Probabilistic seismic assessment of structures considering soil uncertainties

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Abstract. This paper studies soil properties uncertainty and its implementation in the seismic response evaluation of structures. For this, response sensitivity of two 4- and 12-story RC shear walls to the soil properties uncertainty by considering soil structure interaction (SSI) effects is investigated. Beam on Nonlinear Winkler Foundation (BNWF) model is used for shallow foundation modeling and the uncertainty of soil properties is expanded to the foundation stiffness and strength parameters variability. Monte Carlo (MC) simulation technique is employed for probabilistic evaluations. By investigating the probabilistic evaluation results it's observed that as the soil and foundation become stiffer, the soil uncertainty is found to be less important in influencing the response variability. On the other hand, the soil uncertainty becomes more important as the foundation-structure system is expected to experience nonlinear behavior to more sever degree. Since full probabilistic analysis methods like MC commonly are very time consuming, the feasibility of simple approximate methods' application including First Order Second Moment (FOSM) method and ASCE41 proposed approach for the soil uncertainty considerations is investigated. By comparing the results of the approximate methods with the results obtained from MC, it's observed that the results of both FOSM and ASCE41 methods are in good agreement with the results of MC simulation technique and they show acceptable accuracy in predicting the response variability.

Keywords: soil uncertainty; soil structure interaction; seismic response; probabilistic assessment

1. Introduction

Seismic analyses are usually performed for certain models which are simulated considering specific characteristics and neglecting variable parameters. A significant portion of the difference between the structural responses at the time of real earthquake and at the time of numerical seismic simulation is due to the assumptions considered for numerical modeling, analysis and design which may differ from the real state of the problem. Uncertainties are indispensable part of structural engineering and can be from different sources. Earthquake uncertainty arises from two main sources: 1) choosing a proper seismic intensity measure (IM) for structural analysis and design purposes and 2) selecting a set of earthquake records which can properly represent the seismological characteristics of a specific region. Hence, earthquake uncertainties may affect both the seismic response of structures and the amplification function of seismic motions that hit the structure (Kwon and Elnashai 2006, Bazzurro and Cornell 2004, Assimaki et al. 2003). Structural uncertainty plays a considerable role in the seismic assessments. This type of uncertainty appears in the geometric and material properties, structural modeling and construction detailing parameters (Lagaros and Mitropoulou 2013, Liel et al. 2008, Kwon and Elnashai 2006, Cornell et al. 2002). For instance, Jalayer et al. (2010) developed a

*Corresponding author, Associate Professor E-mail: msoltani@modares.ac.ir methodology for investigating the uncertainties in the construction detailing and material properties on the basis of a case-study existing building. They studied the structural performance in terms of seismic demand and capacity regarding the code-based definitions in the presence of uncertainties and developed a method for updating the probabilistic distribution of the uncertain parameters as well as the structural reliability variables.

Geotechnical uncertainty in the seismic evaluation of structures studies can be attributed to the uncertainty of soil material properties and the soil structure interaction modeling and specifications. SSI is well known to have an important effect on the seismic response of structures. Soil properties -as a component of SSI- make an important contribution to the foundation stiffness and strength characteristics. The inherent uncertainty of the soil material and its nonlinear behavior under dynamic loading of earthquake in combination with geometric nonlinearity of foundation can greatly complicate the foundation-structure system performance.

Soil as a natural material experiences a wide variation in its mechanical and physical properties. Darendeli (2001) investigated various soil samples from different geologic locations and gathered information around the variability of physical and mechanical properties of different soil types. Rieck and Houston (2001) studied the soil properties uncertainty as well as soil column depth randomness in their investigation of the seismic response of varied soil column in SSI analyses. Jones *et al.* (2002) studied and gathered the results of investigations made by numerous researchers around the soil material properties variability in the frame of a report for estimating the uncertainty in

geotechnical properties. The variability of the soil properties can influence the dynamic behavior of the foundation and the structure it supports. Raychowdhury and Hutchinson (2010) and Raychowdhury and Jindal (2014) investigated the effect of soil and foundation modeling uncertainties on the response of shallow foundations and the importance of each uncertain parameter. Tang and Zhang (2011) assessed the probabilistic demand of shear walls considering the effect of soil properties uncertainty by investigating mid-rise shear walls with flexible foundations. Moghaddasi et al. (2012) complemented a probabilistic study using the Monte Carlo methodology for investigating the effect of SSI on the response of structures considering the uncertainty of soil and structural parameters and input ground motions; they also assessed the sensitivity of the structural response to the varying model parameters. There are other researches implemented in the areas of seismic response investigation of structures considering soil properties uncertainty (Raychowdhury 2009, Foye et al. 2006, Ray Chaudhuri and Gupta 2002, Jin et al. 2000).

