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Seismic structural demands and inelastic deformation ratios: Sensitivity analysis and simplified models

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Abstract. Modern seismic codes rely on performance-based seismic design methodology which requires that the structures withstand inelastic deformation. Many studies have focused on the inelastic deformation ratio evaluation (ratio between the inelastic and elastic maximum lateral displacement demands) for various inelastic spectra. This paper investigates the inelastic response spectra through the ductility demand μ, the yield strength reduction factor R_y, and the inelastic deformation ratio. They depend on the vibration period T, the post-to-preyield stiffness ratio α, the peak ground acceleration (PGA), and the normalized yield strength coefficient η (ratio of yield strength coefficient divided by the PGA). A new inelastic deformation ratio C_η is defined; it is related to the capacity curve (pushover curve) through the coefficient (η) and the ratio (α) that are used as control parameters. A set of 140 real ground motions is selected. The structures are bilinear inelastic single degree of freedom systems (SDOF). The sensitivity of the resulting inelastic deformation ratio mean values is discussed for different levels of normalized yield strength coefficient. The influence of vibration period T, post-to-preyield stiffness ratio α, normalized yield strength coefficient η, earthquake magnitude, ruptures distance (i.e., to fault rupture) and site conditions is also investigated. A regression analysis leads to simplified expressions of this inelastic deformation ratio. These simplified equations estimate the inelastic deformation ratio for structures, which is a key parameter for design or evaluation. The results show that, for a given level of normalized yield strength coefficient, these inelastic displacement ratios become non sensitive to none of the rupture distance, the earthquake magnitude or the site class. Furthermore, they show that the post-to-preyield stiffness has a negligible effect on the inelastic deformation ratio if the normalized yield strength coefficient is greater than unity.

Keywords: deformation ratio; yield strength reduction factor; ductility; inelastic spectra; earthquakes; seismic response

1. Introduction

Performance based seismic design relies on simplified methods which are calibrated on the basis of sophisticated dynamic analyses. Amongst the existing simplified methods, the Displacement Coefficient Method (DCM) defined in FEMA-440 (2005) and the Capacity Spectrum Method (CSM) specified in ATC-40 (1996) are the most widely used. Their design target displacement is related to either the inelastic response spectrum or the equivalent linear system response. It is, in fact, not easy and not always enough accurate to derive the peak response from elastic spectra. Reduction factors are then used to develop simplified reduction coefficient spectrum, as adopted in displacement coefficient method (FEMA-440 2005). The values of these reduction factors are derived from numerical fitting.

Thus, the Inelastic Displacement Ratio (Inelastic Deformation Ratio), known as C_1 in DCM method, is considered as the most influent reduction factor (FEMA-440 2005). Therefore, the role of inelastic deformation ratio on performance evaluation for existing structures and the seismic design assessment for new structures has been widely investigated (Whittaker *et al*. 1998, Miranda 2000, Ruiz-Garcia and Miranda 2003, Chopra and Chintanapakdee 2004, Ruiz-Garcia and Miranda 2004, Ruiz-Garcia and Miranda 2006, Zhai *et al*. 2007, Mehanny and Ayoub 2008, Hatzigeorgiou and Beskos 2009, Malaga-Chuquitaype and Elghazouli 2012, Massumi and Monavari 2013, Ruiz-García and González 2014, Sang *et al*. 2014, Yazdani and Salimi 2015, Zerbin and Aprile 2015, Skrekas *et al.* 2014, Kazaz 2016, Chikh *et al.* 2017). Furthermore, several authors have proposed empirical expressions for the

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inelastic deformation ratio (Newmark and Hall 1982, Krawinkler and Nassar 1992, Miranda 2000, Ruiz-Garcia and Miranda 2003, Chopra and Chintanapakdee 2004).

Consequently, numerous research works have been devoted to specific case studies and sensitivity analyses. Actually, the inelastic deformation ratio of Single Degree of Freedom (SDOF) systems has been investigated under various earthquake ground motions:

- 20 horizontal components during 10 ground motions recorded on either stiff soil or soft rock sites (Whittaker *et al.* 1998);
- 124 records on different types of soil conditions (Miranda 1991, 1993), 264 records in firm sites (Miranda 2000) and 216 earthquakes accelerations time histories records with magnitude ranging from 5.8 to 7.7 (Ruiz-Garcia and Miranda 2003, Akkar and Miranda 2004) collected during12 earthquakes in California;
- Experimental results from 152 earthquake simulator tests (Matamoros *et al.* 2003);
- 118 earthquake ground motions recorded on bay-mud sites of the San Francisco Bay Area and on soft soil sites located in the former lake-bed zone of Mexico City (Ruiz-Garcia and Miranda 2006);
- 573 ground motions (Zhai *et al.* 2013);
- and 20 earthquake ground motions recorded on very soft soil sites of the old bed-lake of Mexico City (Ruiz-Garcı́a and Gonzalez 2014).

