Vault macro-element with equivalent trusses in global seismic analyses

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Abstract. This paper proposes a quick and simplified method to describe masonry vaults in global seismic analyses of buildings. An equivalent macro-element constituted by a set of six trusses, two for each transverse, longitudinal and diagonal direction, is introduced. The equivalent trusses, whose stiffness is calculated by fully modeled vaults of different geometry, mechanical properties and boundary conditions, simulate the vault in both global analysis and local analysis, such as kinematic or rocking approaches. A parametric study was carried out to investigate the influence of geometrical characteristics and mechanical features on the equivalent stiffness values. The method was numerically validated by performing modal and transient analysis on a three naves-church in the elastic range. Vibration modes and displacement time-histories were compared showing satisfying agreement between the complete and the simplified models. This procedure is particularly useful in engineering practice because it allows to assess, in a simplified way, the effectiveness of strengthening interventions for reducing horizontal relative displacements between vault supports.

Keywords: vault; macro-element; equivalent stiffness; truss; churches

1. Introduction

Seismic assessment of historic buildings is the key to prevent possibly unrecoverable damages on the cultural heritage. Often, the architecture of historic structures is enriched by vaults of different typologies, which may have both structural and non-structural roles. The vaults geometry can be complex: from the simplest barrel vault, derived by the arch, to the more complex cross, dome or fan vaults. Their seismic vulnerability, especially when the building technique is in folio type (brick disposed along the shortest edge), is a well-known issue. Structural engineers are aware of difficulties in taking them into account when a numerical model has to be implemented. Finite elements modeling requires special efforts for two aspects: (i) the time needed to build the geometric model, generally intersection of irregular solids, (ii) the increased computational time in the global model due to the large number of elements and (iii) a possible numerical instability of the solution could emerge, due to the formation of local collapses in the vaults, which would not allow the convergence the global analyses. For that reasons, often the vaults have been modeled as equivalent diaphragms (Marseglia *et al.* 2014), but the identification of an equivalent thickness and boundary conditions arise uncertainties (Giresini *et al.* 2015). In many professional cases, the structural behaviour of the vaults is also simply neglected in global analysis and vaults are modeled only as equivalent masses. Nevertheless, the stiffening role of vaults in both local and global seismic assessment is relevant and needs to be properly considered. With this purpose, the method proposed in the present paper, particularly useful in engineering practice, is based on the possibility to model the masonry vaults as a system of equivalent trusses in terms of stiffness. This procedure, based on the generalization of the "macro-element approach", was first introduced in (Giresini *et al.* 2014) to perform complex numerical analyses in a simplified but effective way: the proposal of modeling vaults as equivalent macro-element arises from the methods of substituting masonry portions with frames (Magenes and La Fontana 1998) or macro-elements (Caliò *et al.* 2012, Caddemi *et al.* 2014, Caddemi *et al.* 2015, Giresini 2015a, Andreini *et al.* 2014, Pantò *et al.* 2016). Besides it, experimental tests on masonry cross vaults have been recently carried out to identify their stiffness in multiple directions (Rossi 2015).

In the present paper, the method to identify equivalent vaults as horizontal macro-elements is described and the aspects more critical are commented. Abaci of vaults as equivalent trusses (VET) are derived for several types of vaults from parametric analyses. Afterwards, the method is implemented in a whole three-naves *basilica*-type church. The simplified models, where the vaults are represented by

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equivalent trusses, are compared with full models, where the vaults are completely modeled with finite elements. With the simplified models is therefore possible not only to interpret the role of the vault in the as-built structure, but also rapidly performing checks to improve the seismic response by strengthening measures. Indeed, the computational time is strongly reduced and the attention is mostly focused on the effect of the vaults in the adjacent structures and the influence of strengthening techniques (i.e., steel tie-rods, FRP reinforcements, extradosal fill etc. Krajewski and Hojdys 2015). However, it could be noticed that, whether the structural behavior of arches or vaults have to be investigated, more non-linear analyses are needed, such as limit analysis in 2D-3D models (Alecci *et al.* 2016, D'Ayala and Tomasoni 2011, Milani *et al.* 2008). Moreover, recently advanced numerical tools were proposed to simulate the behaviour of the monumental buildings (Pantò *et al.* 2016). In this work the single vaults are analyzed in both linear and non-linear ranges, but only a linear analysis is performed for the comparison between the two church models. However, this approach can be extended to more significant non-linear models capable of taking into account this relevant masonry characteristic.

2. Vault Equivalent Trusses (VET)

2.1 Definition of trusses equivalent to vaults

The generic vault may be analyzed as system of equivalent trusses, whose in-plane dimensions are L_1 and L_2 (Fig. 1(a)). The equivalence is established in terms of stiffness, calculated by imposing displacements to a set of six hinged rigid rods in different directions. The plane of the trusses is chosen as that defined by the abutments. Fig. 1(a) displays the stiffness K and the equivalent steel area A of the trusses; the subscripts l, t and d respectively indicate the longitudinal, transverse and diagonal directions. The signs (+) and (-) stand for tensile and compression state. Finally, N and F symbolize internal force and reaction force, respectively.

For the definition of the diagonal truss stiffness, a shear-type mechanism is assumed (Fig. 1(b)). Nodes C and D are horizontally moved by δ_1, inducing compression in B-D and tension in A-C diagonal. Assuming that both tension and compression states are active, the stiffness K_d is given by

$$K_d = \frac{2N_{d,d}^-}{\delta_{d,d}^-} = \frac{2N_{d,d}^+}{\delta_{d,d}^+} = = \frac{|F_1|}{\cos\alpha_d} \cdot \frac{1}{|\delta_1|\cos\alpha_d}$$
$$= \frac{|F_1|}{|\delta_1| \cdot \cos^2\alpha_d} \tag{1}$$

where F_1 is the sum of reaction forces generated by the imposed displacement δ_1, and α_d the angle between the horizontal direction and the diagonal one in the deformed configuration.

For small displacements it can be assumed $\alpha \cong \alpha_d$. Due to the masonry poor tensile strength, rigorously only the truss in compression should be considered. Nevertheless, in

the linear case and for small displacements, the material anisotropy can be neglected. Moreover, the need to consider both states arises once that modal analysis is requested, since the latter regards linear range.

The transverse (longitudinal) truss stiffness can be analogously obtained, i.e., by imposing displacements in transverse (longitudinal) direction. Fig. 1(c) depicts the closing mechanism in transverse direction. Nodes C and D are vertically moved by δ_2, causing the sum of reaction forces F_2. Diagonal trusses are compressed and their stiffness can be expressed as

$$K_t = \frac{N_t^-}{\delta_2^-} \tag{2}$$

The relationship between the internal force in the transverse truss N_t^- and the reaction force F_2 is

$$\frac{F_2}{2} = N_t^- + N_{d,t}^- \cos\alpha_t \tag{3}$$

where α_t is the angle between the horizontal direction and the diagonal one in the deformed configuration. By substituting Eq. (3) in Eq. (2) one obtains

$$K_t = \frac{\frac{F_2}{2} - N_{d,t}^- \cos\alpha_t}{\delta_2^-} \tag{4}$$

The diagonal internal force $N_{d,t}^-$ depends on the stiffness K_d

$$N_{d,t}^- = \frac{K_d}{2}\delta_{d,t}^- = \frac{K_d}{2}\delta_2^- \sin\alpha \tag{5}$$

The transverse stiffness is therefore

$$K_t = \frac{\frac{F_2}{2} - \frac{K_d}{2}\delta_2^- \sin\alpha \cos\alpha_t}{\delta_2^-} \tag{6}$$

Assuming $\alpha \cong \alpha_t$ and considering that $\sin 2\alpha = 2\sin\alpha\cos\alpha$, it follows

$$K_t = \frac{F_2}{2\delta_2^-} - K_d\frac{\sin 2\alpha}{4} \tag{7}$$

Therefore, the stiffness K_t (or the corresponding longitudinal one K_l) is known once that the diagonal stiffness is determined.

The equivalent stiffness can be translated in terms of equivalent cross sectional area A_{eq} or equivalent diameter d_{eq}. For instance, the equivalent area and the diameter $d_{d,eq}$ of the diagonal trusses are expressed by

$$A_{d,eq} = \frac{K_d L_d}{E_s} \qquad d_{d,eq} = 2\sqrt{\frac{K_d L_d}{\pi E_s}} \tag{8}$$

where L_d is the length of the diagonal and E_s the steel Young's modulus. By considering the equivalent stiffness in longitudinal and transverse directions the equivalent steel area and diameter can be analogously calculated.

