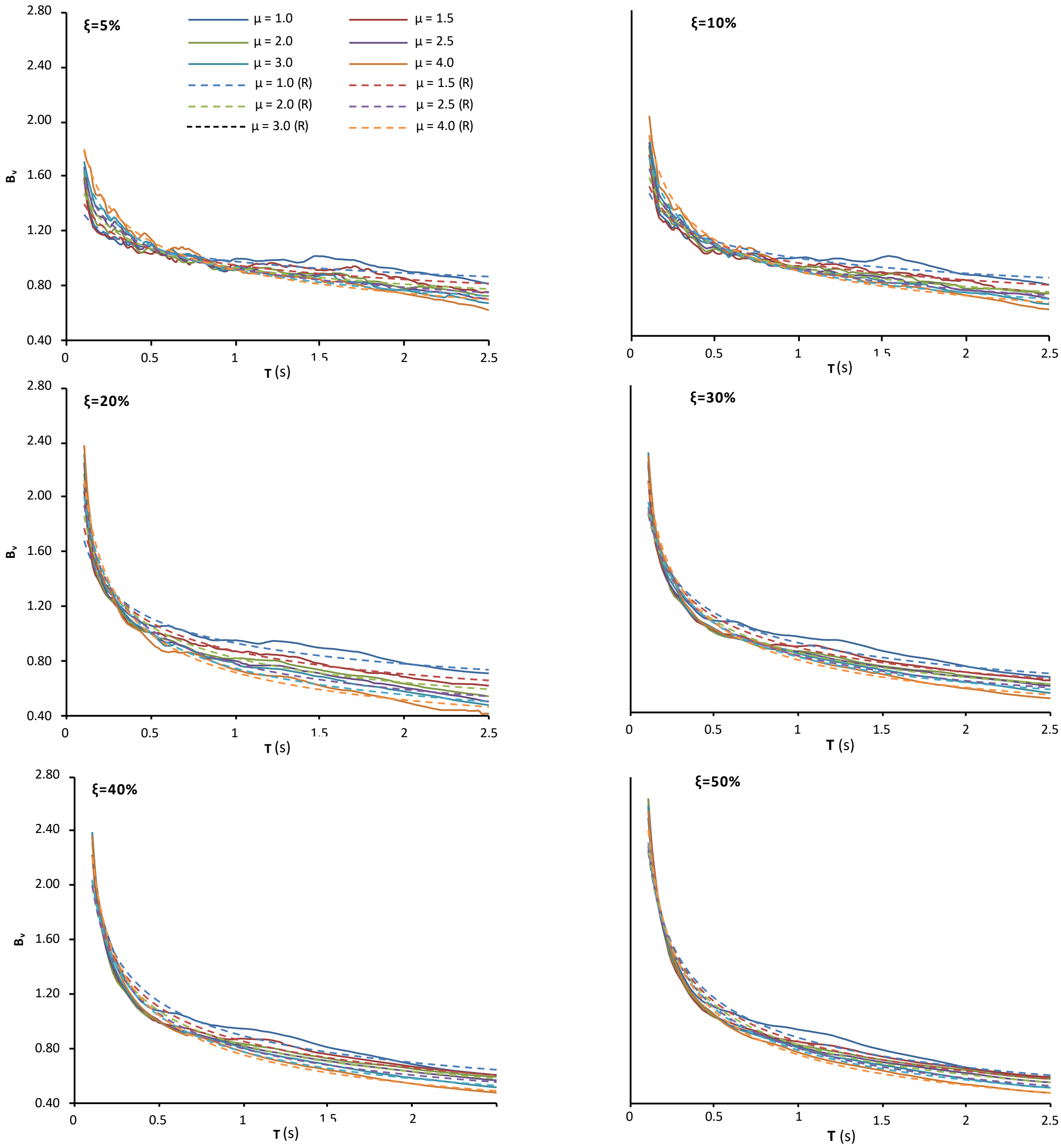


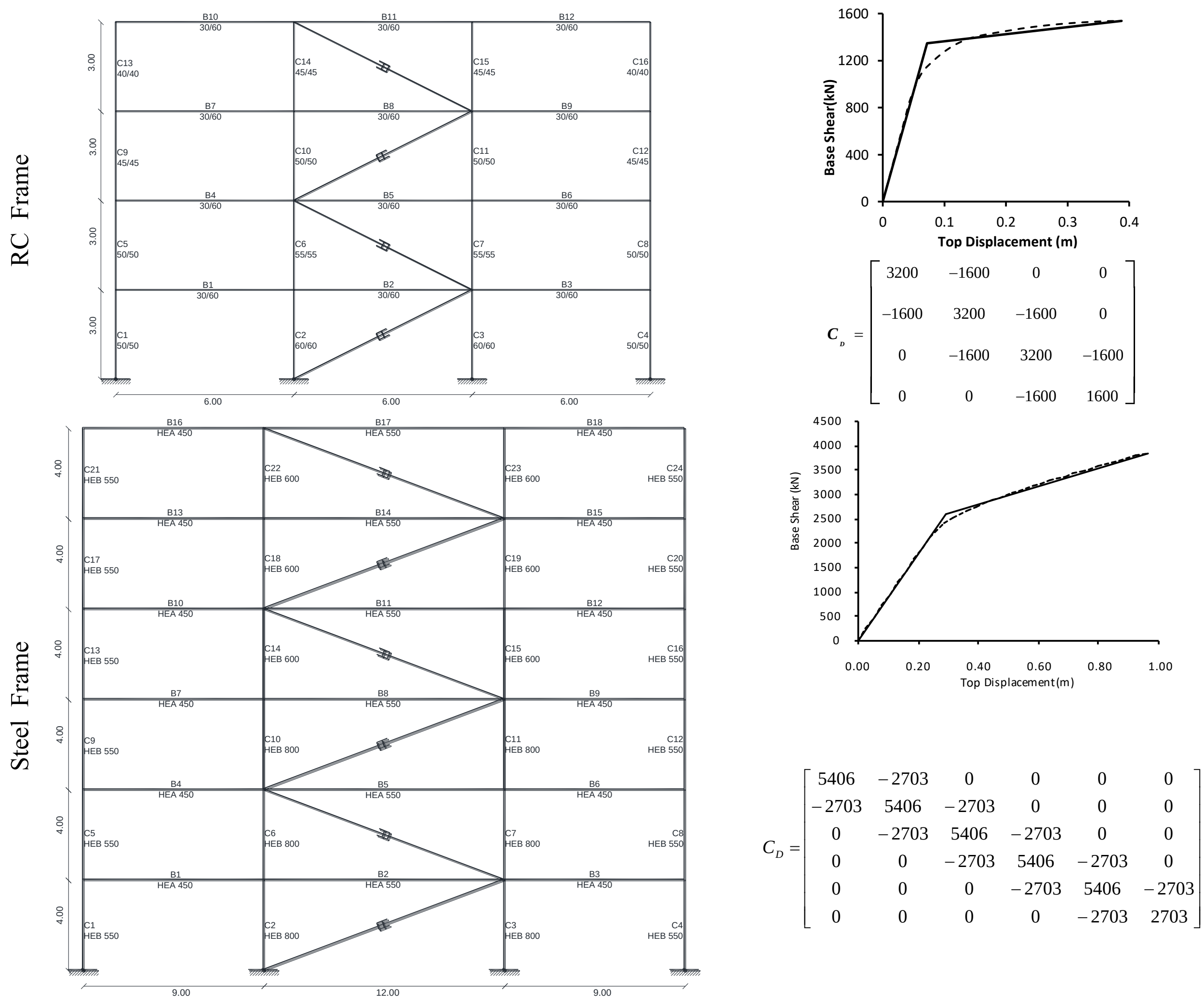
Fig. 5 Corrective factor B_v for different ductility and damping levels

Regarding the inelastic spectra, a number of different R–μ–T relationships have been presented in the literature (Miranda and Bertero 1994, Vidic *et al.* 1994, Hidalgo and Arias 1990, Riddell and Newmark 1979, Chopra 2001). However, as noticed above, the viscous damping ratio of all these models was assumed to be between 1-10%. By the implementation of passive dissipation control systems the damping of the equivalent SDOF system can reach 30% of the critical damping. Thus, in order to examine the effect of the high damping on the inelastic constant ductility spectra, analyses with the same set of ground motions were performed for damping ratio with the range ξ = 5-50% and ductility values μ = 1.5, 2, 2.5, 3 and 4. Subsequently, the main reduction factor was determined for each level of damping and ductility. The main reduction factors for the inelastic spectra for damping values ξ = 0.05-0.5 are presented in Fig. 4.

The single expression of the reduction factor spectra for different levels of damping and ductility are based on the Hidalgo and Arias (1990) relationship and is given by the following equation

$$R_{\mu} = 1 + \frac{T}{aT_0 \exp(b\mu T) + \dfrac{T}{c\mu - 1}} \qquad (24)$$

where T_o is the period that corresponds to the beginning of the constant spectral velocities area and a, b and c rely on the damping ratio and are listed in Table 3.



