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Nonlinear seismic analysis of a super 13-element reinforced concrete beam-column joint model

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Abstract. Several two-dimensional analytical beam column joint models with varying complexities have been proposed in quantifying joint flexibility during seismic vulnerability assessment of non-ductile reinforced concrete (RC) frames. Notable models are the single component rotational spring element and the super element joint model that can effectively capture the governing inelastic mechanisms under severe ground motions. Even though both models have been extensively calibrated and verified using quasi-static test of joint sub-assemblages, a comparative study of the inelastic seismic responses under nonlinear time history analysis (NTHA) of RC frames has not been thoroughly evaluated. This study employs three hypothetical case study RC frames subjected to increasing ground motion intensities to study their inherent variations. Results indicate that the super element joint model overestimates the transient drift ratio at the first story and becomes highly un-conservative by under-predicting the drift ratios at the roof level when compared to the single-component model and the conventional rigid joint assumption. In addition, between these story levels, a decline in the drift ratios is observed as the story level increased. However, from this limited study, there is no consistent evidence to suggest that care should be taken in selecting either a single or multi component joint model for seismic risk assessment of buildings when a global demand measure such as maximum inter-storey drift is employed in the seismic assessment framework.

Keywords: beam-column joint; reinforced concrete; super-element joint model; scissors joint model; seismic analysis

1. Introduction

In the present wake of performance based earthquake engineering (PBEE), the assessment of the vulnerability of a structural system to withstand seismic forces has been addressed by employing probabilistic models to quantify the level of uncertainties associated with the estimation of the seismic demand imposed on a structure given an intensity of ground shaking (Liel *et al*. 2009). In order to reduce the dispersion in the modelling uncertainties associated with structural components, past researches have emphasized the importance of modelling the behaviour of beam-column connections in a bid to predict the seismic demand efficiently, Park (2010). Moehle and Mahin (1991) noted that beam-column joints of reinforced concrete (RC) buildings, typical of the pre 1970 regime, have exhibited significant strength and stiffness deterioration during earthquakes and may lead to the global

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collapse of the structural system. The joint region of these buildings is believed not to be adequately confined and lacks the capacity design method, a concept that most seismic codes have emphasized (Park *et al.* 1995). This may lead to undesirable failure modes such as joint failing in pure shear (Clyde *et al.* 2000, Pantelides *et al.* 2002), or in combination with either yielding of beam reinforcement (Karayannis *et al.* 2008, Masi *et al.* 2013), column reinforcement (Barnes *et al.* 2008, Sanchez *et al.* 2009) or both (Beres *et al.* 1992). Hence, studies on major retrofitting strategies which can improve the seismic performance of beam-column connections, mainly exterior joints, has been a thematic research area of study (Li *et al.* 2015, Bansal *et al.* 2016, Li *et al.* 2015). Also, most experimental test programs have aimed at evaluating the effect of some major physical design parameters, notably the joint aspect ratio (Wong 2005, Dhakal *et al.* 2005), beam longitudinal reinforcement level (Hakuto *et al.* 2000, Alire 2002), joint transverse reinforcement ratio (Hamil 2000, Lee and Lee 2000) and column axial load (Pantelides 2002, Masi *et al.* 2013), on joint shear and deformation capacities. The correlation between these influential parameters and joint shear strength has been extensively evaluated in the works of Kim and LaFave (2007) and Jeon (2013). However the role of column axial load on joint shear strength remains debatable across various experimental studies. Some studies have reported either a positive correlation (Paulay *et al* 1989, Clyde *et al.* 2000, Pantelides *et al.* 2002), negative correlation (Vollum 1998) or statistically insignificance (Kurose et. al 1988, Pantazopoulou and Bonacci 1992) of column axial load on joint shear capacity. Even though modern seismic design codes such as EC8 (2004) considers the effect of column axial load in estimating joint shear strength, current researches (Kim and LaFave 2009, Jeon 2013) which have assembled a large experimental database of tested joint sub-assemblages, have argued that the impact of column axial load can be neglected without losing the reliability and strength of the suggested empirical joint shear models. More so, most researches have stressed that the effect of column axial load on joint shear strength is highly influenced by either the failure mode (Masi *et al.* 2013), shear demand level (Fu *et al.* 2000) and the column-beam flexural capacity ratios (Kim and LaFave 2007).

Shear and bond-slip deformation are primarily the inelastic mechanisms that beam-column joint experience under strong ground motions (Celik and Ellingwood 2008, Favvata and Karayannis 2014). It is believed that the concentration of these mechanisms, primarily the shear deformation within the joint, may lead to early strength deterioration of members, hence not allowing for full flexural capacity of the framing members to be utilized (Zhou and Zhang 2014). Kien *et al.* (2012) noted that the joint shear demand may be a much more significant metric for assessing the seismic performance of RC beam-column connections, rather than the limiting requirement of column-beam flexural capacity ratios as per modern seismic code provisions. Kwak *et al.* (2004) also emphasised that non- linear dynamic analysis of RC frames which considers the effect of anchorage slip, may account for about 50% of the total deformation. Researchers (Alath and Kunnath 1995, Kwak *et al.* 2004, Shin and LaFave 2004, Favvata *et al.* 2008) have attempted to simulate these mechanisms by providing analytical models that can be easily incorporated into computer simulations of RC frames. The behaviour of the joint region under seismic forces is now more popularly simulated by use of single component models, that is, rotational springs (Birely *et al.* 2012) ,whose constitutive relations can easily by calibrated experimentally or defined analytically. Favvata and Kayarannis (2014) noted when an appropriate behavioural constitutive joint response envelope is defined the seismic demand of RC frames can be accurately estimated. Recently (Borghini *et al.* 2016) accounted for joint damage in vulnerability analysis of RC frames by introducing a link element positioned at the beam-column interface. However the cyclic degradation in strength and stiffness of RC joints was not explicitly account which may lead to overestimation of the lateral drift capacity.

