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Modeling of SH-waves in a fiber-reinforced anisotropic layer

Rajneesh Kakar*

163/1, C-B, Jalandhar-144022, India

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Abstract. In this paper we investigate the existence of SH-waves in fiber-reinforced layer placed over a heterogeneous elastic half-space. The heterogeneity of the elastic half-space is caused by the exponential variations of density and rigidity. As a special case when both the layers are homogeneous, our derived equation is in agreement with the general equation of Love wave. Numerically, it is observed that the velocity of SH-waves decreases with the increase of heterogeneity and reinforced parameters. The dimensionless phase velocity of SH-waves increases with the decreases of dimensionless wave number and shown through figures.

Keywords: heterogeneity; fiber reinforced medium; SH-waves

1. Introduction

A continuum model is used to explain fiber-reinforced composites as they are widely used in different engineering applications including aviation, automotive and engineering structures due to their high stiffness, lightweight, strength and damping properties. Reinforced materials are superior to the structural materials in applications because reinforced composite has characteristic property where its components act together as a single anisotropic unit till they remain in the elastic condition. Also, fiber-reinforced composite concrete structures are significant due to their low weight and high strength. Earth can be chosen as a composite material with horizontally preferred direction perpendicular to the propagation of wave with different properties. During an earthquake, the artificial structures on the surface of the earth are excited which gives rise to violent vibrations in some cases. They act like a single unit in the elastic condition such that relative displacement can be absent between them. Thus propagation of SH-waves in fiber-reinforced medium can help us to understand earthquake engineering and seismology. These waves are also useful to understand the anisotropic crustal region near the Earth's surface.

Surface waves are very important in the study of earthquake, geophysics and geodynamics. SH-waves cause more destruction to the structure than that of the body waves due to its slower attenuation of the energy. The supplement of surface wave analysis and other wave propagation problems to anisotropic elastic materials has been the subject of many studies; see, for example, Delfim (2012), Kristel *et al*. (2012), Ogden and Singh (2011), Ogden and Singh (2014), Pakravan *et al*. (2014), Selvamani and Ponnusamy (2013). Many authors have studied the propagation of

*Corresponding author, Professor, E-mail: rajneesh.kakar@gmail.com

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SH-wave by considering dissimilar forms of asymmetry at the interface. Singh (2007) obtained the reflection coefficients from free surface of an incompressible transversely isotropic fiber-reinforced elastic half-space for the case when outer slowness section is re-entrant. Singh and Yadav (2013) studied fiber-reinforced elastic solid half-space with magnetic field by taking the concept of reflection of plane waves. Chattopadhyay *et al.* (2010, 2012) used Green's function technique to study propagation of SH-waves and heterogeneity on the SH-waves in viscoelastic half-spaces. Also Chattopadhyay *et al.* (2014) discussed the influence of heterogeneity and reinforcement on propagation of a crack due to SH-waves. Gupta and Gupta (2013) studied the effect of initial stress on wave motion in an anisotropic fiber reinforced thermoelastic medium. Sahu *et al.* (2014) showed the effect of gravity on shear waves in a heterogeneous fiber-reinforced layer placed over a half-space. Kundu *et al.* (2014) analyzed SH-wave in initially stressed orthotropic homogeneous and an inhomogeneous half space.

In this paper, we study SH-wave propagation in fiber-reinforced layer placed over a heterogeneous elastic half-space. The heterogeneity is caused by consideration of exponential variation in rigidity and density in the lower elastic layer. The dispersion relation for propagation of said waves is derived with Whittaker function and the method of separation of variables. Standard frequency equation of SH-waves is obtained in the special cases in closed form. The influence of heterogeneity and reinforcement is discussed and is represented by a graph. It is observed that inhomogeneity and reinforcement have a remarkable effect on the phase velocity of SH-waves.

2. Formulation of the problem

Let H be the thickness of the steel fiber reinforced silica fume concrete layer placed over inhomogeneous half-space. We consider *x*-axis along the direction of wave propagation and *z*-axis vertically downwards (Fig. 1).

The variations of heterogeneous rigidity and density in the lower layer are taken as