$$C_D = \begin{bmatrix} 3200 & -1600 & 0 & 0 \\ -1600 & 3200 & -1600 & 0 \\ 0 & -1600 & 3200 & -1600 \\ 0 & 0 & -1600 & 1600 \end{bmatrix}$$

$$C_D = \begin{bmatrix} 5406 & -2703 & 0 & 0 & 0 & 0 \\ -2703 & 5406 & -2703 & 0 & 0 & 0 \\ 0 & -2703 & 5406 & -2703 & 0 & 0 \\ 0 & 0 & -2703 & 5406 & -2703 & 0 \\ 0 & 0 & 0 & -2703 & 5406 & -2703 \\ 0 & 0 & 0 & 0 & -2703 & 2703 \end{bmatrix}$$

Fig. 6 Examined frame structures

The damping level did not affect significantly the form of the reduction spectra for the construction of the constant ductility inelastic spectra. It is obvious that for damping ratios higher than 20% the reduction remains constant.

It can be seen by Fig. 4, that for low damping ratios ($\xi = 0.05$), the assumption of equal displacements, comparing the elastic and the inelastic response, of long period structures is verified. On the other hand, assuming high damping ratios ($\xi > 0.10$), that assumption leads to non-conservative results($R < \mu$). The above remark is taken into account by the c factor of Eq. (24), as for long period structures the strength reduction factor take values $R = c \cdot \mu$.

2.2.3 Velocity corrective factors

In order to design the energy dissipation systems the estimation of the devices forces are of great importance. The prediction of the dampers ends' relative velocity is necessary to calculate the damping force as viscous dampers are velocity-depended devices. The most simplified method to calculate it is by using the pseudo-velocity spectra (PS_v) of the equivalent SDOF oscillator.