Theoretical developments, derived from the bilinear capacity curve (Pushover curve), have been recently devoted to this inelastic deformation ratio (Chikh *et al.* 2017). For illustrative purposes, a sensitivity analysis is performed in the present study, in the case of inelastic deformation of SDOF bilinear systems, in order to:

- investigate the ground motions magnitude and source location effects (for small as well as for large magnitudes and rupture distance);
- investigate both materials and structural behaviors effects, i.e., effects of normalized yield strength coefficient (η), post-to-pre yield stiffness ratio (α) and vibration periods (T);
- and develop a simplified model of the inelastic deformation ratio according to a set of selected governing parameters (η, α, T).

2. Theoretical backgrounds

2.1 Ductility factor μ and yield strength reduction factor R_μ for bilinear system

The ductility demand (μ) is a dimensionless quantity defined as a ratio between the ultimate displacement x_m and the yield displacement x_y (Chopra and Chintanapakdee 2004), (see Fig. 1)

$$\mu = \frac{x_m}{x_y} \tag{1}$$

The Strength Reduction Factor is defined as the ratio between the elastic strength demand, f_0, and the inelastic strength demand, f_y, which represents also the ratio between

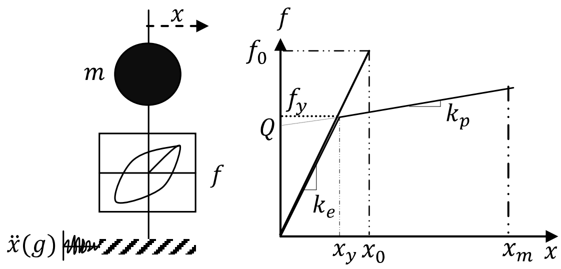

Fig. 1 Elastic behavior of an (SDOF) and its corresponding bilinear system

the elastic displacement and the yield displacement x_y (Chopra and Chintanapakdee 2004), (see Fig. 1)

$$\mathrm{R_y} = \frac{\mathrm{f_0}}{\mathrm{f_y}} = \frac{\mathrm{x_0}}{\mathrm{x_y}} \tag{2}$$

2.2 Inelastic deformation ratio and governing parameters

A sensitivity analysis is proposed in order to evaluate the inelastic deformation ratio for inelastic SDOF systems. The inelastic deformation ratio requires first to normalize the motion equation, according to the yield displacement, (Chikh *et al.* 2017). This latter depends on the mass (m), the elastic stiffness (k_e), the post-yield stiffness (k_e), the yield strength (Q) and the post-to-pre-yield stiffness ratio (α) which define the mechanical characteristics of the SDOF system (Chikh *et al.* 2017), (see Fig. 1)

$$\begin{cases} \ddot{\mathcal{M}}(\mathrm{t}) + 2\xi\omega\dot{\mathcal{M}}(\mathrm{t}) + \alpha\omega^2\mathcal{M}(\mathrm{t}) + \omega^2(1-\alpha)\mathrm{z}(\mathrm{t}) = -\frac{\omega^2(1-\alpha)}{\eta}\bar{\ddot{\mathrm{x}}}_\mathrm{g}(\mathrm{t}) \\ \ddot{\mathfrak{R}}_\mathrm{y}(\mathrm{t}) + 2\xi\omega\dot{\mathfrak{R}}_\mathrm{y}(\mathrm{t}) + \omega^2\mathfrak{R}_\mathrm{y}(\mathrm{t}) = -\frac{\omega^2(1-\alpha)}{\eta}\bar{\ddot{\mathrm{x}}}_\mathrm{g}(\mathrm{t}) \end{cases} \tag{3}$$

$\mathcal{M}(\mathrm{t})$ instantaneous value of ductility demand

$\mathfrak{R}_\mathrm{y}(\mathrm{t})$ instantaneous value of yield strength reduction factor