Theiss (2005) used a rotational spring with rigid links to assess the impact of joint response on a case study reinforced concrete frame. It was concluded that the inclusion of the joint model in nonlinear time history analysis (NTHA) may have significant impact on the maximum inter-story drift demand levels. Similarly findings were noted in previous works of Calvi *et al.* (2002) and Pampanin *et al.* (2003) that assessed the relevance of joint shear damage by employing single component joint models in seismic risk assessment. Celik and Ellingwood (2008) having assessed the performance of four computer simulation joint modelling schemes, concluded that the rotational spring with rigid end zones approach of characterizing joint behaviour, produced the best correlation between the simulated base shear-drift responses compared to observed experimental responses. The study further proposed a constitutive relation that can be used to implement this approach, and later generated fragility functions as part of the seismic risk assessment of RC frames in regions of low to moderate seismicity. Park (2010) sequentially performed both nonlinear static and dynamic analysis on two prototypes RC building and concluded that for unreinforced joints in which the shear mode of failure precedes beam reinforcement yielding, the inclusion of joint flexibility using a single-component rotational spring model is considerable and essential for simulating seismic responses. Hassan (2011) in his assessment of seismic vulnerability of unreinforced exterior RC joints compared the approach of localizing all the inelastic mechanisms in one single rotational spring to one that decouples the shear and bond deformation by providing two springs, i.e., a rotational spring and a bar-slip spring. It was indicated that both approaches were able to predict the maximum shear strength of tested sub-assemblages, with a marginal variation in the estimation of the post peak drift capacities. However, in assessing the adequacy of the conventional assumption of modelling the joint region as rigid, the ultimate shear capacities and pinching behaviour of tested sub-assemblages were not represented appropriately. This emphasizes the need to incorporate joint flexibility in the seismic risk assessment of RC frames. Park and Mosalam (2013) developed analytical and semi-empirical joint shear strength models by using an experimental database of unreinforced exterior joint, and adopting recommendation of ACI 352-02 and ASCE 41, extended the developed models to account for interior and roof joints. Using a rotational spring element to define the finite joint region, these shear strength models was used to develop a constitutive relation for the envelope curve, which was evaluated on three hypothetical RC frames in order to explore the degree of flexibility unreinforced joints impose in generating fragility functions. The RC frames showed an increase in maximum inter-story drift caused by joint rotation, propagating as the spectral acceleration increases. This proves the relevance of modelling beam-column joints in earthquake simulation and vulnerability assessment of non-seismically designed reinforced concrete buildings.

Moreover, as evident from lessons learnt from past earthquakes and researches that have explicitly account for joint damage in seismic risk of RC frames, the anticipated failure mode may change from an expected floor mechanism to an undesirable failure mechanism. This observation has been numerically investigated by Favvata *et al.* (2008) on an eight storey hypothetical RC frame, where development of plastic hinges in beams, could not be achieved when strength and stiffness degradation of the joint region was explicitly modelled. Consequently, they concluded that ductility demand especially in columns of open base floors, is reduced due to joint deterioration.

According to some researchers (Theiss 2005, Hassan 2011), even though the rotational spring approach of simulating joint response has been promising in the past decade, one notable drawback has been its inability to capture the joint kinematics, such as simulating the horizontal translation between the upper and lower columns framing into the joint element. In the view of the present authors, there is a genuine concern on the ability of a single component beam-column joint element to adequately simulate the different expected inelastic failure mechanism. Hence, the need to develop joint models that can explicitly capture more realistic inelastic mechanisms (anchorage, shear and interface shear transfer deformations) by adopting a multi-component joint element formulation to simulate more realistic behaviour of beam column joints is warranted.

One of the early works on multi-component modelling of the joint region was carried out by Biddah and Ghoborrah (1999) where two rotational springs connected in series was proposed to represent joint shear distortion and anchorage failure. A trilinear and bilinear representation of the load-deformation behaviour was employed to simulate the shear and bar-slip deformations respectively. This model was able to account for hysteretic strength degradation in its formulation, but could not admit the accelerated stiffness deterioration or pinching which reduces the energy dissipation capacity of the joint element. Youseff and Ghoborrah (2001) later proposed that two diagonal translational springs could be employed to represent joint shear behaviour and three translational springs at each phase of the joint element to simulate bar-slip deformation appropriately. Lowes and Altoontash (2003) proposed a thirteen element beam column joint model that can efficiently and accurately predict joint response for various beam-column joint configurations and geometry. This initially proposed model has since been subjected to extensive calibration and modifications. Altoontash (2004) provided a simplification by simulating the joint behaviour of two-dimensional reinforced concrete frames, with five components that required a one dimensional constitutive material model for its definition. Mitra and Lowes (2007) later made some modifications in the element definition of the parent model and proposed a framework for the calibration of the joint shear panel component as well as the bar-slip springs. All these modifications have evolved with the prime aim of providing a good fit of the observed responses of experimentally tested beam-column joint sub-assemblages that are usually subjected to quasi-static reverse cyclic loading. (Zhang *et al.* 2016) has made significant modification of the parent model, by removing the column bar-slip springs and replacing the beam bar-slip springs with zero length elements. By modelling the anchorage failure with zero length rotational springs, the quantification of the seismic demand for complex beam cross-sections with varying bond-slip behaviour can be performed; an inherent limitation of the parent model. Other works on super element formulation of joint region, have aimed at reducing the number of calibration parameters needed to define the constitutive hysteretic behaviour under cyclic loading. Chao-Lie and Bing (2015) proposed a nine component super-element model that uses the modified *"Bouc-Wen-Baber-Noori (BWBN) model"*, a one dimensional hysteretic law to define the load-deformation response. This modified hysteresis model, *BWNB,* was developed to characterize the strength and stiffness degradation including pinching effect of RC joint with limited transverse reinforcement (Piyali and Bing 2013). Even though this model showed a fairly good agreement with experimental results of non-ductile exterior and interior sub-assemblages, the extension to other types of joint configurations, such as joints with transverse beams or slab-beam-column joint subassemblies is of concern since calibration of the analytical modelling parameters excluded these types.

The thirteen element beam column joint model under nonlinear time history analysis of RC frames has not given much great attention. This has been attributed to the fact that multi-component joint models have the possibility of causing numerical divergence during frame analysis, Park (2010). More so, there is the perception that modelling demands in terms of calibration of each spring element can be computationally expensive, and may not assure accuracy of the analysis. This study focuses on exploring the impact and differences in seismic demands of the aforementioned joint models implemented in the nonlinear time history analysis of three case study RC frames subjected to a suite of historical ground motions. In other words, the question of whether the multi-component joint model can greatly influence the estimation of the structural performance is addressed by comparing it with the seismic demand of both rigid and single component joint models.