Once that the vault type is specified, together with geometric dimensions and mechanical parameters, VETs can be obtained by experimental tests or numerical simulations. In this paper, finite element models are used to
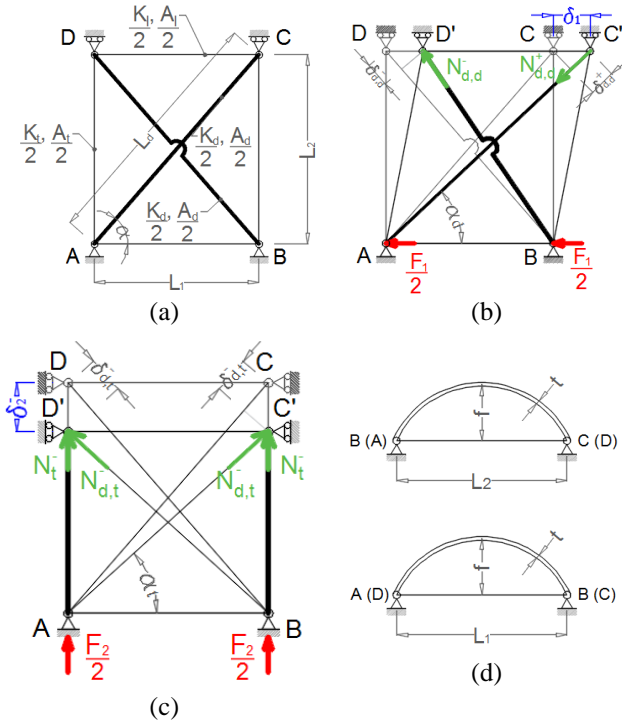
(a)

(b)

(c)

(d)

Fig. 1 Method for the determination of vault equivalent trusses (VET): (a) system of equivalent trusses in transverse, longitudinal and diagonal direction for a generic vault with in-plane dimensions L_1 and L_2; (b) diagonal truss stiffness obtained by shear-type deformation; (c) transverse (longitudinal) truss stiffness obtained by closing mechanism; (d) rise f and thickness t of the vault
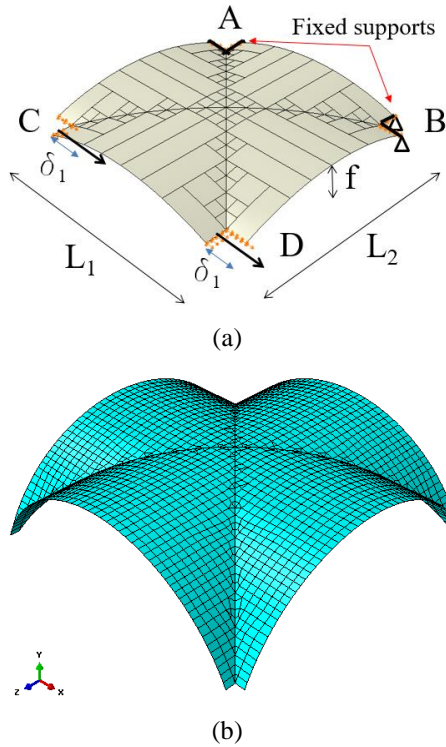

(a)

(b)

Fig. 2 Diagonal truss stiffness determination for a squared cross vault with different geometries: (a) boundary conditions and (b) model mesh

calculate equivalent stiffness. Clearly, the possible presence of backfill strongly influences the stiffness value, and therefore it must be taken into account in the VET definition.

2.2 Linear parametric analysis for different vaults types and geometries

A parametric analysis of several vaults types and geometries is performed to draw charts useful in modeling vaults as equivalent trusses (VETs). Indeed, once that geometry of the vault, mechanical properties and boundary conditions are known, the stiffness values can be directly obtained by these charts and speed up the modeling process. To validate the method, the vaults fully modeled with finite elements have been compared to the trusses in global analysis. Further considerations on the validation process can be found in §3.1 and the comparisons are described in detail in § 4.1.

Cross and dome vaults are analyzed, since they are very widespread in historical buildings and churches. For the sake of simplicity, only in-plane squared vaults are considered, namely $L_1 = L_2 = L$ (Fig. 1(a)), assuming the lack of backfill. Mechanical properties taken into account are those of brick masonry for existing buildings (NTC2008) and in particular: elastic modulus 1500 MPa, Poisson coefficient 0.15. All vaults have specific weight 18 kN/m^3.

The adopted parameters are the in-plane length L, between 300 and 600 cm, the ratio rise to in-plane length f/L, between 0 and 0.5, and the thickness t equal to 3, 6, 12 cm (Fig. 1(a)). Generally speaking, the boundary conditions strongly affect the equivalent stiffness, but for diagonal truss stiffness the results have been shown to be not so sensitive depending on the number of nodes where displacements are applied (Giresini *et al.* 2014).

Therefore, even though Sabouret's cracks cause a detachment of the vault from the adjacent structure, the value of diagonal stiffness does not appreciably changes. A first analysis is performed by applying displacements to the corners C and D, while A and B are hinged (Fig. 2(a)). The numerical analysis is performed with Abaqus CAE 6.12 (Abaqus CAE 6.12, User's and Theory Manuals); the finite elements are shell quadratic elements with 9 integration points through thickness and the average mesh size is 10-15 cm (Fig. 2(b)).

The analysis is first performed by assuming linear material and the equivalent stiffness and steel diameters are obtained by Eqs. (1)-(7)-(8). The characterization of the diagonal trusses for a cross vault is depicted in Fig. 3.

The equivalent diagonal stiffness decreases whether the in-plane length increases for the same rise to length f/L ratio - with a nearly linear behavior - and naturally is much lower for thinner vaults. Moreover, for lower f/L ratios and same thickness t, the diagonal stiffness is higher since the vault shape is closer to a flat diaphragm. Concerning the equivalent steel diameter d, it does not appreciably change by increasing L and with f/L constant. Indeed, d is proportional to the squared root of the product $K \cdot L$.

Longitudinal/transverse stiffness values for a squared
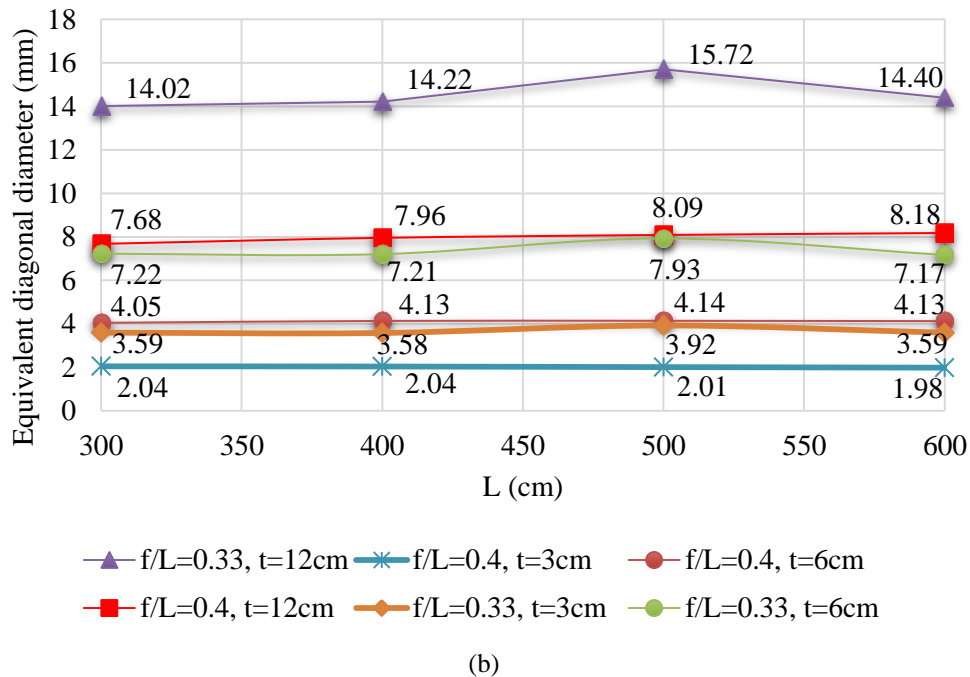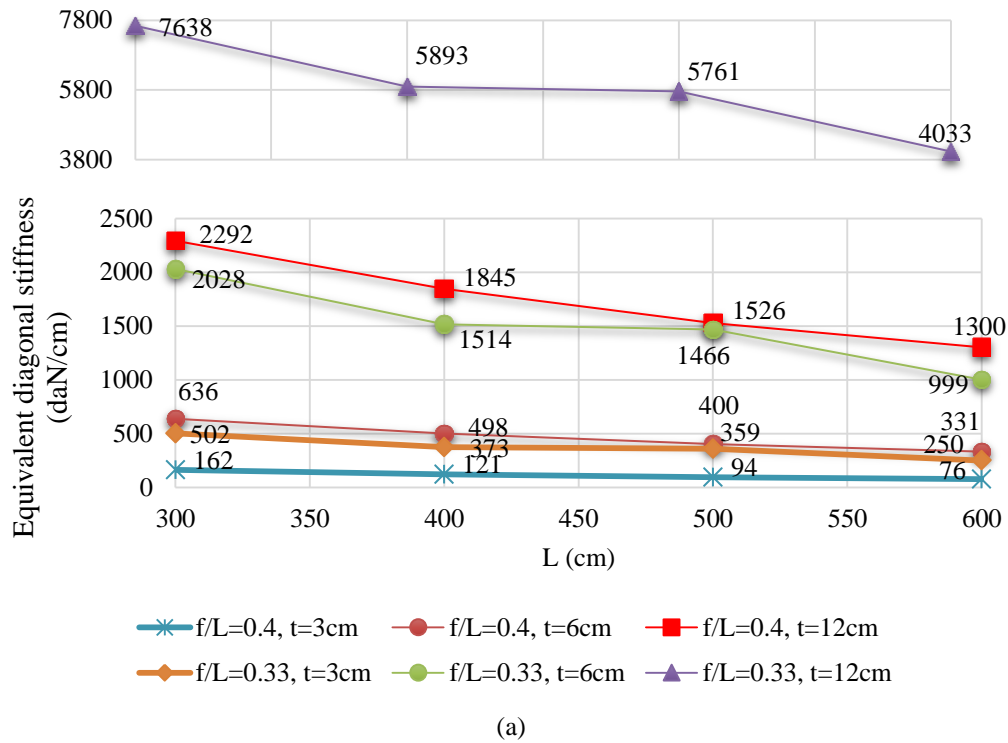
(a)