The motivation behind this research work was to examine the importance of the soil parameters uncertainty in the evaluation of soil structure interaction and seismic performance of structures. The variable parameters include foundation stiffness and strength determined based on the uncertain soil properties that are defined as random variables. The evaluations are done on the basis of the seismic response assessment of two shear walls as the most common lateral load bearing systems. At the first place, probabilistic response of the shear walls was evaluated considering soil uncertainties through the Monte Carlo simulation technique as a full probabilistic and accurate analytical approach. Then, the efficiency of the First Order Second Moment approximate method and the ASCE41 proposed approach for the probabilistic studies was evaluated by comparing their results with the exact results of the MC simulations. Since probabilistic assessments are generally time consuming and costly, the simple methods are beneficial for minimizing the analytical efforts. The SSI was incorporated by Beam on Nonlinear Winkler Foundation model and the shear wall was modeled using fiber-based nonlinear beam-column elements with the aid of OpenSees software. Seismic response quantities were obtained by performing nonlinear time history analysis for the foundation-shear wall conjunct system using a real earthquake record as the input seismic excitation.

2. Soil properties uncertainty

The soil material uncertainties arise from the estimation of the basic stiffness and strength parameters of soil. The strength and stiffness of soil are the most important parameters used in geotechnical engineering since many of design regulations rely on these parameters as the foundation design criteria. In the present work, the soil properties were considered as being random variables and their variability was expanded to the variability of the loaddeformation characteristic (stiffness and strength) parameters of foundation.

Table 1 Soil properties mean, median and coefficient of variation

Soil properties	Symbol	Mean	Median	CV (%)
Maximum shear modulus (kg/cm ²)	G_0	1700	1525	50
Material unit weight (kg/cm ³)	γ	0.002	0.0019	10
Poisson's ratio	v	0.35	0.0034	20

The studied soil was a stiff, incohesive and relatively dense sandy deposit. The mean, median and Coefficient of Variation (CV) of the considered uncertain soil parameters are presented in Table 1. CV is defined as the ratio of the standard deviation to the mean value of a particular parameter. The values of the characteristic parameters of soil and their variability range were determined based on the available information provided for the same soil type in different research works accomplished in both field and laboratory in Refs. (Jones et al. 2002, Darendeli 2001, Rieck and Houston 2001). Soil parameters were lognormally distributed. A perfect correlation was assumed between the maximum soil shear modulus -as the initial stiffness in the soil backbone curve-, G_0 , and the friction angle, φ , through Mohr-Coulomb theory and the soil backbone curve. Excluding the soil friction angle and the maximum shear modulus, the other soil parameters were assumed independent of each other. Therefore, the variability of φ was considered within the variability of G_0 values (for more explanations please see Hamidpour and Soltani 2016).

3. Input seismic excitation

The seismic response of the RC shear wall was obtained from nonlinear dynamic time history analysis of the foundation-shear wall system subjected to a record of the Northridge earthquake (1994). The record had the magnitude of 6.7 and the distance to rupture zone of 24.2 km and was recorded at Vasquez Rocks Park station (here known as VRP record). The record was actually selected based on the observed results in the supplementary study accomplished by the authors to investigate the effect of soil uncertainties on the ground motions intensity (Hamidpour and Soltani 2016). According to the study which considered the same soil properties uncertainty as this research, the VRP record showed the closest result to the average result obtained out of 20 earthquake records. In the mentioned study the records were applied at the bedrock level, 30 meters underground, and then were recorded on the surface after passing upward through the soil medium which was given the mean value of the soil characteristic parameters (G_0, γ, ν) . In order to record the motions on the surface, the soil medium was modeled in COM3, a finite element code developed at the University of Tokyo, Japan (Maekawa and Ishida 2010).

By recording the surface motions and averaging the PGA and spectral responses of single degree of freedom



Fig. 1 Time history and spectral acceleration of input VRP record used in analyses

oscillator, it was observed that VRP record had the closest response to the average, hence was used for seismic evaluation of the above the ground structure. The VRP record was chosen as the input seismic excitation and considered constant in the shear wall-foundation evaluations. This selection also helped to eliminate the probable effect of record to record variability of input seismic excitations and to exclusively evaluate the response variability due to the uncertainty of soil properties. Acceleration time history and spectral acceleration of VRP record on the soil surface are displayed in Fig. 1. This record had the maximum PGA of 0.395 g and maximum spectral acceleration of 1.54 g.

4. Monte Carlo simulation technique

Monte Carlo simulation technique is a series of computational algorithms based on repetition of stochastic sampling for calculating the result. In this technique the uncertain problem is converted to a set of problems with certain and definite responses so that, they can be answered utilizing simple and regular solutions. Gathering and combining the responses of the substituted definite problems, the answer of the primitive uncertain problem is obtained. MC is accomplished in several steps including defining a domain of possible input values, choosing random input values from the defined domain, performing computational operations on the selected inputs and finally calculating the result. Combining the definite results, a set of responses is obtained which demonstrates the probable consequence of the early uncertain problem. MC is known as an exact statistical approach and its result can be the explainer of the real state of a problem. It's also known as an approach that can obtain a solution with acceptable error of probability for geotechnical studies (Farah et al. 2013).

