

# A multimodal adaptive evolution of the N1 method for assessment and design of r.c. framed structures

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**Abstract.** This paper presents a multimodal adaptive nonlinear static method of analysis that, differently from the nonlinear static methods suggested in seismic codes, does not require the definition of the equivalent Single-Degree-Of-Freedom (SDOF) system to evaluate the seismic response of structures. First, the proposed method is formulated for the assessment of r.c. plane frames and then it is extended to 3D framed structures. Furthermore, the proposed nonlinear static approach is re-elaborated as a displacement-based design method that does not require the use of the behaviour factor and takes into account explicitly the plastic deformation capacity of the structure. Numerical applications to r.c. plane frames and to a 3D framed structure with in-plan irregularity are carried out to illustrate the attractive features as well as the limitations of the proposed method. Furthermore, the numerical applications evidence the uncertainty about the suitability of the displacement demand prediction obtained by the nonlinear static methods commonly adopted.

**Keywords:** pushover analysis; multimodal procedure; adaptive procedure; displacement based design; inelastic response of structures

## 1. Introduction

In the last decade, the Nonlinear Static Method (NSM) of analysis has gained a great appreciation among structural engineers also in practical applications. The main reason of its success is that it can provide a more extensive and detailed information on the seismic response of structures than that achievable by linear methods of analysis: lateral force method of analysis and modal response spectrum analysis. Furthermore, the NSM has become the preferred tool for the seismic assessment of existing structures. In fact, the uncertainty on the value of the behaviour factor to be used for these structures makes the linear methods of analysis based on reduced design spectra unreliable. Finally, the NSM is relatively easy to be handled compared to the more complex Nonlinear Dynamic Analysis (NDA). The NSM is a promising tool also for the design of new structures; for instance, it can be used to verify the design based on linear methods of analysis and to validate the value of the behaviour factor given by seismic codes. The scientific community, by contrast, is still sceptical about the massive use of the nonlinear static method of analysis. The main criticism arises from the theoretical bases of this method of analysis, which are not universally established, and from the knowledge that the NSM may lead to significant errors in the prediction of the displacement demand of in-plan irregular structures unless it is properly adjusted (Bhatt and Bento 2011, Bosco *et al.* 2013,

Chopra and Goel 2004, Fujii 2014 and 2016, Kreslin and Fajfar 2012, Moghadam and Tso 2000). These considerations suggest the use of NDA, which is the most advanced method of analysis available today, but that should be used very carefully because of important critical issues. In fact, the use of NDA requires a complex and very refined numerical model, whose seismic response is influenced by many parameters. The results of NDA are very sensitive to the stiffness and the strength of the structural members, as well as to the mass and its distribution in the structure. The results of NDA are highly dependent on the input ground motion. The selection of the ground motions, the number of ground motions considered, and the treatment of the results, may affect significantly the prediction of the seismic response (Shi *et al.* 2013, Tanganelli *et al.* 2016). Further critical points are the uncertainties in the modelling of damping, the sensitivity of the results to the software adopted for the numerical analysis (Chopra 2004) and the time-stepping method used for numerical integration of the equations of motion (Hancock 2006). All these issues and, in addition, the large amount of results that need to be processed in nonlinear dynamic analyses, establish a serious limitation on the use of NDA.

## 2. Review of previous studies

The nonlinear static method of analysis was developed to predict explicitly the nonlinear response of structures as an alternative to the common linear methods of analysis and to the more complex nonlinear dynamic analysis. In past years, a great effort has been devoted to improve the NSM

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and today many versions of the NSM are available in literature. A state of the art on nonlinear static method of analysis is presented here. The scope of this Section, which is not totally comprehensive, is to present the development over time of this method of analysis and to evidence the critical issues that have not been solved yet.

The first NSM is the *Capacity Spectrum Method* (CSM) developed by Freeman *et al.* (1975) in the Seventies. According to this method, a pushover analysis is performed and the structure is pushed up the attainment of a predefined limit state, for instance the collapse of the structure. The base shear - roof displacement relationship (*pushover curve*) is used to define an equivalent SDOF system. This system is elastic and overdamped. The stiffness of this system is reduced to take into account the effect of yielding. In particular, the stiffness of the SDOF is evaluated as the slope of the line connecting the points of the pushover curve corresponding to the zero-displacement and the peak displacement demand (Freeman 1998, Freeman 2004). The damping ratio of the equivalent SDOF system, which is assumed larger than the nominal one to take into account the beneficial effect of energy dissipation in reducing the displacement demand, is related to the ductility demand of the structure. Different formulations for the evaluation of the equivalent of damping ratio are available in literature (Blandon 2004, Chopra 1995, Freeman 2004, Newmark *et al.* 1982, Priestley 2003). The displacement demand of the equivalent SDOF system can be evaluated also by a graphic procedure. In particular, the pushover curve is idealized by a bilinear relationship and scaled to obtain the *capacity curve* of the equivalent SDOF system. The capacity curve is superimposed to the overdamped response spectrum in the ADRS (Acceleration Displacement Response Spectrum) format and the intersection of the two curves provides the displacement demand. Finally, the displacement demand of the equivalent SDOF system is transformed back to the roof displacement demand of the MDOF system. Fajfar *et al.* (1988, 1996, 1999, 2000) developed the *N2 method*, which is a nonlinear static method based on inelastic demand spectra. The name of the method underlines that it predicts the nonlinear (N) response of the structure by two (2) numerical models (the MDOF and SDOF systems). The procedure of the N2 method is similar to that of the CSM method, but it adopts an inelastic equivalent SDOF system instead of the elastic overdamped SDOF system of CSM. The assumed force-displacement relationship of the equivalent SDOF system is bilinear and coincides with the capacity curve derived from the pushover curve of the MDOF system in case of CSM. The period of the SDOF system is that corresponding to the stiffness of the elastic branch of the capacity curve. If the period of the SDOF system is larger than the transition period  $T_C$  that separates the constant acceleration branch of the spectrum from the constant velocity branch, the equal displacement rule applies (Newmark and Hall 1982) and the displacement demand of the SDOF system is calculated by the elastic spectrum. Otherwise, the displacement provided by the elastic spectrum should be increased based on the ductility demand (Fajfar 1999). Also the N2 method can be applied by a graphic procedure, but in this case the displacement

demand of the equivalent SDOF system is provided by the intersection of the capacity curve and the inelastic response spectrum in the ADRS (Acceleration Displacement Response Spectrum). The *N1 method* proposed by Bosco *et al.* (2009), which is a variant of the N2 method, does not require the definition of the equivalent SDOF system. According to this method, the roof displacement of the MDOF structure is determined by modal response spectrum analysis. Then, this displacement is increased to account for the reduction of stiffness of the structure caused by yielding. The obtained displacement is further increased if the period of the mode of vibration that mostly contributes to the seismic response is smaller than  $T_C$ . The criteria for the evaluation of the reduced stiffness and the increase of the displacement demand if the period is smaller than  $T_C$  are the same used in the N2 method for the definition of the equivalent SDOF system. If the modal response spectrum analysis is restricted to the first mode of vibration, the N1 method provides the same displacement demand of the N2 method applied by the first mode load pattern. Giorgi and Scotta (2013) proposed a variant of the N1 method to obtain the same prediction of the N2 method also when the constant lateral load pattern is applied.