(b)

Fig. 3 Linear parametric analysis results for cross vaults: (a) equivalent diagonal stiffness - in-plane length relationship obtained by Eq. (1) and (b) equivalent diagonal steel diameter - in-plane length relationship for different rise to length ratios

cross vault having L=300 cm are displayed in Fig. 4. These values are plotted against the rise to length ratio f/L, with a lower limit value 0, corresponding to a flat diaphragm with plane stress. The stiffness rapidly drops for higher f/L ratios with non-linear behavior. When the ratio f/L is constant, the longitudinal/transverse stiffness is markedly higher for thicker vaults. For instance, if f/L =0.1, from t=3 cm to t=12 cm one has a stiffness three orders of

magnitude higher. The graph also shows the possible mistake can be made comparing a generic vault with a flat diaphragm. Moreover, in § 4.1 a sensitivity analysis is performed to compare global analysis behavior of a church, where vaults are modeled first in their actual configuration and then as equivalent trusses or diaphragms.

Another possible parametric analysis considers different vaults boundary conditions. Generally, vaults are connected
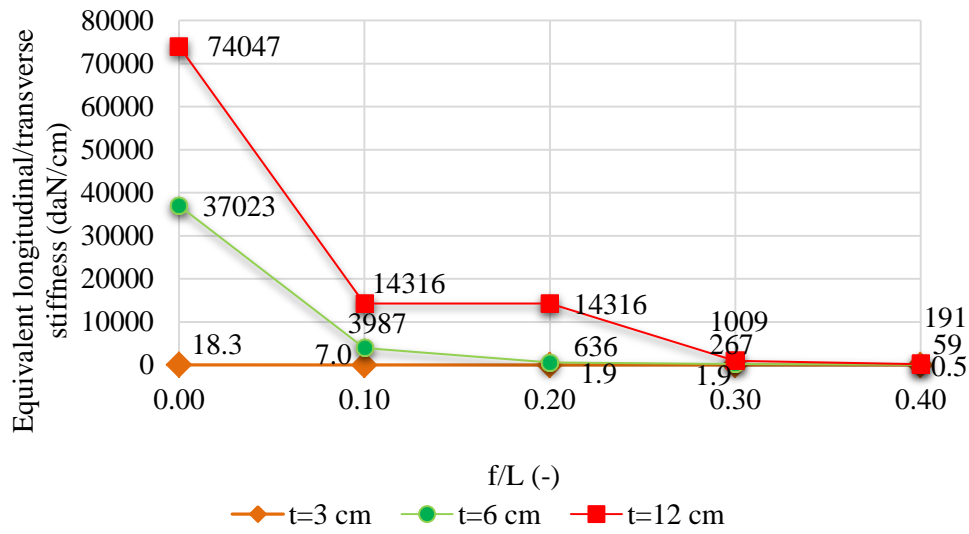
Fig. 4 Linear parametric analysis results for squared cross vaults with L = 300 cm: equivalent longitudinal/transverse stiffness - rise/length relationship obtained by Eq. (7)
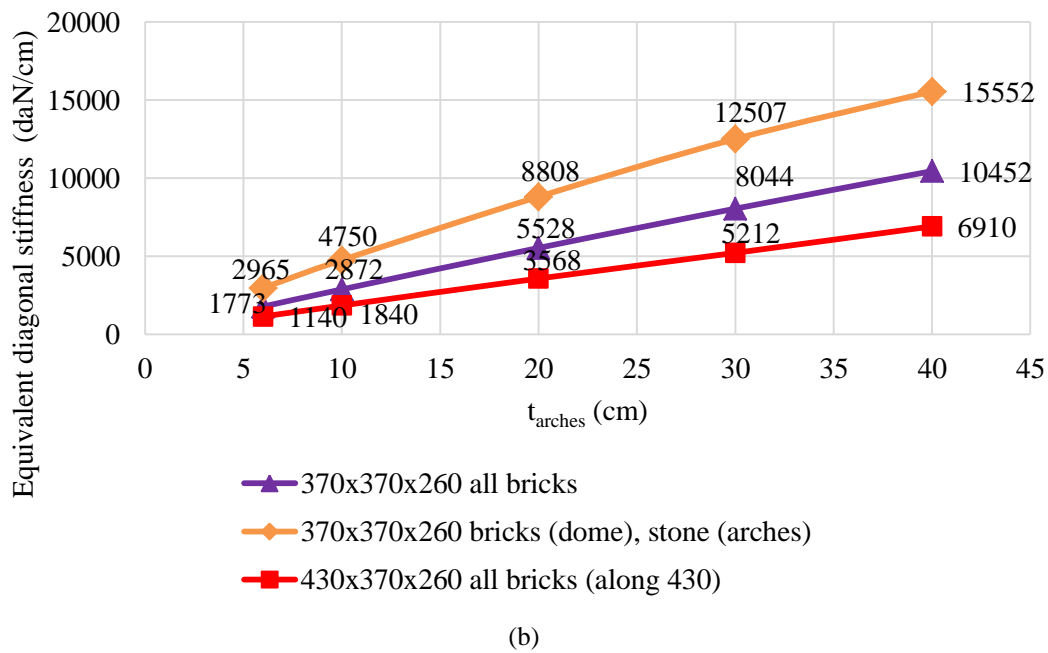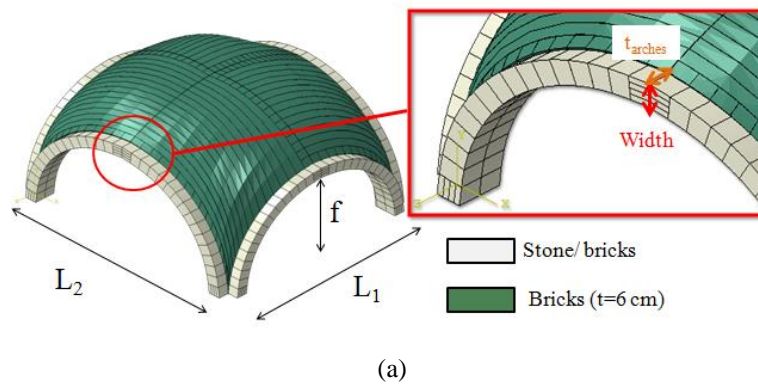

(a)

(b)