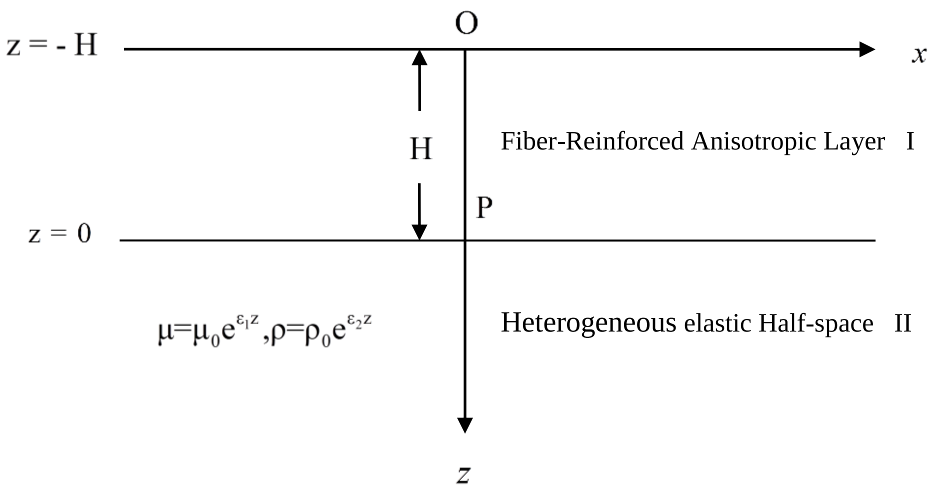

Fig. 1 Geometry of the problem

$$\left.\begin{aligned}\mu &= \mu_0 e^{\varepsilon_1 z}\\ \rho &= \rho_0 e^{\varepsilon_2 z}\end{aligned}\right\} \tag{1}$$

where ε_1 and ε_2 are inhomogeneous parameters of lower half-space and having dimension that are inverses of length, density and rigidity vary exponentially with space variable z, which is perpendicular to the direction of wave i.e., x-axis. We have also assumed that the shear velocity in the lower half-space is constant and independent of the depth z.

3. Boundary conditions

The displacement components and stress components are continuous at z=-H, and at z=0, therefore the geometry of the problem leads to the following conditions

(1) At z=-H, the stress component $\tau_{23}=0$.
(2) At z=0, the stress component of the layer and half space is continuous, i.e., $\tau_{23}=\sigma_{23}$.
(3) At z=0, the velocity component of both the layer is continuous, i.e., $u_2=v_2$.

4. Solution of the problem

4.1 Solution for the upper layer

The constitutive equations for a fiber reinforced linearly elastic anisotropic medium with respect to a preferred direction $\vec{a}$ (Belfield *et al.* 1983) are

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha\left(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j\right) + 2\left(\mu_L - \mu_T\right)\left(a_i a_k e_{kj} + a_j a_k e_{ki}\right) + \beta\left(a_k a_m e_{km} a_i a_j\right) \tag{2}$$

where, $e_{ij} = \dfrac{1}{2}(\mu_{i,j} + \mu_{j,i})$ are components of strain; $\alpha, \beta\ ?\mu_L - \mu_T$ are reinforced anisotropic elastic parameters; λ, μ_T are elastic parameters. Preferred direction of fibers are given by $\vec{a} = \left(a_1, a_2, a_3\right), a_1^2 + a_2^2 + a_3^2 = 1.$ If $\vec{a}$ has components that are (1, 0, 0) so that the preferred direction is the z-axis normal to direction of propagation. Relation (2) in the presence of initial compression simplifies as given below

$$\left.\begin{aligned}
\tau_{11} &= \left(\lambda + 2\alpha + 4\mu_L + \beta - 2\mu_T\right)e_{11} + \left(\lambda + \alpha\right)e_{22} + \left(\lambda + \alpha\right)e_{33}\\
\tau_{33} &= \left(\lambda + \alpha\right)e_{11} + \lambda e_{22} + \left(\lambda + 2\mu_T\right)e_{33}\\
\tau_{22} &= \left(\lambda + \alpha\right)e_{11} + \left(\lambda + 2\mu_T\right)e_{22} + \lambda e_{33}\\
\tau_{23} &= 2\mu_T e_{23}\\
\tau_{13} &= 2\mu_T e_{13}\\
\tau_{12} &= 2\mu_T e_{12}
\end{aligned}\right\} \tag{3}$$

The equations of motion in upper half are

$$\left.\begin{aligned}\frac{\partial\tau_{11}}{\partial x}+\frac{\partial\tau_{12}}{\partial y}+\frac{\partial\tau_{13}}{\partial z}=\rho\frac{\partial^2u_1}{\partial t^2}\\ \frac{\partial\tau_{21}}{\partial x}+\frac{\partial\tau_{22}}{\partial y}+\frac{\partial\tau_{23}}{\partial z}=\rho\frac{\partial^2u_2}{\partial t^2}\\ \frac{\partial\tau_{31}}{\partial x}+\frac{\partial\tau_{32}}{\partial y}+\frac{\partial\tau_{33}}{\partial z}=\rho\frac{\partial^2u_3}{\partial t^2}\end{aligned}\right\} \tag{4}$$

For SH-wave propagation along the *x*-axis, we have

$$u_1=0,\qquad u_3=0,\qquad u_2=u_2(x,z,t) \tag{5}$$

Taking transversely isotropic and setting a_2=0 we get from Eq. (3) as

$$\left.\begin{aligned}&\tau_{12}=\mu_T\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right)\frac{\partial u_2}{\partial x}+\mu_T\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3\frac{\partial u_2}{\partial z}\\ &\tau_{23}=\mu_T\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\right)\frac{\partial u_2}{\partial z}+\mu_T\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3\frac{\partial u_2}{\partial x}\\ &\tau_{11}=\tau_{22}=\tau_{33}=\tau_{23}=\tau_{13}=0\end{aligned}\right\} \tag{6}$$