2. Methodology

The following provides a framework for assessing the variations in the seismic demand of two explicit joint models investigated at quasi-static analysis of structural components (sub-assemblages) as well as nonlinear time history analysis of RC frames. The null hypothesis is given as that, inclusion of different joint models in frame analysis does not matter in the estimation of nonlinear seismic demand, quantified here by using the inter-story drift ratio. Three different hypothetical RC frames conditioned on their natural vibrational periods are subjected to nonlinear time history analysis at various classes of ground motion intensities to gain knowledge into whether and under what conditions this assumption of equivalence holds. The magnitude, source to site distance and epsilon (if available from seismic hazard disaggregation) are primarily the metrics that quantify the degree of ground shaking and subsequent selection of records, Iervolino (2004). The classes of records, index, R-1, R-2 and R-3 consists of 10 historical ground motions with magnitude ranging from; 5-5.5 to represent low intensities ,5.5-6.5 to represent moderate intensities, 6.5-7.5 to represent high intensities, with all having a maximum source to site distance of 50km. These records have been matched to the PEER NGA-West2 Spectrum with strike-slip fault type and magnitude conditioned on the mid-point on the selected in each class. Only one component of the horizontal motion for each record is selected for nonlinear time history analysis.

The conventional rigid joint assumption is referred in here as centreline model. Explicit joint region representations which admit strength and stiffness loss through either a single rotational spring component (scissor model) or a multi component idealization (super element model) are also considered. These three different joint models are incorporated into each of the three

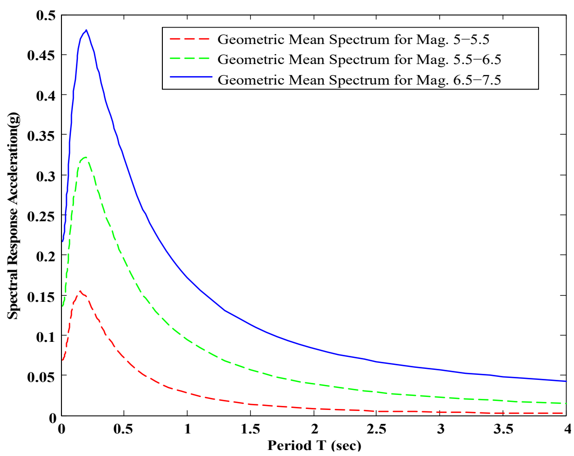


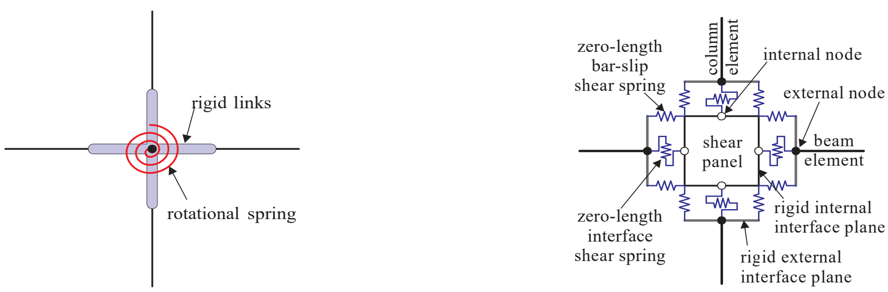
Fig. 1 Target Response Spectrum for various class of record sets

hypothetical RC frames, thus producing nine case study RC frames. Each of these frames are subjected to the three classes of records to yield twenty seven combinations and finally 270 runs of nonlinear dynamic analysis. The centreline model serves as the baseline model from which relative ratios of the inter-story drift ratio for each joint element formulation is then computed from a record set given a particular prototype RC frame. The mean of these ratios in each record set is then computed alongside the sample standard deviation. A two-sided hypothesis test is then performed on each joint model type with a particular record set. Finally results are pooled together to addresses the issue of whether it matters in the selection of a joint model scheme in estimating structural demand under dynamic loading.

2.1 Joint modelling validation and RC frame simulation

The use of analytical joint models that can be implemented in computer simulations requires the definition of the constitutive (shear stress-strain) relations that evolves under cyclic loading. One primary source of such relationship is by subjecting sub-assemblages to quasi-static reverse cyclic loading and monitoring the key points at which significant stiffness changes appear throughout the loading history. Celik and Ellingwood (2008) collected 26 beam-column joint test and suggested ranges of joint shear stress-strain values which can be used in defining the backbone curve for the joint panel zone. Kim and LaFave (2009) assembled a database of 341 test specimen of different configurations and performed a Bayesian parameter estimation on the major design parameters that are needed to define the monotonic backbone curve. This shear stress-strain relation excluded the effect of column axial load, which can significantly influence the mode of failure as noted by (Masi *et. al* 2013).The developed unified stress-strain equation from their research was adopted in this research to define the backbone curve for the panel zone of both joint models. In order to account for bond slip, a reduction in the moment capacities of the beams framing into the joint or using recommendations provided by FEMA 356(2000) to modify the strength of the longitudinal reinforcement steel located in the plastic hinge zone of the adjoining beams and column can be adopted. A strength reduction factor of 0.5 on the moment capacity of the beam framing into the joint was selected in order to simulate anchorage failure mechanism (Bracci *et al*. 1994, Jeon *et al.* 2012).The interface shear transfer failure mechanism was modelled assuming elastic and stiff shear spring elements (Mitra and Lowes 2007).