Fig. 5 Linear parametric analysis of dome vault with different adjacent arches materials: (a) vault geometry with indication of the arch thickness and width; (b) equivalent diagonal truss stiffness depending on arch thickness and material obtained with Eq. (1)
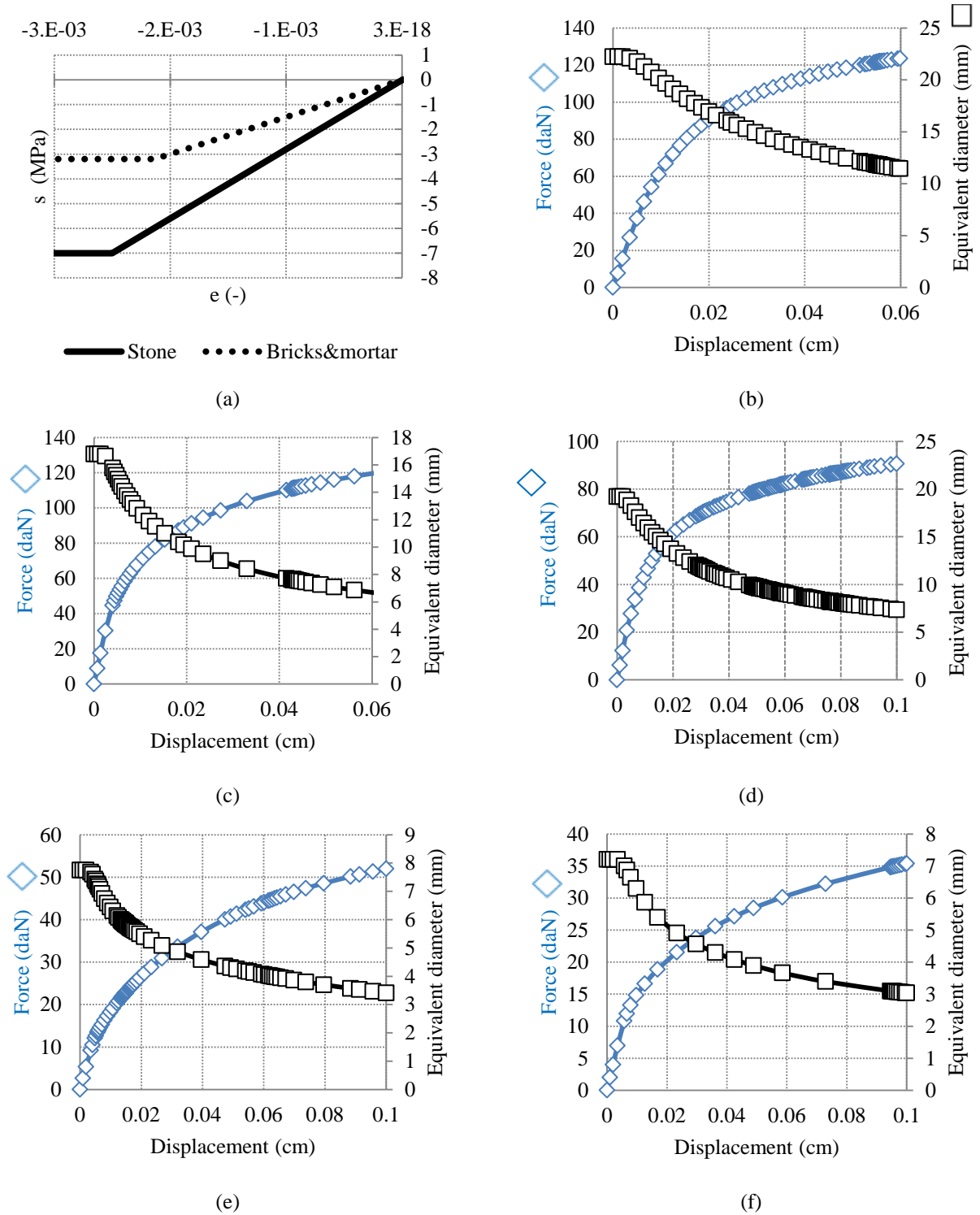
(a)

(b)

(c)

(d)

(e)

(f)

Fig. 6 Non-linear static analysis of dome vaults with f/L=0.5: (a) compression stress-strain relationship (tensile strength 0.01 MPa); (b) equivalent diagonal diameter and sum of reaction forces (RF) for DV1 $L_1 = L_2$=370 cm; (c) equivalent transverse/longitudinal diameter and sum of RF in closing mechanism (as in Fig. 1(c)) for DV1; (d) equivalent diagonal diameter and sum of RF for DV2 $L_1 = 430, L_2$=370 cm loaded along the longer side; (e) equivalent transverse diameter and sum of RF in closing mechanism for DV2 along the shorter side; (f) equivalent transverse diameter and sum of RF in closing mechanism for DV2 along the longer side

to arches along the edges (Fig. 5). A dome vault (whose geometry is that of the vault of the case study presented in § 4.1) is analyzed to investigate the effect of different

materials of adjacent arches.

Three cases are considered: (i) in-plane squared vault $L_1=L_2$=370 cm, with all bricks; (ii) same in-plane squared

vault with vault made by bricks and stone arches; (iii) vault with L_1=370 cm and L_2=430 cm, made all by bricks. All the side arches have ratio f/L equal to 0.5 and the central dome vault is for all cases 6 cm thick. The second dome vault is not completely symmetric, since the side arches along the longitudinal direction have width of 40 cm and along the transverse direction width of 20 cm. Besides the width of the side arches, Fig. 4(a) shows the thickness of the dome vault, in the examined cases constant and equal to 40 cm. The bricks masonry elastic modulus is 1500 MPa, and the specific weight 18 kN/m^3. The stone masonry elastic modulus is 1800 MPa while the assumed specific weight is 22 kN/m^3 (average values for existing masonry buildings obtained from (NTC2008)). For all cases, the crown height is 260 cm measured from the pendentives.

The considered boundary conditions are pinned nodes at the pendentives and displacements imposed to the whole side arch profile.

The values of diagonal truss stiffness are displayed in Fig. 4(b): the diagonal stiffness strongly decreases with shape ratio L_1/L_2 different from 1. The case (iii)-where displacements are applied along the longer direction- is more flexible than case (i).

By contrast, if the vault is squared in-plane, stiffer side arches cause higher equivalent diagonal stiffness, as expected. In addition, the arch thickness strongly influences the diagonal stiffness.

2.3 Non-linear parametric analysis for different vaults types and geometries

An additional analysis is performed to define the stiffness variation when a non-linear masonry behavior is considered. An example is provided for the two dome vaults DV1 and DV2 (section 2.2) with arch thickness of 40 cm and stone arches. A brittle failure is assumed for tensile state, with a tensile strength equal to 0.01 MPa for both brick and stone materials. Compressive strengths are assumed equal to 3.2 MPa for brick and 7.0 MPa for stone respectively. The maximum strain in compression is assumed equal to 3E.-3. The compressive elastic-plastic behavior, implemented in the Concrete Damaged Plasticity model in Abaqus (Abaqus CAE 6.12, User's and Theory Manuals), is shown in Fig. 6(a).

The non-linear analysis results are displayed in Fig. 6(b)-(c) for the squared dome vault DV1, and in Fig. 6(d)-(e)-(f) for the asymmetric DV2. In all cases, it can be noticed that the equivalent steel diameter markedly drops for small displacement, of the order of millimeter, due to the about null tensile strength assumed. Concerning the diagonal stiffness, if the vault is not squared in-plane, the initial stiffness is much lower but the rate of reduction is less rapid than for DV1 (Fig. 6(b)-(d)). The convergence is obtained for a lower displacement value for DV1, 0.06 cm against 0.1 cm of DV2.

Referring to the longitudinal-transverse stiffness in closing mechanism - the opening mechanism has negligible stiffness if nearly zero tensile strength is assumed - stiffness values are much lower than diagonal ones, especially for the asymmetric vault. The initial stiffness is one order of magnitude higher for DV1, but the reduction rate at the same common maximum displacement (=0.06 cm) is similar. The equivalent truss in longitudinal direction is slightly more flexible than in the transverse one for the asymmetric DV2, due to the more marked bending effect in the longer direction.

3. VET analysis for seismic vulnerability assessment

The idea to model vaults as macro-element made by equivalent trusses support the need to evaluate the role of the vaults in the seismic response of historic buildings. The main purpose is to reduce modeling and computational time, especially for complex and large structures. Vaults are flexible diaphragms of different geometries, mechanical properties and boundary conditions with adjacent structures. Vaults equivalent trusses (VETs) can be implemented in both global and local analysis in commercial programs or may be included in local analysis, as rocking approaches or kinematic analysis for façades or other structural elements restrained by vaults. The latter ones are the tool indicated by the Italian codes for assessing seismic vulnerability of existing masonry structures. In this paragraph, some criteria to include VETs in the analysis of historic masonry structures are discussed.
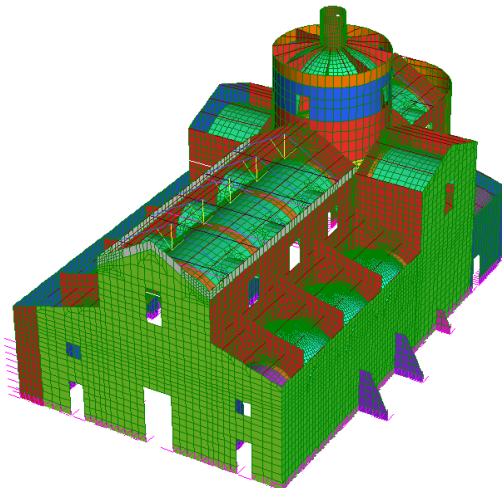
3.1 VET in global analysis of historic buildings

The validation process of inserting VET in place of complete vaults can be numerically carried out by comparing full-modeled church (as those displayed in Fig. 7) to VET simplified models, where equivalent trusses substitute the vaults. Naturally, it is not simple to identify for each vault the boundary conditions with adjacent elements to obtain realistic stiffness values, but numerical comparisons can be performed by assuming a perfect connection between nodes of the vault and nodes of the adjacent structures. Indeed, the purpose here is of numerically validating the method, whereas the stiffness abaci can take into account more in-depth different boundary conditions.
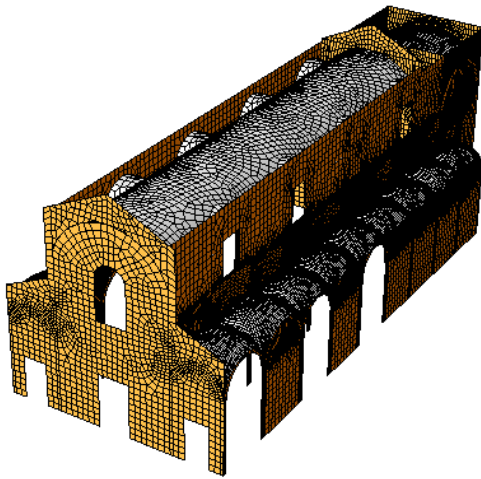
Two types of comparisons are performed: (i) modal analysis and (ii) global time-history analysis. In the first case, natural frequencies and modal participation factors are considered; indeed, the modal analysis, although simple and linear, can give preliminary information on the dynamic behavior. In the time-history analysis, relative displacements of vaults support in different directions are monitored. Also in this case, only a linear analysis has been performed, due to the uncertainties in defining non-linear behavior for large structures and to the spirit of simplification that leads the method. Moreover, the aim is to evaluate displacements peaks that lead to stress peaks in vaults and masonry panels, in order to identify the most critical portions.