ξ damping ratio

ω circular frequency

$\alpha = \frac{\mathrm{k_p}}{\mathrm{k_e}}$ post-to-pre-yield stiffness ratio

$\mathrm{z(t)}$ represents the dimensionless variable that characterizes the Bouc-Wen model of hysteresis (Wen 1976),

$\bar{\ddot{\mathrm{x}}}_\mathrm{g}(\mathrm{t}) = \frac{\ddot{\mathrm{x}}_\mathrm{g}(\mathrm{t})}{\mathrm{A_g}}$ represents the normalized ground acceleration with respect to the PGA, and

η represents the normalized yield strength coefficient.

The inelastic deformation ratio (C_η) defined as the inelastic vs. linear SDOF system deformations ratio is expressed as (Chikh *et al.* 2017)

$$\mathrm{C_\eta} = \frac{\mu = \mathcal{M}_{\max}(\mathrm{t})}{\mathrm{R_y} = \mathfrak{R}_{\mathrm{y},\max}(\mathrm{t})} \tag{4}$$

Where: μ = the peak ductility factor (peak ductility demand); R_y= peak yield strength reduction factor;

The inelastic deformation ratio (C_η) is influenced by several governing parameters such as the normalized yield strength coefficient η (Mahin and Lin 1983, Benazouz *et al*. 2012, Chikh *et al*, 2017) characterizing the system strength relative to the Peak Ground Acceleration (PGA) value A_g

$$\eta = \frac{q \cdot g}{A_g} \qquad (5)$$

$$q = \frac{Q}{mg} \qquad (6)$$

Where: Q = yield strength; q = yield strength ratio; m = SDOF system mass and g= gravity acceleration.

The investigated parameters are then expressed as functions of the post-to-pre-yield stiffness ratio (α) and the normalized yield strength coefficient (η). The novelty of the proposed approach consists in the fact that the inelastic deformation ratio (C_η), is evaluated whatever are the ductility or the strength reduction factor values. There is no restriction as it requests only fixed ductility ratios or only fixed strength reduction factor values that are considered as control parameter. In this study, the inelastic deformation ratio C_η is defined as μ/R_y which use sa new control parameter called 'normalized yield strength coefficient' η (Chikh *et al.* 2017).

3. Applications and sensitivity analysis

3.1 Sensitivity analysis

The instantaneous ductility $\mathcal{M}(t)$ and yield strength reduction factor $\mathfrak{R}_y(t)$ depend on the damping (ξ), the hardening and yielding parameters (α) and (q), the structural period (T) or circular frequency (ω), the ground motion (*PGA* as well as soil effect and rupture distance) and their combined effects (η). A sensitivity analysis is performed out in order to study the influence of these governing parameters on the inelastic deformation ratio C_η.

3.2 Selected ground motions

A selected set of ground motions is collected from PEER (Pacific Earthquake Engineering Research Center) Strong Motion Database (Chopra and Chintanapakdee 2003, PEER 2011). These seven subsets of ground motions, each containing 20 records, correspond to:

- a first category containing four sets denoted LMSR (Large Magnitude Short Distance), LMLR (Large Magnitude Large Distance), SMSR (Small Magnitude Short Distance) and SMLR (Small Magnitude Large Distance), representing four combinations of large magnitude (M= 6.6 - 6.9) or small magnitude (M= 5.8 - 6.5) and short rupture distance (R = 13 to 30 km) or large distance (R = 30-60 km);
- and a second category concerning mainly the sites classes (soils: B, C and D) of NEHRP (National Earthquake Hazard Reduction Program) (FEMA 1997) for magnitudes ranging between 6.0 and 7.4, and horizontal distances to the fault rupture ranging within the interval11 to118 kms.