After that, the velocity values are distributed to the structure storeys based on the 1st mode. The PS_v can be computed by the displacement spectra given the relationship

Table 5 Earthquakes and applied scale factors

Earthquake	Component	Scale Factors
Northern Calif-01	315	x1 x1.25 x1.50 x1.75 x2 x3
Kern County	21	x1 x1.25 x1.50 x1.75 x2 x3
Northern Calif-05	314	x1 x1.25 x1.50 x1.75 x2
San Fernando	291	x1 x1.25 x1.50 x1.75 x2x3
Santa Barbara	250	x1 x1.25 x1.50 x1.75 x2

$$PS_V = \omega \cdot S_d \tag{25}$$

while for the inelastic systems,

$$PS_V = \left(\omega_{el} / \sqrt{\mu}\right) \cdot S_d \tag{26}$$

where ω_{el} is the structures' natural circular frequency of vibration and $\mu = 1$ for elastic seismic response.

However, even for elastic systems, the assumption of the equivalence between the velocity spectra (S_v) and the pseudo velocity spectra (PS_v) is valid for oscillators with period values near to T = 0.5s (Sadek *et al.* 2000). For period values larger than T = 0.5s and as the damping ratio

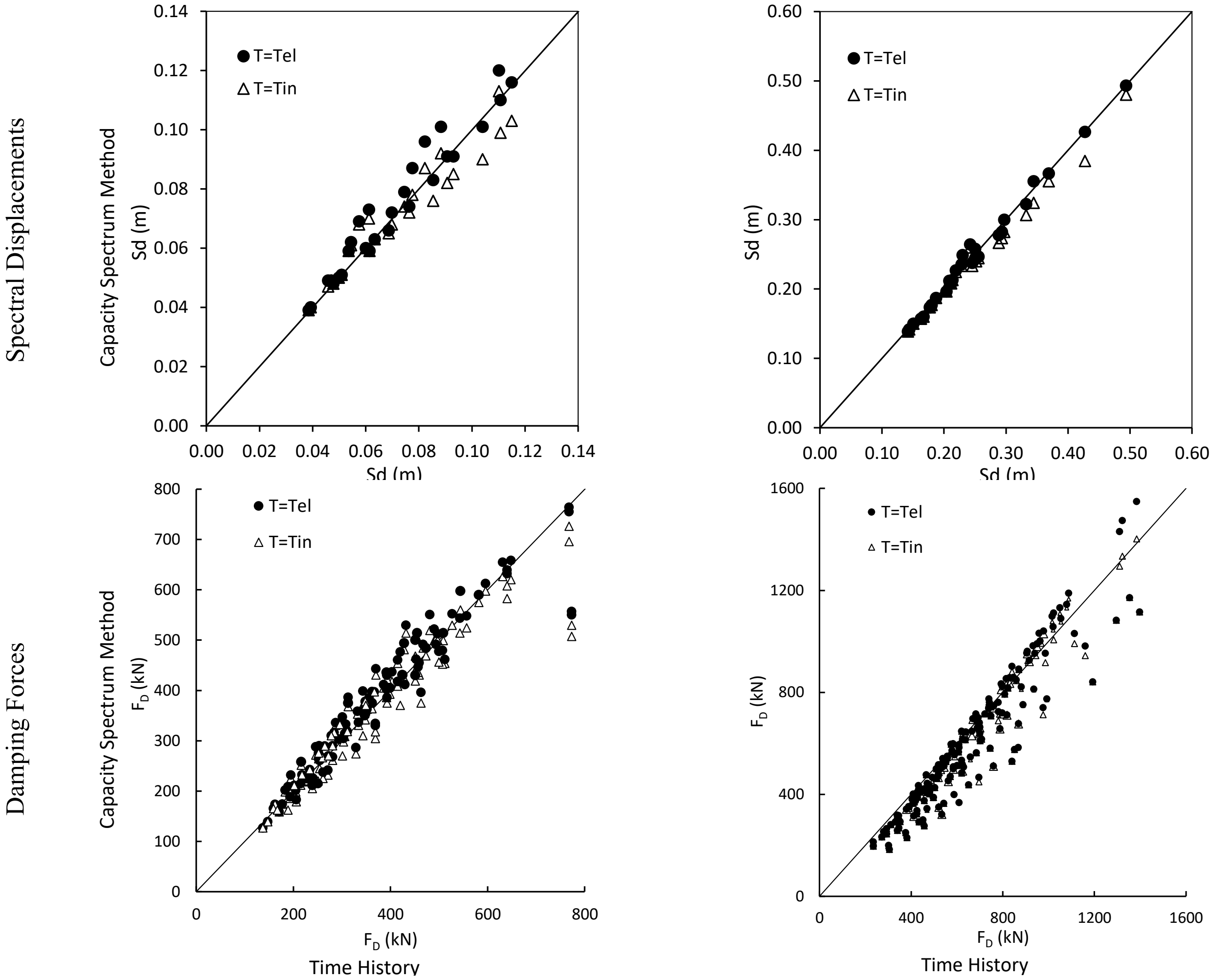


Fig. 7 Comparison of time history analyses spectral displacements and dampers' forces to the proposed method ones for the constant and adaptive damping assumptions (4-storey frame left and 6-storey frame right)

increases, this assumption leads to underestimation of the developed velocities, whereas for lower periods it leads to overestimation. Owing to this, a corrective factor (B_v) is introduced by using the ground motions of Table 1. B_v is equal to the pseudo velocity spectra divided by the velocity spectra (Eq. (27)).

$$B_v = \frac{PS_V}{S_V} = \frac{\omega_{el} / \sqrt{\mu}\, S_d}{S_V} \tag{27}$$

The expression of the B_v calculated by a regression analysis is presented in Eq. (28), and is demonstrated for different values of damping ratios and ductility in Fig. 5.

$$B_v = (a_1\mu^2 + a_2\mu + a_3) T^{a_4\mu^2 + a_5\mu + a_6} \tag{28}$$

where α_1-α_6, are coefficients listed in Table 4 for different damping ratio levels.