In the formulations of the BNWF model, the foundation stiffness and strength parameters are determined based on the soil shear modulus, friction angle, Poisson's ratio and material unit weight. According to the MC simulation methodology, for each soil parameter a domain of random values was generated and then the foundation stiffness and strength parameters were calculated based on these random values. The random domain was generated with the aid of a simple code written in MATLAB programming language by employing the mean, standard deviation and probability distribution (lognormal) for each soil parameter. The probabilistic seismic response of shear wall was investigated by analyzing the random samples of shear wall-foundation models with variable foundation stiffness and strength parameters that were generated based on the variable soil properties.

Since we needed to determine the probability distribution of the structural responses, stochastic samples were analyzed to produce sufficient number. In order to ensure the sufficient number of the analyzed samples which makes the results more accurate and reliable, a convergence test for the obtained results of the MC simulations was performed. Convergence test is mainly performed for the mean and standard deviation (STD) of the analyses results. In the test, the cumulative mean and STD of the results are calculated while the samples are increasingly generated and analyzed. At each step, the cumulative mean and STD values are divided by the total mean and STD values of the results and a ratio is obtained. The consequence of this test is demonstrated in a two-axis plot with one axis for the obtained ratio and the other for the number of analyzed samples. By the time when by increasing the number of the analyzed samples the ratio tends to 1, the number of the samples are considered sufficient and the analyses results are considered reliable. Once the seismic analysis results were available, the probability distribution of the responses could be determined.

5. First Order Second Moment method

First Order Second Moment is known as an approximate method in statistical engineering and is used for calculating the response of uncertain problems. In the FOSM method the uncertain problem is defined as a mono- or multivariable function as Eq. (1), where Y represents the uncertain problem as a function of the random variable, x which is characterized with its mean, μ_x and variance, σ_x^2 . Ensuring the availability of the function's derivatives at any point of x, solution of the uncertain problem is acquired by finding a relationship between the moments of the uncertain problem and the moments of the input variables. Using this method, obtaining two first moments is much simpler than the exact solution of the problem. Thus, the mean and variance of the input varying parameter are used to calculate the mean and variance of the uncertain problem.

Developing the FOSM method based on the partial derivative or Taylor series expansion is the most popular form of the method. Hence, the function is rewritten on the basis of the Taylor series expansion method about the point x_0 as presented in Eq. (2). The first and the second moment of the function, μ_Y and σ^2_Y , represent the mean and variance of the problem Y in Eqs. (3) and (4), respectively. Term $(dg/dx)_0$ in the equations represents the sensitivity of the response to the changes of the value of variable x

$$Y = g(x) \tag{1}$$

$$Y \approx g_0 + (dg/dx)_0 (x - x_0)$$
 (2)

$$\mu_{Y} \approx g_{0} + (dg/dx)_{0}(\mu_{x} - x_{0})$$
(3)

$$\sigma_Y^2 \approx g_0^2 + (dg/dx)_0^2 \sigma_x^2 + 2g_0(dg/dx)_0 (\mu_x - x_0) - \mu_Y^2$$
(4)

In case of $x_0=\mu_x$, Eqs. (3) and (4) are expressed as Eqs. (5) and (6) that can be used for calculating the mean and variance of the problem by knowing the corresponding quantities of the random variable as follows

$$\mu_Y = g(\mu_x) \tag{5}$$

$$\sigma_Y^2 \approx (\frac{dg}{dx})_0^2 \sigma_x^2 \tag{6}$$

For multi-variable functions, in which the vector of the random variables is as $x=[x_1, x_2, ..., x_n]$ with the mean vector of $\mu_x=[\mu_1, \mu_2,..., \mu_n]$ and variance of $\sigma_x^2=[\sigma_1^2, \sigma_2^2,..., \sigma_n^2]$, considering VC[x] as the variance-covariance matrix of variables, Eqs. (5) and (6) are written as below

$$\mu_{Y_i} = g(\mu_1, \mu_2, ..., \mu_n), \qquad i = 1, 2, ..., n$$
(7)

$$\sigma_{Y_i}^2 \approx (\frac{dg}{dx})_n^2 \sigma_i^2, \qquad i=1,2,...,n$$
 (8)

where

$$\sigma_Y^2 \approx \nabla g(x) V C[x] \nabla^T g(x) \tag{9}$$

$$\nabla g(x) = \left[\frac{\partial g}{\partial x_1}, \frac{\partial g}{\partial x_2}, \dots, \frac{\partial g}{\partial x_n} \right]$$
(10)

$$diag(VC[x]) = \left[\sigma_1^2, \sigma_2^2, \dots, \sigma_n^2\right]$$
(11)

$$\frac{\partial g}{\partial x_i} = \frac{g(\mu_i + \Delta x_i) - g(\mu_i - \Delta x_i)}{2\Delta x_i}, \quad i = 1, 2, \dots, n$$
(12)

In the formulations, $\Delta x_i = \sigma_i$ and σ_i is the standard deviation of the random variables. The diagonal entries of the variance-covariance matrix for the independent variable parameters, as in the case of our considered soil parameters, contain the variance of variables while the off-diagonal entries are equal to zero.