3.3 Adopted methodology

The inelastic displacement ratio C_η is computed for SDOF systems, with a viscous damping ratio equal to 5% and a bilinear hysteretic behavior, for the selected ground motions sets. The sensitivity analysis is conducted as follows:

1. Select the ground motion.
2. Select and fix the damping ratio ξ and the post-to-preyield stiffness ratio α.
3. Select the period T and the normalized yield strength coefficient η within the range 0.02 to 3 seconds for T and for seven values {0.25, 0.5, 0.75, 1.0, 1.5, 2.0 and 2.5} of η.
4. Determine the instantaneous ductility $\mathcal{M}(t)$ and strength reduction factor $\mathfrak{R}_y(t)$ as response of the system according to the values selected for T, η, α, and ξ. From $\mathcal{M}(t)$ and $\mathfrak{R}_y(t)$, derive the peak ductility factor μ and yield strength reduction factor R_y.
5. Repeat steps 3 and 4 for a range of T, resulting in the $\mu(T,\eta,\alpha,\xi)$ and $R_y(T,\eta,\alpha,\xi)$ values, η value being specified in step 3.
6. Repeat steps 3 to 5 for several values of η.
7. Calculate the inelastic deformation ratio $C_\eta(T,\eta,\alpha,\xi)$.
8. Repeat steps 2 to 7 for each ground motion.
9. Compute the mean value of $C_\eta(T,\eta,\alpha,\xi)$ corresponding tothe whole ground motions and for each value of T and η.

3.4 Mean values of the ductility demand and the yield strength reduction factor

As reported in previous research studies, the mean spectra shows in general three main regions (T<0.6s, 0.6s≤T≤3s, T>3s) (Chopra and Chintanapakdee 2004, Benazouz *et al.* 2012, Chikh *et al.* 2017).

The present results confirm also that, in the case of the ground motions subset "LMSR", the mean values of the ductility demand (μ) and the yield strength reduction factor (R_y), i.e., the response spectrum, is also divided into three main regions sensitive to acceleration, velocity and displacement, (see Fig. 2):

- Acceleration-sensitive region for systems having periods less than T_b=0.6s. This region is also divided into two sub-regions separated by T_a=0.1s. Thus, the response spectrum can be easily idealized by a series of approximating straight lines.
- Velocity-sensitive region corresponds to periods ranging between T_b and 3 sec.
- Displacement-sensitive region for systems having periods larger than 3 sec.

3.5 Inelastic deformation ratio sensitivity

The results obtained for the inelastic deformation ratio mean value show that, (see Fig. 3):

- In the acceleration sensitive region for $T \leq T_b$ and normalized yield strength coefficient $\eta < 1$, C_η ratio decreases rapidly with an increase of η. The spectrum can then be easily idealized by a series of straight lines.
- For $T \geq T_b$ and systems having $\eta \geq 1$, C_η ratio is generally constant and insensitive to η. In the velocity-sensitive region, C_η≈1 remaining also almost insensitive

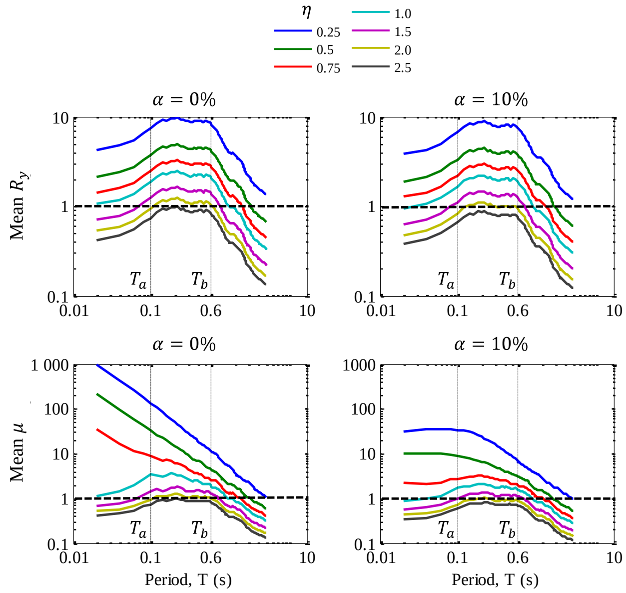

Fig. 2 Mean ductility demand μ and strength reduction factor R_y for inelastic systems computed for LMSRground motionsset ($\eta = 0.25, 0.5, 0.75, 1.0, 1.5, 2.0, 2.5$ from top line to bottom line)