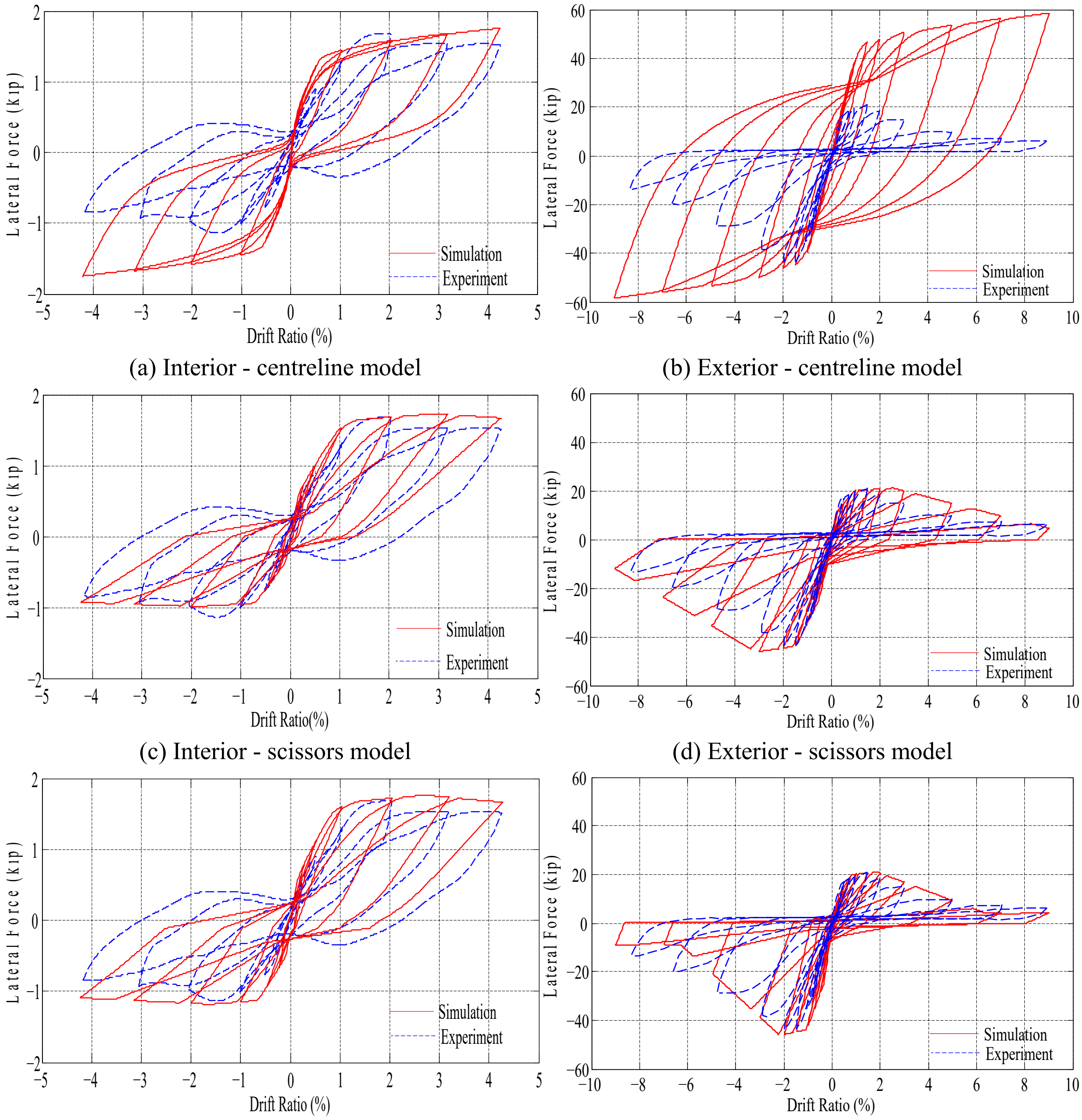
Fig. 2 shows a schematic diagram of the two joint models under investigation. The computer program OpenSees (McKenna, 2010), an open source computational platform was selected for



(a) Scissors joint model (Alath and Kunnath 1995)

(b) Super element joint models (Lowes and Altoontash 2003)

Fig. 2 Analytical joint modelling schemes



(a) Interior - centreline model (b) Exterior - centreline model

(c) Interior - scissors model (d) Exterior - scissors model

(e) Interior - super element model (f) Exterior - super element model

Fig. 3 Experimental versus simulated base shear-drift response for the various analytical joint modelling schemes

implementation of the joint models and performing the dynamic analyses of the RC frames. This program contains a wide range of material models and elements that can be directly applied to the beam-column joint element formulation, and also considers material and geometric nonlinearities that beams and column experience. The *"Pinching4"* material model was used to simulate the behaviour of the shear panel zone of zero length rotational spring of the scissors and super element model respectively. This hysteretic model is able to capture strength and stiffness degradation as well as unload-reload paths under cyclic loading using a tri-linear envelope. In addition to the element definition of the scissors model, four rigid links were placed within the finite area of the joint to take into account the flexural rigidity of the joint.