According to the results depicted in Fig. 5, it can be observed that as the demanded ductility and the effective damping ratio increase, the corrective factor B_v is increased for stiff structures and decreased for long-period ones. Particularly, for structures with damping ratio $\xi = 0.05$ and ductility $\mu = 1$ the B_v ranges from 1.3 to 0.9, while for demanded ductility $\mu = 4$, ranges from 1.7 to 0.7. In addition, for structures with damping ratio $\xi = 0.20$, B_v take values from 2.25 to 0.6 and from 2.4 to 0.5, for demanded ductility $\mu = 1$ and $\mu = 4$ respectively.

3. Verifying the proposed methodology

To examine the performance of the CSM by using inelastic constant ductility-high damping spectra to MDOF systems with linear viscous dampers, two frame buildings were considered. A four-storey reinforced concrete frame building, designed according to EC-2 and EC-8 provisions, and a six-storey steel frame building, designed according to EC-3 and EC-8 provisions, were analyzed (Fig. 6). Regarding the damping systems, elastic viscous dampers were assumed with a damping constant of C = 2000kN s/m, placed at 26.56° from the horizontal for the RC building, and C = 3000 kN s/m, placed at 18.56° from the horizontal for the steel one. The dampers are implemented at the central opening of each floor for both cases.

Performing the process described by the Eqs. (16a)-(16c) the matrix $\mathbf{C_D}$ is calculated (Fig. 6). Applying Eq. (7) introduced by FEMA for elastic response and the proposed

Table 6 Interstorey drifts ratios $Drift_{Push}/Drift_{TH}$ of the 4-Storey RC frame

Ground motion	Storey	x1	x1.25	x1.5	x1.75	x2.00	x3
		$Drift_{push}/Drift_{TH}$					
Northern Calif-01	1	0.99 (0.99)	0.97 (0.97)	0.95 (0.94)	0.94 (0.90)	1.00 (0.89)	0.89 (0.77)
	2	0.98 (0.98)	0.96 (0.96)	0.98 (0.97)	1.01 (0.97)	1.12 (1.00)	1.15 (1.00)
	3	1.02 (1.02)	1.01 (1.00)	1.04 (1.03)	1.08 (1.02)	1.19 (1.07)	1.24 (1.08)
	4	0.94 (0.94)	0.93 (0.92)	0.94 (0.93)	0.93 (0.88)	0.88 (0.79)	1.00 (0.87)
		Average = 1.01 (0.95)			Std. Deviation = 0.09 (0.08)		
Kern County	1	0.99 (0.99)	1.03 (1.02)	1.07 (1.07)	1.16 (1.13)	1.13 (1.08)	0.78 (0.71)
	2	0.98 (0.98)	1.00 (1.00)	1.06 (1.06)	1.17 (1.14)	1.16 (1.11)	1.01 (0.91)
	3	1.00 (1.00)	1.02 (1.02)	1.08 (1.08)	1.2 (1.17)	1.19 (1.13)	1.13 (1.02)
	4	0.89 (0.89)	0.91 (0.91)	0.96 (0.96)	1.06 (1.03)	1.01 (0.96)	0.98 (0.88)
		Average = 1.04 (1.01)			Std. Deviation = 0.10 (0.10)		
Northern Calif-05	1	0.95 (0.95)	0.88 (0.88)	0.84 (0.81)	0.81 (0.75)	0.79 (0.71)	-
	2	0.95 (0.95)	0.92 (0.92)	0.95 (0.92)	0.99 (0.91)	1.01 (0.91)	-
	3	0.98 (0.98)	0.98 (0.98)	1.02 (0.99)	1.07 (0.99)	1.11 (0.99)	-
	4	0.88 (0.88)	0.87 (0.87)	0.88 (0.85)	0.80 (0.74)	0.61 (0.55)	-
		Average = 0.91 (0.88)			Std. Deviation = 0.11 (0.12)		
SanFernando	1	0.96 (0.96)	0.92 (0.92)	0.95 (0.90)	1.00 (0.92)	0.96 (0.87)	0.80 (0.70)
	2	0.96 (0.96)	0.97 (0.96)	1.05 (0.99)	1.16 (1.06)	1.15 (1.05)	1.03 (0.91)
	3	1.01 (1.00)	1.03 (1.02)	1.11 (1.05)	1.24 (1.13)	1.25 (1.13)	1.11 (0.98)
	4	0.91 (0.91)	0.91 (0.90)	0.84 (0.80)	1.01 (0.92)	1.00 (0.91)	0.89 (0.79)
		Average = 1.01 (0.95)			Std. Deviation = 0.11 (0.10)		
Santa Barbara	1	1.06 (1.06)	1.04 (1.02)	0.83 (0.82)	0.80 (0.73)	0.77 (0.68)	-
	2	1.02 (1.02)	1.10 (1.08)	0.94 (0.92)	0.98 (0.90)	1.00 (0.89)	-
	3	1.01 (1.01)	1.15 (1.13)	1.03 (1.02)	1.08 (0.99)	1.12 (0.99)	-
	4	0.87 (0.87)	0.99 (0.98)	0.94 (0.92)	0.93(0.85)	0.94 (0.83)	-
		Average = 0.98 (0.94)			Std. Deviation = 0.10 (0.11)		
		Total Average = 0.99 (0.95)			Total Std. Deviation = 0.11 (0.10)		

Eq. (11), the effective damping of the examined frames were estimated from both equations equal to ξ_{eff} = 20.8% and ξ_{eff} = 16%, for the RC frame and the steel frame respectively. By this, the two methods seem to be equivalent despite the fact that the calculations are based on different assumptions.