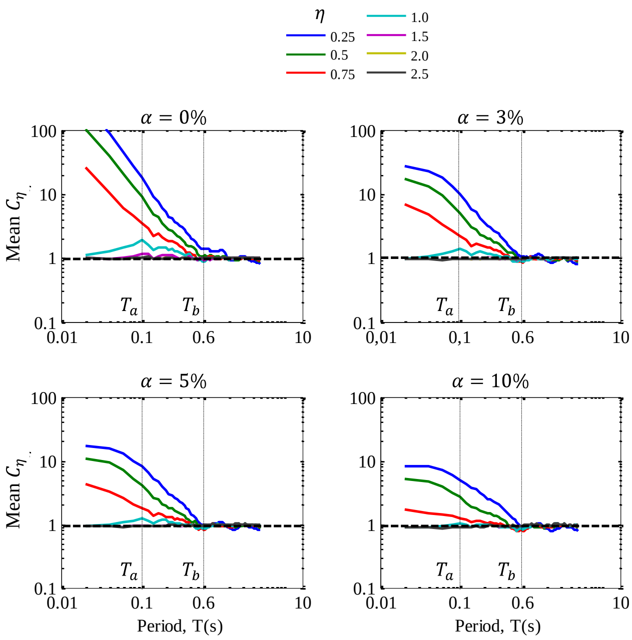

Fig. 3 Mean inelastic deformation ratio C_η for inelastic systems computed for LMSR ground motions set ($\eta = 0.25, 0.5, 0.75, 1.0, 1.5, 2.0, 2.5$ from top line to bottom line)

to η. Therefore, the maximal displacement equals the elastic displacement, i.e., x_m=x_0 for any value of C_η: the system behaves then as an elastic system. These observations, for $T \geq T_b$, confirm those reported by previous studies (Miranda 2000, Ruiz-Garcia and Miranda 2003, Chopra and Chintanapakdee 2003, Benazouz *et al.* 2012).

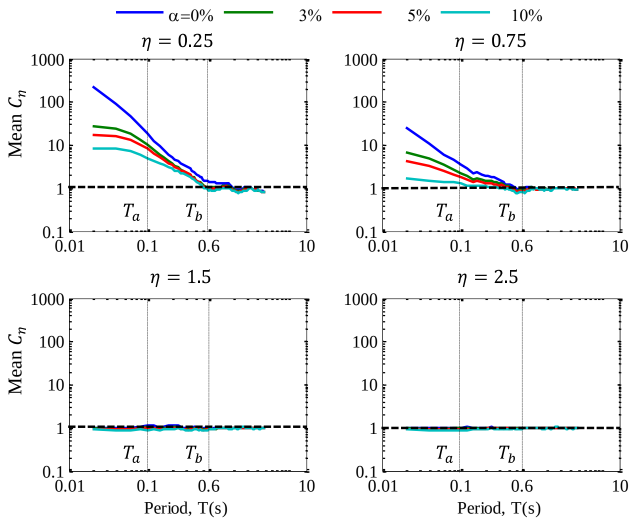

Fig. 4 Influence of post-to-pre-yield stiffness ratio α onthe inelastic deformation ratio C_η for inelastic systems subjected to LMSR ensemble of ground motions

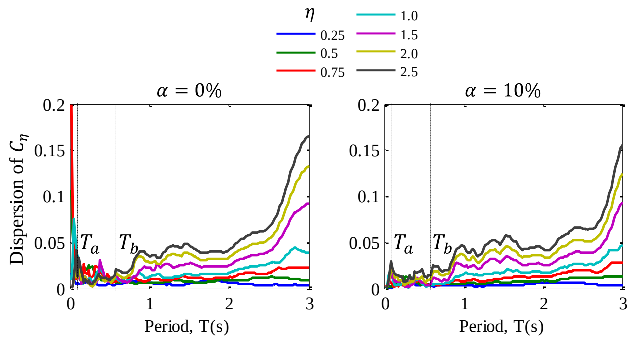

Fig. 5 Scatterness (c.o.v.) of C_η for $\alpha = 0\%$ and $10\%$for LMSR ground motions set (η=0.25, 0.5, 0.75, 1.0, 1.5, 2.0, 2.5 from top line to bottom line).

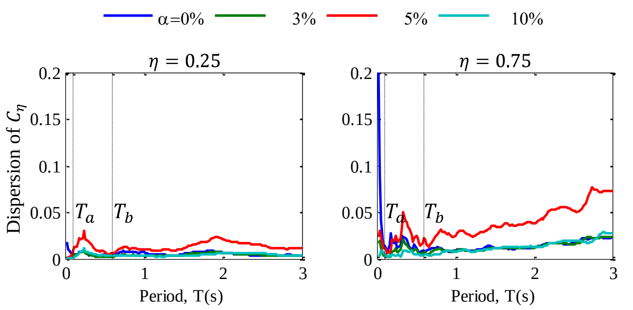

Fig. 6 Scatterness (c.o.v.) of C_η for η=0.25 and 0.75

4. Parametric study

4.1 Influence of the post-to-pre yield stiffness ratio

The influence of α (the post-to-pre-yield stiffness ratio) and η (the normalized yield strength coefficient) upon C_η (the inelastic deformation ratio) is plotted in Fig. 4.