Substituting Eq. (6) in Eq. (4), we get

$$\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right)\frac{\partial^2u_2}{\partial x^2}+2\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3\frac{\partial^2u_2}{\partial x\partial z}+\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\right)\frac{\partial^2u_2}{\partial z^2}=\frac{\rho}{\mu_T}\frac{\partial^2u_2}{\partial t^2} \tag{7}$$

In order to solve Eq. (7), we take

$$u_2(x,z,t)=\xi(z)\mathrm{e}^{ik(x-ct)} \tag{8}$$

Here, k is wave number; c is the phase velocity of simple harmonic waves of wave length $2\pi/k$. From Eq. (7) and Eq. (8), we get

$$\left\{1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\right\}\frac{\partial^2\xi(z)}{\partial z^2}+\left\{2\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3ik\right\}\frac{\partial\xi(z)}{\partial z}+\left\{\frac{\rho}{\mu_T}\omega^2-\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right)k^2\right\}\xi(z)=0 \tag{9}$$

where, $\omega=kc$ is the angular frequency, k the wave number and c is the phase velocity.

Let the solution of Eq. (9) is

$$\xi(z)=\mathrm{M}e^{-ikXz}+\mathrm{N}e^{-ikYz} \tag{10}$$

where, X and Y are arbitrary constants given by

$$X = \frac{\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3 + \sqrt{\left(\frac{\mu_L}{\mu_T}-1\right)^2 a_1^2a_3^2 + \left\{1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\right\}\left\{\frac{c^2}{c_\rho^2}-\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right)\right\}}}{1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2} \tag{11}$$

$$Y = \frac{\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3 - \sqrt{\left(\frac{\mu_L}{\mu_T}-1\right)^2 a_1^2a_3^2 + \left\{1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\right\}\left\{\frac{c^2}{c_\rho^2}-\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right)\right\}}}{1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2} \tag{12}$$

and $c_\rho = \sqrt{\frac{\mu_T}{\rho}}$ is the shear velocity.

Therefore, the equation of displacement of the upper reinforced medium is the solution of Eq. (7) and is given by

$$u_2(x,z,t) = \left(\mathrm{M}e^{-ikXz} + \mathrm{N}e^{-ikYz}\right)\mathrm{e}^{ik(x-ct)} \tag{13}$$

4.2 Solution for the lower half-space

The equations of motion for SH-wave in lower layer (Biot 1965) are

$$\left.\begin{aligned}
\frac{\partial\sigma_{11}}{\partial x} + \frac{\partial\sigma_{12}}{\partial y} + \frac{\partial\sigma_{13}}{\partial z} &= \rho\frac{\partial^2 v_1}{\partial t^2}\\
\frac{\partial\sigma_{21}}{\partial x} + \frac{\partial\sigma_{22}}{\partial y} + \frac{\partial\sigma_{23}}{\partial z} &= \rho\frac{\partial^2 v_2}{\partial t^2}\\
\frac{\partial\sigma_{31}}{\partial x} + \frac{\partial\sigma_{32}}{\partial y} + \frac{\partial\sigma_{33}}{\partial z} &= \rho\frac{\partial^2 v_3}{\partial t^2}
\end{aligned}\right\} \tag{14}$$

Using SH-wave conditions $v_1=v_3=0$, $v_2=v_2(x,z,t)$, the Eq. (14) can be reduced to

$$\frac{\partial\sigma_{21}}{\partial x} + \frac{\partial\sigma_{23}}{\partial z} = \rho\frac{\partial^2 v_2}{\partial t^2} \tag{15}$$

The stress-strain relations are

$$\left.\begin{aligned}
&\sigma_{11} = \sigma_{12} = \sigma_{13} = \sigma_{22} = \sigma_{31} = \sigma_{32} = \sigma_{33} = 0\\
&\sigma_{21} = 2\mu e_{xy} = 2\mu\frac{1}{2}\left(\frac{\partial v_2}{\partial x} + \frac{\partial v_1}{\partial y}\right)\\
&\sigma_{23} = 2\mu e_{yz} = 2\mu\frac{1}{2}\left(\frac{\partial v_3}{\partial y} + \frac{\partial v_2}{\partial z}\right)
\end{aligned}\right\} \tag{15}$$

Using Eq. (1) in Eq. (15), we get

$$\left.\begin{aligned}\sigma_{21} &= \mu_0 e^{\varepsilon_1 z}\frac{\partial v_2}{\partial x}\\ \sigma_{23} &= \mu_0 e^{\varepsilon_1 z}\frac{\partial v_2}{\partial z}\end{aligned}\right\} \tag{16}$$