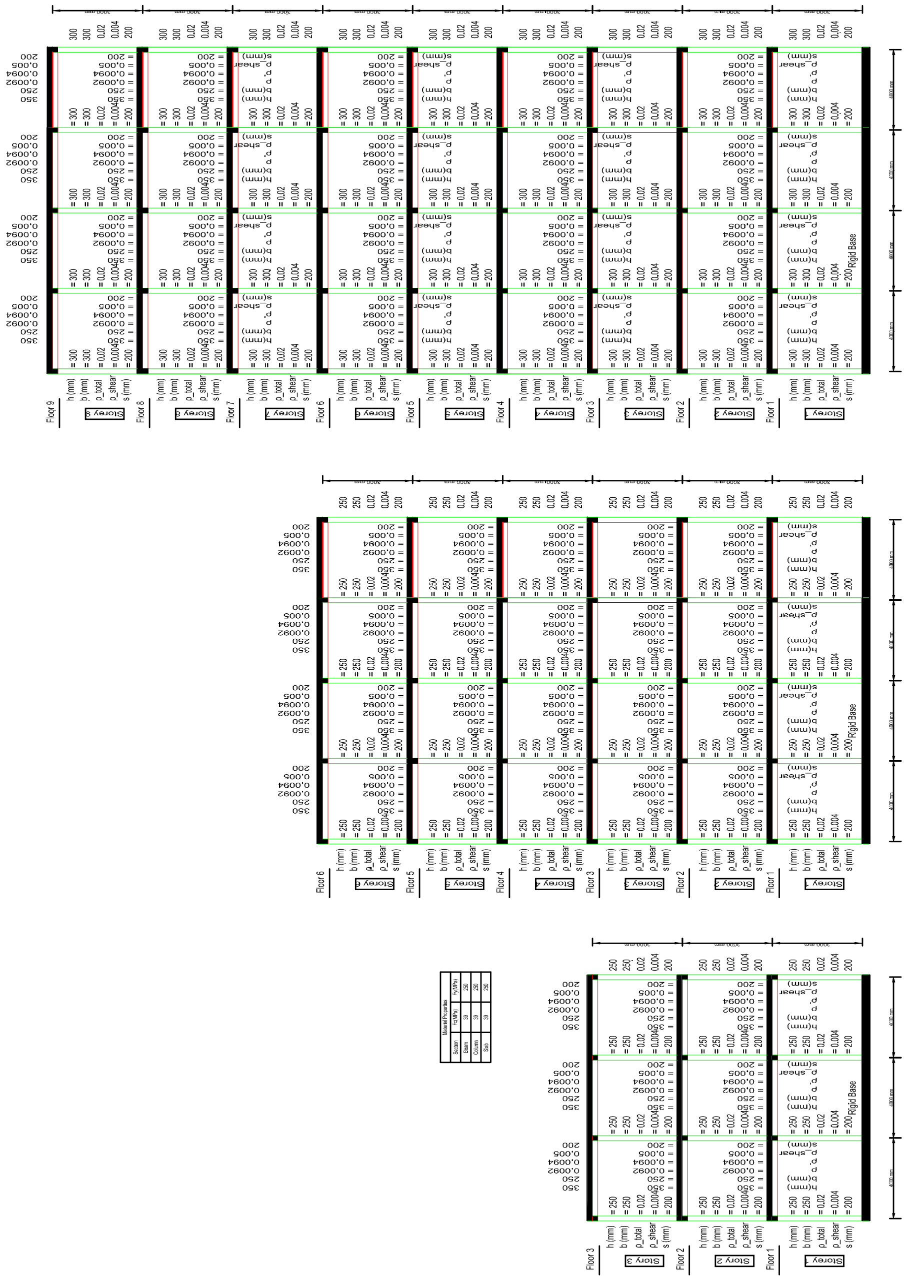


Fig. 4 Geometry of hypothetical RC frames

In validating the accuracy of the selected joint shear strength model, experimental results of an interior sub assemblage of (Bracci *et al.* 1994) and that of an exterior joint of substandard details from (Pantelidis *et al.* 2002) were employed. The analytical and experimental base shear-lateral drift responses of sub-assemblies are shown below in Fig. 3.

A comparative assessment of the hysteretic responses under simulated and reported experimental results (Fig. 3), confirms that the scissors and super element joint modelling schemes are able to fairly capture the rate of energy dissipation, as well as the strength and stiffness degradation, better than the centreline joint model. This emphasizes that the underlining assumption of reinforced concrete moment resisting frames that are not seismically designed to be rigidly connected at the joint, as invalid because it tends to over-estimate and under-estimate the stiffness and drift ratios respectively.

Nonlinear time history analysis was performed on three, six and nine story hypothetical RC frames (Fig. 4) that possessed fundamental periods of vibration of 0.56, 1.14, 1.96 seconds respectively. These frames were intended to represent an internal frame of an office building with similar symmetric floor plans with 4 m bay spacing. The design was of a weak column-strong beam approach and lacked seismic action considerations. The characteristic compressive strength and yield strength of reinforcing bar were 30 MPa and 250 MPa respectively. Gravity loads from tributary floor areas were idealised as uniformly distributed loads of magnitude 15 KN/m on beam members and appropriate lumped masses assigned at nodes connecting beams and column elements. It is worth noting that for the super element joint model which requires the definition of four nodes, the total lumped mass was distributed equal among these external nodes. It is also worth noting, that for RC frames with explicit modelling of joint region (scissors joint model), there were marginal increase in their first modal periods (0.58, 1.20, 2.03) when compared with the centreline model. However for the super element model, due to the presence of several springs, their first modal period is significantly affected and becomes very large. Displacement based beam-column element with distributed plasticity, each with five Gauss- Legendre integration points was used to model the frame elements. The concrete was modelled using the *"Concrete02"* material object, as well as considering the increase in the strength of concrete due to confinement by adopting the modified Kent-Park material model. The constitutive behaviour of the steel reinforcement was modelled with the uniaxial *"steel02"* material object which employs a bilinear response envelope and Menegotto-Pinto (1973) curves to describe the cyclic behaviour as well as account for bauschinger effect. Fibre modelling was used to integrate the different stress-strain responses of the reinforcing steel, confined and unconfined concrete as well as consider the spread of inelasticity along the length and across the section of the member.

2.2 Analysis

A structural response quantity that is closely related to its degree of damage is required to assess the vulnerability of buildings to seismic action (Sozen 1981). The inter-story drift ratio has been one of the most widely used damage indices for assessing the seismic performance of RC frame components and is used here as the engineering demand parameter. Each of the considered hypothetical RC frames that incorporate the three joint models has been analysed by running the three classes of record sets. The post processing phase consists of obtaining, $IDR_{if|j,k,l}$, which represents the peak inter-story drift at a particular story, f, for record, i, belonging to a particular class of record, j, for RC frame, k, with joint model, l. These responses were monitored at critical floor levels in order to address the hypothesis that the structural demand is equal irrespective of the joint model used. The peak in time drift ratio at the first, roof and at any story level was used in this study. To test this equivalency statistically, a two phase process was adopted.

In the first phase, the inter-story drift ratio for the joint models in the treatment group(scissor and super-element joint models) were normalized by using the demand from the centreline model(control group) to investigate the degree to which it underestimates or overestimate the responses from the conventional approach. This parameter is given as

$$\alpha = \frac{IDR_{ex(i)}}{IDR_{im}} \tag{1}$$

where IDR_{im}, is defined in here exclusively as the drift due to implicit modelling (centreline model), while $IDR_{ex(i)}$, is the drift due to explicit modelling; i is 1 for scissor model and 2 for super element model.

In the second phase, a ratio of the estimated means of the normalized quantity, α, in a particular class of record set, for the scissors and super-element joint model, is then defined as Z

$$Z = \frac{N_1}{N_2} \tag{2}$$

Where N_1 and N_2, is defined as the mean of the normalized drift responses of the scissors and super element joint model respectively within a particular record set. This quantity is desirable, because it can be used to address the issue of, under what conditions the assumption of equivalence in engineering demand parameter holds.

A two sided hypothesis test was made on the null, H_o, defines as

H_o: the mean of the normalized responses are equal

The theoretical lognormal probability density function is typically used to describe the distribution of drift responses in vulnerability assessment, (Shome1 1999, Iervolino 2004 and others).In the generation of fragility functions for lightly reinforced beam-column joints, Piyali and Bing (2014) , showed how consistent and efficient this probability model can be used to fit an empirical cumulative distribution of observed responses. Under this assumption of log-normality in the peak drift responses with an unknown standard deviation, the test statistics computed is required to follow a student-t distribution, Rice (2007). This test statistics is calculated as

$$t = \frac{\ln(N_1) - \ln(N_2)}{B_{1-2}} = \frac{\ln(Z)}{B_{1-2}} \tag{3}$$

Where $B_{1\text{-}2}$ is the standard error of Z, and can be estimated as

$$B_{1-2} = S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \tag{4}$$

$$S_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 - n_2 - 2}} \tag{5}$$

Where s, s_2, are the standard deviation of the natural logarithms of α in the scissors and super-element joint models; S_p is the pooled sample standard deviation of the logarithms of α; n_1 and n_2

are the number of records in each record set. The number of degrees of freedom for the student-t distribution is given as (n_1+n_2-2). Typical values used in here are 18, 58 and 178 depending on the chosen pair of normalized responses being compared.

