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Numerical simulation of elastic-plastic stress concentration in fibrous composites

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Abstract. In the present study an elastic-plastic strain analysis is carried out for fibrous composites by using numerical modeling. Application of homogeneous transversely-isotropic model was chosen based on problem solution of a square plate with a circular hole under uniaxial tension. The results obtained in this study correspond to the solution of fiber model trial problem, as well as to analytical solution. Further, numerical algorithm and software has been developed, based on simplified theory of small elastic strains for transversely-isotropic bodies, and FEM. The influence of holes and cracks on stress state of complicated configuration transversely-isotropic bodies has been studied. Strain curves and plasticity zones that are formed in vicinity of the concentrators has been provided. Numerical values of effective mechanical parameters calculated for unidirectional composites at different ratios of fiber volume content and matrix. Content volume proportions of fibers and matrix defined for fibrous composite material that enables to behave as elastic-plastic body or as a brittle material. The influences of the fibrous structure on stress concentration in vicinity of holes on boron/aluminum D16, used as an example.

Keywords: modeling; FEM; stress; strain; elastic-plastic; fiber; composite; hole; crack; boron/aluminum

1. Introduction

The study of construction strain processes of complicated configuration materials with different physical and mechanical properties is one of the most important problems in durability and reliability calculations of construction elements (Kaminsky 2007). Composite materials have wide application of in modern industry, especially in engineering, aviation. The latest technologies allow creating composite materials with predictable mechanical properties. Because of the great interest in composite materials from industries and academic institutions, research of construction elements strain became one of the most up-to-date issues to be investigated.

Very often due to reasons, caused by structural, technological or economic needs, it's necessary to disrupt constructions' integrity while creation, i.e. by making of various cavities and cracks. Such inclusion is becoming a kind of stress concentrators. Local stress concentrations, occurring in the immediate vicinity of the concentrators, significantly affect the strength and bearing capacity of constructions. Consideration of these stresses is essential in determining the possibility of further operation of construction (Zeng and Fatemi 2001).

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Investigation of durability issue in areas with sharp stress increase is extremely important. Analyzing the distribution of stresses and strains allows determining of plasticity existence zone, defining its shape, size, location, as well as stresses and strain intensity. The importance of this problem is caused by need of solving practical issues of constructions analysis and has theoretical and practical significance in determining the efficiency of such elements.

Awareness of material behavior during strain enables designing of constructions more accurately and efficiently. Stress concentration is one of the major factors defining durability of constructions. Investigation of various concentrators types influence on composite materials behavior and determining of stress concentration reduction ways are the main tasks of modern mechanics.

Construction elements made from fibrous composites represent the special interest. Consideration as transversely-isotropic material with some efficient parameters is one of the ways of studying the behavior of such materials. This approach allow to describe static behavior of composite constructions with sufficient accuracy of investigated material, while all geometric dimensions of material considerably exceed characteristic size of structural inhomogeneity.

At the same time, it allows you to consider the factor of composite materials structural inhomogeneity, which significantly affects the behavior of construction elements (Khaldjigitov and Adambaev 2004).

Fibrous composite materials of boron, carbon and epoxy fiber are widely used in practice. They have brightly expressed anisotropic properties that have to be considered while analyzing constructions of these materials. The mechanical behavior of the composite is determined by matrix and reinforcing elements properties, as well as connection durability between them (Rodriguez-Ramos *et al*. 2005).

Determination of critical loadings and the study of geometry influence of existing cavities and cracks in elements of constructions made of composite materials by methods of mechanics as against the engineering approaches require, first of all, mathematical modeling of strain process (Masayuki Kamaya 2006, Nagaraj *et al*. 2008).

An elastic-plastic stress analysis aluminum metal – matrix laminated plates with square hole was carried out by Aktas (2005). A stress analysis for long silicon carbide fiber reinforced magnesium metal matrix composite with a square hole by using finite element technique is carried out in study (Okumus 2011). The results show that the intensity of the residual stresses is maximal near the open square hole and the yield points in symmetric laminates are higher than those in antisymmetric laminates. The plastic regions at the plate edges expand in the direction of the fiber, but at the border of the hole expand toward the diagonal of the hole. Authors (Sayman *et al*. 2000) investigated elastic-plastic stress analysis of aluminum metal matrix composite laminated plates under in-plane loading. Elastic as well as elastic-plastic finite element analyses of the two geometries were performed in study (Zeng and Fatemi 2001). Notch root strains and stresses were predicted by employing the linear rule, Neuber’s rule and Glinka’s rule relationships under both monotonic loading conditions. The predicted results are compared with those from elastic–plastic finite element analyses and strain gauge measurements. Effects of notch constraint and the material stress–strain curve on the notch root stress and strain predictions are also discussed. In paper (Hitham e*t al*. 2005) is researched with the effect of the notch depth on the newly defined strain-concentration factor (SNCF) under static tension. Emphasis has been put on the elastic-plastic SNCF of shallow notches. The strain distributions at the net section have been obtained using the finite element method (FEM). The FEM calculations have been performed up to a strain level close to that when the notch tensile strength is attained. Attempts have been made to predict the axial strain at the notch root under static and cyclic tensile loading (Harkegard and Mann 2003). This prediction was made using the strain-concentration factor (SNCF), referred to as the conventional SNCF here, through Neuber's and Glinka's rule (Livieri and Nicoletto 2003) or linear rule (Zeng and Fatemi 2001).

In connection with this, in order to have reliable construction elements strength evaluation it is relevant to use the modern computer technologies for composite materials (Van Der Meer and Sluys 2009).

For the description of elastic-plastic strain process of fibrous composites based on averaging method different versions of the plasticity theory are proposed, in which the composite material is replaced by a homogeneous anisotropic medium (Sebastian *et al*. 2005).