However, according to Eq. (7), the effective damping must be revaluated according to the effective period when the structure responds inelastically. Assuming that when CSM are performed based on inelastic spectra, any alteration of the effective damping due to the shift of the effective period is taken into account through the level of ductility, the consideration of an adaptive effective damping seems redundant. In order to evaluate that assumption, both constant and changing effective damping are considered.

To assess the effective damping calculation with a higher value of accuracy the Performance Point is defined by using the ground motion inelastic spectra and not the approximate relationships B(T, ξ) and R–μ–T presented above. For this reason, a total of 28 time history analyses (TH) were performed for each frame. Table 5 presents the 5 ground motions along with their scale factors used to verify

Table 7 Interstorey drifts ratios $Drift_{Push}$ / $Drift_{TH}$ of the 6 - Storey Steel frame

Ground motion	Storey	x1	x1.25	x1.5	x1.75	x2.00	x3
		$Drift_{push}/Drift_{TH}$					
Northern Calif-01	1	0.88 (0.88)	0.88 (0.88)	0.90 (0.90)	0.98 (0.97)	0.97 (0.95)	0.91 (0.88)
	2	0.93 (0.93)	0.93 (0.93)	0.94 (0.94)	1.03 (1.01)	1.02 (1.00)	0.91 (0.88)
	3	0.97 (0.97)	0.97 (0.97)	0.97 (0.97)	1.04 (1.02)	1.04 (1.03)	0.90 (0.87)
	4	1.02 (1.02)	1.02 (1.02)	1.02 (1.02)	1.10 (1.08)	1.13 (1.11)	1.00 (0.97)
	5	1.06 (1.06)	1.06 (1.06)	1.09 (1.09)	1.19 (1.17)	1.23 (1.20)	1.16 (1.13)
	6	1.09 (1.09)	1.09 (1.09)	1.13 (1.13)	1.25 (1.22)	1.28 (1.26)	1.35 (1.31)
		Average = 1.04 (1.03)			Std. Deviation = 0.12 (0.11)		
Kern County	1	0.98 (0.98)	1.05 (1.05)	1.04 (1.02)	1.11 (1.04)	1.14 (1.04)	1.13 (1.10)
	2	0.97 (0.97)	1.03 (1.03)	0.99 (0.97)	1.01 (0.95)	1.02 (0.93)	0.99 (0.96)
	3	0.96 (0.96)	1.00 (1.00)	0.92 (0.9)	0.93 (0.88)	0.94 (0.86)	0.90 (0.88)
	4	0.95 (0.95)	1.00 (1.00)	0.94 (0.92)	0.96 (0.91)	0.97 (0.89)	0.93 (0.91)
	5	0.94 (0.94)	1.01 (1.01)	1.00 (0.98)	1.05 (0.99)	1.09 (0.99)	1.05 (1.02)
	6	0.94 (0.94)	1.01 (1.01)	1.02 (1.01)	1.14 (1.07)	1.23 (1.12)	1.26 (1.22)
		Average = 1.02 (0.98)			Std. Deviation = 0.08 (0.07)		
Northern Calif-05	1	0.92 (0.92)	0.93 (0.93)	0.93 (0.92)	0.94 (0.9)	0.95 (0.90)	-
	2	0.94 (0.94)	0.98 (0.98)	0.99 (0.97)	0.98 (0.94)	0.98 (0.93)	-
	3	0.97 (0.97)	0.98 (0.98)	0.97 (0.95)	0.94 (0.90)	0.93 (0.89)	-
	4	1.00 (1.00)	0.98 (0.98)	0.97 (0.95)	0.96 (0.92)	0.95 (0.91)	-
	5	1.02 (1.02)	0.95 (0.95)	0.97 (0.95)	0.97 (0.93)	0.99 (0.94)	-
	6	0.92 (0.92)	0.92 (0.92)	0.96 (0.94)	1.01 (0.97)	1.07 (1.02)	-
		Average = 0.96 (0.94)			Std. Deviation = 0.03 (0.03)		
San Fernando	1	0.89 (0.89)	0.89 (0.89)	0.90 (0.90)	0.94 (0.93)	0.99 (0.97)	1.12 (1.01)
	2	0.94 (0.94)	0.94 (0.94)	0.94 (0.94)	0.99 (0.98)	1.04 (1.02)	1.00 (0.90)
	3	0.98 (0.98)	0.98 (0.98)	0.97 (0.97)	1.01 (1.00)	1.05 (1.03)	0.91 (0.82)
	4	1.01 (1.01)	1.01 (1.01)	1.01 (1.01)	1.06 (1.05)	1.01 (0.99)	0.93 (0.84)
	5	1.03 (1.03)	1.03 (1.03)	1.04 (1.04)	1.11 (1.10)	1.04 (1.02)	1.04 (0.93)
	6	1.04 (1.04)	1.04 (1.04)	1.05 (1.05)	1.12 (1.11)	1.05 (1.03)	1.21 (1.09)
		Average = 1.01 (0.99)			Std. Deviation = 0.07 (0.07)		
Santa Barbara	1	0.86 (0.86)	0.87 (0.87)	0.97 (0.96)	1.00 (0.99)	1.06 (1.02)	-
	2	0.92 (0.92)	0.92 (0.92)	1.02 (1.01)	1.04 (1.03)	1.00 (0.97)	-
	3	0.98 (0.98)	0.98 (0.98)	1.09 (1.08)	0.95 (0.94)	0.93 (0.9)	-
	4	1.04 (1.04)	1.06 (1.06)	1.09 (1.08)	0.93 (0.92)	0.92 (0.89)	-
	5	1.09 (1.09)	1.10 (1.10)	0.99 (0.98)	0.94 (0.93)	0.95 (0.92)	-
	6	1.12 (1.12)	1.12 (1.12)	0.98 (0.97)	0.95 (0.94)	0.98 (0.95)	-
		Average = 1.00 (0.99)			Std. Deviation = 0.07 (0.08)		
		Total Average = 1.01 (0.99)			Total Std. Deviation = 0.08(0.08)		