From Eq. (14), Eq. (1) and Eq. (16), we get

$$\mu_0 e^{\varepsilon_1 z}\frac{\partial^2 v_2}{\partial x^2} + \varepsilon_1 \mu_0 e^{\varepsilon_1 z}\frac{\partial v_2}{\partial z} + \mu_0 e^{\varepsilon_1 z}\frac{\partial^2 v_2}{\partial z^2} = \rho_0 e^{\varepsilon_2 z}\frac{\partial^2 v_2}{\partial t^2} \tag{17}$$

In order to solve Eq. (17), we take

$$v_2(x,z,t) = \zeta(z)\,\mathrm{e}^{ik(x-ct)} \tag{18}$$

From Eq. (17) and Eq. (18), we get

$$\frac{\partial^2 \zeta(z)}{\partial z^2} + \varepsilon_1\frac{\partial \zeta(z)}{\partial z} - k^2\left(1 - \frac{e^{\varepsilon_2 z}}{e^{\varepsilon_1 z}}\frac{c^2}{c_2^2}\right)\zeta(z) = 0 \tag{19}$$

where, $c_2 = \sqrt{\dfrac{\mu_0}{\rho_0}}$ is the shear velocity.

Now substituting $\zeta(z) = \chi(z)e^{\frac{-\varepsilon_1 z}{2}}$ in Eq. (19) to eliminating term $\dfrac{\partial \zeta(z)}{\partial z}$, we obtain

$$\chi''(z) + k^2\left[\frac{e^{\varepsilon_2 z}}{e^{\varepsilon_1 z}}\frac{c^2}{c_2^2} - \left(1 - \frac{\varepsilon_1^2}{4k^2}\right)\right]\chi(z) = 0 \tag{20}$$

Introducing the dimensionless quantities $\in = \sqrt{1 - \dfrac{\varepsilon_2}{\varepsilon_1}\dfrac{c^2}{c_2^2} + \dfrac{\varepsilon_1^2}{4k^2}}$ and $\eta = \dfrac{2 \in k\left(1+\varepsilon_1 z\right)}{\varepsilon_1}$ in Eq. (20), we get

$$\frac{d^2\chi(\eta)}{dz^2} + \left(\frac{R}{\eta} - \frac{1}{4}\right)\chi(\eta) = 0 \tag{21}$$

where, $R = \dfrac{c^2\left(\varepsilon_1 - \varepsilon_2\right)}{2c_2^2\varepsilon_1^2 \in}$ Eq. (21) is the well known Whittaker's equation [Whittaker and Watson (1990)].

The solution of Whittaker's Eq. (21) is given by

$$\chi(\eta) = \mathrm{A}\mathrm{W}_{\mathrm{R},\frac{1}{2}}(\eta) + \mathrm{B}\mathrm{W}_{-\mathrm{R},\frac{1}{2}}(-\eta) \tag{22}$$

where A and B are arbitrary constants and $\mathrm{W}_{\mathrm{R},\frac{1}{2}}(\eta), \mathrm{W}_{-\mathrm{R},\frac{1}{2}}(-\eta)$ are the Whittaker function.

Now Eq. (22) satisfying the condition $\lim_{z\to\infty}\zeta(z)\to 0$ i.e., $\lim_{z\to\infty}\chi(\eta)\to 0$ →0 may be taken as

$$\chi(\eta) = \mathrm{AW}_{\mathrm{R},\frac{1}{2}}(\eta) \tag{23}$$

Hence the displacement for the SH-wave in the lower layer is

$$v_2(x,z,t) = \mathrm{AW}_{\mathrm{R},\frac{1}{2}}\left[\frac{2\in k}{\varepsilon_1}(1+\varepsilon_1 z)\right]e^{-\frac{\varepsilon_1 z}{2}}e^{ik(\mathrm{x}-\mathrm{ct})} \tag{24}$$

5. Dispersion relation

The dispersion relation for SH-waves can be obtained by using boundary conditions given in section 3. Therefore, the displacement for the SH-waves in the in-homogeneous half-space using boundary conditions (1), (2) and (3) in Eq. (13) and Eq. (14) becomes (taking Whittaker's function $\mathrm{W}_{\mathrm{R},\frac{1}{2}}(\eta)$ up to linear terms in η)

$$\mu_T\left\{\mathrm{M}\left[\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2X\right)e^{iXkH}+\mathrm{N}\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2Y\right)e^{iYkH}\right]\right\}=0 \tag{25}$$

$$\begin{aligned}&\mu_T ik\left\{\mathrm{M}\left[\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2X\right)+\mathrm{N}\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2Y\right)\right]\right\}= \\ &\mathrm{A}\mu_0\left(\frac{2\in k}{\varepsilon_1}\right)e^{-\frac{2]\in k}{\varepsilon_1}}\left[1+(1-R)\frac{\in k}{\varepsilon_1}\right]\left[\frac{(1-R)\in k}{1+(1-R)\frac{\in k}{\varepsilon_1}}-\left(\in k-\frac{\varepsilon_1}{2}\right)+\varepsilon_1\right]\end{aligned} \tag{26}$$

$$\mathrm{M}+\mathrm{N} = \mathrm{A}\left(\frac{2\in k}{\varepsilon_1}\right)e^{-\frac{2\in k}{\varepsilon_1}}\left[1+(1-R)\frac{\in k}{\varepsilon_1}\right] \tag{27}$$