The NSMs are generally based on a pushover analysis performed by an invariant load pattern. This represents an important shortcoming, as underlined by many researchers, for instance Krawinkler (1996). Two main limitations are ascribed to these methods: they neglect the effects of higher modes of vibration and do not consider that yielding gradually modifies the dynamic properties of the structure. Based on this consideration, many authors developed new procedures. Sasaki *et al.* (1998) propose a Multi Modal Procedure (MMP) that accounts for the collapse mechanisms promoted by the effects of higher modes of vibration. According to this procedure, pushover analysis has to be performed several times considering the load patterns proportional to the modes of vibration of interest. After, each pushover curve is scaled and superimposed to the elastic overdamped spectrum to evaluate how the structure behaves in each mode of vibration and where yielding takes place. Chopra and Goel (2002, 2003) developed the Modal Pushover Analysis (MPA), a nonlinear static method that can take into account the contributions to the seismic response of all the modes of vibration. Furthermore, for linear systems, the MPA provides the same prediction of the modal response spectrum analysis. According to this method, the modes of vibrations are determined and pushover analysis is performed by load patterns proportional to the modes of vibrations. An equivalent inelastic SDOF system is determined for each mode of vibration: the mass is equal to the modal mass associated to the mode of vibration and the force-displacement relationship is assumed as the bilinear idealization of the pushover curve obtained for the relevant load pattern. For each SDOF system, the peak response of the structure caused by the relevant mode of vibration is determined following the procedure of the N2 method. Finally, combining these peak responses by the CQC or SRSS rule leads to the total response. The MPA considers the effect of higher modes of vibration, but it neglects the

effect of the gradual yielding of the structure on its dynamic properties.

Bracci *et al.* (1997) are among the firsts who proposed adaptive load patterns to consider the modification of the dynamic properties of the structure due to gradual yielding. Based on their NSM, the structure is pushed by horizontal forces with heightwise distribution which is inverted triangular at the beginning. Then, it is updated at each step of the loading process by a function depending on the ratio of the storey shear to the storey lateral strength. Loading is stopped if the storey drift reaches the limit value or the structure becomes unstable because yielding. Gupta and Kunnath (2000) proposed a multimodal adaptive method called Adaptive Spectra-based Pushover Procedure (ASP). According to this NSM, a pushover analysis is performed for each mode of vibration of interest. The load patterns are proportional to the modal shapes and are updated at each step to consider the effect of yielding on the dynamic properties of the structure. The effect of each distribution of forces is weighted by the pseudo-acceleration evaluated by the elastic response spectrum for the period of the relevant mode of vibration. The total response at the end of the step is obtained by the SRSS combination rule. The analysis is stopped when the storey drift reaches the limit value or the collapse mechanism is attained. Antoniou and Pinho, starting from the study of Elnashai (2001), developed two variants of an adaptive NSM. The two variants are different for the loading vector, which can be a force loading vector or a displacement loading vector. The second variant (Antoniou and Pinho 2004), called Displacement-based Adaptive Pushover (DAP), utilizes a vector of normalised displacements determined by the combination of the modal storey drifts, which are weighted by the corresponding spectral displacements. The modal storey drifts and the spectral displacements are updated at each step taking into account the current level of damage of the structure. The proper combination rule (SRSS or CQC) should be used. Based on the comparison between the storey drifts provided by DAP and those determined by nonlinear dynamic analysis, the authors of DAP suggest the displacement loading vector derived from the storey drifts. The displacement demand is determined by a procedure based on CSM (Casarotti and Pinho 2007). At each step of the analysis, an equivalent SDOF system is determined to evaluate the current displacement demand.

The NSMs presented before were developed for planar structures and generally are not suitable to predict the displacements of in-plan irregular buildings with large torsional response. Moghadam and Tso (2000) proposed a nonlinear static method for in-plan irregular framed buildings. A modal response spectrum analysis is used for the determination of the displacement demand of each frame and to evaluate the heightwise distribution of the horizontal forces to be used for pushover analysis. Each frame is analysed separately by pushover analysis and its seismic response is provided by the pushover analysis stopped at the attainment of the roof displacement demand. Fajfar *et al.* (2005, 2012) developed a variant of the N2 method specifically improved for in-plan asymmetric buildings (*extended N2 method*). According to this NSM,

the horizontal forces are applied to the mass centres  $M$  and a pushover analysis is carried out until the displacement of the centre of mass of the roof reaches the displacement demanded by the assigned peak ground acceleration. Then, a modal response spectrum analysis is performed and the results are scaled so that the roof displacement of  $M$  equals that provided by the pushover analysis. The obtained results are used to define two sets of correction factors: one set is to adjust the in-plan distribution of floor displacements and the other is applied to the heightwise distribution of storey drifts. The correction factors of the first set are calculated at the roof level as the ratio of the displacement at an arbitrary location to the displacement of  $M$ . The correction factors of the second set are calculated at the centre of mass of each storey as the ratio of the storey drift obtained by modal response spectrum analysis to the storey drift obtained by pushover analysis. If a correction factor (in plan or in elevation) is smaller than 1.0, the value 1.0 is used instead. The two sets of correction factors are used to adjust the results of the pushover analysis. The correction factor determined for in-plan displacements also applies to storey drifts. Chopra and Goel (2004) extended the MPA to 3D building structures. According to this procedure, a pushover analysis of the structure has to be performed for each mode of vibration. The loading vectors contain lateral forces and torques proportional to the modes of vibration. Each pushover curve is converted to the idealised bilinear relationship and the equivalent SDOF system is determined. Based on the same theory presented for plan-symmetric buildings, the peak modal responses are determined and combined by the CQC rule to evaluate the total response. Fujii (2014) proposed an extension of the NSM to asymmetric buildings that are regular in elevation. The method is based on the use of two equivalent SDOF models that capture the response of two modes of vibration. The properties of the first equivalent SDOF model are determined using the pushover analysis considering the change in shape of the first mode at each nonlinear stage. The second mode vector is then determined considering the orthogonal condition of the mode vectors. The bi-directional horizontal seismic input is simulated by the response spectra of the two horizontal components assumed to be the same. The contribution of each modal response is estimated based on the unidirectional response in the principal direction of each horizontal component. Finally, the drift demand is determined by four pushover analyses considering the combination of bi-directional excitations. This method has been also successfully tested on buildings with setbacks (Fujii 2016). Bosco *et al.* (2012) proposed the corrective eccentricity method, which predicts the response of asymmetric buildings as the envelope of the results of two pushover analyses. For each pushover analysis, the lateral forces are applied to points of the deck shifted from the centres of mass. The distance between the points where the lateral forces are applied and the centres of mass (constant along the height of the building) is named corrective eccentricity and is calculated by means of analytical expressions. The effectiveness of this method has been also tested on multi-storey buildings that are regular in elevation (Bosco *et al.* 2015a and 2015b). The DAP by

Antoniou and Pinho has been tested on 3D structures (Meireles *et al.* 2006). Furthermore, Adhikari (2009) proposed two variants of DAP specifically developed for in-plan asymmetric buildings. The loading vector used according to the first variant, called DAP2, contains horizontal displacements along two orthogonal directions applied to the centres of mass of the deck. Furthermore, the results of pushover analysis are adjusted by a torsional corrective factor conceptually similar to that used in the extend N2 method. The loading vector used by the second variant of DAP, which is called DAP3, contains rotations in addition to horizontal displacements to predict properly the torsional component of the response.