Table 8 Total Average and Std. Dev. values of the analyses

		Sd_{Push}/Sd_{TH}	$Drift_{Push}/Drift_{TH}$	$F_{D,Push}/F_{D,TH}$
4-Storey	Average	1.04 (0.99)	0.99 (0.95)	1.03 (0.99)
	Std. Dev.	0.08 (0.08)	0.11 (0.10)	0.09 (0.09)
6-Storey	Average	0.99 (0.98)	1.01 (0.99)	0.90 (0.89)
	Std. Dev.	0.03 (0.04)	0.08 (0.08)	0.12 (0.14)

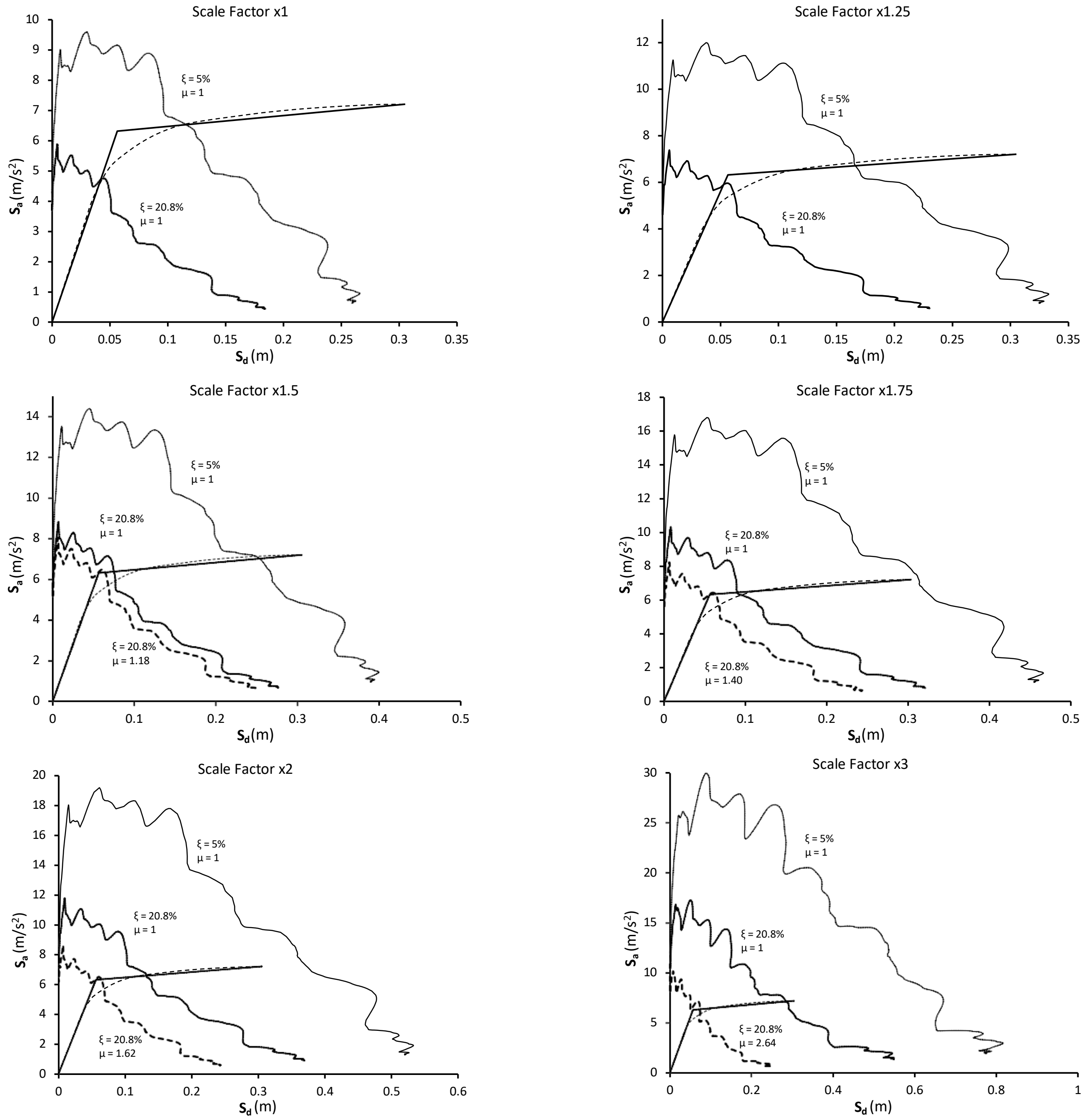


Fig. 8 Displacement demand determination of the 4-storey frame

the proposed methodology. Incremental factors were used in order to investigate the method performance at different levels of ductility.

The results of the CSM are presented in Fig. 7 and Tables 6-8. In Tables 6-8 the results are listed in pears, where the first value is calculated according to the constant damping and the second in the parenthesis according to the adaptive damping assumption. Fig. 7 depicts the peak spectral displacements Sd_{Push} and Sd_{TH} and dampers forces $F_{D,Push}$ and $F_{D,TH}$ obtained by the TH analyses, as well as the estimated displacements by the CSM analyses. In Tables 6 and 7, the ratios of each floors' interstorey drifts, $Drift_{Push}/Drift_{TH}$ are presented.