The results show that α:

- Has no effect on C_η when $\eta \geq 1$.
- Influences C_η when $\eta < 1$, which influence decreases

when α increases as long as the period T is smaller than T_b. Beyond this limit T_b, α has negligible effect on C_η.

The coefficient of variation (c.o.v.) is calculated for each ground motions set. For the LMSR set, the coefficient of variation (c.o.v.) for C_η is plotted *vs.* the period T for various values of α (ranging within [0..10%]) and η (ranging within [0.25..2.5]) (Figs. 5-6). The results show that this c.o.v. is smaller than 20% for the whole cases investigated, i.e.:

- very small (almost zero) c.o.v. over a wide range of periods T within $[T_a..T_b]$.
- the parameters η and α have slight effect on the c.o.v. as long as the systems period T respects $T \le T_a$ or $T \ge T_b$.

4.2 Influence of earthquake magnitude and distance

To investigate the influence of the earthquake magnitude and the rupture distance, the mean value of C_η is plotted vs. the period T for α=5% and η in {0.25; 0.75}, for a large set of earthquakes, i.e., LMSR, LMLR, SMSR and SMLR (Fig. 7). The results show that both the rupture distance and the magnitude have the same and small influence on C_η spectra. This conclusion confirms existing studies (Baez and Miranda 2000, Miranda 2000, Ruiz-Garcia and Miranda 2003, Chopra and Chintanapakdee 2003).

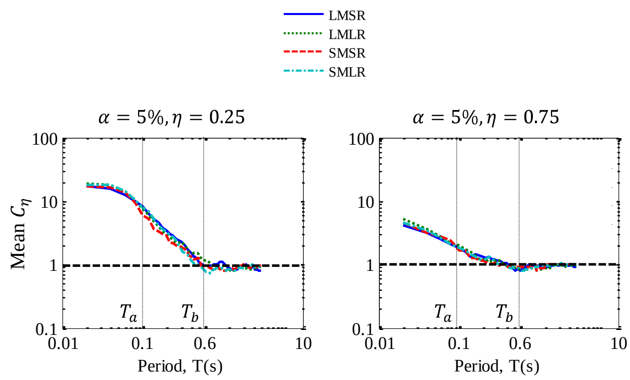

Fig. 7 Comparison of the inelastic deformation ratio C_η for LMSR, LMLR, SMSR, and SMLR ground motions with α=5% and η in {0.25; 0.75}

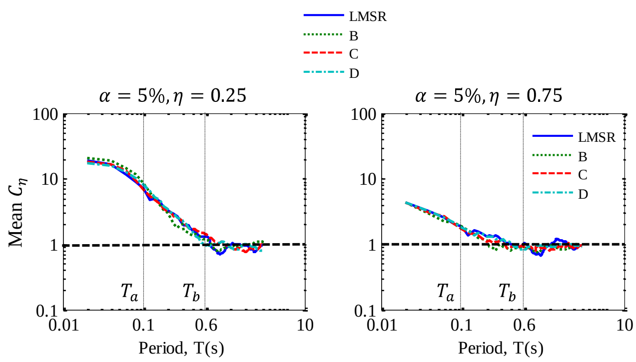

Fig. 8 Comparison of the inelastic deformation ratio C_η for LMSR and NEHRP site classes B, C, and D ensembles of ground motions for fixed α (α=5%) and two values of η=0.25 and 0.75

4.3 Influence of firm site classes

The sensitivity of the ratio C_η to the site classes B, C, and D (Fig. 8) shows that C_η is slightly influenced by the site class and distances to the fault rupture, which confirms other studies results (Miranda 1991, 1993, Ruiz-Garcia and Miranda 2003, Chopra and Chintanapakdee 2003,Ruiz-Garcia and Miranda 2006).