### 3. Proposed method for 2D frames

As highlighted in Section 2, several NSMs can be found in literature. However, research on this topic is still vigorous and great efforts come from the scientific community to propose improved NSMs. The nonlinear static method proposed in this paper may be considered as a multimodal adaptive evolution of the N1 method (Bosco *et al.* 2009). The basic idea of this evolution was proposed by Ghersi *et al.* (2013). A list of the main features of the proposed method, called Advanced N1 method (AN1), is reported below:

- the pushover curve does not need to be idealised by a bilinear relationship;
- AN1 method does not require neither the definition of the control node nor the definition of the equivalent SDOF system;
- a member-by-member modelling with plastic hinges assigned at member ends is adopted for pushover analysis;

- the pushover analysis is performed applying horizontal displacement loading rather than forces;
- the displacement pattern is multimodal and adaptive.

In this section, the proposed method is formulated with reference to 2D frames and is applied to a r.c. frame case study.

#### 3.1 Formulation of the method

AN1 method involves the use of a modal response spectrum analysis with incremental seismic input. The modal response spectrum analysis of the structure, which is re-performed at each step of the analysis, provides the peak horizontal displacements for each mode of vibration. The displacements are combined by the proper combination rule and used for the pushover analysis. The use of a displacement pattern, instead of forces, reduces the number of unknowns involved in the equations of equilibrium. Including the contributions of the relevant modes of vibration into the displacement pattern accounts for the effect of the higher mode of vibration, but the use of any modal combination rule (SRSS or CQC) makes the sign missed. Hence, the sign of the peak response of the first mode of vibration, which is generally the most important, is applied to the total response. Updating the numerical model, any time the modal response spectrum analysis is re-performed, accounts for progressive variation of dynamic characteristics of the structure in the inelastic range of its behaviour. In the following, the step-by-step procedure for the application of AN1 method is described in details with reference to the prediction of seismic response of planar frames. The flowchart of the procedure is shown in Fig. 1.

1. The single (i-th) step of the procedure starts with a modal response spectrum analysis (*analysis B*). At the first step, all the members of the numerical model are

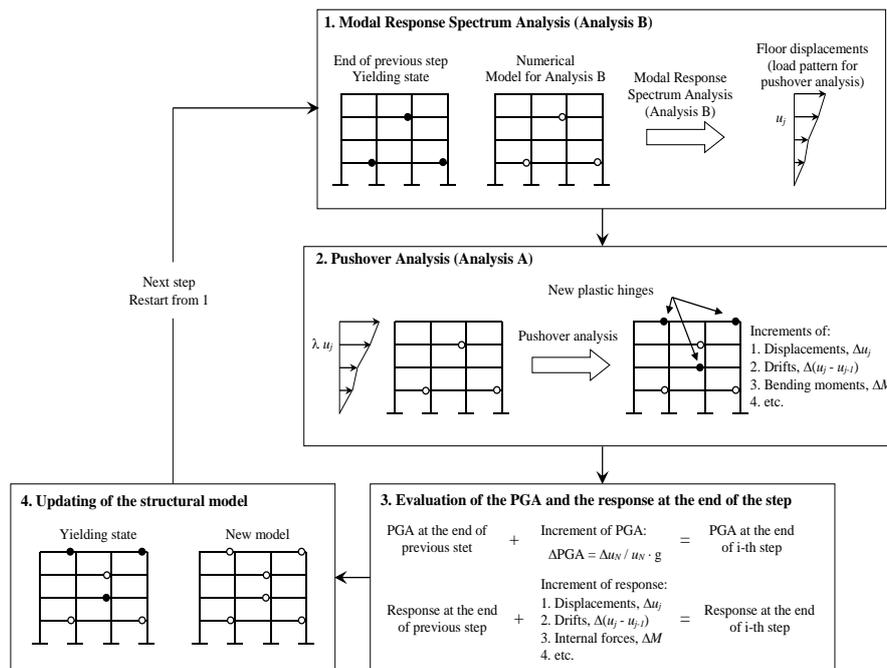


Fig. 1 Flowchart of the proposed nonlinear static procedure

connected by rigid joints, otherwise pin are introduced where yielding has taken place. The modal response spectrum analysis of the frame (analysis B<sub>i</sub>) provides the peak modal responses in terms of horizontal displacements. The elastic response spectrum corresponding to the considered soil type and to a reference value of PGA is used. Here, the reference value of PGA is assumed equal to the gravity acceleration *g*. The peak modal displacements are combined according to the CQC rule to obtain the displacement pattern that will be used for the subsequent pushover analysis (analysis A).

2. The horizontal displacements are proportionally increased up to the end of the step (analysis A<sub>i</sub>). Yielding of this model takes place when the internal forces provided by the analysis A<sub>i</sub> cumulated to those attained at the end of the previous step equal the yielding value. This pushover analysis turns into a linear static analysis if the end of the step is assumed at the formation of the first plastic hinge. Pushover analysis can continue until a number of plastic hinges providing a non-negligible reduction of the lateral stiffness of the frame occurs; this choice may be recommended for highly statically undetermined frames.

3. The PGA and the response corresponding to the end of the *i*-th step are calculated. In particular, the increase of the peak ground acceleration ΔPGA of the *i*-th step, calculated as the ratio of the increase of top displacement to the one provided by the modal response spectrum multiplied by *g*, is cumulated to the PGA of the previous step. This is rigorous if the step terminates when the first plastic hinge forms, while it is approximate if more plastic hinges occur within the step. Increases of displacements, drifts, plastic rotations, internal forces evaluated by the pushover analysis A<sub>i</sub> are cumulated to those of the previous step to obtain the total response of the structure at the end of the step.

4. The structural model is updated to account for

yielding and the procedure is iterated. The structural model is updated by replacing each yielded cross-section with a hinge or a spring. The use of hinges is admitted if a rigid-perfectly plastic relation is used to model the nonlinear behaviour of the end cross-sections of the structural members. Otherwise, rotational springs with stiffness equal to the residual one due to hardening has to be used.

The procedure proceeds cumulating the results of the incremental steps until a prefixed limit state is attained. Sometimes, the structure may become unstable before to attain the prefixed limit state. In this case, the displacement pattern to be used in the analysis A<sub>i</sub> cannot be determined by modal response spectrum analysis, but it coincides with the displacement field consistent with the collapse mechanism. In this stage, the structure behaves as a SDOF with zero stiffness. The horizontal displacements increase and it is assumed that the increase of the displacement of the centroid of the storey masses equals the design ground displacement *d<sub>g</sub>* calculated according to EuroCode 8 (EC8, CEN 2003)

$$d_g = 0.025 \text{ PGA } T_C T_D \quad (1)$$

According to EC8, the spectral displacement *S<sub>De</sub>* is equal to *d<sub>g</sub>* only for SDOF system with period longer than 10 s. Instead for systems with period shorter than 10 s and larger than *T<sub>E</sub>* (upper period of the constant branch of the elastic displacement response spectrum, which ranges from 4.5 to 6 s depending on the soil type), *S<sub>De</sub>* is larger than *d<sub>g</sub>* and is calculated as follows

$$S_{De} = 0.025 \text{ PGA } S T_B T_C \left[ 2.5 \eta + (1 - 2.5 \eta) \cdot \frac{T - T_E}{10 - T_E} \right] \quad (2)$$

In Eq. (2), *S* is the soil factor,  $\eta$  is the damping correction factor, while *T<sub>B</sub>* and *T<sub>C</sub>* are the lower and the upper limits of the period of the constant branch of the elastic spectral acceleration response spectrum, respectively. In practical applications, in case the structure becomes unstable, the authors suggest to consider the period never larger than 5 s and to evaluate the spectral displacement by Eq. (2). This assumption accounts for the residual stiffness provided by infills and partition walls to the structure.