As expected, concerning the effective damping ratio calculation, when the performance point corresponds to ductility value $\mu = 1$ the CSM results are similar. When the structure develops displacement ductility $\mu > 1$ the CSM results start to differ between each other. As the demanded ductility increases, the variation of the results are notably increases too. Regarding the assumption of the changeable effective damping it could lead to non-conservative results, as the usage of the effective period of the equivalent SDOF oscillator overestimates the effective damping in the case of high ductility demands. This is observed by the ratios of the peak displacements Sd_{Push}/Sd_{TH} for large displacements that results in 0.99 instead of 0.94 for the 6-storey frame and 1.01 instead of 0.91 for the 4-storey frame. The same results are implied observed Table 8 with the total average values of the analysis.

Moreover, the alteration of the effective damping leads to even more non-conservative results in terms of interstorey drifts. As inter-srorey drifts constitute a crucial criteria that define the structural members' seismic demand levels, this underestimation is of major importance (Tables

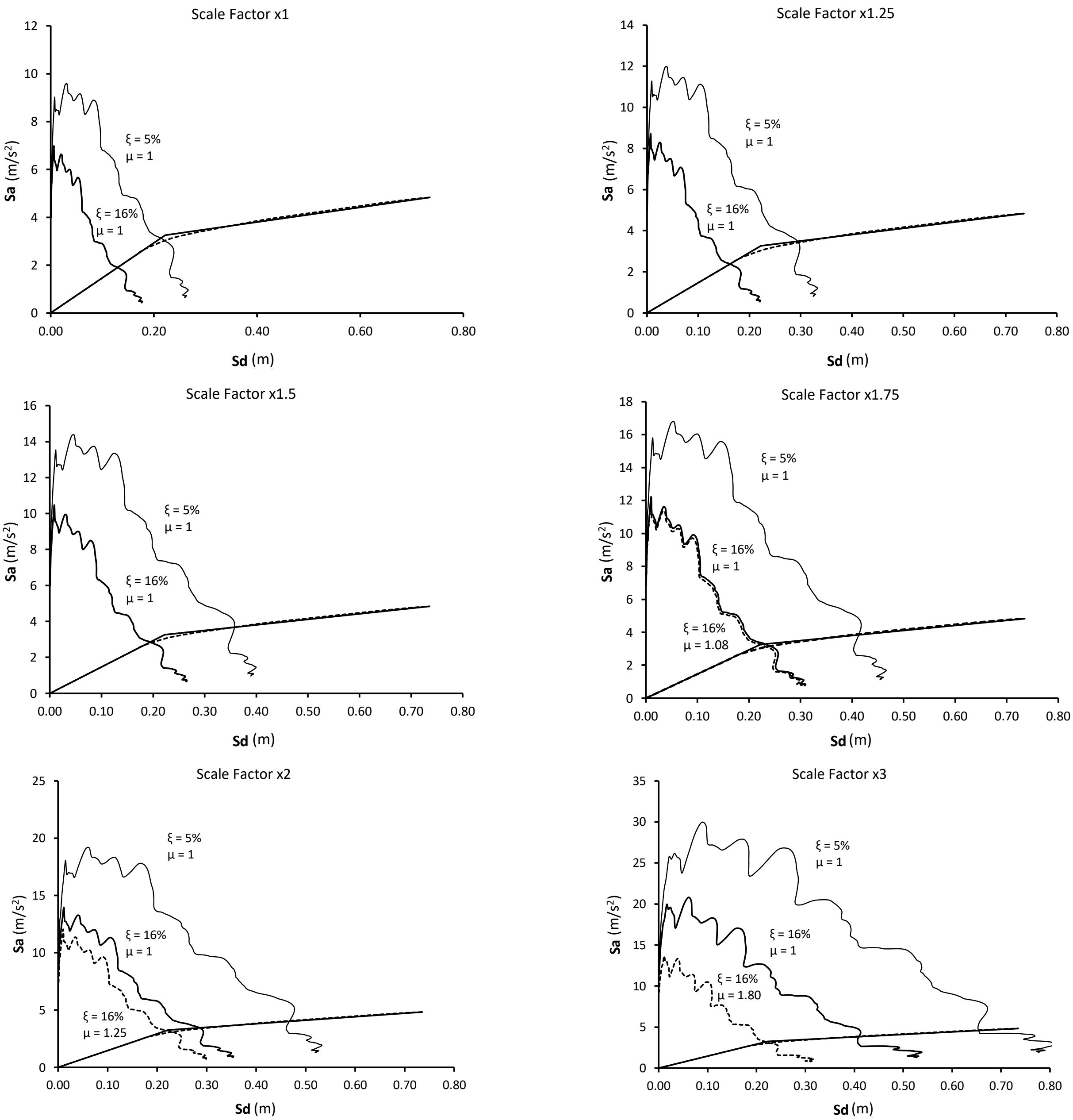


Fig. 9 Displacement demand determination of the 6-storey frame

6-7). In this view, as the CSM is based on relationships that combine the strength demand reduction factor (R) with the ductility level (μ) without considering an equivalent SDOF elastic system with stiffness K_{eff}, the constant effective damping methodology forms a more compatible method.

Considering that elastic viscous dampers are implemented in the structure it could be essential to specify the damping forces of the dampers. The spectral velocity S_v of the inelastic spectra corresponding to the period of the oscillator has been used to calculate the damping forces. The damper forces are given by Eq. (4). The ratios of the damping forces obtained by the CSM by them obtained by the TH analyses $F_{D,Push}/F_{D,TH}$ are presented for each analysis in Fig. 7. Comparing the results of the TH analyses with both assumptions of effective damping, the results are very satisfactory in the case of 4-storey frame but it seems to be underestimated in the case of 6-storey frame. This variation may be due to the contribution of higher modes.