Composite materials mechanics is one of very rapidly developing research areas, which obtained significant theoretical and experimental results. However, non-linear strain processes of composite materials with concentrators are not well investigated yet. Modern developments in mathematical modeling of transversely-isotropic materials' elastic-plastic strain process cannot be considered as complete. Wide implementation of composite materials has led to the emergence of new fields in science related to the study of elastic-plastic materials strain (Lee *et al*. 2012).

In this paper the fibrous composites was studied on the basis of averaging and assuming that they are a homogeneous transversely-isotropic elastic-plastic materials with effective mechanical parameters. Computational algorithm developed on the basis of a simplified plasticity strain theory of transversely-isotropic media, proposed by Pobedrya (1984), and FEM. It allows applying small elastic-plastic strains theory to solve specific application tasks of composite materials strain.

The purpose of this paper is to describe numerical study of nonlinear strain process for fibrous composite with concentrators.

2. Problem statement

Quasi-static problem of the theory of transversely isotropic-bodies' small elastic strains is solved by equilibrium equations

$$\sum_{j=1}^{3}\frac{\partial\sigma_{ij}}{\partial x_j}+X_i=0, \quad x_i\in V \tag{1}$$

based on the generalized Hooke's law

$$\sigma_{ij}=C_{ijkl}\varepsilon_{kl} \tag{2}$$

Cauchy relations

$$\varepsilon_{kl}=\frac{1}{2}\left(\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k}\right) \tag{3}$$

and boundary conditions

$$u_i\big|_{\Sigma_1}=u_i^o, \quad x_i\in\Sigma_1 \tag{4}$$

$$\sum_{j=1}^{3}\sigma_{ij}n_j\Big|_{\Sigma_2} = S_i^o, \quad x_i \in \Sigma_2 \tag{5}$$

where u_i - components of the displacement vector;
S_i, X_i - surface and volume forces,
Σ_1, Σ_2 - parts of the surface Σ of V volume;
n_j - external surface normal Σ_2 of V volume;
C_{ijkl} - tensor of elastic constants.

3. Solution method

In reinforced composites investigation, when stiffness of reinforcing elements significantly exceeds the stiffness of binding agents, simplified strain theory of plasticity could be used. It allows applying theory of small elastic-plastic strain for solution of specific applied tasks. Simplification is based on assumption that the simple stretching in the axis direction of the composite's transversal isotropy and in perpendicular direction to it, plastic strains do not occur.

For transversely isotropic solids - the ratio between the stresses and strains is presented in the form of the stress tensor decomposition on the spherical and deviatory parts (Khaldjigitov 2003)

$$\sigma_{ij} = \tilde{\sigma}(\delta_{ij} - \delta_{i3}\delta_{j3}) + \sigma_{33}\delta_{i3}\delta_{j3} + \frac{P_u}{p_u}p_{ij} + \frac{Q_u}{q_u}q_{ij} \tag{6}$$

where P_u, Q_u and p_u, q_u - stress and strain tensor intensity (respectively plane isotropy and isotropy transversal axis)

$$P_u = \sqrt{\frac{1}{2}P_{ij}P_{ij}} = \frac{\sqrt{2}}{2}\sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2}\,,$$
$$p_u = \sqrt{\frac{1}{2}p_{ij}p_{ij}} = \frac{\sqrt{2}}{2}\sqrt{(\varepsilon_{11} - \varepsilon_{22})^2 + 4\varepsilon_{12}^2}\,, \tag{7}$$

$$Q_u = \sqrt{\frac{1}{2}Q_{ij,}Q_{ij}} = \sqrt{\sigma_{13}^2 + \sigma_{23}^2}\,,$$
$$\mathrm{q_u} = \sqrt{\frac{1}{2}q_{ij}q_{ij}} = \sqrt{\varepsilon_{13}^2 + \varepsilon_{23}^2}\,. \tag{8}$$

The stress tensor ratio

$$P_{ij} = \sigma_{ij} + \tilde{\sigma}\left(\delta_{i3}\delta_{j3} - \delta_{ij}\right) + \sigma_{33}\delta_{i3}\delta_{j3} - \left(\sigma_{i3}\delta_{j3} + \sigma_{3j}\delta_{i3}\right) \tag{9}$$

$$Q_{ij} = \sigma_{i3}\delta_{j3} + \sigma_{3j}\delta_{i3} - 2\sigma_{33}\delta_{i3}\delta_{j3}\,, \quad \tilde{\sigma} = (\sigma_{11} + \sigma_{22})/2 \tag{10}$$

In disclosing these ratios look like

$$P_{11}=(\sigma_{11}-\sigma_{22})/2,\, P_{22}=(\sigma_{22}-\sigma_{11})/2,\, P_{12}=P_{21}=\sigma_{12}$$

$$Q_{13}=Q_{31}=\sigma_{13},\, Q_{23}=Q_{32}=\sigma_{23}$$

Similarly prescribed ratio of strain tensor

$$\varepsilon_{ij}=\tilde{\theta}(\delta_{ij}-\delta_{i3}\delta_{j3})+\varepsilon_{33}\delta_{i3}\delta_{j3}+p_{ij}+q_{ij}$$

where

$$p_{ij}=\varepsilon_{ij}+\frac{\tilde{\theta}}{2}\left(\delta_{i3}\delta_{j3}-\delta_{ij}\right)+\varepsilon_{33}\delta_{i3}\delta_{j3}-\left(\varepsilon_{i3}\delta_{j3}+\varepsilon_{3j}\delta_{i3}\right) \tag{11}$$

$$q_{ij}=\varepsilon_{i3}\delta_{3j}+\varepsilon_{3j}\delta_{3i}-2\varepsilon_{33}\delta_{i3}\delta_{j3}\,,\,\tilde{\theta}=\varepsilon_{11}+\varepsilon_{22} \tag{12}$$

It is assumed that the transverse isotropy axis coincides with the axis OZ.