6. ASCE41 proposed approach for consideration of foundation uncertainty

According to ASCE41 (2013) regulations, while it is recognized that the load-deformation behavior of foundations is nonlinear, an equivalent elasto-plastic representation of the load-deformation behavior is recommended because of the difficulties in determining soil properties and the probable variability of soil which supports the footing. In order to take such variability into account, the upper- and lower-bound approach for defining stiffness and capacity limit states of foundation is considered. According to ASCE41, the sources of this uncertainty may include variations due to rate of loading,



Fig. 2 Idealized elasto-plastic load-deformation behavior of soils proposed by ASCE41

assumed elasto-plastic soil behavior, level of strain, cyclic loading and variability of the soil properties. According to ASCE41, stiffness and capacity values of the upper- and lower-bound approach are twice and half the calculated stiffness and capacity values respectively as depicted in Fig. 2. It is to be noted that the upper- and lower-bound represent stiffer and softer soil models respectively in comparison to the calculated one.

7. Analytical model of shear wall

7.1 RC shear wall modeling scheme

Precise assessment of the shear wall requires an accurate and practical analytical modeling method which is able to simulate the material and the behavioral characteristics of the structure. The effective analytical model should be simple to use and accurate for response determination. Various numerical models which are mostly based on finiteelement methods have been developed to simulate the behavior of RC shear walls under dynamic loadings. Finiteelement-based models can generally simulate the nonlinear load displacement behavior of different structural element types under various loading conditions (Ayoub and Filippou 1998).

In the present study, nonlinear displacement-based beam-column element was used for modeling of RC shear wall (Orakcal et al. 2004) depicted in Fig. 3. In this model, the shear wall is simulated by an equivalent beam column element located at the central axis of the wall. The rotation of the beam-column element occurs around this central axis. The mentioned modeling scheme was implemented in software OpenSees. Material properties of the RC member were defined by assigning perpendicular sections called Fiber elements to the equivalent beam-column element. In the Fiber element, the cross section of the member is divided into small cells each belonging to a fiber. Geometrical characteristics and location of each cell are determined in two perpendicular directions. In the Fiber element the reinforced concrete member is modeled as a set of fibers in which, based on the location of each fiber it receives the properties of concrete or steel. The existing

stresses in the member include axial stress (axial and flexural forces) and shear stress (shear and torsional forces), hence, the overall distortion is divided into two main parts including flexural and shear distortion (Mazzoni *et al.* 2006). The fiber element model is a standard modeling scheme extensively used for simulation of RC members. It has been previously validated by simulation of cyclic tests of shear walls in the laboratory representing reasonable agreement between experimental and numerical analyses results (Briely *et al.* 2008, Thomsen and Wallace 2004).

In the modeling of the shear walls, gravity loads and story masses were equal to 15 and 50 tons respectively and assumed to be the same at the all story levels. Lumped masses were defined at each story level and gravity loads were allocated to these lumped masses in the equivalent beam-column element.

7.2 RC shear wall design and structural details

The two shear walls were designed according to displacement-based seismic design procedure (DBD) proposed by Priestley (2004) and their structural details were determined according to the regulations of Iranian Reference for Concrete Structures. Displacement-based design is a method of the performance-based design (PBD) and its concept is to use the earthquake induced displacements for seismic design of structural elements. In this method, structural properties and details are determined such that the shear wall reaches the defined target displacement. The method is independent of the displacement spectra used as the earthquake induced displacement for seismic design. Hence, in the present study the average displacement spectrum of 7 earthquake records which were the closest in their displacement spectra among 20 records was chosen as the design spectrum. The shear walls were designed as being a cantilever to reach the interstory drift ratio of 2% as the design criteria. The obtained shear force and bending moment at the base of the shear wall were used to design the cross section of the walls regarding the seismic regulations of Iranian Reference for Concrete Structures. The concrete material had the compressive strength of 280 kg/cm² and material unit weight of 2500 kg/cm³. The steel reinforcement of shear walls had yielding stress of 4000 kg/cm² and primary elastic modulus of 2.1×10^6 kg/cm².

The walls had rectangular cross section and square boundary elements at two ends of their length. The geometrical properties and the structural details of shear walls were considered constant at the height of the walls. This assumption was made to eliminate the complexity of

Table 2 Structural details of shear walls

Shear Wall	Height (m)	Length (m)	Thickness (cm)	Boundary Element Size (cm×cm)	Uniform Reinforcement of the Web	Reinforcement Ratio of Boundary Element (%)
4- story	12	2	20	30×30	$\Phi 10 @ 30 \text{ cm}$	2.5
12- story	36	4	30	50×50	Φ12 @ 25 cm	2.5



Fig. 3 Schematic models of Beam-column element of shear wall and BNWF modeling scheme of foundation

the nonlinear behavior of the walls due to the lack of clear plastic hinge location. Hence, in the case of plastic hinge formation it will be located at the base of the wall where the forces are maximum. The geometrical properties of walls and boundary elements including length, thickness, and reinforcement details are displayed in Table 2.