5. Theoretical modeling of the inelastic deformation ratio C_η

Several studies have been devoted to develop a theoretical model of the inelastic deformation ratio C_η for inelastic SDOF systems. Fig. 9 and Table 1 provide a comparative study of existing models (Newmark and Hall 1982, Krawinkler and Nassar 1992, Miranda 2000, Chopra and Chintanapakdee 2004), in the case of elastoplastic and bilinear systems under LMSR ground motions. However, it is still challenging to develop a simple equation able to include the whole governing parameters: T, α and η.

After running simulations for a wide range of the governing parameters values, a nonlinear regression analysis, thanks to the software DataFit (Oakdale Engineering), leads to the simplified model for C_η in the case of bilinear and elastic-perfectly plastic systems

$$\eta \le 1 \quad \begin{cases} C_\eta = a b^{\frac{1}{T}} T^{\eta - c} & T \le 1\text{sec} \\ C_\eta = 1 & T > 1\text{sec} \end{cases} \tag{7}$$

$$\eta > 1 \qquad C_\eta = 1 \qquad \text{for all values of } T \tag{8}$$

Where: a, b and c are derived from the non-linear regression analysis for each specific α value and each of the three specific intervals for T, see Table 2. The theoretical predictions of this simplified model show the sensitivity of C_η ratio to post-to-pre yield stiffness ratio α, see Fig. 10. This new model is helpful in seismic evaluation of structures, especially those having periods falling in the sensitive accelerations intervals.

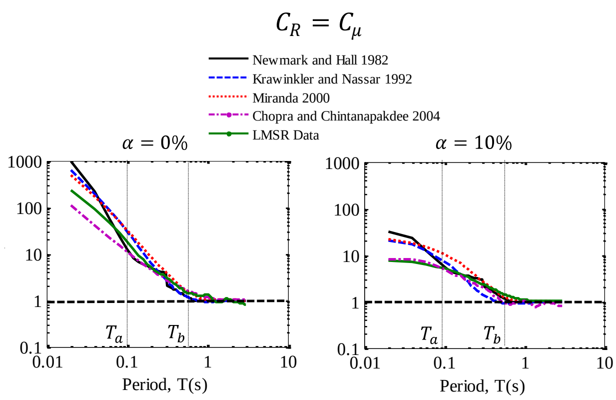

Fig. 9 Theoretical values of C_R, C_μ and LMSR data for η=0.25, α=0% (Elastoplastic systems) and α=10%

Table 1 Existing models for the inelastic deformation ratio

Researcher	Inelastic system	C_μ	C_R	Parameters Definition
Miranda 2000	EP (Elastic Perfectly Plastic)	$C_\mu = \left[1 + \left(\frac{1}{\mu} - 1\right) exp(-12\, T\mu^{-0.8})\right]^{-1}$		
Newmark and Hall 1982	EP	$C_\mu = \begin{cases} \mu & T < T_a \\ \mu/\sqrt{2\mu - 1} & T_b < T < T_{c'} \\ 1 & T > T_c \end{cases}$	$C_R = \begin{cases} \infty & T < T_a \\ (R_y^2 + 1)/2R_y & T_b < T < T_{c'} \\ 1 & T > T_c \end{cases}$	T_a, T_b and T_c inelastic design spectra (Newmark and Hall 1982) $T_{c'} = T_c\sqrt{2\mu - 1}/\mu$
Krawinkler and Nassar1992	Bilinear	$C_\mu = \mu[c(\mu - 1) + 1]^{-1/c}$	$C_R = \frac{1}{R_y}\left[1 + \frac{1}{c}(R_y^c - 1)\right]$	$C(T, \alpha) = \frac{T^a}{1 + T^a} + \frac{b}{T}$ a=1 and b=0.42 for $\alpha = 2\%$; a=0.8 and b=0.29 for $\alpha = 10\%$.
Chopra and Chintanapakdee 2004	Bilinear	$C_\mu = 1 + \left[(L_\mu - 1)^{-1} + \left(\frac{a}{\mu^b} + c\right)\left(\frac{T}{T_c}\right)^d\right]^{-1}$	$C_R = 1 + \left[(L_R - 1)^{-1} + \left(\frac{a}{R_y^b} + c\right)\left(\frac{T}{T_c}\right)^d\right]^{-1}$	$L_\mu$is given by Eq. (7a) (Chopra and Chintanapakdee 2003). a=105, b=2.3b c=1.9c d=1.7 using the data for four (LMSR, LMLR, SMSR and SMLR). $L_R$is given by Eq. (7a) (Chopra and Chintanapakdee 2003)For LMSR a=63, b=2.3b c=1.7 and d=2.3. For four ensembles of ground motion a=61, b=2.4, c=1.5 and d=2.4.
Ruiz-Garcia and Miranda 2003	EP		$C_R = 1 + \left[\frac{1}{a(T/T_s)^b} + \frac{1}{c}\right](R - 1)$	a=50; b=1.8; c=55 and T_s = 0.75, 0.85 or 1.05 for NEHRP site class B, C or D, respectively