Based on the overall results in terms of peak displacements, interstorey drifts and damping forces (Table 8), it can be easily recognizable that the introduced CSM estimates in an acceptable grade, the nonlinear seismic response of structures equipped with viscous dampers.

In order to evaluate also the approximate relations of reducing the elastic spectra due to high damping (Eqs. (20)-(22)), as well as the R–μ–T relations (Eq. (23)) for the construction of the high damping constant ductility spectra, this process was applied for the mean spectrum for each scale factor. The results are displayed in Table 9. The calculation of the performance point can be observed in Figs. 8 and 9. A graphic method was applied by defining the ductility of the demand spectra that crosses the bilinear

Table 9 Results using the R – μ – T relationships

Scale Factor		x1	x1.25	x1.5	x1.75	x2.00	x3.00	Average
S_d / $S_{d,Ave}$	*4-storey*	0.91	0.95	0.99	1.00	1.03	1.41	*1.05*
	6-storey	0.87	0.88	0.88	0.91	0.94	0.92	0.90
S_d / $S_{d,Max}$	*4-storey*	0.80	0.83	0.87	0.84	0.82	1.29	*0.91*
	6-storey	0.80	0.81	0.80	0.80	0.80	0.80	0.80
μ	*4-storey*	1.00	1.00	1.18	1.39	1.63	2.64	-
	6-storey	1.00	1.00	1.00	1.08	1.25	1.80	-
Bv	*4-storey*	1.06	1.06	1.04	1.02	1.01	0.94	-
	6-storey	0.84	0.84	0.84	0.83	0.81	0.75	-
Sv	*4-storey*	0.40	0.51	0.60	0.67	0.73	1.00	-
	6-storey	0.59	0.75	0.89	1.03	1.13	1.47	-
F_d / $F_{d,Ave}$	*4-storey*	0.90	0.95	1.00	0.98	0.96	0.95	0.96
	6-storey	0.85	0.86	0.87	0.89	0.88	0.85	0.87
F_d / $F_{d,Max}$	*4-storey*	0.82	0.84	0.90	0.87	0.86	0.89	0.86
	6-storey	0.73	0.75	0.77	0.79	0.79	0.77	0.77

capacity spectra at the yield point. Using the proposed relations leads to notable estimation of the top displacement compared with the average and the maximum results obtained by the TH analyses. The results of both methods estimate with a satisfactory accuracy the TH results.

Moreover, the viscous dampers forces are calculated using the Eqs. (27)-(28), to correct the PS_v values. By the CSM, the ductility of the equivalent SDOF system are available and as such the dampers forces are given by the following equation

$$F_{D,i} = \Gamma^* \left(\omega_{el} \sqrt{\mu} S_d / B_v \right) \varphi_{ij} C_i \cos\theta_i \qquad (26)$$

The B_v factors for each ductility levels are listed to the Table 9. Moreover, Table 9 displays the ratio of the viscous dampers' median forces calculated by the corrected PS_v, by the average ($F_{d,Ave}$) and maximum ($F_{d,Max}$) forces computed by the TH analyses. Finally, the total average of the ratios F_d /$F_{d,Ave}$ and F_d /$F_{d,Max}$ are shown.

It can be observed that the PS_v values are similar to S_v in the case of 4 - storey frame. Mentioned above, for structures period near to 0.5s the PS_v are nearly equal to S_v. Thus, as the period of the examined structure is T = 0.61s, the result was expected to be the present. On the other hand, in the case of 6 - storey frame, due the longer period compared to the RC frame (T = 1.69 s) the PS_v are less than S_v, thus the application of the velocity corrective factors is necessary.

4. Conclusions

In the present study capacity spectrum method with the use of high damping constant ductility spectra to assess the response of RC frame buildings with viscous dampers was investigated. The definition of the structures' effective damping ratio (ξ_{eff}) and a method of reducing the elastic spectra corresponding to 5% damping ratio in order to result in inelastic constant ductility and high damping spectra were the main objectives of this paper. Moreover, applications of the proposed methodology on a RC 4-storey and 6-storey steel frame with viscous dampers were presented.

Regarding the reducing of the elastic spectra to construct high damping spectra, a continuum nonlinear expression was indicated, which can be applied to the whole range of the spectra periods. A particular feature of the proposed relationships is that it could describe the descending reducing factor beyond the constant spectral acceleration area.

As done for the elastic spectra a continuous R – μ – T relationship was proposed for the inelastic spectra taking into consideration the effect of the damping level. The fact that has to be mentioned is that the damping level does not notably affect the reduction spectra. In fact, the reduction remains constant for damping ratio values higher than 20%.

Once the viscous dampers are velocity-depended devices, the accurate estimation of the dampers ends' relative velocity are of great importance in order to calculate the damping forces. Owing to this, an expression that relates the S_v with the PS_v is presented by introducing a corrective factor (B_v) which is affected by the damping ratio and the demanded ductility of the structure.

Applying the proposed CSM based on constant ductility inelastic spectra indicates that, the assumption of the modified effective damping depending on the ductility level could result in damping overestimation that leads to non-conservative results according to the structural assessment.

Throughout the analysis of two frame building equipped with viscous dampers, the CSM seems to estimate with great accuracy the top displacement, the inter storey drifts and the dampers forces. Following that, the whole calculation scheme provides an efficient, simplified, and easy to apply method which evaluates the nonlinear response of structures with supplemental viscous damping.

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