8. Analytical model of shallow foundation

8.1 Shallow foundation modeling

Probably the easiest way to evaluate the SSI effects is to model a significant part of the soil around the embedded structure and apply free-field motion to the complete system of soil, foundation and structure. This method is known as direct analysis method and requires large computational workload and long analysis time which is not feasible for probabilistic analyses. As the law of superposition is implicitly assumed valid in SSI analysis, it is computationally more efficient to use the substructure method in which the force-displacement of the contact area is interpreted as a generalized spring-dashpot system (Wolf 1985).

In this research, foundation is modeled using Beam on Nonlinear Winkler Foundation model which is developed based on the substructure method for shallow foundations. The BNWF model has been found to show reasonable results in SSI effects evaluations (Gajan et al. 2008). The BNWF model consists of an elastic beam-column element to capture the structural footing behavior and a series of horizontal and vertical springs to capture the translational and rotational deformation of foundation (Fig. 3). The vertical springs (q-z springs) intend to model the vertical and rotational flexibility and resistance of the foundation while in horizontal direction, the springs aim to capture the passive soil resistance (p-x springs) and sliding-friction of foundation with respect to the soil on which the footing lies (t-x springs). The springs have nonlinear inelastic behavioral model and are independent of each other.

For the objective of this study, the footing was fixed in the horizontal direction because no horizontal transition was considered and only vertical springs for capturing the rotation and settlement of foundation were defined. Loaddeformation characteristics of q-z springs were defined based on a nonlinear backbone curve resembling a bilinear behavior -a linear region and a nonlinear region with decreasing stiffness- in software OpenSees (Gajan *et al.* 2008). The bilinear backbone curve is characterized via the soil type specific constants defined by the user in software OpenSees controlling the elastic and inelastic region range and shape. The q-z spring considers reduced strength in tension so is capable of modeling the limited capacity of soil to carry tension loads.

The moment-rotation behavior of foundation was simulated by distribution of the vertical springs along the footing length. In order to consider the enhanced rotational stiffness at two ends of the footing, two parameters were taken into account: (1) the stiffness intensity ratio $(R_k = K_{end}/K_{mid})$ and (2) the end length ratio $(R_e = L_{end}/L)$, where the K_{end} and K_{mid} are the stiffness of the foundation at extreme ends and middle region of footing and L_{end} and L represent the enhanced stiffness length at extreme ends and total length of the footing, respectively. The values of R_k and R_e were determined based on the analytical equations developed by Harden (2005) for the enhanced end stiffness as a function of the footing aspect ratio (B/L). Varying vertical stiffness at the footing length was considered to match the equations proposed by Gazetas (1991) for foundation stiffness calculations. For more details about the BNWF model please see Gajan et al. (2008).

The ultimate load bearing capacity of foundation was calculated on the basis of Terzaghi's theory through the general bearing capacity equation and factors of Meyerhof. The foundation capacity was calculated by specifying the footing dimensions and soil friction angle in Eqs. (13)-(18). In the formulations, N_c , N_q and N_γ are bearing capacity factors, F_{cs} , F_{qs} and $F_{\gamma s}$ are shape factors, F_{cd} , F_{qd} and $F_{\gamma d}$ are depth factors, F_{ci} , F_{qi} and $F_{\gamma i}$ are inclination factors and q_{ult} , c, γ , D_f , and B represent ultimate load bearing capacity of foundation, soil cohesion, soil material unit weight, foundation depth and breadth, respectively.

Foundation surface stiffness of vertical translation was determined through the proposed formulation by Gazetas (1991) in Eq. (19). The formulation is proposed for shallow foundations on a semi-infinite half-space and the input parameters include soil shear modulus at small strains, G, Posisson's Ratio, v, footing length, L and breadth, B. The proposed formulation by Gazetas is also adopted by ATC40 (1996) for determining the stiffness of shallow foundations. It also worth to mention that soil properties were considered spatially constant along the foundation length.

$$q_{ult} = cN_c F_{cs} F_{cd} F_{ci} + \gamma D_f N_q F_{qs} F_{qd} F_{qi} + 0.5\gamma.5 \gamma F_{\gamma s} F_{\gamma d} F_{\gamma i} (13)$$

$$N_q = e^{(\pi \tan \varphi)} \tan^2(45 + \frac{\varphi}{2})$$
 (14)

$$N_{\gamma} = (N_q - 1)\tan(1.4\varphi) \tag{15}$$

$$F_{qs} = 1 + (\frac{B}{L})\tan\varphi \tag{16}$$

$$F_{\gamma s} = 1 - 0.4(\frac{B}{L})$$
 (17)

$$F_{qd} = 1 + 2\tan\varphi(1 - \sin\varphi)^2(\frac{D}{B}) \tag{18}$$

$$K_{z} = \frac{GL}{1 - \nu} (0.73 + 1.54(\frac{B}{L})^{0.75})$$
(19)