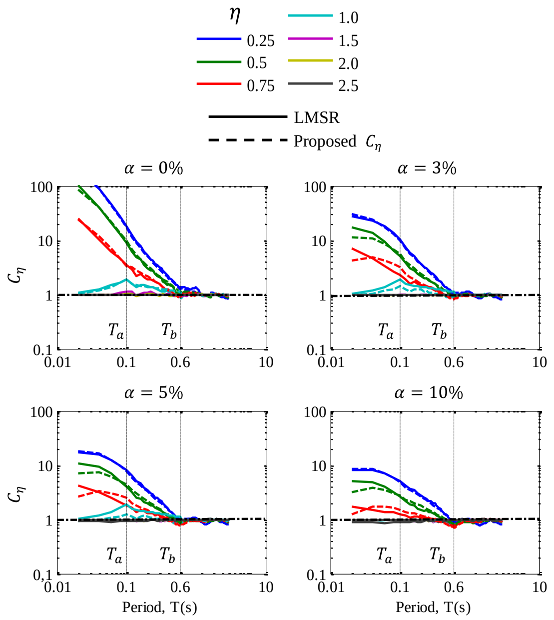

Fig. 10 Comparison between simplified model for C_η and LMSR simulations for different η values (η=0.25, 0.5, 0.75, 1.0, 1.5, 2.0, 2.5 from top line to bottom line)

Table 2 Parameters in Eq. (7) for each value of T and α

α(%)	Fitting parameters	$T \leq 0.1$	$0.1 < T \leq 0.6$	$0.6 < T < 1$
0	a	0,158	0,658	0,799
	b	0,98	1,107	1,36
	c	2,369	1,196	0,247
3	a	0,382	0,586	1,99
	b	0,965	1,055	0,508
	c	1,818	1,203	1,203
5	a	0,494	0,563	2,509
	b	0,968	1,014	0,394
	c	1,589	1,298	1,448
10	a	0,499	0,565	3,494
	b	0,97	0,943	0,272
	c	1,366	1,391	1,817

6. Conclusions

The present paper presents the development of ductility and strength reduction factors based on normalized yield strength coefficient which is a key issue for seismic demand evaluation. The purpose is also to establish a simplified theoretical expression of the inelastic deformation ratio (C_η) according to T, α and η. For this purpose, the results obtained for 2,352,000 analyses are reported. They correspond to:

- seven (7) sets of ground motions, each containing twenty (20) earthquakes,
- seven (7) values of the normalized yield strength coefficient (η)
- four (4) values of post-to-pre yield stiffness ratio (α)
- and three hundred (300) values of vibration periods (T).

The following conclusions can be drawn from the obtained results,:

- the accuracy of C_η depends strongly on the systems inelastic properties, i.e., α, η and PGA (peak ground acceleration).
- The parameter α (post-to-pre yield stiffness ratio) has a great influence on (C_η) for structures with small periods ($T \leq 0.6$ s). However, α has no influence and can be neglected when $\eta \geq 1$.
- For systems having a normalized yield strength coefficient $\eta \leq 1$, C_η ratio is very sensitive to η as it rapidly decreases when η increases, to reach an asymptotic value ($C_\eta=1$) when ($\eta>1$).
- The inelastic displacement ratios are insensitive to the rupture distance, the earthquake magnitude and site class, for the selected accelerograms.
- The new theoretical curves for C_η, proposed after fitting, are close to the LMSR references. The scatterness of the C_η values remains very small.

This study reviewed the existing research on inelastic deformation ratios, with the purpose to estimate the maximum inelastic displacements according to maximum elastic displacements demands. It presented also a new parameter C_η useful for Pushover curves. To gether with the η coefficient, used as a control parameter, they improve the existing practice as they lead to a rational and improved seismic design approach.

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