Ratios of (6), (9) and (10) have the following matrix representation

$$\begin{pmatrix}\sigma_{11} & \sigma_{12} & \sigma_{13}\\ \sigma_{21} & \sigma_{22} & \sigma_{23}\\ \sigma_{11} & \sigma_{32} & \sigma_{33}\end{pmatrix}=\begin{pmatrix}\tilde{\sigma} & 0 & 0\\ 0 & \tilde{\sigma} & 0\\ 0 & 0 & 0\end{pmatrix}+\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \sigma_{33}\end{pmatrix}+\begin{pmatrix}P_{11} & P_{12} & 0\\ P_{21} & P_{22} & 0\\ 0 & 0 & 0\end{pmatrix}+\begin{pmatrix}0 & 0 & Q_{13}\\ 0 & 0 & Q_{23}\\ Q_{31} & Q_{32} & 0\end{pmatrix}$$

Relationship between stresses and strains intensity is represented as (Khaldjigitov 2002)

$$\begin{aligned}P_u &= 2\lambda_4(1-\pi(p_u))p_u\\ Q_u &= 2\lambda_5(1-\chi(q_u))q_u\end{aligned} \tag{13}$$

where $\pi\left(p_u\right)$ and $\chi(q_u)$ - function of plasticity which value is zero at elastic zone.

For simplified transversely isotropic plasticity theory the relationship between stresses and strains is given by the relations

$$\begin{aligned}\tilde{\sigma} &= \left(\lambda_4+\lambda_7\right)\tilde{\theta}+\lambda_5\varepsilon_{33}\\ \sigma_{33} &= \lambda_5\tilde{\theta}+\lambda_3\varepsilon_{33}\\ P &= P(p);Q=Q(q)\end{aligned} \tag{14}$$

where

$$P=\begin{cases}2\lambda_7\, p, \text{ if } & p<p*\\ 2\,\lambda_7\,\mathrm{p}*+2\lambda_7'(p-p*), \text{ if } & p>p*\end{cases} \tag{15}$$

$$Q=\begin{cases}2\lambda_9\, q, \text{ if } & q<q*\\ 2\,\lambda_9\,\mathrm{q}*+2\lambda_9'(q-q*), \text{ if } & q>q*\end{cases} \tag{16}$$

p^*, q^* - appropriate limit of elastic strain.

Introducing the notations

$$\alpha = \left(1 - \frac{\lambda_7^{'}}{\lambda_7}\right)\left(1 - \frac{p^*}{p}\right), \beta = \left(1 - \frac{\lambda_9^{'}}{\lambda_9}\right)\left(1 - \frac{q^*}{q}\right) \tag{17}$$

expressions for stressed state components in plasticity zone could be written as

$$\begin{aligned}
\sigma_{11} &= (\lambda_4 + 2\lambda_7)\varepsilon_{11} + \lambda_2\varepsilon_{22} + \lambda_3\varepsilon_{33} - \lambda_7\alpha(\varepsilon_{11} - \varepsilon_{22}) \\
\sigma_{22} &= \lambda_4\varepsilon_{11} + (\lambda_4 + 2\lambda_7)\varepsilon_{22} + \lambda_3\varepsilon_{33} - \lambda_7\alpha(\varepsilon_{22} - \varepsilon_{11}) \\
\sigma_{33} &= \lambda_5(\varepsilon_{11} + \varepsilon_{22}) + \lambda_3\varepsilon_{33} \\
\sigma_{12} &= 2\lambda_7\varepsilon_{12} - 2\lambda_7(1-\alpha)\varepsilon_{12} \\
\sigma_{23} &= 2\lambda_9\varepsilon_{23} - 2\lambda_9(1-\beta)\varepsilon_{23} \\
\sigma_{31} &= 2\lambda_9\varepsilon_{31} - 2\lambda_9(1-\beta)\varepsilon_{31}
\end{aligned}$$

Coefficients of λ_i in (15) are associated with mechanical parameters of a transversely isotropic material in the following relationships:

$$\lambda_3 = E'(1-\nu)/l,\ \lambda_4 = E(\nu + k\nu'^2)/[(1+\nu)/l],\ \lambda_5 = E\nu'/l,$$
$$\lambda_7 = G = E/[2(1+\nu)],\ \lambda_9 = G',\ l = 1 - \nu - 2\nu'^2 k,\ k = E/E'.$$

Problem statement of small elastic strains theory of transversely isotropic homogeneous media consists in solving of equilibrium equations

$$\tilde{\sigma}_{,i} + \left(\sigma_{33}^* - \tilde{\sigma}^*\right)_{,3}\delta_{3i} + \left[\frac{P}{p}p_{ij} + \frac{Q}{q}q_{ij}\right]_{,j} + X_i = 0 \tag{18}$$

subject to the following of boundary conditions

$$u_i\Big|_{\sum_1} = u_i^0 \tag{19}$$

$$\left[\tilde{\sigma}^* n_i + (\sigma_{33}^* - \tilde{\sigma}^*)n_3\delta_{3i} + \left(\frac{P}{p}p_{ij} + \frac{Q}{q}q_{ij}\right)n_j\right]\sum\nolimits_2 = S_i^0 \tag{20}$$

where

$\tilde{\sigma}^*, \sigma_{33}^*$ - some unknown functions, which are determined in problem solution process;

$\varepsilon_{33} \equiv 0, \tilde{\theta} \equiv 0$ - conditions of material's transversal incompressibility;

$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ - relationship between strains and displacements.

4. Computer modeling

Software provides users with universal tools for preparation process routine tasks automation, data processing and storage (Korobeynikov and Babichev 2007). Computer modeling of composite materials behavior allows to:

- project new composite materials;
- investigate the behavior of composites;
- visualize the calculation results.

ARPEK software package – is one of such tools which developed to automate the spatial structural elements calculation. Solution of problem is performed based on the FEM. Complex of programs operates in interactive mode. Users can directly participate in particular problem solution process.