8.2 Foundation design and dimensions

Footings of two shear walls were designed for design forces generated at the walls' base. The dimensions of the footings were determined on the basis of the ultimate strength method proposed by ASCE7 (2010). The regulations allow the soil stress to reach 60% of the soil ultimate strength, q_{ult} , and consider no uplift limitation. In the case of 12-story shear wall, the regulation requirements were fully achieved in the footing design. On the other hand, in the case of 4-story shear wall the dimensions of the footing were determined so that the soil stress reaches its design stress threshold (0.6 q_{ult}). The aim of this design assumption is to consider the most critical situation of foundation in terms of nonlinear behavior under dynamic loading. This assumption provided a model in which studying of the uncertainties effect on the shear wall response became possible while the foundation was in its worst and weakest condition in terms of the soil stress. The dimensions of the shear walls' footings were obtained as follows: length of 4 m, breadth of 3 m and depth of 1 m for 4-story shear wall and length of 7 m, breadth of 5 m and depth of 1.7 m for 12-story shear wall. It was also assumed that both footings were buried 1 m underground.

9. Probabilistic seismic assessment of shear walls

Probabilistic analyses results of the shear walls are presented in three parts: 1) the results of the MC simulation technique for estimating the variability of the shear walls' responses considering soil properties uncertainty, 2) comparing the results of MC and FOSM approximate method and 3) comparing the results of MC and ASCE41 proposed approach. By investigating the results obtained in the first part, the mean and variability range of the seismic responses of shear walls and the importance of the soil uncertainty considerations were determined. In the second and third part, the accuracy of the FOSM and ASCE41 approximate and simple methods for investigating the effects of soil uncertainties was evaluated.

9.1 Results of the MC simulation technique

In the MC simulation technique, as previously explained, a set of random samples of foundation-shear wall system was generated and analyzed. In the simulations, the stiffness and strength of foundation determined based on the soil properties were the only parameters that differed from one sample to another. Parameters such as RC shear wall characteristics, footing dimensions and the input seismic excitation were assumed constant in all of the simulations.

The interested seismic responses of the shear walls for studying are the inter-story drift ratio excluding the effect of foundation rotation, shear wall structural displacement excluding the effect of foundation rotation, shear wall rotational displacement due to foundation rotation and total displacement of wall (the summation of the structural and rotational displacements). Total number of 70 samples of each shear wall with different foundation properties was analyzed. The number of the analyzed samples was found sufficient by performing convergence tests for the mean and standard deviation of the results which are presented for the inter-story drift ratio in Figs. 4 and 5, for instance.

The structural responses are displayed against soil G_0 since the shear modulus was found to have the most influence on both the stiffness and strength parameters of foundation among the parameters G_0 , γ and ν . The increase in G_0 value results in the increase in foundation stiffness and strength. In other words, G_0 is used as the representative parameter of the soil uncertainty and the sensitivity of the structural responses is assessed based on its variability. It is observed for 4- and 12-story shear wall according to Figs. 4 and 5 that the value of the inter-story drift ratio and the shear wall structural displacement excluding the effect of foundation rotation increased with increasing the G_0 , while the shear wall displacement due to foundation rotation decreased. This indicates that as the soil became stiffer due to larger G_0 value, the structure of the wall mainly contributed to the total displacement since it's reasonable to have a limited foundation rotation (and rotational displacement) on the stiff soil in comparison to the soft soil. According to the graphs, the increase in the values of the inter-story drift and the decrease in the rotational displacement were more tangible in the first half of the G_0 extent while they were almost constant in the second half. It could be concluded that the further increase in G_0 value which results in a more rigid foundation, soil parameters variability did not necessarily affect the responses. In other words, soil uncertainty influenced the seismic response of the shear walls whose footings laid on soft soil more than those whose footings were supported by stiff soil.

Similar to the soil and foundation properties, lognormal probability distribution was observed for the seismic responses of the shear walls. The mean and the standard deviation of two shear walls' responses are presented in Table 3. According to the obtained results, despite the similar total displacement of two shear walls, the inter-story drift ratio and the shear wall structural displacement were



Fig. 4 Seismic responses of 4-story shear wall using MC simulation technique and convergence test result

very smaller in the case of 4-story compared to 12-story while the rotational displacement was found to show inversed result. This observation could be explained by examining the dynamic behavior of the shear wallfoundation conjunct system. As the result of the different footing sizes, wall heights and dynamic behavior of two walls, it could be concluded that whenever there was the possibility of greater rotation of foundation -such as soft soil or small footing size making the foundation more flexible- main part of the wall total displacement belonged to the rotational displacement. When the foundation rotates, by neglecting the structural deformation of wall the wall rotates like a solid element in accordance with its foundation thus, the rotational displacement makes the main contribution to the wall total displacement and results in limited structural deformation.