Now eliminating M, N and A from the Eq. (25), Eq. (26) and Eq. (27), we obtain

$$\begin{vmatrix} \left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2X\right)e^{iXkH} & \left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2Y\right)e^{iYkH} & 0 \\ 1 & 1 & -\left(\frac{2\in k}{\varepsilon_1}\right)e^{-\frac{2\in k}{\varepsilon_1}}\left[1+(1-R)\frac{\in k}{\varepsilon_1}\right] \\ \begin{array}{c} ik \\ \times\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2X\right)\end{array} & \begin{array}{c} ik \\ \times\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2Y\right)\end{array} & \begin{array}{c}\frac{\mu_0}{\mu_T}\left(\frac{2\in k}{\varepsilon_1}\right)e^{-\frac{2\in k}{\varepsilon_1}}k\left[1+(1-R)\frac{\in k}{\varepsilon_1}\right] \\ \times\left[\frac{(1-R)\in}{1+(1-R)\frac{\in k}{\varepsilon_1}}-\in+\frac{\varepsilon_1}{2k}\right]\end{array} \end{vmatrix}=0 \tag{28}$$

On simplifying Eq. (28), we get

$$-\frac{\mu_0\left(\frac{2\in k}{\varepsilon_1}\right)e^{-\frac{2\in k}{\varepsilon_1}}\left[\frac{(1-R)\in k}{\left[1+(1-R)\frac{\in k}{\varepsilon_1}\right]}-\in k+\frac{\varepsilon_1}{2}\right]\left[1+(1-R)\frac{\in k}{\varepsilon_1}\right]}{2\mu_T ik}$$
$$\times\left[\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2X\right)e^{iXkH}-\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2Y\right)e^{iYkH}\right] \quad (29)$$
$$+\left(\frac{2\in k}{\varepsilon_1}\right)e^{-\frac{2\in k}{\varepsilon_1}}\left[1+(1-R)\frac{\in k}{\varepsilon_1}\right]$$
$$\times\left[\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2X\right)\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2Y\right)\right]\left[e^{iXkH}-e^{iYkH}\right]=0$$

Solving Eq. (29), we get

$$\tan\left[\frac{kH}{\left[1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\right]}\sqrt{\left[\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3\right]^2-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\left[\frac{c^2}{c_1^2}-\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right)\right]}\right] \quad (30)$$
$$=\frac{\mu_0}{\mu_T}\frac{1}{\sqrt{\left[\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3\right]^2-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\left[\frac{c^2}{c_1^2}-\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right)\right]}}\left[\in-\frac{(1-R)\in k}{\left[1+(1-R)\frac{\in k}{\varepsilon_1}\right]}-\frac{\varepsilon_1}{2k}\right]$$

Eq. (30) is the dispersion equation of SH-wave propagation in a fiber-reinforced anisotropic layer over a heterogeneous isotropic elastic half-space.

Case-1

For inhomogeneous reinforced medium over an homogeneous half space, we take $\varepsilon_1=0$ and $\varepsilon_2=0$, therefore, Eq. (30) reduces to

$$\tan\left[\frac{kH}{\left[1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\right]}\sqrt{\left[\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3\right]^2-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\left[\frac{c^2}{c_1^2}-\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right)\right]}\right] \quad (31)$$
$$=\frac{\mu_0}{\mu_T}\frac{1}{\sqrt{\left[\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3\right]^2-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\left[\frac{c^2}{c_1^2}-\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right)\right]}}\sqrt{\frac{c^2}{c_1^2}-1}$$

Case-2

For homogeneous reinforced medium over an inhomogeneous half space, we take $a_1=1$,

a_2=a_3=0 then $\rho \to \frac{\mu_L}{\mu_T}$ and $\mu_L \to \mu_T \to \mu_1$, therefore, Eq. (30) reduces to

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right]=\frac{\mu_0}{\mu_1}\frac{1}{\sqrt{\frac{c^2}{c_1^2}-1}}\left[\in-\frac{(1-R)\in k}{\left[1+(1-R)\frac{\in k}{\varepsilon_1}\right]}-\frac{\varepsilon_1}{2k}\right] \tag{32}$$

Case-3

For homogeneous reinforced medium over an homogeneous half space, we take ε_1=0, ε_2=0, a_1=1, a_2=a_3=0 then $\rho \to \frac{\mu_L}{\mu_T}$ and $\mu_L \to \mu_T \to \mu_1$, therefore, Eq. (30) reduces to

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right]=\frac{\mu_0}{\mu_1}\frac{\sqrt{1-\frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2}-1}} \tag{33}$$

Eq. (33) is the classical dispersion equation of SH-waves given by Love (1911) and Ewing *et al.* (1957).