Computer modeling and solution of applied tasks provides:

- formation of modules series for building body's finite element model;
- performance of computational model modules;
- visualization of the calculation results.

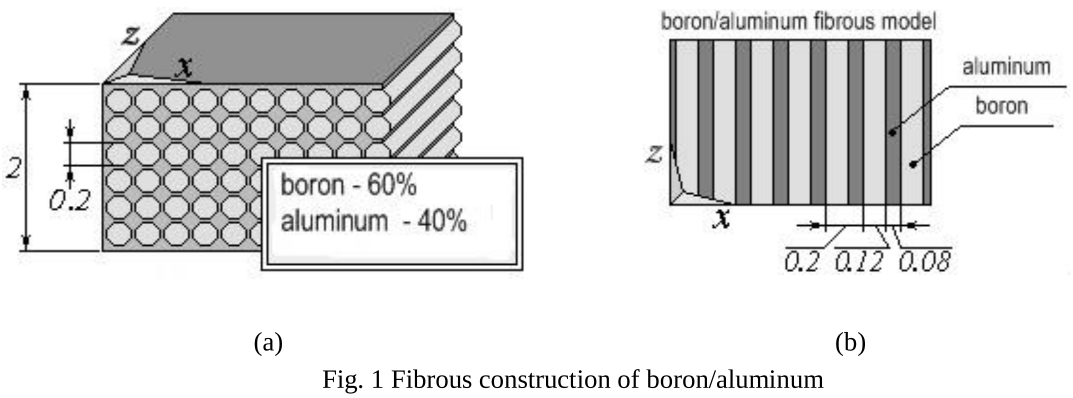
Furthermore, opportunity of new modules integration, developed by the user.

The software consists of the following stages

- pre-processing (APKEM module - building of construction's finite element model);
- solving (NERPEK module - includes state equation coefficients calculation, construction and solving of equations system, elastic and elastic-plastic construction parameters calculation, plasticity zone determination, residual strain and secondary plastic strain area);
- post-processing (TASVIR module – visualization of finite element model and calculation results).

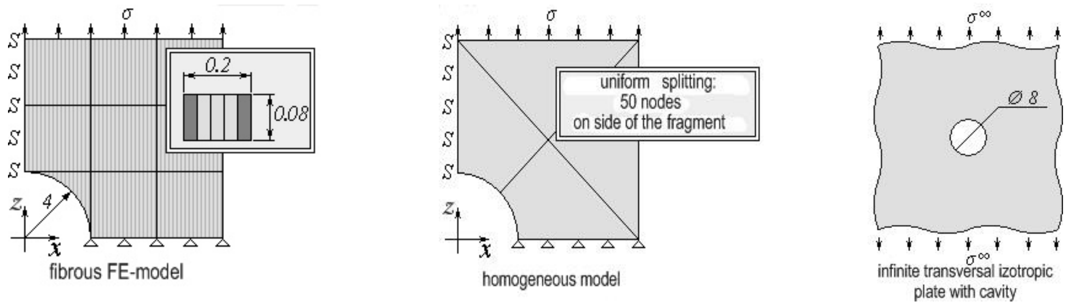
This software structure allows to:

- perform a computational experiment and determine the optimal parameters of composite materials by determining fiber volume content, construction elements' mechanical and geometrical parameters;
- investigate the influence and interaction of concentrators on stress state constructions;
- obtain a visual picture of investigated object's stress state distribution.



(a) (b)

Fig. 1 Fibrous construction of boron/aluminum



(a) (b) (c)

Fig. 2 Representation of solved problems

5. Research object and model

The finite element model, which explicitly reproduces the fibrous structure of the material, is used for validation of the homogeneous model (Karpov 2002). Three - dimensional scheme with characteristic size of boron/aluminum structure is shown in Fig. 1. Its two-dimensional analogue of preserving the transverse size of fiber and the percentage of composite components are also given.

Fig. 2 represents three test problems, solutions are compared assuming both composite components deformed elastically. First two problems are for uniaxial tension of a square plate with a circular hole. The first problem was solved by the fiber model (Fig. 2(a)) consisting of finite element packages with a single fixed thickness. Each package consists of five identical elements (the length is 1/50, width is 1/100 of hole's radius). Duralumin characteristics are specified for the two edge elements and characteristics of boron for three internal elements.

The second problem is solved using homogeneous transversely-isotropic material model with the effective elastic characteristics of boron/aluminum (Fig. 2(b)). In these problems, uniaxial tension was implemented by specifying uniformly distributed normal stress at the nodes of the upper border. The third problem is the problem of uniaxial tension of infinite transversely-isotropic plate with a circle hole (Fig. 2(c)). Comparing of these three problems was carried out for ε_{zz}, σ_{xx}, τ_{xz} values, as fibrous model continuous solutions could be obtained only for them.

Graphs for the first two values are presented in Fig. 3. Similar graph for τ_{xz} is not completed because stress contours does not cross OX and OZ axis.

These graphs show that the continuous solutions for first two models are almost identical. Difference could be observed only within one millimeter from the hole's edge (first five fibers). Homogeneous model solution is slightly larger in absolute value than on fibrous model solution.

The reason for above is that the structural model has a characteristic size and ratio of size and hole diameter influences stress concentration in the intact fibers. Deviations from analytical solutions are due to plate finite dimensions, which results influence of plate's edges on the stress-deformed state. With increasing of homogeneous plate size, problem solution is drawn near to analytical solution.

For illustrative purposes, stress components' calculations results for fiber plate in the hole vicinity are shown in Fig. 4, and strains in Fig. 5. Solutions correspond to σ_{zz} - external tensile stress equal to 10 MPa.

Fig. 4(a) illustrates that largest concentration of tearing stress lays not on OX axis, as in the homogeneous model, but near the end of last fiber that has been cropped by circle hole.