3.2 VETs in local analysis

VETs in local analysis can be useful to model vault

(a)

(b)

Fig. 7 Finite elements models of historic churches: (a) Santa Gemma church in Goriano Sicoli (Andreini *et al.* 2010); (b) San Frediano church in Pisa
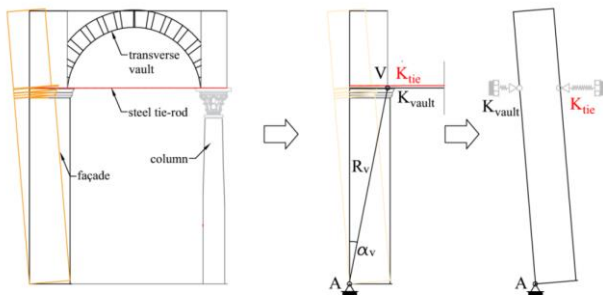

Fig. 8 Vaults equivalent trusses connected to masonry façade (adapted from Giresini *et al.* 2015)

connected to masonry panels that can be assimilated to rigid blocks. Generally, existing masonry buildings are regarded as set of macro-elements, each of them composed by rigid blocks due to the poor connections between structural elements.

Masonry façades, for instance, can be studied as rigid

blocks oscillating around a horizontal hinge. Evidently, possible vaults connected play role in the seismic response. These kind of structures can be studied with kinematic linear/nonlinear approaches (NTC2008) and rocking analysis (Giresini *et al.* 2016, DeJong and Dimitrakopoulos 2014). A spring with equivalent stiffness, obtained with the procedure illustrated in § 2.1, represents the vault connected to the rigid block (Fig. 8). The VET-model might be easily expanded with other constitutive laws: the stiffness can be different in tension and in compression but constant, or may vanish for a limit displacement, or can follow a specific constitutive law (Giresini and Sassu 2016). In addition, strengthening measures, like steel tie-rods acting only in tension, might be modeled with this procedure.
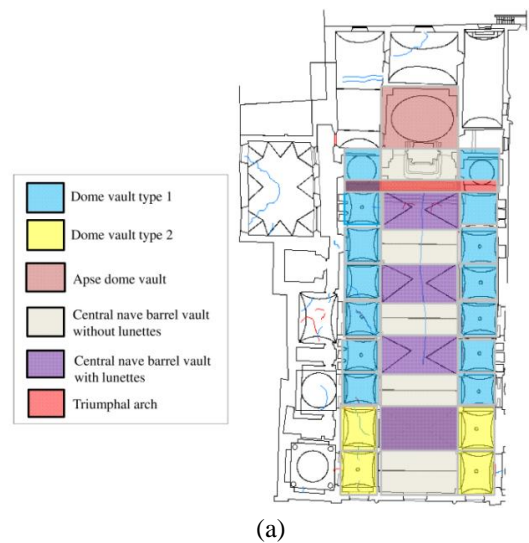
4. Numerical case study

In this paragraph, an application of the use of VETs in global and local analysis is performed. An historic basilica church is analyzed with two FE models: one with fully-modeled vaults and another with VETs in place of the vaults, comparing the results of the dynamic analyses.

4.1 Global analysis of a historic basilica church

The church of San Frediano in Pisa (Italy), was considered to validate the method of substituting equivalent trusses in global analysis. The plan of the church is shown in Fig. 9(a).

The structure is a *basilica*-type church with three naves, covered by dome vaults and barrel vault with lunettes in the central nave. The church has dimensions 41×15×16 m (length×width×maximum tympanum height) and it is considered structurally isolated. The church is made up by ashlar stone (façade and walls), granite (columns), bricks and mortar (vaults).

(a)

Fig. 9 Vaults typologies in San Frediano church: (a) in-plane disposition; (b) lateral naves and apse vaults with dimensions in cm; (c) central nave barrel vault subdivided in 8 barrel vaults
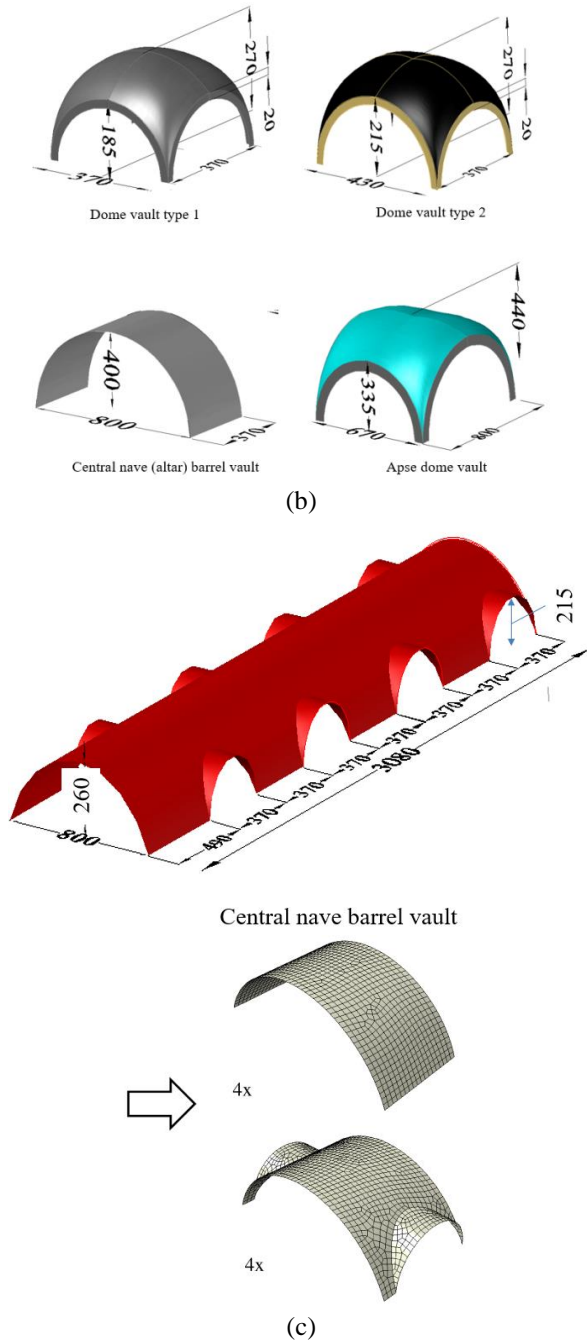
Dome vault type 1

Dome vault type 2

Central nave (altar) barrel vault

Apse dome vault

(b)

Central nave barrel vault

4x

4x

(c)

Fig. 9 Continued

dimensions 430×370×270 cm in the lateral naves, labeled dome vault type 2 or DV2.

• apse dome vault of dimensions 670×800×440 cm;

• altar barrel vault of dimensions 370×800×400 cm;

• central nave barrel vault, modeled as set of smaller barrel vaults, with and without lunettes, of dimensions 430/370×800×260 cm.

Table 1 Vaults equivalent trusses stiffness of the vaults in San Frediano church. Diag=diagonal direction, long=longitudinal direction (X direction, Fig. 10(a)), transv=transverse direction (Y direction, Fig. 10(a))

		Vault	K_{eq} (daN/cm)	d_{eq} (mm)	$K_{eq}/2$	d_{eq} ($K_{eq}/2$)
Lateral naves	DV1	diag	15552	22.2	7776	15.7
		long	12555	16.8	6278	11.9
		transv	12555	16.8	6278	11.9
	DV2	diag	10060	18.6	5030	13.1
		long	9882	14.9	4941	10.5
		transv	13663	17.5	6832	12.4
Apse	-	diag	56445	59.7	28223	42.2
		long	138719	74.8	69360	52.9
		transv	54208	51.3	27104	36.3
Central nave	800×370	Barrel_lun_diag	62206	57.7	31103	40.8
		Barrel_lun_long	172806	62.3	86403	44.0
		Barrel_lun_transv	15240	27.2	7620	19.2
		Barrel_diag	81209	65.9	40604	46.6
		Barrel_long	248612	74.7	124306	52.8
		Barrel_transv	9626	21.6	4813	15.3
	800×430	Barrel_lun_diag	54977	55.0	27488	38.9
		Barrel_lun_long	152189	63.0	76095	44.5
		Barrel_lun_transv	17441	29.1	8720	20.6
		Barrel_diag	74086	63.9	37043	45.2
		Barrel_long	223165	76.3	111583	53.9
		Barrel_transv	12611	24.7	6306	17.5
Complete longitudinal barrel vault			21413	63.2	10707	44.7

The in-plan disposition of the vaults and their geometrical dimensions are displayed in Fig. 9. The following mechanical properties were considered: Young modulus 50000 MPa (granite), 1500 MPa (bricks and mortar), 2800 MPa (stone); Poisson coefficient 0.15 for all materials; specific weight 27 kN/m³ (granite), 18 kN/m³ (bricks and mortar), 22 kN/m³ (stone). There are five typologies of 6 cm thick vaults in the San Frediano church:
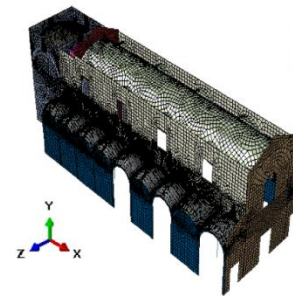
• n°14 dome vaults of dimensions 370×370×270 cm (plan longitudinal length, plan transverse width and rise at the crown respectively) in the lateral naves, labeled dome vault type 1 or DV1.