Examining the inter-story drift ratio, a parameter that served as the design criteria in shear wall design, it was observed that the CV of the drift decreased to 13% for 12story compared to 37.5% for 4-story. In the case of the walls' total displacement as an important parameter which contributes in the seismic performance of structure at the time of earthquake, although the total displacements of two walls were almost the same, the CV decreased to 1.1% for 12-story compared to 7.3% for 4-story. According to the observed results, it could be concluded that the soil uncertainty had more effect on the response of 4-story which in addition to the shear wall dynamic behavior could be attributed to the critical condition of the 4-story shear wall foundation. Therefore, the soil properties variability can affect the settlement, translation and rocking behavior of foundation, which can cause the variability of the seismic response of structure. The effect of soil properties



simulation technique and convergence test result

Table 3 Inter-story drift and displacements of 4- and 12story shear walls

Shear wall	Inter-story drift ratio (%)		displacement		Rotational displacement (cm)		Total displacement (cm)	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
4-story	0.57	0.214	4.78	1.624	16.56	0.804	21.34	1.56
12-story	0.833	0.108	20.5	2.083	3.465	2.225	23.97	0.262

uncertainty should be included in the seismic performance evaluations, especially for the structures with weak or insufficient foundations as well as structures which are expected to experience the nonlinear behavior to more severe degree during the earthquake.

9.2 Comparison between the results of FOSM and MC simulation

Although the results of the MC simulation represent the real condition of a problem, its application is not feasible for practical cases, especially for problems with large number of degrees of freedom. On the other hand, the FOSM approximate method is very time- and cost-saving and needs a few number of simulations compared to MC to calculate the problem response and its variability.

According to the MC simulation results presented in Fig. 4 and Fig. 5, the diversity of the responses was more tangible in the first half of the G_0 extent. Therefore, we put the responses into two ranges of G_0 including smaller and larger G_0 quantities with border value of $G_0=1600$ kg/cm². The FOSM analysis was performed for the samples whose results varied within the range of the smaller G_0 values (smaller than 1600 kg/cm²); because in this range, responses varied significantly due to G_0 changes while in the range of the larger G_0 values (larger than 1600 kg/cm²), responses were almost constant. It should be mentioned that the variability of the responses in the smaller G_0 values range covers the total extent of the responses' variability (see Fig. 4 and Fig. 5). According to the MC results, the shear wall responses were lognormally distributed. The lognormal distribution was observed for the responses which sit in the range of the smaller G_0 quantities as well.

In this study, by investigating the results of FOSM, it was found that for the functions with lognormal distribution, the mean value in FOSM formulations, μ_x , should be substituted by the median, and the standard deviation of the variables, σ_x , should be determined for *ln* of the parameters' quantities, $\sigma_{ln(x)}$. In this way, the FOSM method can properly predict the median and variance of the results in lognormally distributed functions.

In the simulations of FOSM, foundation characteristics were determined by taking single soil parameter as being variable at a time while the others were given their constant median value. The FOSM analysis results cover three responses including median, lower- and upper-bound responses as the limit states. On the basis of the statistical concepts, percentiles 16 and 84 are defined as the lower and upper limit states of functions with lognormal probability distribution and represent the variability amplitude. It's noteworthy that they are the equivalents of mean ± STD values in normally distributed functions. In the FOSM analyses in this study, 7 samples were analyzed including a sample in which all the parameters were given their median, and 6 samples in each only one parameter was given its percentiles while the two others were given their median (G_0, γ, ν) . The obtained results were used to determine the median and the variance of the responses according to the formulations presented in the FOSM methodology description section.

Since the comparison between MC and FOSM was made in the smaller G_0 range, the median and percentiles of the soil parameters were determined based on the set of exact soil parameter values that belonged to the samples whose results sit in the smaller G_0 range in MC simulations. The FOSM procedure for calculating the median and percentiles of the inter-story drift ratio at the roof of 4-story shear wall is demonstrated below as example:

The standard deviation, σ , of the soil parameters G_0 , γ , ν , considering the *ln* of the values were equal to 0.237, 0.088 and 0.182, respectively. The median and percentiles 16 and 84 of soil parameters were determined in logarithmic space and then converted to normal real quantities in order to be used in foundation stiffness and strength calculations in the simulations.

$$\sigma_{x} = \begin{bmatrix} 0.088 & 0.182 & 0.237 \end{bmatrix}$$
$$VC(x) = \begin{bmatrix} 0.007685 & 0 & 0 \\ 0 & 0.03324 & 0 \\ 0 & 0 & 0.05625 \end{bmatrix}$$

from the analyses results it was found that:

 $\nabla g(x) = [1.408 - 1.2961 0.9138]$

now, according to Eq. (11) the variance and standard deviation of drift could be calculated in logarithmic space:

$$\sigma^2 = 0.0118 \implies \sigma = 0.334$$

The drift value obtained from analysis of the sample in which all the soil parameters were given their median value was equal to 0.00385 whose *ln* value is equal to -5.623 in logarithmic space, the percentiles are determined as:

$$Perc.16 = e^{(-5.623 - 0.334)} = 0.00254$$
$$Perc.84 = e^{(-5.623 + 0.334)} = 0.00505$$