### 3.2 Application of the method to a r.c. frame

The proposed nonlinear static method is applied to a r.c. planar frame (Fig. 2). The frame is 6-storey high, with 3 spans. Beams sustain uniformly distributed gravity loads. Fig. 2 shows the geometrical scheme of the frame, and provides the length of the spans, the inter-storey height, the cross-section size (width×depth in m) of members, and the magnitude of gravity loads. The amount of longitudinal rebars of columns is about 1% of the gross-area of the cross section. Beams are provided with a similar percentage of steel reinforcement. End plastic hinges with rigid-perfectly plastic moment-rotation relationship are adopted for both beams and columns. The yielding moment of beams and the bending-moment-axial-force interaction domain of columns

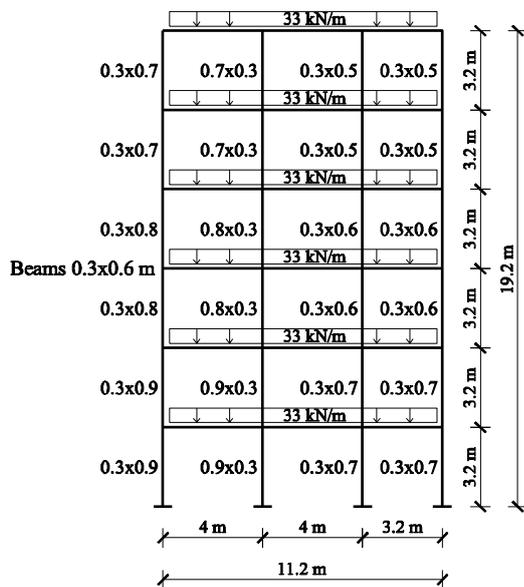


Fig. 2 Analysed 2D frame

are determined assuming the compressive cylinder strength of concrete  $f_c$  equal to 14.2 MPa and the yield strength of the longitudinal rebars  $f_y$  equal to 390 MPa. Two values of plastic bending moments (positive and negative) are considered for beam cross-sections, because their longitudinal reinforcement is unsymmetrical. The ultimate plastic rotation is assumed equal to 0.02 rad for beams and 0.01 rad for columns. Possible shear failure mechanisms of members are not taken into account.

The structure stands on level land in Assergi, Gran Sasso (Italy). The foundation soil is characterised by a medium compressibility and can be classified as soil type B according to the Italian seismic code (Italian Ministry of Public Works 2008). For this site, the Italian seismic code provides a PGA equal to 0.30 g and the elastic response spectrum shown in Fig. 3, which are representative of a seismic input level with probability of exceedance of 10% in 50 years. This probability of exceedance is also suggested in EC8 (CEN 2005) for the verification of Significant Damage limit state of ordinary buildings.

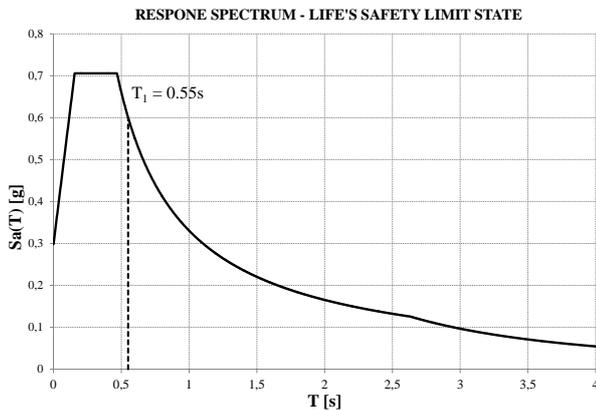


Fig. 3 Elastic pseudo-acceleration response spectrum

The fundamental period of the frame is equal to 0.55 s. The effective modal mass of the first mode of vibration is 78.6% of the total mass denoting that the response of the frame is mainly affected by the contribution of the first mode of vibration.

For each step of the incremental procedure, Fig. 4 shows the displacement profile determined by the elastic analysis  $B_i$ , which is applied in the subsequent pushover analysis  $A_i$ . Fig. 4 also shows the distribution of the plastic hinges occurred at the end of each step. The numerical model obtained by replacing the plastic hinges formed at the end of the  $i$ -th step with pinned joints is used for the analyses of the step  $(i+1)$ . At the end of the 6-th step, the plastic hinges developed within the frame are sufficient to form a mechanism. After that, the ultimate plastic rotation is attained at the bottom cross-section of the first-storey column of the left side of the frame (step 6\*). The increment of PGA corresponding to this step of analysis is calculated by Eq. (2) assuming a period of 5 s and it is equal to  $1.08 \text{ m}\cdot\text{s}^{-2}$ . The sum of the  $\Delta\text{PGAs}$  determined for each single step (Fig. 4) provides the PGA corresponding to the collapse of the frame and it is equal to  $2.63 \text{ m}\cdot\text{s}^{-2}$  (0.27 g). This PGA represents the maximum excitation level that the frame can sustain, i.e., it quantifies the (PGA) capacity of the frame. Instead, if the frame was considered with zero stiffness (i.e., with infinite period) in the step 6\*, Eq. (1) would apply for the evaluation of the increment of PGA for this step and it would return the value  $2.55 \text{ m}\cdot\text{s}^{-2}$ . Therefore, the PGA corresponding to the collapse of the frame would be equal to  $4.10 \text{ m}\cdot\text{s}^{-2}$  (0.42 g).

The prediction of the seismic response of the analysed frame provided by AN1 method is compared to that obtained by the nonlinear static method currently suggested by the Italian building code (Italian Ministry of Public Works 2008, O.P.C.M. 3431 2005) and EC8. Both the two codes have adopted the N2 method developed by Fajfar and

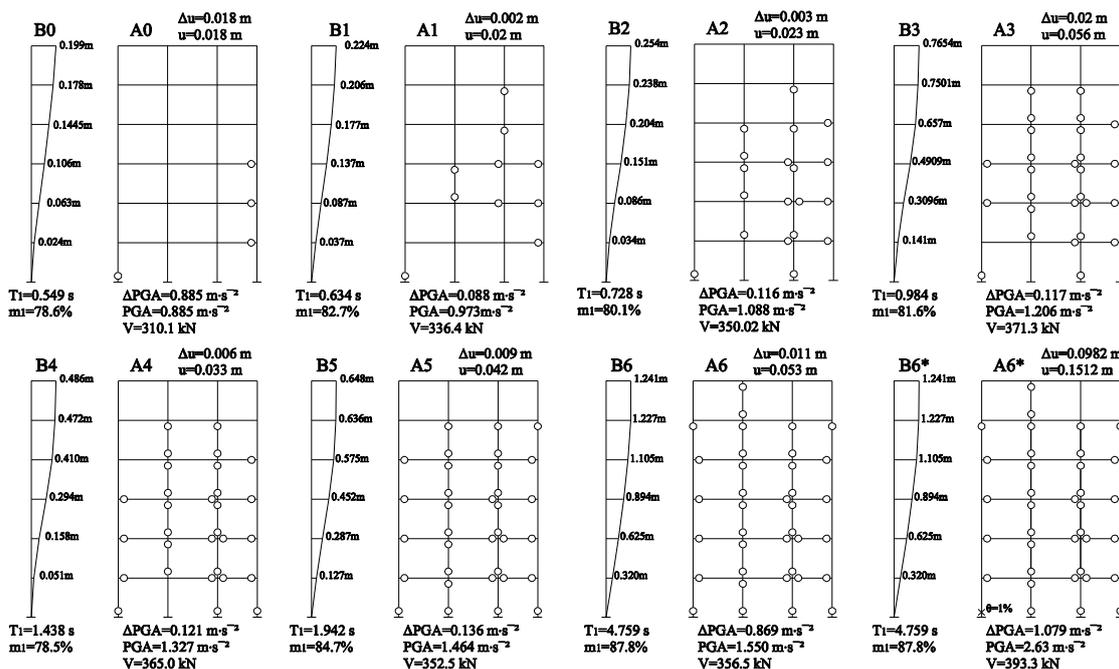


Fig. 4 Results of the incremental steps of the AN1 method and sequence of plastic hinges