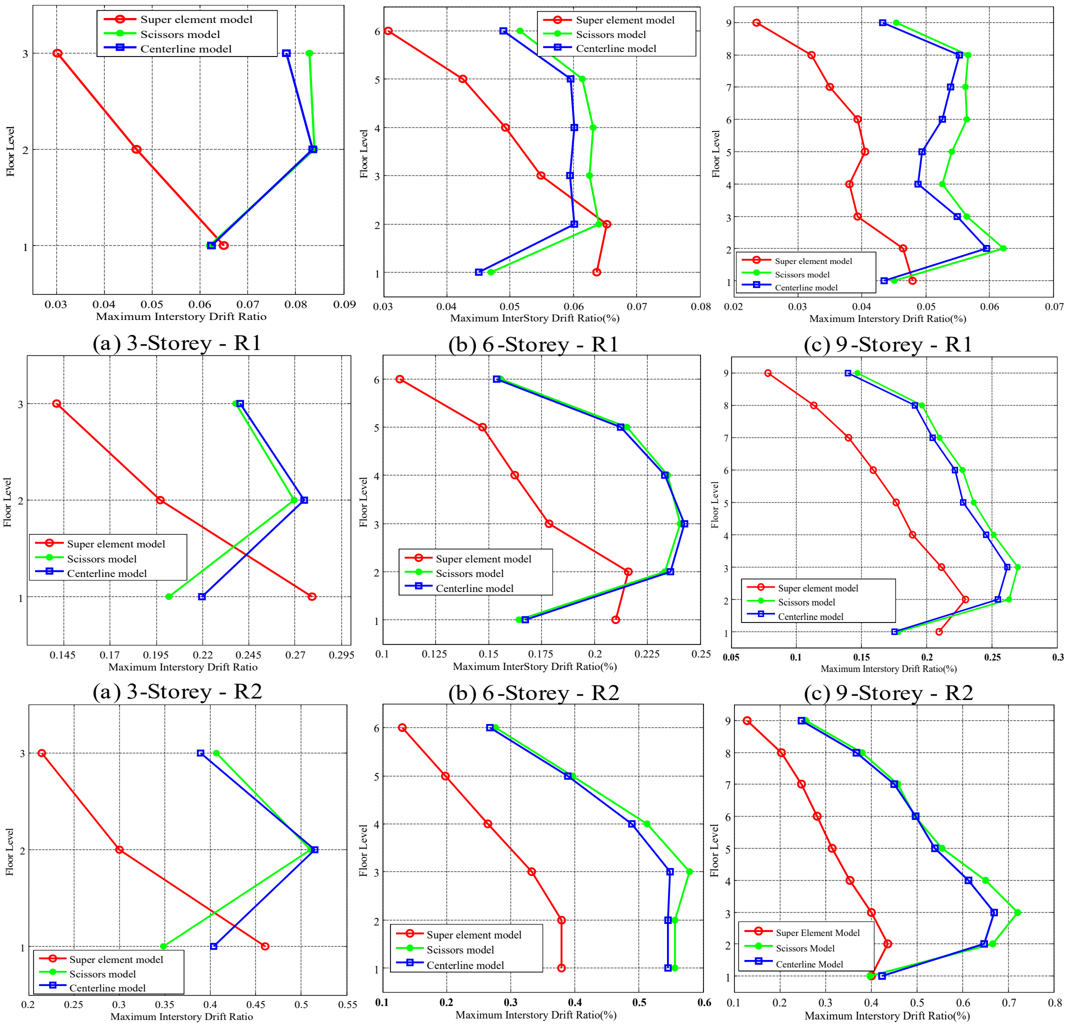
Two statistical significance level of 5% and 10% was adopted in the present study in order to determine whether to accept the hypothesis that the mean of the normalized drift responses from the scissors and super-element joint model are the same. This value corresponds to the probability of making a type 1 error; thus rejecting the null hypothesis when it is in fact true. Given a two-sided test, with the selected level of significance under a student-t distribution with 18 degrees of freedom, the region of acceptance will correspond to ±2.101 and ±1.734 standard error, $B_{1\text{-}2}$, away the mean, which is centred at zero. For the 58 degrees of freedom under a student-t distribution, these quantiles corresponded to ±1.67 and ±2.002 standard deviations away from the expected mean of the sampling distribution. A Gaussian distribution was assumed in the case of 178 degrees of freedom, and the corresponding test statistics at 5% and 10% significance level is ±1.64 and ±1.96 respectively.

From Eq. (3), *t*, the test statistics, is computed and compared to the ranges of acceptance under a given significance level in order to test the hypothesis that the responses from the two joint models investigated are equal under a particular record set. For example, in Table 1, where the centreline model is compared to the scissors joint model for a three story reinforced concrete frame subjected to ground motion characterized as having moderate intensities in this study, *t* is computed as -0.36. With this value being in the range of ±2.101 and ±1.734, the equality of the means of the peak in time drift ratio at the first floor for both joint models may be accepted at the 95% and 90% confidence level for RC frames subjected to moderate ground motion intensities. For cases where the null hypothesis is rejected, a single and double asterisk is used to signify rejection for the 95% and 90% confidence level respectively.

3. Results and discussion

In a preliminary attempt to investigate the impact of the inclusion of the joint models in RC frame simulation, and also the extent to which it overestimates or underestimate the peak in-time drift ratios, the averages of the story-specific peak drift in each class of record (R-1, R2 and R-3) for a particular building configuration was computed. Fig. 5 summarizes the profile of this quantity along the frame height. Significant differences in the mean of the peak drift ratios were observed at the first and roof level, and as such results presented, lay much emphasis on their seismic demand. One other observation was that, whilst the path of drift ratio for the centreline model and the scissors joint model seem to follow the same line, the super element joint model exhibited a decline in the drift ratio from the second story, which propagates monotonically to the roof level. The test of equivalency of the peak inter-story drift ratio for the joint models that are conditioned on their modal period and the intensity of ground shaking were investigated at the first and roof levels as well as the maximum responses irrespective of the floor level.