• n°4 dome vaults adjacent to the main façade of

(a)

Fig. 10 FEM models: (a) church fully modeled - actual; (b) VET model; (c) church with preventive steel tie-rods in complete configuration in lateral naves
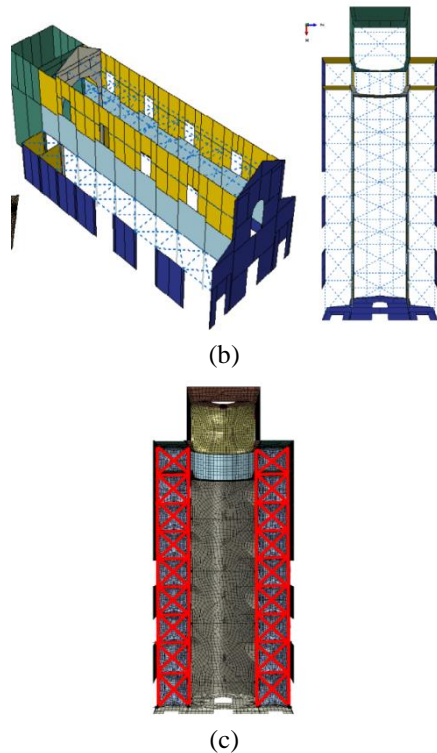
(b)

(c)

Fig. 10 Continued

The diagonal equivalent stiffness for DV2 and for the apse vault have been obtained as average between the case in which it is calculated imposing a displacement along the vault side with 430 cm and that in which the displacement is imposed along the side 370 cm-long. The stiffness values are reported in Table 1.

The simplified model, named VET model, is compared to the so-called full model, where vaults are modeled with finite elements recreating their real geometry, in terms of modal analysis and time-history analysis.

4.1.1 Modal analysis

Two simplified models with VETs have been considered: (i) VETs in place of vaults and additional masses to recreate the removed mass of the vaults and (ii) VETs in place of vaults without additional masses. The diagonal stiffness was fully considered in tension and in compression.

In case (i), the additional masses are concentrated at the extremities of the trusses. 230 modes were necessary to reach about 85% of the total mass for the two main horizontal directions of the building. A sensitivity analysis was also performed by adopting in place of vaults flat diaphragms with variable thickness: 6 cm like in the as-built configuration, and 60 cm with properly reduced specific weight, not to alter the mass distribution. Finally, a model without vaults has been analyzed, by increasing the specific weight to consider the removed masses.

Results for the transverse and longitudinal direction are reported in Table 2 and Table 3, respectively. Those tables list, for each model, the frequency in Z (or X) direction (transverse or longitudinal, Fig. 10(a)) f_z (or f_x), the

percentage of participant mass M_z (or M_x) and in the third column the percentage difference between the equivalent models and the complete model. The VET model without additional masses shows the best correspondence with the complete (actual) model, in terms of natural frequencies and mode shapes. The difference of the first natural frequency is lower than 8% of the first natural frequency along the transverse direction and the corresponding modal mass. In the longitudinal direction, the difference of the first natural frequency is 6% while the corresponding mass is higher. If additional masses are considered with VETs, the natural frequency along the longitudinal direction is acceptable (<10%), while in the transverse direction is 17%.

When the vaults are modeled as horizontal slabs, in transverse direction the agreement is good in terms of both modal masses and natural frequencies (with the exception of natural frequency of the model with 60 cm thick slabs), while a poor agreement in longitudinal direction results.

The model with horizontal slabs is much more rigid than the complete model (Table 3) in longitudinal direction, (a difference of about +18% and +48% for the natural frequencies of the model with 6 cm and 60 cm respectively) and the associated mass much lower. The latter aspect is due to the small activated mass concentrated in the diaphragms, therefore the model with horizontal slabs does not give the global church response in longitudinal direction, where anyway the relative deformations of vaults are lower. By contrast, the longitudinal response is correctly reproduced by the VETs model without additional masses

Table 1 Comparisons between natural frequencies and modal masses in the transverse direction of the building

Model	f_z (Hz)	M_z (%tot)	% Actual f_z	% Actual M_z
Complete model	1.90	42.36	0.00	0.00
Simplified	1.82	38.86	-4.33	-8.27
Simplified + add. masses	1.56	37.63	-17.85	-11.16
Horizontal slabs 6 cm	1.76	42.90	-7.50	1.26
Horizontal slabs 60 cm	2.53	45.80	32.90	8.11
No vaults	0.56	18.53	-70.71	-56.27

Table 2 Comparisons between natural frequencies and modal masses in the longitudinal direction of the building

Model	f_{x1} (Hz)	M_{x1} (%tot)	% Actual f_{x1}	f_{x2} (Hz)	M_{x2} (%tot)	% Actual f_{x2}	Sum M_x (%tot)
Complete model	4.12	36.79	0.00	4.36	11.99	0	48.78
Simplified	3.87	56.06	-6.00	-	-	-	56.06
Simplif. + add.masses	3.75	26.25	-8.89	3.82	21.96	-12.30	48.21
Horizontal slabs 6 cm	4.88	11.25	18.54	4.96	8.12	13.80	19.36
Horiz. slabs 60 cm	6.09	26.82	48.09	7.17	34.77	64.51	61.59
No vaults	3.50	47.82	-15.08	3.51	10.60	-19.35	58.42

(second row of Fig. 11). Moreover, the simplified model has the same deformed shape as the complete model regarding the walls above the colonnade. These relative displacements of the central vault abutments, which cause a brittle failure, are particularly dangerous since they can determine the vault collapse: it is relevant that VETs model can efficiently describe this aspect.

For these reasons, the VET model can be recommended for simplified analysis, while horizontal slabs or simple equivalent masses neglecting the real vault geometry should be avoided.

4.1.2 Transient analysis

Global transient analysis has been performed by applying acceleration time-histories at the base of the church. The linear range is considered, as the interest is focused on small relative displacements and on peak displacements. Since the maximum effects have to be considered, three accelerograms are sufficient to perform dynamic analysis (European Committee for Standardization (2003) Eurocode 8). The accelerograms are spectrum-compatible with response spectrum with return period of 75 years and behavior factor 1.5, soil C and topographic category T1. A fourth accelerogram was considered by doubling the acceleration values to check the response with the same frequency content but different intensity. For further details on the acceleration time-histories, refer to (Giresini *et al.* 2014). Relative displacements of the vaults