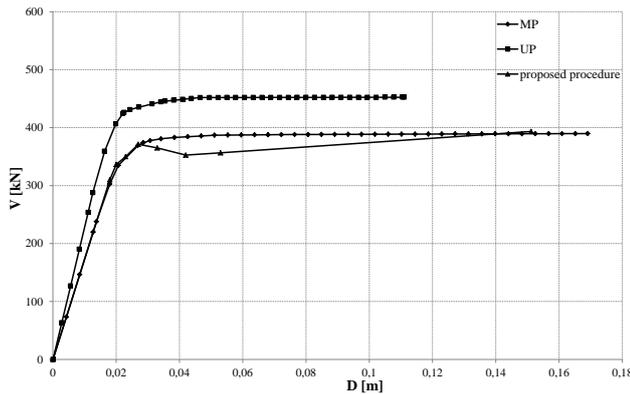


Fig. 5 Comparison of pushover curves

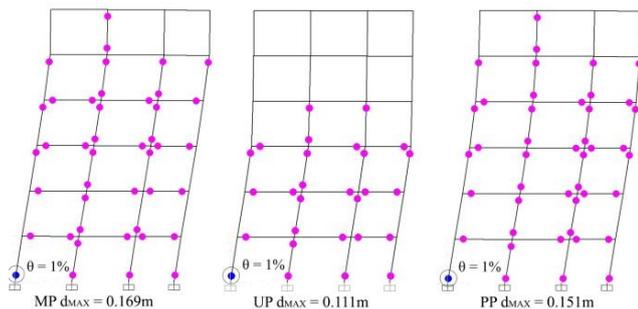


Fig. 6 Deformed shapes and plastic hinge distributions at collapse

his co-workers (Fajfar 1999, Fajfar and Fischinger 1988, Fajfar and Gaspersic 1996). The idealised bilinear relationship of the equivalent SDOF systems is derived from the pushover curve of the frame under the two conditions: (i) the yield force is assumed equal to the base shear force at the formation of the collapse mechanism and (ii) the initial stiffness of the idealized system is determined in such a way that the areas under the actual and the idealized force - deformation curves are equal. The N2 method is applied twice considering two load patterns: a Modal Pattern (MP), proportional to the first mode shape, and a Uniform Pattern (UP), based on lateral forces that are proportional to storey masses.

Fig. 5 compares the pushover curve of the frame determined by AN1 method to those determined by the N2 method and the two load patterns. The frame is pushed by horizontal forces acting from left to right up to the attainment of collapse. This occurs when the plastic rotation of the bottom cross-section of the first-storey column of the left side of the frame reaches the ultimate value of 0.01 rad (Fig. 6). This happens regardless of the NSP considered. Fig. 6 shows the deformed shapes of the frames and the plastic hinges at collapse. The three curves in Fig. 5 are rather close to one another, even though the pushover curve determined by the uniform load pattern evidences an initial stiffness and a base shear strength larger than those obtained by the AN1 method and the modal load patterns. In spite of this, the N2 and the AN1 method predict significantly different seismic capacities. According to the AN1 method the collapse of the frame occurs when the roof displacement  $d_{max}$  attains 151 mm. Instead, the roof displacement

capacity predicted by the N2 method, which is equal to the smallest between those determined by the modal and the uniform load pattern, is equal to 111 mm. The capacity of the frame is different also in terms of PGA. AN1 method provides a PGA corresponding to the roof displacement  $d_{max}$  equal to 0.27 g or 0.42 g depending on whether the residual stiffness of the frame after the attainment of the collapse mechanism is considered or not. Instead, the PGA determined by the N2 method is equal to 0.64 g, which is the smallest value between those determined by the modal (0.85 g) and uniform (0.64 g) load pattern, respectively.

In order to investigate the effectiveness of the analysed NSPs, the responses predicted by the N2 and AN1 method are compared to that determined by incremental nonlinear dynamic analysis (IDA). The modelling adopted for nonlinear dynamic analyses is the same adopted for nonlinear static analysis. As per the hysteresis model of the plastic hinges, the rigid-perfectly plastic hysteresis model is adopted. A suite of seven recorded accelerograms is used for IDA. This suite of accelerograms is selected by means of the REXEL computer program (Iervolino *et al.* 2010) in compliance with the requirements stipulated in EC8 about representation of the seismic excitation by ground acceleration time-histories. Fig. 7 shows the 5% damped response spectra of the seven accelerograms and the average spectrum of the suite of ground motions, together with the spectrum provided by the Italian seismic code for the site in consideration and PGA=0.30 g (target spectrum). Fig. 8 also shows that the average spectrum is close to the target spectrum and does not exceed the lower and upper bounds defined by the spectrum-compatibility conditions of EC8 in the range of periods from 0.2 s to 2.0 s.

Fig. 8 compares the results obtained by the NSPs and the seven IDAs in terms of base shear - roof displacement relationship and PGA - roof displacement relationship. For each accelerogram, the black circle denotes the PGA corresponding to the attainment of the ultimate plastic rotation of a plastic hinge. The frame exhibits elastic behaviour for low displacement demand and the results provided by IDAs lay on the elastic branch of the pushover curves obtained by AN1 method and pushover analysis using the modal load pattern (Fig. 8(a)) regardless of the accelerogram. Instead, when the frame is well excited into the inelastic range of its behaviour, the seven IDAs provide results more scattered and base shear corresponding to a given roof displacement larger than that obtained by the NSPs.

The difference between the base shear obtained by IDAs and NSPs is relatively small. Furthermore, as shown in Fig. 8(a), the NSPs predict conservatively the base shear strength of the frame (equal to the smallest base shear corresponding to the attainment of the displacement capacities determined by the modal and uniform patterns in case of N2 method). A different conclusion is found when the results of IDAs and NSPs are compared in terms of PGA - roof displacement relationship. Fig. 8(b) shows that AN1 method applied without considering the residual stiffness after the formation of the collapse mechanism returns a PGA capacity (0.42 g) very close to that obtained by IDA for four earthquakes over seven. AN1 method

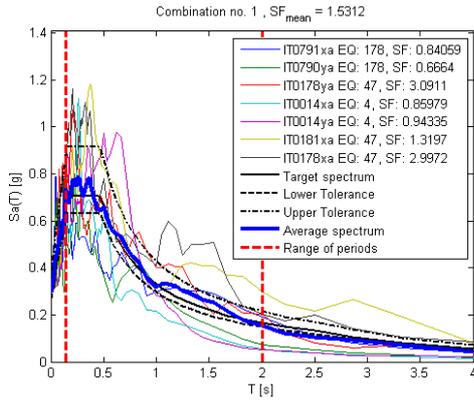


Fig. 7 Spectral shapes

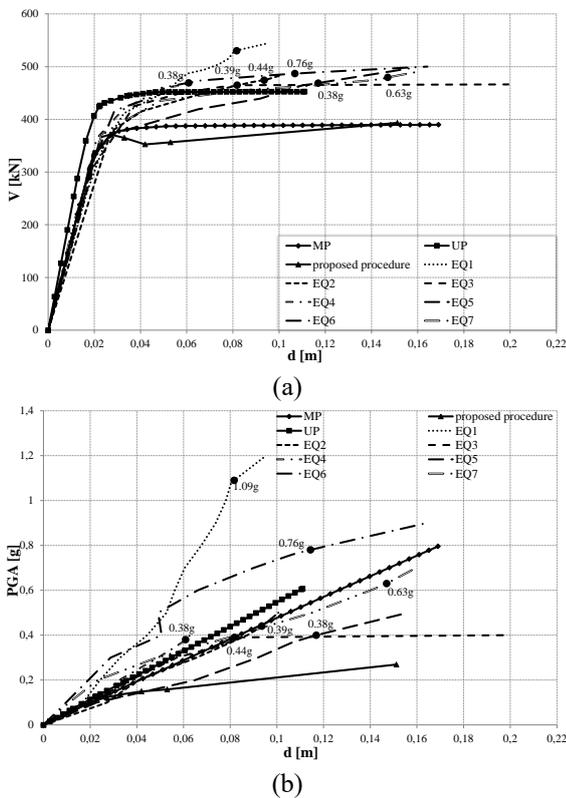


Fig. 8 Results of IDAs (a) vs results of NSPs (b)