5. Numerical analysis

To show the effect of inhomogeneity parameters and steel reinforced parameters on SH-wave propagation in a fiber-reinforced anisotropic layer over a heterogeneous isotropic elastic half-space, we take following parameters.

a. Material Parameters for upper reinforced layer, Gupta (2014).

$$\mu_T = 5.65\times10^9\,N/m^2,\ \mu_L = 2.46\times10^9\,N/m^2,\ \lambda = 5.65\times10^9\,N/m^2, a_3^2 = 0.85,$$
$$\alpha = \text{-}1.28\times10^{10}\,N/m^2,\ \beta = 220.09\times10^9\,N/m^2,\ \rho = 7800\ kg/m^3,\ a_1^2 = 0.15.$$

b. Material Parameters for elastic half-space, Gubbins (1990).

$$\mu_0 = 6.34\times10^{10}\,N/m^2,\ \rho_0 = 3364 Kg/m^3.$$

We have plotted non-dimensional phase velocity $\frac{c}{c_1}$ against dimensionless wave number kH on the propagation of SH-wave in a fiber-reinforced anisotropic layer by using MATLAB software.

The effects of reinforced parameters and inhomogeneity parameters have been shown in Figs. 2-5. Fig. 2 gives the effect of inhomogeneity parameters $l=\frac{\varepsilon_1}{k}$ and $m=\frac{\varepsilon_2}{k}$ in the presence of reinforced parameters on the propagation of SH-waves. The reinforced parameters $r=a_1^2$, $s=a_3^2$

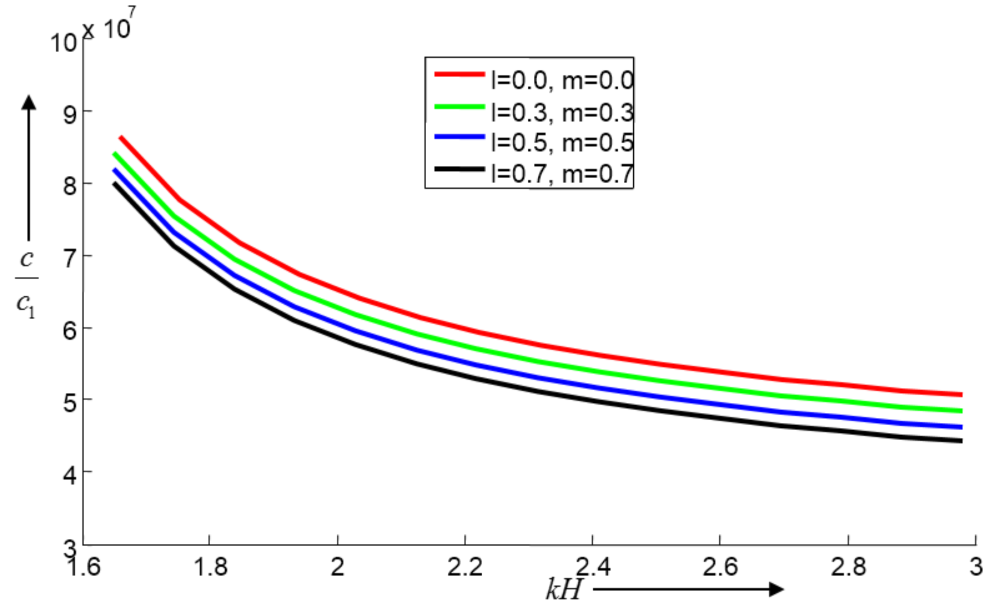

Fig. 2 Dimensionless phase velocity $\frac{c}{c_1}$ against dimensionless wave number kH for different values of $l = \frac{\varepsilon_1}{k}$ and $m = \frac{\varepsilon_2}{k}$ in the presence of reinforced parameter