The tables below are used to provide summaries of the mean and standard deviation of the normalized responses of the scissors and super element joint models. Considering the duality between hypothesis testing and the establishing of confidence intervals, these quantities were used to assess the degree to which the joint models deviate away from the centreline model. The results are pooled for each hypothetical RC frame given a class of record set, and are shown in the last row and column of each table.



(a) 3-Storey - R1 (b) 6-Storey - R1 (c) 9-Storey - R1

(a) 3-Storey - R2 (b) 6-Storey - R2 (c) 9-Storey - R2

(a) 3-Storey - R3 (b) 6-Storey - R3 (c) 9-Storey - R3

Fig. 5 maximum inter-story drift ratio profile for the three RC frames

3.1 First story

In two of the 32 cases of mean normalized drift responses in Table 1, the hypothesis that the equality of the seismic demand of RC frames that includes joint models, can be rejected at the 95% confidence level when compared to the centreline model. In order to assess the degree of equivalence at the explicit joint model level, the ratio of the normalized drift responses as discussed was used. Table 2 shows its distribution for the range of RC frames and record set considered. None of the cases investigated resulted in rejecting the null hypothesis at the 5% significance level. However in three out of 16 cases, the equivalency of the estimates of the peak drift ratios for the scissors and super element joint model may be rejected at the 10% significance level. Also the observed mean of the normalized drift ratio in Tables 1 and 2, is distributed on either side of unity. For the normalized scissors-centreline model comparison, their mean ranges from 1.08 to as low as 0.93, whereas from the super element-centreline model comparison it ranges from 1.44 to 0.65.

Table 1 Ratio of mean and standard deviation of the normalized drift responses at the first floor level

			CM-R1	CM-R2	CM-R3	CM-P
Three Story	SM	Mean	1.00	0.93	0.93	0.95
		SD	0.12	0.20	0.33	0.22
	SEM	Mean	1.11	1.44	1.22	1.26
		SD	0.15	0.23	0.33	0.24
Six Story	SM	Mean	1.04	0.98	1.02	1.02
		SD	0.12	0.14	0.23	0.27
	SEM	Mean	1.44	1.21	0.65	1.10
		SD	0.14**	0.19	0.31	0.25
Nine Story	SM	Mean	1.08	1.02	0.96	1.02
		SD	0.14	0.19	0.16	0.25
	SEM	Mean	1.17	1.33	0.98	1.16
		SD	0.13	0.18	0.22	0.25
Pooled	SM	Mean	1.04	0.98	0.97	1.00
		SD	0.08	0.11	0.15	0.14
	SEM	Mean	1.24	1.33	0.95	1.17
		SD	0.09**	0.16*	0.17	0.14

*CM-R1: Centreline model for record set 1; CM-R2: Centreline model for record set 2; CM-R3: Centreline model for record set 3; CM-P: Centreline model for pooled record set; SM: Scissors model; SEM: Super element model; SD: Standard deviation; ** and * signify the rejection of the null hypothesis at 5% and 10% significance levels respectively

Table 2 Ratio of mean and standard deviation of the means of Z at the first floor level

			SM-R1	SM-R2	SM-R3	SM-P
Three Story	SEM	Mean	1.10	1.54	1.35	1.33
		SD	0.15	0.22*	0.32	0.23
Six Story	SEM	Mean	1.38	1.24	0.64	1.09
		SD	0.14	0.19	0.31	0.25
Nine Story	SEM	Mean	1.09	1.30	1.00	1.13
		SD	0.11	0.18	0.21	0.24
Pooled	SEM	Mean	1.19	1.36	1.00	1.18
		SD	0.08*	0.16*	0.17	0.14

*SM-R1: Scissors model for record set 1; SM-R2: Scissors model for record set 2; SM-R3: Scissors model for record set 3; SM-P: Scissors model for pooled record set; SEM: Super element model; SD: Standard deviation; ** and * signify the rejection of the null hypothesis at 5% and 10% significance levels respectively

3.2 Roof level

A visual inspection in Fig. 5 shows that the average of the peak in-time drift ratio for the super element model decreases appreciably when compared to the scissors and the centreline models at the roof level. In summary, in 13 out of 16 cases at the 5% significance level as well as all cases for the 10% significance level, the equivalency of the normalized super element-centreline model peak drift ratio may be rejected. Hence a better approach to assess the impact of the super element joint model on the seismic demand of the hypothetical frames under study is by establishing confidence intervals on the population parameter (the mean of normalized responses). Under the assumption of a student t-distribution the expected decrease in terms of the peak drift ratio for the super element joint model at the roof level can range from 16%-70% of the drift demand of RC frames modelled under the conventional centreline approach of frame connectivity. However from Table 3, for the scissor-centreline model cases, the equivalency of the peak drift ratios can be accepted at both the 95% and 90% confidence level. It should be interpreted that on average, in 90% or 95% cases, we expected the peak drift ratio of the scissors joint model and centreline model to be equal.

Observing that the scissors joint model dynamic responses approximating the centreline model, we expect the drift demand of the super element joint model to be less than the scissors model. The means of the ratios range from 0.4 to 0.68. On average, using the pooled set of record, a decrease in the range of 20%-70% is expected when compared to the scissors joint model. It is worth noting

Table 3 Ratio of mean and standard deviation of the normalized drift responses at the roof floor level

			CM-R1	CM-R2	CM-R3	CM-P
Three Story	SM	Mean	1.06	1.01	1.07	0.95
		SD	0.12	0.18	0.31	0.22
	SEM	Mean	0.40	0.68	0.60	0.56
		SD	0.12**	0.22*	0.29*	0.23*
Six Story	SM	Mean	1.05	1.01	1.03	1.03
		SD	0.12	0.17	0.24	0.20
	SEM	Mean	0.64	0.67	0.47	0.60
		SD	0.12**	0.22	0.27**	0.21**
Nine Story	SM	Mean	1.05	1.07	1.05	1.06
		SD	0.14	0.22	0.25	0.21
	SEM	Mean	0.55	0.59	0.52	0.55
		SD	0.12**	0.22**	0.27**	0.22**
Pooled	SM	Mean	1.06	1.03	1.05	1.04
		SD	0.10	0.13	0.16	0.12
	SEM	Mean	0.53	0.64	0.53	0.57
		SD	0.10**	0.14**	0.20**	0.14**