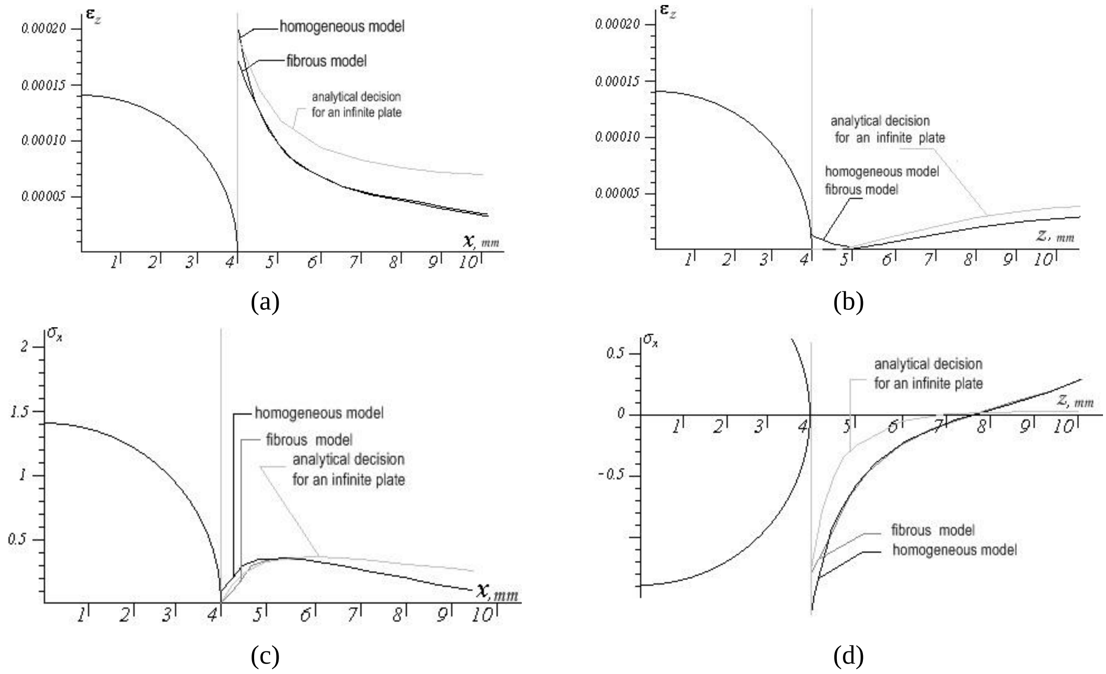
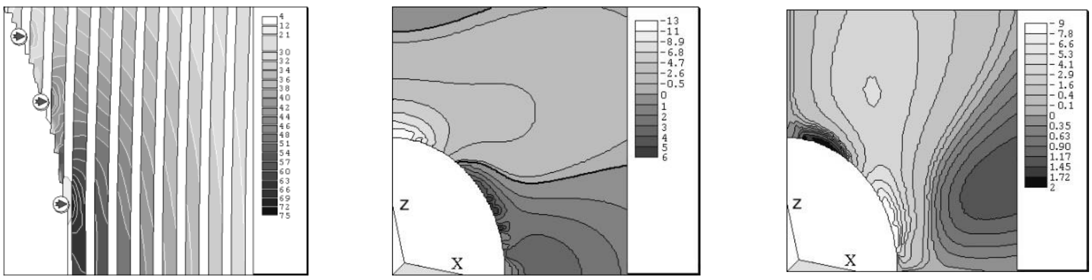
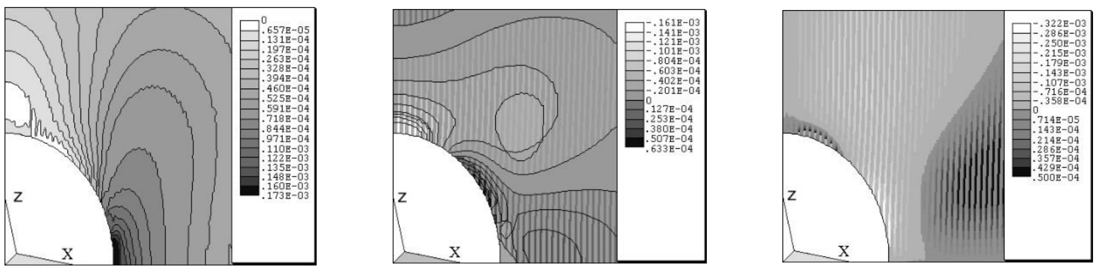


Fig. 3 Graphs for ε_{zz}, σ_{xx} values



(a) (b) (c)

Fig. 4 Stress values distribution of σ_{zz}, σ_{xx}, τ_{xz}



(a) (b) (c)

Fig. 5 Strain values distribution of ε_{zz}, ε_{xx}, γ_{xz}

Similar concentration zones (marked with arrows in the figure) on opposite side of previous fibers cutoff place are observed in other fibers. This happens because of two plate parts interaction located on left and right sides of cropped fibers do not extend beyond cutoff point, even though binding material layer drops below this point.

The stresses σ_{zz} in boron fibers belong to the stresses calculated as a 3/2 using homogeneous model, which equals to the ratio of the Young's modulus of boron to the averaged E_{zz} modulus of boron/aluminum. For duralumin stresses ratio in interlayers to homogeneous model stresses are as ratio of the E_{zz} modulus of duralumin to the averaged E_{zz} modulus of boron/aluminum is approximately equal to 1/4. These results are consistent with the formulas expressing the relationship between local and the averaged stresses in layered composite.

Consequently, homogeneous model can be considered as appropriate and used for further calculations.

6. Numerical experiment

Below carried investigations are related to stress state study of unidirectional fiber composites. All problems solved in three-dimensional setting. This allows realistically take into account the elastic-plastic state of considered constructions.