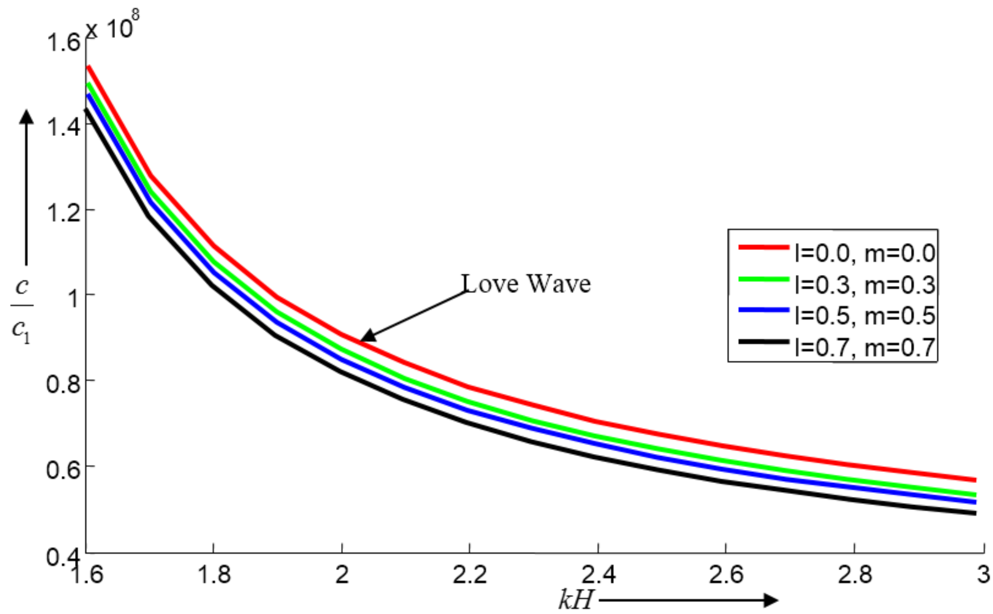

Fig. 3 Dimensionless phase velocity $\frac{c}{c_1}$ against dimensionless wave number kH for different values $l = \frac{\varepsilon_1}{k}$ and $m = \frac{\varepsilon_2}{k}$ in the absence of reinforced parameter

are taken 0.15, 0.85 respectively and various curves are plotted for inhomogeneity parameters 0.0, 0.3 and 0.5 respectively. It is clear from this figure, the phase velocity decreases with increase of inhomogeneity parameters $\frac{\varepsilon_1}{k}$ and $\frac{\varepsilon_2}{k}$. Fig. 3 represents the variation of dimensionless phase velocity $\frac{c}{c_1}$ with dimensionless wave number kH on the propagation of SH-waves for different values $\frac{\varepsilon_1}{k}$ and $\frac{\varepsilon_2}{k}$ in the absence of reinforced parameter. The values of inhomogeneity parameters for curves have been taken as 0.0, 0.1, 0.3 and 0.5, respectively. It is observed from these curves that as the nonhomogeneity parameters $\frac{\varepsilon_1}{k}$ and $\frac{\varepsilon_2}{k}$ in the half-space increases, the velocity of SH-wave decreases. Also, red curve represents the Case-3 i.e., the dispersion of SH-wave in reinforced layer overlying a homogeneous elastic half-space. From Figs. 2 and 3, it is cleared that the SH-wave propagation is more influenced by inhomogeneity parameters $\frac{\varepsilon_1}{k}$ and $\frac{\varepsilon_2}{k}$ in comparison of reinforcement in upper layer. It is also seen that for large value of inhomogeneity parameters, the curves of phase velocities make significant distances from each other. Fig. 4 shows the effect of reinforced parameters a_1^2 and a_3^2 on the propagation of SH-waves. The reinforced parameters (a_1^2, a_3^2) for red curve, green curve, blue curve and black curve have been taken as (0.15, 0.85), (0.25, 0.75), (0.35, 0.65) and (0.45, 0.55), respectively. All the four curves in Fig. 4 are drawn at fixed

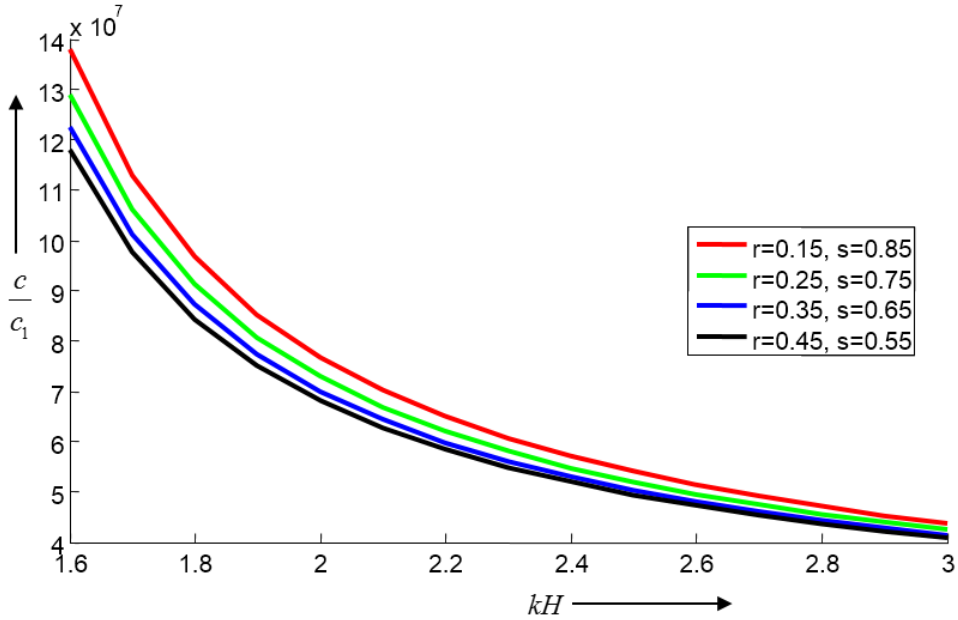

Fig. 4 Dimensionless phase velocity $\frac{c}{c_1}$ against dimensionless wave number kH for different values $r = a_1^2$ and $s = a_3^2$ at constant values of $l = \frac{\varepsilon_1}{k}$ and $m = \frac{\varepsilon_2}{k}$