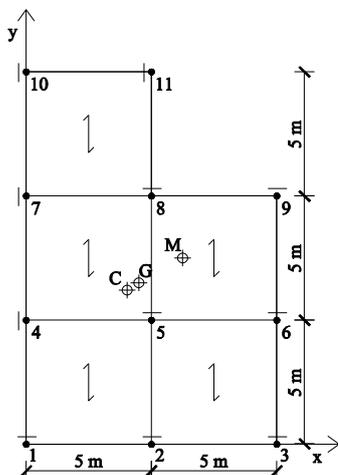


Fig. 9 Plan layout of the analysed 3D frame

applied considering the residual stiffness is conservative providing a PGA capacity (0.27 g) always smaller than that obtained by IDA. Instead, N2 method provides PGA capacity (0.63 g) larger than that determined by IDA for most of the accelerograms considered and is generally unconservative.

#### 4. Extension of the method to 3D structures

The proposed NSP can be applied also to 3D structures modelled by a set of planar frames connected by rigid floor diaphragms. This modelling is commonly used to simulate the seismic behaviour of building structures and it is accepted by seismic codes when the in-plan stiffness of floor decks is much larger than that of vertical resisting elements. Also for 3D structures, the procedure requires modal response spectrum analysis (*analysis B*) and displacement-controlled pushover analysis (*analysis A*) performed in succession. The analysis B is executed on the 3D model and provides the displacement patterns of the planar frames that constitute the 3D structure. The analysis A is executed applying simultaneously these displacement patterns to the planar frames. The horizontal displacements are proportionally increased until yielding occurred in the frames determines a non-negligible reduction of the lateral stiffness of the 3D structure. Analysis A can be stopped at the formation of the first plastic hinge. The numerical model is then updated considering the formation of the new plastic hinges and Analyses B and A are repeated until the attainment of the target limit state. If the collapse mechanism is formed before the attainment of this limit state, the increments of the horizontal displacements will be determined consistently with the collapse mechanism developed. The increment of PGA corresponding to the end of each step is determined as described in Section 3.1 for the planar frame.

As an example, the response of a 3-storey building, whose plan-layout is shown in Fig. 9, is evaluated by N2 method, AN1 method and IDA. Then, the obtained results are compared. The seismic excitation is uni-directional and acts along the *x*-direction. The set of seven accelerograms defined in Section 3.2 is used for IDA. Fig. 9 also shows the position of the geometrical centre *G* of the deck, the centre of mass *M* and the centre of rigidity *C*. In particular, the eccentricity between *M* and *C* is equal to 2.20 m and 1.30 m along the *x*- and *y*-direction, respectively. The building is endowed with hollow clay block-cement mix decks. Rectangular cross-sections are adopted for beams and columns. Flat beams with cross-section size 0.70×0.24 m are used at all storeys. The longitudinal rebars of the beams are determined considering only gravity loads in design. Cross-sections of columns have the same size at all storeys (0.30×0.60 m), but different reinforcements. In particular, the amount of longitudinal reinforcement of the columns is equal to 2%, 1.5% and 1% of the gross area of the cross-section at the first, second and third storey, respectively. The strengths of concrete and steel rebars are the same assumed for the 2D frame in Section 3.2.

The periods of the first two modes of vibration are 0.55 s

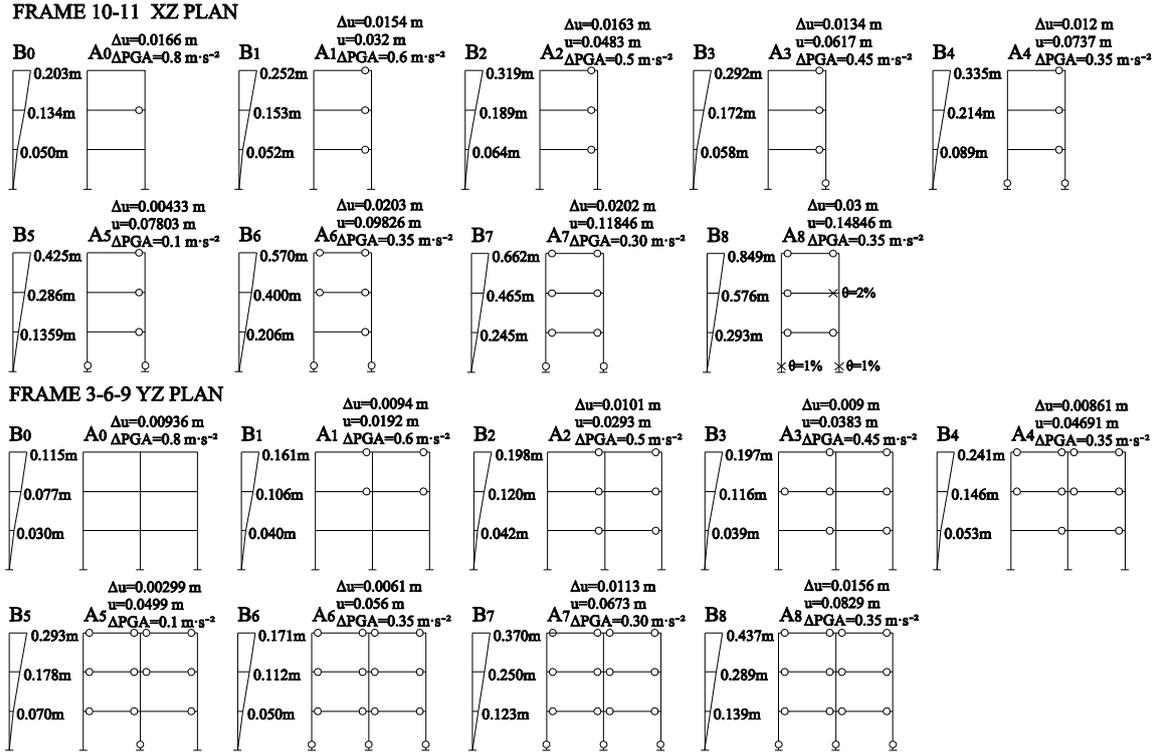


Fig. 10 Evolution of displacement pattern and sequence of plastic hinges determined by the AN1 method