Expressions obtained on the basis of composite materials' asymptotic methods of calculating are used for unidirectional fiber composites effective parameters calculation. Usage of them allows taking into account the radial interaction of components caused by the difference of matrix and fibers Poisson's coefficients.

6.1. Elastic-plastic problem of uniaxial tension (P_{zz} = 850 MPa) of rectangular plate with isolated hole in the center is considered for investigation of fiber volume fraction influence in unidirectional composite material (hole radius R = 0.1 cm). Material fibers are arranged parallel to axis OZ and match up with loading direction. In this investigation boron/aluminum is used as fiber composite. D16 - aluminum alloy's matrix material has an elastic constants E=7.1*10^4 MPa, μ=0.32, hardening coefficient $\bar{\lambda}$=0.5 and the elastic limit σ_s=142 MPa. For boron fiber - E'=39.7*10^4 MPa, μ'=0.21 and tensile strength σ'_s=2.5*10^3 MPa .
Boron/aluminum of different fiber fraction is used as fiber composite, effective modules for them are given in Table 1. Fiber volume fraction influence on deformed state in composite is given in Fig. 6. Strain intensity values distribution in the vicinities of hole in plane of isotropy at v = 25% is given in Fig. 6(a). Plastic strain zones are located in the upper and lower vicinities of hole. Furthermore, small plasticity zones, between 30° and 40° relatively to horizontal diametrical section, are observed in boundaries of hole. Presence of such zones means that at this fiber fraction combined action of fiber and matrix is not performed. Increasing of v-value up to 35% leads to decreasing in hole's plastic strain zone in vertical and their extinction to side edges. Further volume fraction increasing from 45% to 55% leads to reduction of plastic strain areas, which are almost disappear at v = 60% value. This is confirmed by strain intensity values presented in Table 2. Values correspond to a point located directly on the intersection of hole border with vertical diametric section. Thus, at fiber volume fraction of 60% or more unidirectional fiber composites can be considered as a brittle material.

Table 1 Effective modules of fiber composite

v	E' [MPa]	E [MPa]	G' [MPa]	G [MPa]	μ'	μ
25%	$1.5271*10^5$	$0.8934*10^5$	$0.3401*10^5$	$0.3720*10^5$	0.2881	0.2011
35%	$1.8532*10^5$	$0.9964*10^5$	$0.3802*10^5$	$0.4311*10^5$	0.2762	0.1558
45%	$2.1797*10^5$	$1.1261*10^5$	$0.4312*10^5$	$0.5053*10^5$	0.2646	0.1144
55%	$2.5056*10^5$	$1.2948*10^5$	$0.4979*10^5$	$0.5991*10^5$	0.2537	0.0806
60%	$2.6682*10^5$	$1.3992*10^5$	$0.5396*10^5$	$0.6551*10^5$	0.2480	0.0682

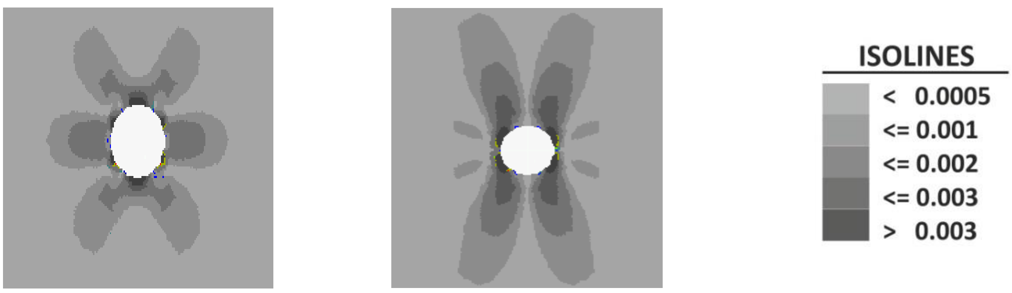
Table 2 Values of strain and stress intensity

v	elasticity		plasticity	
	p_u	P_u [MPa]	P_u	P_u [MPa]
25%	0.00612	455.1	0.00594	332.6
35%	0.00520	447.4	0.00493	341.8
45%	0.00443	448.1	0.00404	355.7
55%	0.00380	455.5	0.00331	378.3
60%	0.00352	461.5	0.00305	396.2

Fig. 6(b) illustrates distribution of strain intensity values along the main axis of transversal isotropy with v = 35%. Elastic strain' increased values appear on the side vicinities of hole contour. However, values become minimal in the vicinity of horizontal diametric cross section (tone filling in Figs. 6(b), 8(a) and 9(a) corresponds to dual strain values).

Strain curves in the plane of isotropy for various values of fiber fraction content in composite are shown in Fig. 7. Lower curve corresponds to v = 25%, and further in accordance with Table 2. Strength characteristics of the composite are increasing with fiber volume fraction increasing. But at the same time, it increases the fiber density and decreases elastic characteristics of the composite.

6.2. Problem of elastic-plastic uniaxial tension (P_{zz} = 300 MPa) of rectangular plate with concentrators in the form of cracks are being solved. Boron/aluminum is used as fiber composite. Effective mechanical parameters for boron/aluminum are as follows: E=160*10^3 MPa, μ=0.32, E'=260*10^3 MPa, μ'=0.254, G'= 51*10^3 MPa, G=E/(2*(1+ μ)).