Complete model – f_X=4.12 Hz, M_X=36.79%

Complete model – f_Z=1.90 Hz, M_Z=42.36%

Simplified – f_X=3.87 Hz, M_X=56.06%

Simplified – f_Z=1.82 Hz, M_Z=38.86%

Simplified+masses – f_X=3.75 Hz, M_X=26.25%

Simplified+masses – f_Z=1.56 Hz, M_Z=37.63%

Horizontal slabs of thickness 6 cm – f_X=4.88 Hz, M_X=11.25%

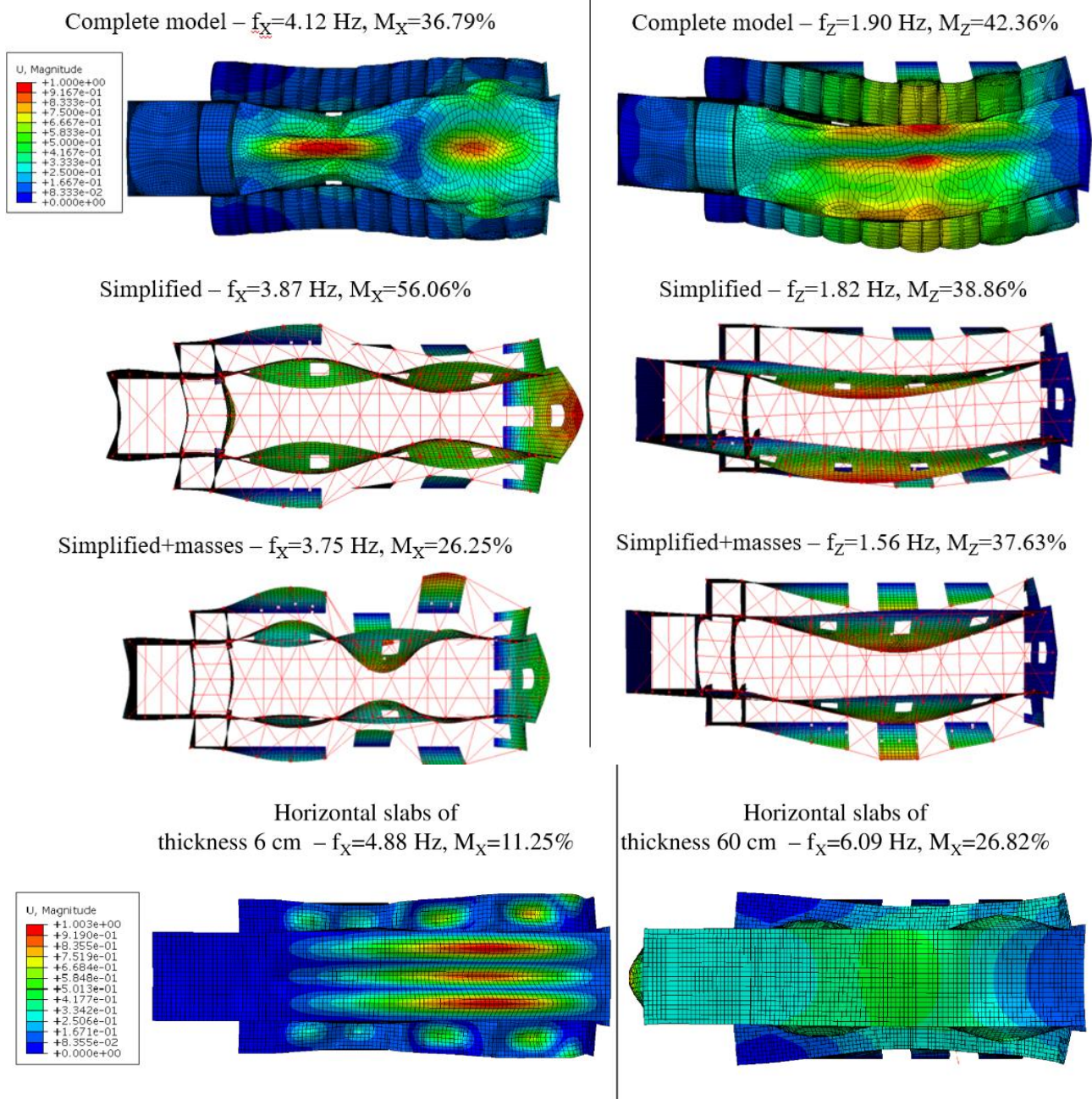Horizontal slabs of thickness 60 cm – f_X=6.09 Hz, M_X=26.82%

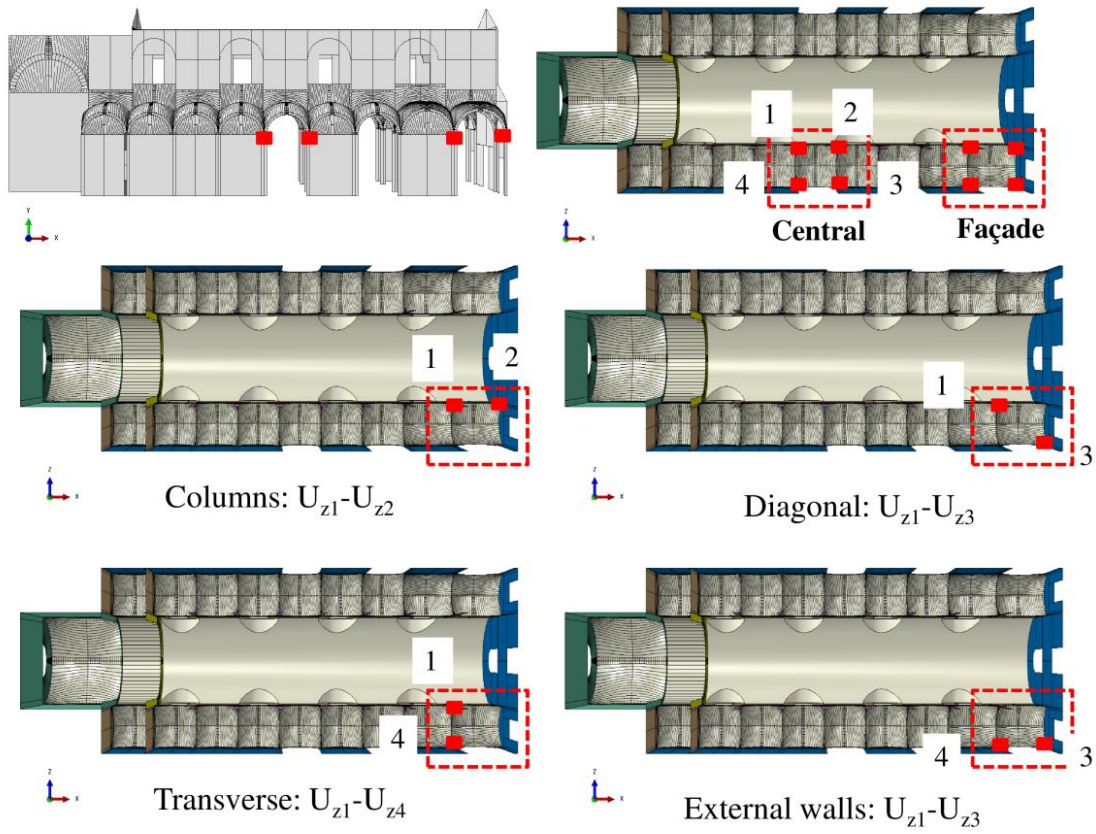Fig. 11 Mode shapes for the different vaults modeling

Fig. 12 Points where relative displacements are monitored in the time-history analysis

supports/pendentives in the lateral naves are monitored in different directions: relative transverse displacements between columns, between column and external wall in diagonal direction, between column and external wall in transverse direction, between vaults support in external walls (with openings, Fig. 12). Each combination of these four points has been checked both for a vault in central area and for a vault adjacent to the façade, for a total of eight combinations for each time-history. The results of time history analyses are commented only in case of analysis in transverse direction, since, along that, relative displacements are larger and vaults play a more relevant role. For simplicity, the interaction of effects in the two main directions has been neglected.

Generally, the VET model correctly reproduces the peak relative displacements, even though not always the history displacement is similar. Nevertheless, this should be expected, due to the strong simplification introduced. Figs. 13-14-15 display an excellent agreement between maximum relative displacements of the complete model and those of VET model.

The displacement peaks are always higher in the simplified model, and this is in favor of safety. Moreover, it confirms the modal analysis results where the VET model is more flexible and "time saving" than the complete one. Nevertheless, transverse displacements are generally not correctly reproduced by the simplified model and underestimated, probably because the connection between

transverse point (1 and 4 in Fig. 12) is direct, while in the real model the arch is more flexible.

An useful application of time-history analyses with simplified models is that aimed to assess the benefits offered by strengthening measures. In particular, steel tie-rods with 20 mm of diameter have been implemented in numerical simulations as preventive safety features (De Falco *et al.* 2013).

Also in this case, a complete model has been analyzed with steel tie-rods as axial connectors CONN3D2, defined by a stiffness value (Abaqus CAE 6.12). The complete retrofitted model has been compared to the simplified one, where the equivalent trusses have an increased stiffness considering the effect of the steel tie-rods.

The comparisons are presented in terms of two design ratios:

• R: ratio between maximum displacements with steel tie-rods and without them for the full modeled church

$$R = \frac{\delta_{max,TIE-RODS}}{\delta_{max}} \quad (9)$$

• R_{VET} : ratio between maximum displacements with steel tie-rods and without them for the simplified church with VETs

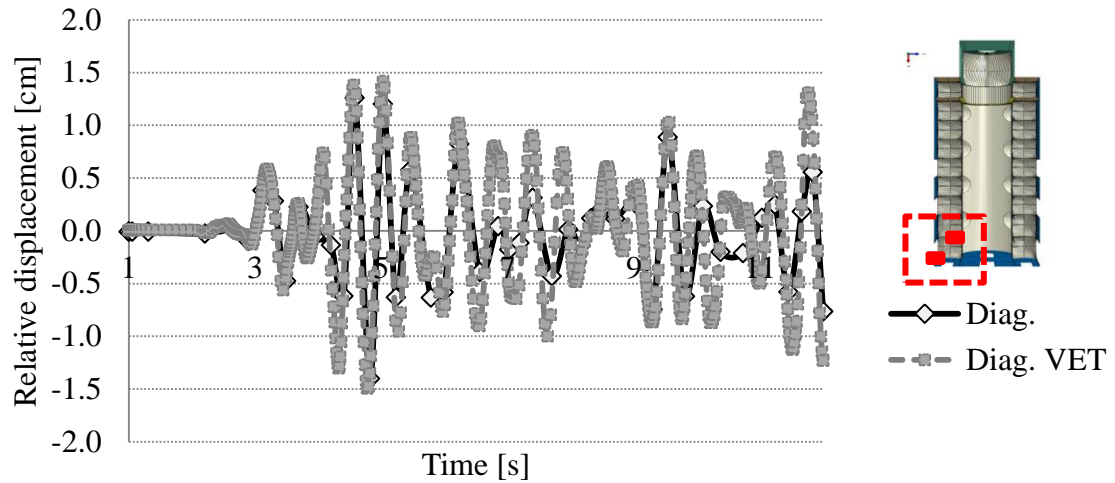$$R_{VET} = \frac{\delta_{max,TIE-RODS}^{VET}}{\delta_{max}^{VET}} \quad (10)$$