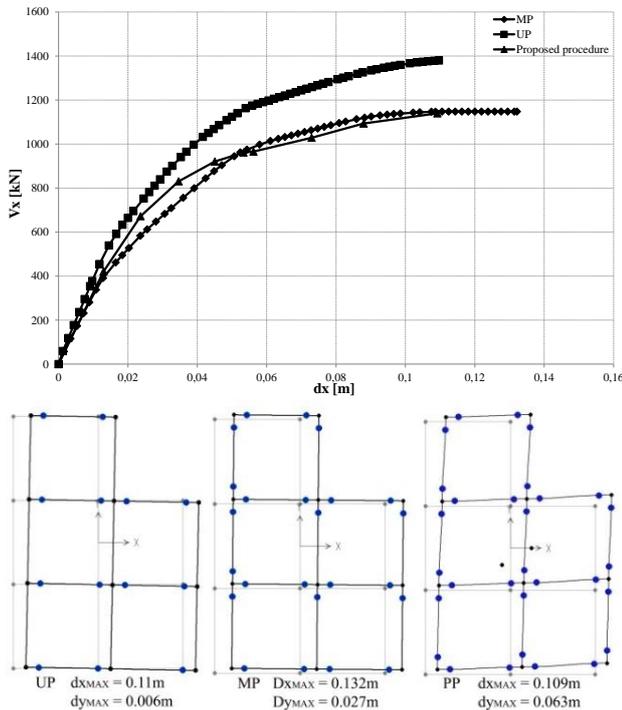


Fig. 11 Pushover curves (displacement in  $x$ -direction) and in-plan displacements at collapse determined by the analysed NSMs

plan. Beams and columns are modelled by elastic members with concentrated plasticity at their ends. The moment-rotation relationship of the plastic hinges is rigid-plastic. The ultimate value of the plastic rotation is assumed equal to 2% and 1% for beams and columns, respectively. Fig. 10 shows the evolution of the displacement pattern and the progression of the plastic hinges for two outermost frames of the building analysed by AN1 method. The frame 10-11 is arranged along the  $x$ -direction, while the frame 3-6-9 is aligned with the  $y$ -direction. The pushover analysis terminates before a collapse mechanism is formed, when three plastic hinges of the frame 10-11 attain their ultimate plastic rotation (Fig. 10). The PGA corresponding to the collapse of the building is equal to 0.39 g. The pushover curve, in terms of base shear in  $x$ -direction vs displacement of the mass centre of the top storey, determined by AN1 method is compared to those obtained by N2 method considering the MP and UP pattern (Fig. 11). When the N2 method is applied, seismic forces parallel to the  $x$ -direction are applied in the centres of mass.

The pushover curves are close to one another. In spite of this, the PGA capacities are different. In fact, the PGA capacities determined by the N2 method are equal to 0.55 g and 0.50 g for the MP and UP load pattern, respectively. These PGA capacities are significantly larger than that determined by AN1 method (0.39 g). Fig. 11 also compares the in-plan distributions of top storey displacements obtained by N2 and AN1 method. This comparison points out that the torsional component of the seismic response predicted by N2 method is smaller than that obtained by AN1 method regardless of the load pattern assumed (MP or UP).

and 0.60 s. The effective modal mass is 59.0% and 20.0% of the total mass for the first and second mode of vibration, respectively. Both the modes of vibration are coupled denoting that the analysed building is irregular in-



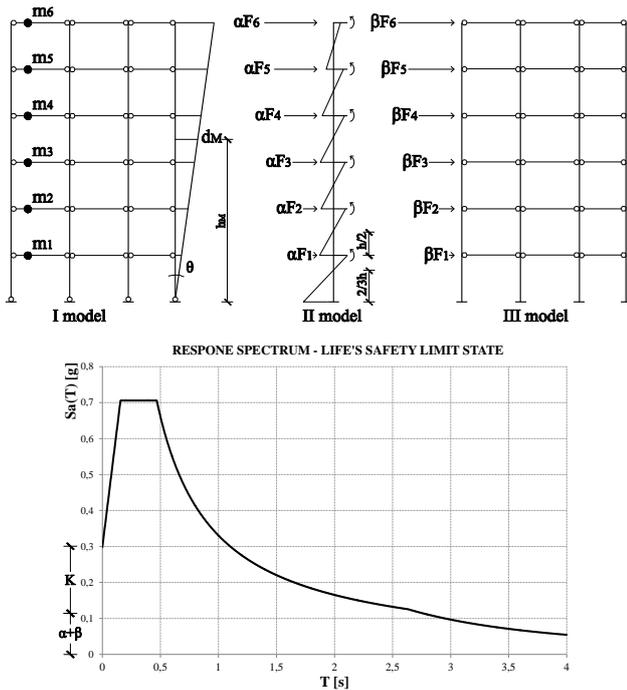


Fig. 13 Numerical models for the application of the design method

calculated. After that, Eq. (1) can be used to evaluate the increment of PGA ( $\Delta PGA_3$ ) that determines the increase of displacement  $d_g$  from the formation of the collapse plastic mechanism. For the same purpose, Eq. (2) can be used alternatively in order to take into account the residual stiffness provided by non-structural elements. In this case, the authors suggest the use of a period of 5 s.

The value  $\Delta PGA_3$ , which is a rate  $K$  of the design value of PGA (Fig. 13), is subtracted from PGA. The obtained value ( $PGA - \Delta PGA_3$ ), equal to  $(1-K)PGA$ , has to be filled by the two values of PGA that lead the frame to the end of

the elastic range of its behaviour (first stage, second numerical model) and to the yielding of the bottom cross-sections of the first storey columns (second stage, third numerical model). Fig. 13 shows that this value of PGA can be divided in two parts  $\Delta PGA_1 = \alpha(1-K)PGA$  and  $\Delta PGA_2 = \beta(1-K)PGA$ . The coefficients  $\alpha$  and  $\beta$  have to be fixed so that their sum is one. The choice of the values of  $\alpha$  and  $\beta$  determines if the behaviour of the frames to be designed will be more or less ductile. In fact, a large value of  $\beta$  leads to an early yielding of the frame and to a long transition stage from the formation of the plastic hinges at the beam ends to the attainment of the collapse mechanism. This prefigures a strong exploitation of the plastic deformation capacity of the frame and, therefore, a very ductile behaviour. Instead, if  $\beta$  is small, the frame yields for a larger value of PGA and this condition is quickly followed by the formation of the collapse mechanism. In this case, the obtained behaviour is less ductile. The authors suggest to set  $\alpha$  and  $\beta$  to 0.3 and 0.7 for high ductility frames, while  $\alpha=0.9$  and  $\beta=0.1$  may be assumed for medium ductility frames.

After that  $\alpha$  and  $\beta$  have been set, the seismic response of the second numerical model by the lateral force method of analysis stipulated in EC8 is determined. The response spectrum for the evaluation of the seismic force is determined by means of the  $\Delta PGA_1$ . The fundamental period of vibration  $T_1$  can be estimated by the empirical equations provided in seismic codes. The bending moments of beams obtained by this analysis combined to those provided by gravity loads are used to design the size of beam cross-sections and their longitudinal rebars. The verification of the shear strength of beam cross-sections as well as the design of beam shear reinforcements can be performed by the relevant capacity design criterion provided in EC8.

The response of the third numerical model is also determined by the lateral force method of analysis.