(a) (b)

Fig. 6 Distribution of strain intensity values

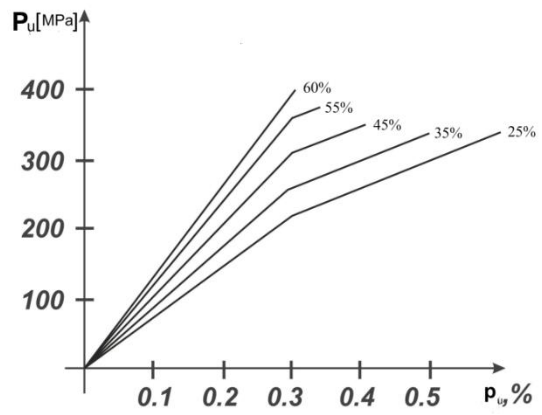
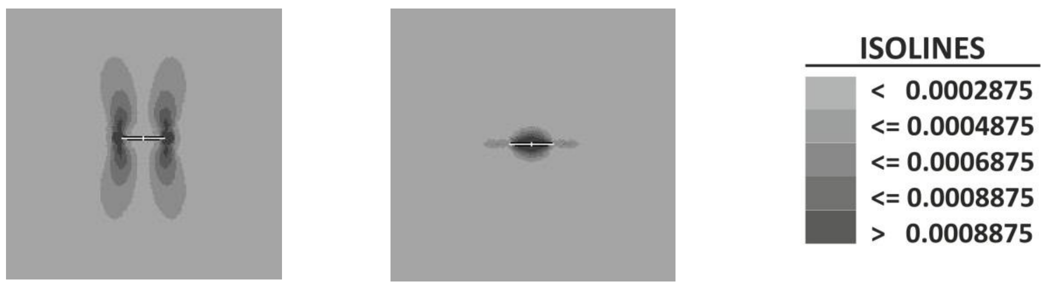


Fig. 7 Strain diagram

Problem of uniaxial tension plate with a horizontal isolated linear crack in the center with length l = 0.1 cm is being considered. Intensity values distribution of strains q_u and p_u is shown in Fig. 8. Maximum intensity values of strains q_u are concentrated in the vicinity of the crack tip, but they do not reach ultimate tensile strength (Fig. 8(a)). Strain intensity distribution investigation shows that the plasticity zones are concentrated in crack area (Fig. 8(b)).



(a) (b)

Fig. 8 Intensity values distribution of strains q_u and p_u

(a) (b)

Fig. 9 Distribution of q_u and p_u values

Table 3 Strain and stress intensity values

crack	p_u	P_u [MPa]	σ_{xx} [MPa]	σ_{zz} [MPa]
central	0.000390	47.2	188.2	805.4
laterals	0.001468	142.8	361.8	769.4

Further, uniaxial tension problem of plate with horizontal rectilinear cracks (l = 0.05 cm), located on plate's side edges is considered. Intensity values distribution of strain q_u is shown in Fig. 9(a). Distortion of construction sides and disclosure of crack edges are observed under tension. Increased values of q_u concentrated in area of crack tips and spread vertically. Elastic-plastic strains - p_u are formed in the vicinities of the crack tips (Fig. 9(b)).

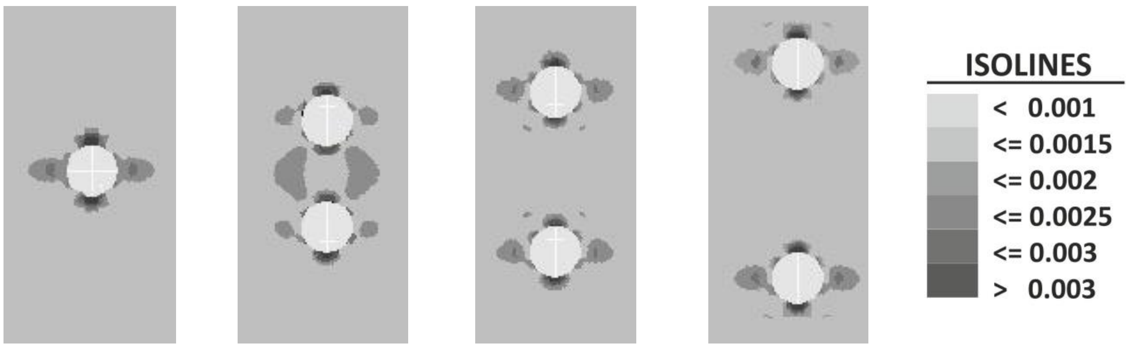
Component values of stress - strain state in crack tips under uniaxial tension of plate with isolated crack and of plate with lateral cracks are shown in Table 3.

6.3. In this paragraph stress reducing process by changing of contour shape with minimal distortion stress are being studied. Improvements in stresses distribution and increased constructional strength could be achieved by constructional changes (Neuber 1946).

In the present paper elastic-plastic stress-strain state of fibrous boron/aluminum plates is being investigated. It is stretched uniaxial in direction of fiber. Circular hole was cut off in the center of plate for constructional purposes. Rectangular plate's dimensions are: height - 1 cm, width - 0.5 cm, thickness - 0.1 cm. Boron fiber volume fraction - 35%, hole radius R = 0.05 cm, external loading P_{zz} = 950 MPa.

Analysis of solution results are given below. As it is known, distribution of stresses in the plate in isolated holes presence is considerably distorted. Increased stress is observed in vicinity of hole (Fig. 10(a)). Second hole addition to existing hole is assumed (Figs. 10(b)-10(d)).

Computational experiment is carried out for investigating of two vertically positioned holes influence. It turns out that additional hole also causes stress increase in surrounding area. However, it is known that holes interference reduces overall stress. The values p_u for distance between centers of holes h = 0.2 cm decrease in external points to 7.7%, and in internal - to 26.7% (Fig. 10(b)). It is interesting to note that the elastic problem values are, accordingly, 6.7% and 32.7%. Increasing stresses in this case are less than in case with isolated hole (Tables 4 and 5).