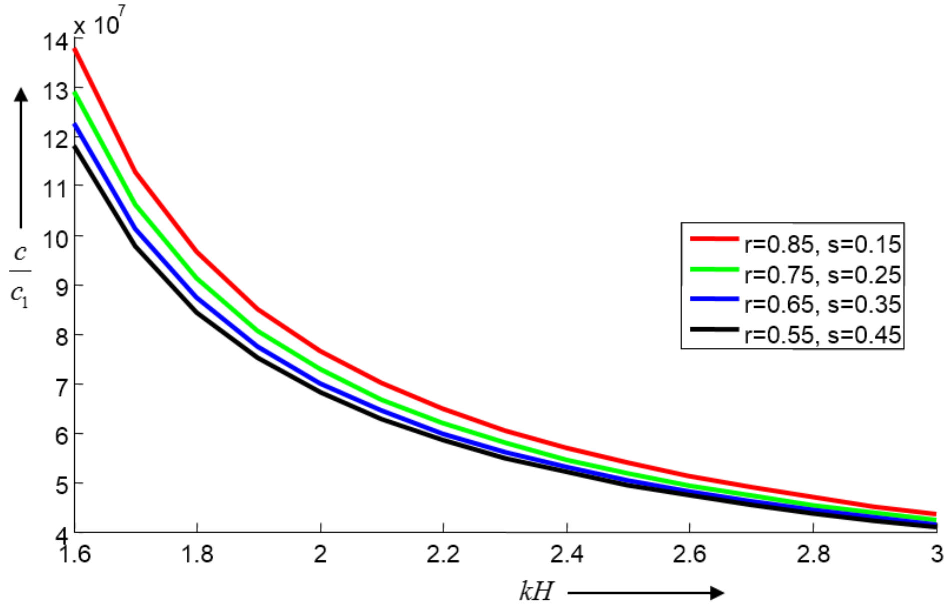

Fig. 5 Dimensionless phase velocity $\frac{c}{c_1}$ against dimensionless wave number kH for different values $r = a_1^2$ and $s = a_3^2$ at zero values of $l = \frac{\varepsilon_1}{k}$ and $m = \frac{\varepsilon_2}{k}$

values of the nonhomogeneity parameters $\frac{\varepsilon_1}{k} = 0.3$ and $\frac{\varepsilon_2}{k} = 0.3$, respectively. It is seen from the diagram that as a_1^2 increases as well as a_3^2 decreases, the velocity of SH-wave decreases. In Fig. 5, the curves show the effect of reinforced parameters on the propagation of SH-wave in a fiber-reinforced anisotropic layer in absence of inhomogeneity parameter in lower half-space. All the four curves in are drawn at zero values of the nonhomogeneity parameters $\frac{\varepsilon_1}{k}$ and $\frac{\varepsilon_2}{k}$, respectively. It is seen from the diagram that as a_1^2 decreases as well as a_3^2 increases, the velocity of SH-wave decreases. Thus from Figs. 4 and 5, we observed that as the reinforcement of the upper layer increases, the phase velocity of SH-wave decreases. Hence the phase velocity of SH-wave is more dependent on reinforced parameters (a_1^2, a_3^2).

6. Conclusions

In this problem we have taken two layers; fiber-reinforced anisotropic layer upper layer and hetrogeneous lower with exponential variation in rigidity and density. We have employed Whittaker function and separation of variable method to find the dispersion of SH-waves in fiber-reinforced layer placed over a heterogeneous elastic half-space. Displacement in the upper fiber-reinforced layer is derived in closed form and the dispersion curves are drawn for various values of inhomogeneity and reinforced parameters. In a particular case, the dispersion equation coincides with the well-known classical equation of Love wave when the upper and lower layer is homogeneous. The above results may be used to study surface wave propagation in fiber reinforced medium. This validates the solution.

From above numerical analysis, it may be conclude that:

(1) In entire figures, dimensionless phase velocity of SH-waves decreases with increase of dimensionless wave number.

(2) The dimensionless phase velocity of SH-wave shows remarkable change with heterogeneity and reinforced parameters.

(3) It is observed as the depth increases the velocity of SH-wave decreases.

(4) The velocity of SH-wave decreases with the increase of as the reinforced parameter of upper layer and inhomogeneous parameter of the lower half-space, which is the property of seismic wave propagation in caster layer.

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