Fig. 13 Relative transverse displacements in diagonal direction (vault adjacent to the main façade): comparison between complete model and VET model - TH2 (TH: acceleration time-history)
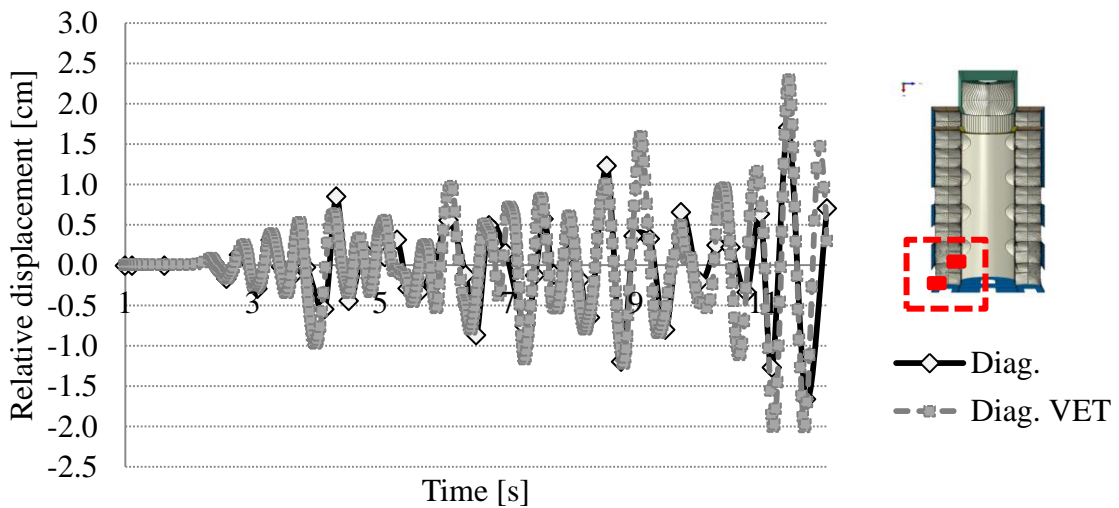

Fig. 14 Relative transverse displacements in diagonal direction (vault adjacent to the main façade): comparison between complete model and VET model - TH3 (TH: acceleration time-history)
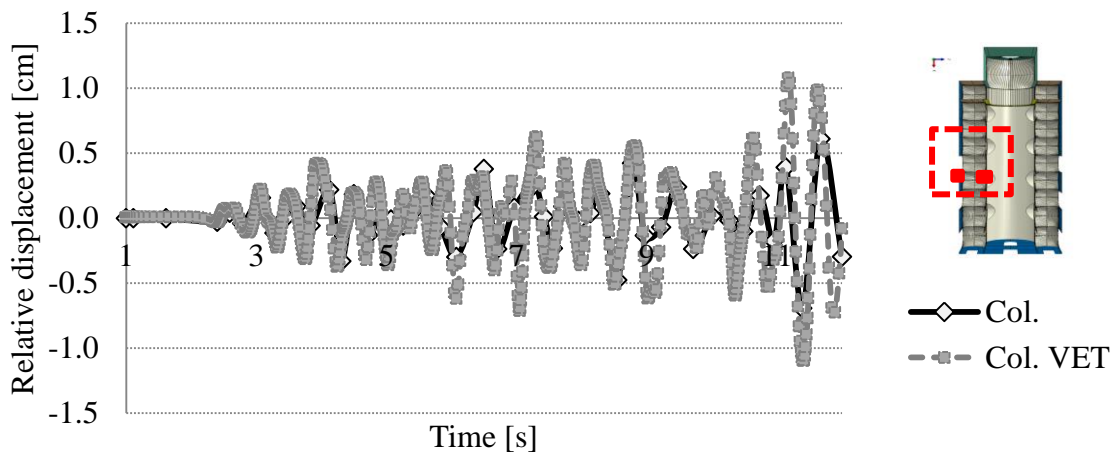

Fig. 15 Relative transverse displacements between columns top (central part of lateral nave): comparison between complete model and VET model - TH3 (TH: acceleration time-history)

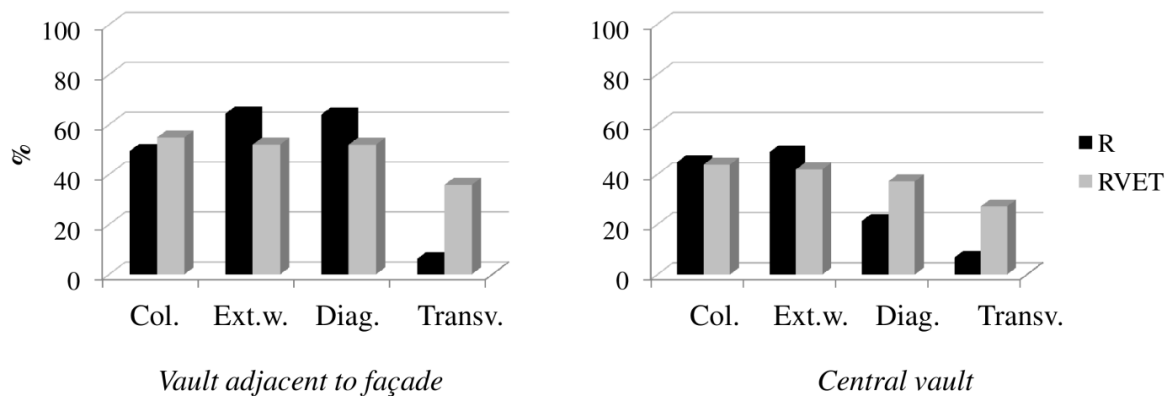*Vault adjacent to façade*   *Central vault*

Fig. 16 Design ratios in the complete model and in the VET model for the different monitoring point: average values of relative displacement reduction in the four time-histories

The main interest of the structural designer is indeed, among others, to assess the reduction of relative displacements of the vaults to avoid the moving away of the pendentives, for which the thrust is not contrasted and collapse mechanisms could take place.

The same time-histories analysis have been performed for the retrofitted configuration and the average of maxima displacements plotted for each monitoring position (Fig. 12). The results are displayed in Fig. 16. It is worthy to notice that the agreement is very good for the vault adjacent to the façade, with differences lower than 10% of rate reduction. In addition, the reduction for the VET model (in red in the figure) is generally lower than that for the complete model, and this is in favor of safety. Nevertheless, the method is not able to completely describe the response in the transverse direction for both areas. The strong difference between R and R_{VET} is due to an accumulated effect: from one side $\delta_{max,TIE-RODS}^{VET} > \delta_{max,TIE-RODS}$ and from the other $\delta_{max}^{VET} < \delta_{max}$. This results in $R_{VET} \gg R$ and therefore if the transverse relative displacement is considered, the VET model is not in favor of safety.

Finally, in all those analyses the computing resources by VETs models with respect to complete FE models are very low: computational time is less than 10% for VETs compared with complete models and in most cases this aspect is crucial for designers, joined to the simplicity in use. The VET approach then leads to an estimate of the structural response and is a step forward in comparison to the general approach where equivalent slabs are used.

4. Conclusions

The problem of assessing seismic vulnerability of historic buildings with masonry vaults is of great relevance. Structural designers need to have simple but effective numerical tools to perform analysis of global and/or local model of historic masonry structures. A macro-element with a system of trusses equivalent to vaults has been proposed to reduce complexity and modeling and computational time. Indeed, often masonry structures have many and complex vaults, whose role cannot be neglected in the seismic

behavior. The method to identify the equivalent stiffness and steel diameter, that can be easily included in commercial programs, is described focusing on the common vaults features found in the architectural heritage. Geometrical and mechanical properties, boundary conditions and backfill influence the stiffness determination. A parametric analysis was performed to account for them, both in linear and non-linear range. Whether a brittle behavior in tension is assumed for masonry, the equivalent stiffness markedly drops for small relative displacements. The way to use vaults equivalent trusses in both global and local analysis - rocking and kinematic - has then been illustrated. A case study was analyzed to validate the equivalence between full models, where vaults are modeled with finite elements recreating their geometry, and simplified models with VETs. Modal analysis showed a very good agreement between complete and simplified model. This analysis also highlighted that, whether vaults are not modeled at all or they are modeled as diaphragms, results are not compatible with those of the reference model. Time-histories analysis was also carried out to compare relative displacements histories, generally with good agreement. This method was also demonstrated to be rapid and effective to evaluate the reduction rate of maxima displacements with a configuration of steel tie-rods in the lateral naves of the church.

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