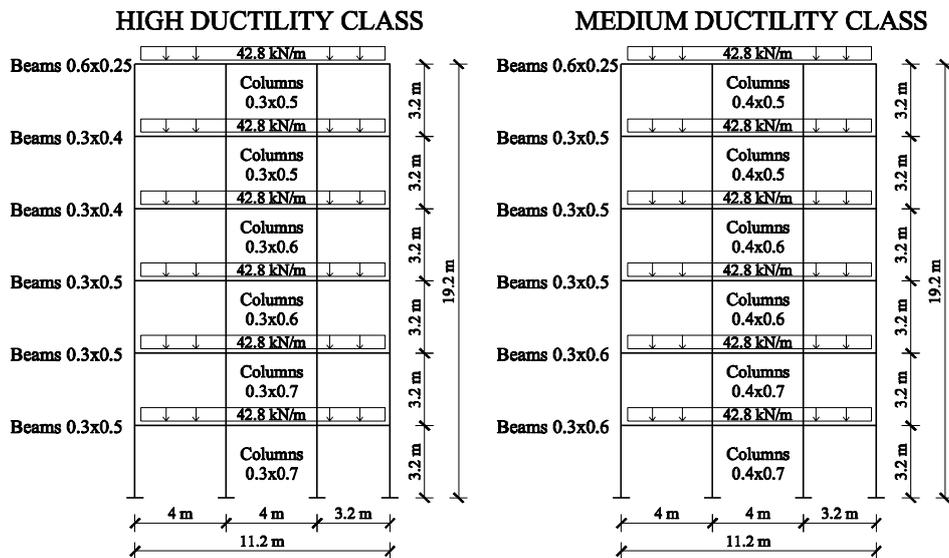


Fig. 14 Description of the HDC and MDC frames

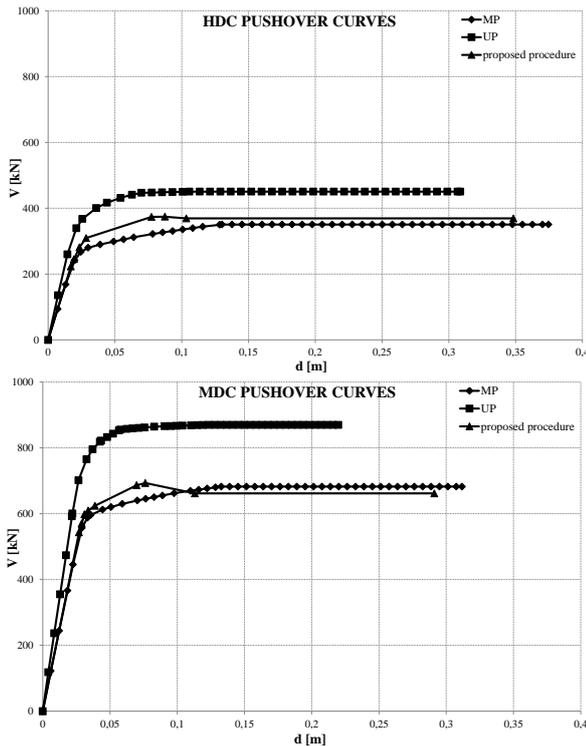


Fig. 15 Pushover curves of the HDC and MDC frames

However, in this case, the  $\Delta PGA_2$  and an elongated fundamental period  $T_{e1}$  are used. The period  $T_{e1}$  may be estimated as 0.1 times the total height of the frame in meters. The internal forces of columns determined by the analysis of the third numerical model are added to those obtained by the second numerical model to evaluate the design values of bending moment and axial force of the columns. This internal forces are used to size the columns and their longitudinal rebars. Instead, the capacity design criterion of EC8 may be used to check the shear resistance of the cross-section and to design the stirrups.

After that all the structural members have been designed, the nonlinear static method proposed in Section 3 can be used to assess the seismic response of the frame. If necessary, some cross-sections or steel reinforcements may be adjusted based of the results of the nonlinear analysis.

The proposed design method can be extended to 3D frames. In this case, the design can be performed separately for the two principal axes of the structure. For each of the two cases, the decks of the structure are constrained to move along the relevant principal axis and the design is performed according to the same procedure adopted for the 2D frame. Then the design internal forces can be increased according to the torsional provisions of EC8 or the method developed by Kreslin and Fajfar (2012).

### 5.2 Application of the design method to a r.c. frame

The planar r.c. frame presented in Section 3.2 is designed by the proposed method to withstand a seismic excitation with PGA equal to 0.30 g and the elastic response spectrum shown in Fig. 3. The design is performed twice considering two levels of prefixed ductility: high ductility

and medium ductility levels. The coefficients  $\alpha$  and  $\beta$  are set to 0.7 and 0.3 for the design of the high ductility (HDC) frame, while they are set to 0.9 and 0.1 for the medium ductility (MDC) frame. The ultimate plastic rotation of the bottom cross-sections of the first storey columns is assumed equal to 0.013 rad and 0.010 rad for the HDC and MDC frame, respectively. Based on these values of ultimate plastic rotation, the rates ( $K$ ) of PGA that lead the frame from the formation of the collapse plastic mechanism to the attainment of the ultimate plastic rotation are 0.68 PGA (0.20 g) and 0.52 PGA (0.16 g). Therefore, considering the values of  $\alpha$  and  $\beta$  assumed for the HDC frame, the rates of the design PGA used for the analysis of the first, second and third structural model are 0.07 g, 0.03 g and 0.20 g, respectively. Instead, in case of the MDC frame, the values of  $\Delta PGA$  adopted for the three structural models are 0.13 g, 0.01 g and 0.16 g. Fig. 14 shows the size of the cross-sections obtained for the two frames, along with the geometrical configuration and the gravity loads. Details on steel rebars may be found in Pellicchia (2012). The AN1 method is used to evaluate the maximum PGA that the two frames can endure. Fig. 15 shows the pushover curves determined for the two frames. The comparison between the two curves evidences that the design has led to a frame with low strength and large ductility capacity in case of the HDC frame. Instead, the MDC frame is less ductile but possesses larger lateral strength. The PGA capacity obtained for the HDC and MDC frame are 0.38 g and 0.40 g, which are both larger than the design value 0.30 g.

The seismic response of the two frames is also predicted by the N2 method. The pushover curves shown in Fig. 15 confirm the differences in terms of lateral strength and ductility between the two frames evidenced by the application of the AN1 method. However, the PGA capacity determined by the N2 method is much larger than that predicted by the AN1 method. In case of the HDC frame, the N2 method provides a PGA at the attainment of the ultimate plastic rotation equal to 0.64 g. This PGA is the smallest value between those determined by the modal (0.80 g) and uniform (0.64 g) load pattern. A larger difference is observed for the MDC frame. Indeed, in this case, the PGA provided by the N2 method is 1.13 g (1.28 g for modal pattern and 1.13 g for the uniform pattern).

## 6. Conclusions

The paper presents a multimodal adaptive nonlinear method for prediction of seismic response of r.c. framed structures. This method can provide important advantages with respect to those suggested in seismic codes. In particular, the proposed method (i) accounts for the contribution of the higher modes of vibration to the seismic response, (ii) considers the modification of the dynamic properties of the structure due to gradual yielding, and (iii) provides the PGA associated to each point of the pushover curve without requiring the definition of the equivalent SDOF system.

The proposed design method is applied to some case study r.c. structures. Both 2D and 3D frames are considered.

The results provided by the proposed method are compared to those obtained by the N2 method, as implemented in EC8. The comparison shows that the two methods provide similar pushover curves, but the AN1 method leads to a PGA capacity generally lower than that obtained by the N2 method.

The paper also proposes a displacement-based design method. This method does not require the use of the behaviour factor and is based on the analysis of three structural models that simplify the response of the structure in three stages of behaviour. The method allows the designer to predetermine the ductility of the structure to be designed. The method is applied to design two r.c. frames with high and medium ductility. The assessment of the maximum PGA that the two frames can sustain by the AN1 method shows that the proposed design allows the achievement of the prefixed level of safety.

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