*CM-R1: Centreline model for record set 1; CM-R2: Centreline model for record set 2 ; CM-R3: Centreline model for record set 3; CM-P: Centreline model for pooled record set; SM: Scissors model; SEM: Super element model; SD: Standard deviation ; ** and * signify the rejection of the null hypothesis at 5% and 10% significance levels respectively

Table 4 Ratio of mean and standard deviation of the means of Z at the roof floor level

			SM-R1	SM-R2	SM-R3	SM-P
Three Story	SEM	Mean	0.38	0.67	0.58	0.54
		SD	0.14**	0.21*	0.28*	0.22**
Six Story	SEM	Mean	0.61	0.66	0.46	0.58
		SD	0.14**	0.22*	0.28**	0.20**
Nine Story	SEM	Mean	0.53	0.55	0.50	0.53
		SD	0.14**	0.21**	0.27**	0.21**
Pooled	SEM	Mean	0.50	0.63	0.51	0.55
		SD	0.10**	0.14**	0.19**	0.13**

*SM-R1:Scissors model for record set 1; SM-R2: Scissors model for record set 2; SM-R3: Scissors model for record set 3; SM-P: Scissors model for pooled record set; SEM: Super element model; SD: Standard deviation; ** and * signify the rejection of the null hypothesis at 5% and 10% significance levels respectively

Table 5 Ratio of mean and standard deviation of the normalized drift responses

			CM-R1	CM-R2	CM-R3	CM-P
Three Story	SM	Mean	1.03	0.99	1.03	1.02
		SD	0.11*	0.17	0.33	0.21
	SEM	Mean	0.77	1.11	0.95	0.94
		SD	0.14	0.21	0.33	0.23
Six Story	SM	Mean	1.05	1.00	1.04	1.03
		SD	0.10	0.14	0.24	0.24
	SEM	Mean	1.02	0.86	0.63	0.84
		SD	0.13	0.18	0.32	0.23
Nine Story	SM	Mean	1.09	1.03	1.06	1.06
		SD	0.16	0.18	0.19	0.27
	SEM	Mean	0.85	0.95	0.66	0.82
		SD	0.13	0.17	0.23*	0.26
Pooled	SM	Mean	1.05	1.01	1.04	1.03
		SD	0.08	0.10	0.16	0.14
	SEM	Mean	0.88	0.97	0.75	0.87
		SD	0.09	0.11	0.17	0.14

*CM-R1: Centreline model for record set 1; CM-R2: Centreline model for record set 2; CM-R3: Centreline model for record set 3; CM-P: Centreline model for pooled record set; SM: Scissors model; SEM: Super element model; SD: Standard deviation; ** and * signify the rejection of the null hypothesis at 5% and 10% significance levels respectively

that the underline difference in the two joint formulations is the explicit localization of bond deterioration, by using bar-slip components in the super element joint model. In addition to this, the effect of anchorage failure which is represented by using a reduced load-deformation response

Table 6 Centre Ratio of mean and standard deviation of the means of Z

			SM-R1	SM-R2	SM-R3	SM-P
Three Story	SEM	Mean	0.75	1.11	0.95	0.94
		SD	0.14*	0.21	0.33	0.23
Six Story	SEM	Mean	0.97	0.86	0.61	0.81
		SD	0.13	0.18	0.32	0.23
Nine Story	SEM	Mean	0.78	0.93	0.63	0.78
		SD	0.13*	0.17	0.23*	0.26
Pooled	SEM	Mean	0.83	0.97	0.73	0.84
		SD	0.09**	0.11	0.18*	0.14

*SM-R1: Scissors model for record set 1; SM-R2: Scissors model for record set 2; SM-R3: Scissors model for record set 3; SM-P: Scissors model for pooled record set; SEM: Super element model; SD: Standard deviation; ** and * signify the rejection of the null hypothesis at 5% and 10% significance levels respectively

envelope in the scissors joint model, is absent in the super element joint model. Thus the variation in global deformability of RC frames using the two approaches can be attributed to the appropriateness of these bar-slip springs to simulate the bond stress distribution which depends on the element damage state, as suggested by Paulay *et al.* (1978).

3.3 General case

The maximum inter-story drift ratio observed in any story of the building is basically the quantity is used in fragility assessment of RC frames. Table 5 shows that the estimate of this quantity in about 94% of the cases are equal for the explicit joint models when compared to the centreline approach at the 10% significance level. However upon comparing the scissors joint model with the super element model in Table 6, for low intensity ground motions, the equivalency cannot be accepted at both the 5% and 10% significance level. However for records in the moderate magnitude range, the hypothesis of that the responses of the super element joint model being equal to the scissors and centreline model can be accepted at both the 90% and 95% confidence level.

5. Conclusions

Three joint modelling schemes used for simulating the behaviour of joint response under nonlinear seismic analysis of RC frame structures were investigated. Hysteretic responses from quasi-static reverse cyclic loading of interior and exterior sub-assemblies showed that the single component approach (scissors joint model) and the multi-component approach (super element joint model) is relatively in good agreement than the centreline modelling scheme. To evaluate the seismic performance of the various joint models, the demand from three hypothetical frames with different modal period was performed for three classes of records. Based on this investigation, there is a large deviation in the drift responses of super element joint models at the roof level when compared to the scissors and centreline model. The equivalence in the drift responses for the joint modelling approaches also did not yield a significant correlation with selected class of records with varying intensities. For seismic risk assessment of RC building the maximum inter-story drift at any story height is preferred for generating fragility functions. Hence, from this study, there is no consistent evidence to suggest that care should be taken in selecting either a single or multi component joint model for seismic risk assessment of buildings when the maximum inter-story drift ratio is used as the engineering demand parameter. It is worth noting that a couple of research studies have highlighted the role played by infills on the seismic responses, column ductility demand and failure modes (Magenes and Pampanin 2004, Karayannis *et al.* 2011,). The impact of infill walls on seismic risk was not explicitly accounted; hence these conclusions are limited to RC frames which do not possess them. It is recommended that, even though the super element model provides a greatly deal of transparency in its formulation, that is, the decoupling of joint inelastic mechanisms which makes it useful for performing sensitivity analysis, the scissors joint model which lumps all the primary inelastic mechanisms, is simple to implement and computationally efficient for performing non-linear time history analysis of RC frames.

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