A similar procedure was employed for the other responses at all story levels. The FOSM results are presented in Fig. 6 and Fig. 7 depicting the shear wall deformation profile including median and percentiles 16 and 84 obtained from MC and FOSM in comparison. The graphs display the deformation profile of the shear walls at the time when the maximum displacement occurred at the wall's roof. According to the figures, the FOSM approximate method can estimate the median responses well. In the response variability estimation, FOSM was found to be more accurate for the 12-story compared to the 4-story shear wall. This could be attributed to the fact that designing of two shear walls for similar target drift caused the shorter one to experience nonlinear behavior to the greater degree. Furthermore, due to the critical condition of the 4-story shear wall's foundation the complexity of its dynamic behavior increases and thus, these nonlinearities of system could not be covered by the limited number of simulations. On the other hand, in the case of the 12-story, FOSM was found to show very good congruence in the results with the MC simulation. Despite the observed



Fig. 6 Comparison between the results of MC and FOSM for 4-story shear wall



Fig. 7 Comparison between the results of MC and FOSM for 12-story shear wall

approximations of FOSM method in estimating the responses, it still predicts the seismic responses of the shear walls with acceptable accuracy and can be used instead of the MC simulation with less analytical effort for considering the soil and foundation uncertainties.

9.3 Comparison between the results of ASCE41 and MC simulation

A comparison between the results of the MC simulation and the approximate approach proposed by ASCE41 was also made in this paper. The results of the two samples simulated based on the lower- and upper-bound foundation models of ASCE41 were compared to the percentiles 16 and 84 of the whole MC probabilistic responses presented in Fig. 4 and Fig. 5. Determining the lower- and upper-bound of the foundation capacity and stiffness by doubling and halving the calculated values as proposed in ASCE41, indicates that the procedure considers 100% increase and 50% decrease of the calculated values as the limit states of variability amplitude that could be attributed to the consideration of lognormal distribution of foundation stiffness and strength. In this study, it was also found that the calculated foundation stiffness and strength should be determined considering the median value of the soil properties. According to the observations, in this way the proposed lower- and upper-bound load-deformation properties determined based on the calculated one will be more proper and precise explainer of the soil-foundation uncertainty.

Fig. 8 and Fig. 9 compare the results of the MC and ASCE41 approaches for 4- and 12-story shear walls, respectively. According to the results, the ASCE41 approach can accurately predict the variability of the responses demonstrating good agreement with the results of MC simulation. Similar to the FOSM results, the ASCE41 approach estimated the response variability of 12-story more precisely than 4-story shear wall. Therefore, the proposed approach of the ASCE41 by defining the lower-



Fig. 8 Comparison between the results of MC and ASCE41 for 4-story shear wall



Fig. 9 Comparison between the results of MC and ASCE41 for 12-story shear wall

and upper-bound of foundation stiffness and strength is well appropriate for consideration of foundation uncertainties.

10. Conclusions

The implementation of the soil properties uncertainty through the SSI effects in the seismic assessment of structures and the importance of considering such uncertainty was studied in this paper. A comprehensive probabilistic assessment was performed utilizing Monte Carlo simulation technique for the exact evaluation of the response variability considering the effect of soil uncertainty. Then, the efficiency of the approximate methods of First Order Second Moment and ASCE41 for considering the uncertainty of soil properties was evaluated. The study was complemented on the basis of the probabilistic seismic assessment of two RC shear walls supported by shallow foundations.

It was observed that soil uncertainties can affect the settlement, translation and rocking behavior of foundation, which can cause the variability of the seismic response of structure and play an important role in the probabilistic assessments. The soil properties variability showed to have more influence on the response variability of the structure supported by less stiff soil and flexible foundation in comparison to the one established on stiffer soil. This was obtained by examining the rate of the response variability of shear walls on the basis of the changes in the soil and foundation stiffness and strength as a function of the soil shear modulus. It was also observed that the soil properties variability influenced the response variability more as the foundation and structure were expected to experience the nonlinear behavior to greater degree under seismic loading. For example, the inter-story drift ratio varied with the CV of 37.5% for 4- and 13% for 12-story shear wall, respectively, knowing the lower wall was more susceptible to nonlinear behavior than the higher one. Furthermore, it was observed

that with increasing the soil stiffness, the main contribution of the wall deformation belonged to the structural displacement while in the less stiff soil, displacement of the wall due to foundation rotation constituted the major portion of the wall total displacement.

The comparison made between the results of approximate methods (FOSM and ASCE41) and the exact method (MC) revealed that discussing generally, the FOSM and ASCE41 approaches could estimate the variability of the responses with acceptable accuracy. However, both approximate methods found to show better results for the higher shear wall in this study. According to the presented results, it could be concluded that the approximate methods lead to more reliable results if applied to structures which remain in the linear behavioral region when subjected to seismic loads. However, in structures which are expected to experience the nonlinear behavior to more severe degree both in the structure and the foundation- the approximate methods may not be able to efficiently evaluate the probabilistic seismic behavior.

The acquired results investigating the FOSM and ASCE41 approximate approaches are of great importance in the scope of the probabilistic analyses. Because of the limited number of the required simulations and the time saving feature of the approximate methods, the need for such methods is recognized when it's not feasible in terms of time and cost to perform full probabilistic analyses. They showed reasonable results in probabilistic assessment of the shear walls under the consideration of this study. The outcome of this research is specific for the structural models and uncertain parameters under consideration and for other purposes further studies might be required.

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