(a) (b) (c) (d)

Fig. 10 Distribution of strain intensity values

Table 4 Parameter values in external points of hole contours

hole	elastic problem		elastic-plastic problems	
	p_u	P_u [MPa]	p_u	p_u [MPa]
isolated	0.00524	452.0	0.00449	322.7
2-vertical, h=0.2	0.00489	421.8	0.00414	307.7
2-vertical, h=0.3	0.00520	448.8	0.00451	323.8
2-vertical, h=0.4	0.00560	482.7	0.00507	347.7

Table 5 Parameter values in internal points of hole contours

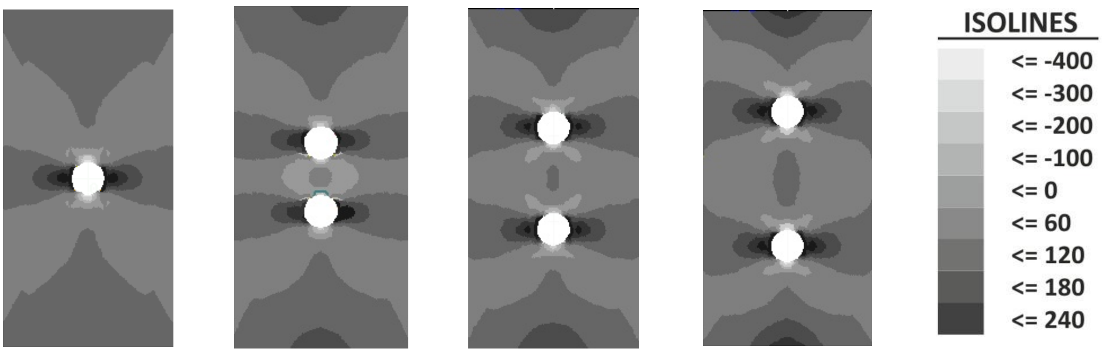
hole	elastic problem		elastic-plastic problems	
	p_u	P_u [MPa]	p_u	p_u [MPa]
isolated	0.00524	452.0	0.00449	322.7
2-vertical, h=0.2	0.00353	304.2	0.00325	269.4
2-vertical, h=0.3	0.00468	403.5	0.00433	315.9
2-vertical, h=0.4	0.00527	454.0	0.00491	340.9

Single stress concentrator is formed by two vertically arranged holes. With distance increasing of holes from each other at h = 0.3 and 0.4 cm their interference disappears (Figs.10(c) and 10(d)).

This phenomenon could be explained using the power flow idea. External forces create a flow that spreads along the construction. Pressure line (power flow) is rejected by the second hole. Influence of hole after rejection of passing power flow cannot grow anymore (Neuber 1946).

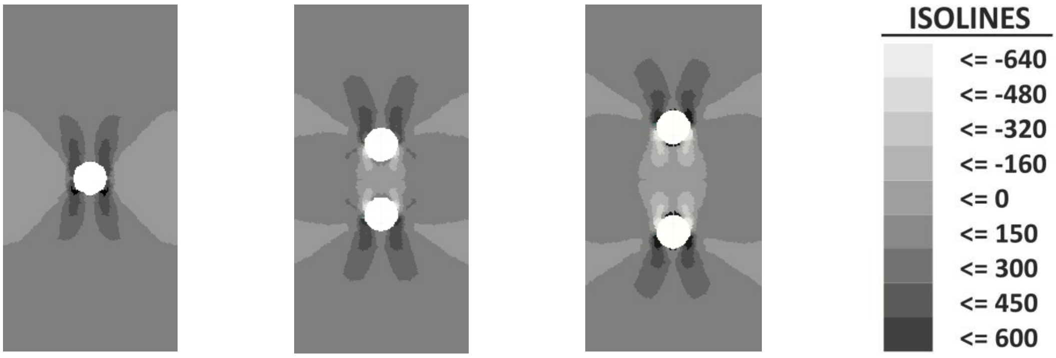
To study of unloading holes effect - stress stated component distribution is studied. The formation of a plasticity zone in the plane of isotropy is determined mainly by σ_{xx} stress component (Fig. 11). Other parameters' influences are insignificantly.

Stress components τ_{zx} values distribution is given in Fig. 12. Maximum τ_{zx} values concentrated on holes' sides at an angle from $\pi/6$ to $\pi/4$ relative to the horizontal diametric section. In the vicinities of isolated holes τ_{zx} has negative values (Fig. 12(a)). Two vertical holes form both negative and positive zones (Figs. 12(b) and 12(c)).



(a) (b) (c) (d)

Fig. 11 Stress components σ_{xx} values distribution



(a) (b) (c)

Fig. 12 Stress components values τ_{zx} distribution

6.4. Stress-strain state of fibrous material infinite strip is investigated. The fibers are arranged along the axis OZ. Boron fiber volume fraction is equal to 35%. The plate is stretched in fibers' direction (P_{zz} = 950 MPa). Center of hole system, having radius R = 0.05 cm is located on the OX axis. The distance between the hole centers l = 0.5 cm, strip height 1 cm, thickness 0.1 cm.

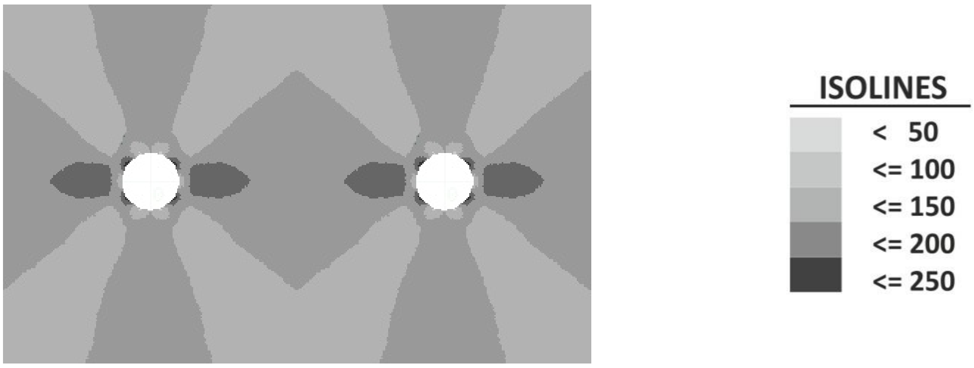


Fig. 13 Stress intensity P_u values distribution

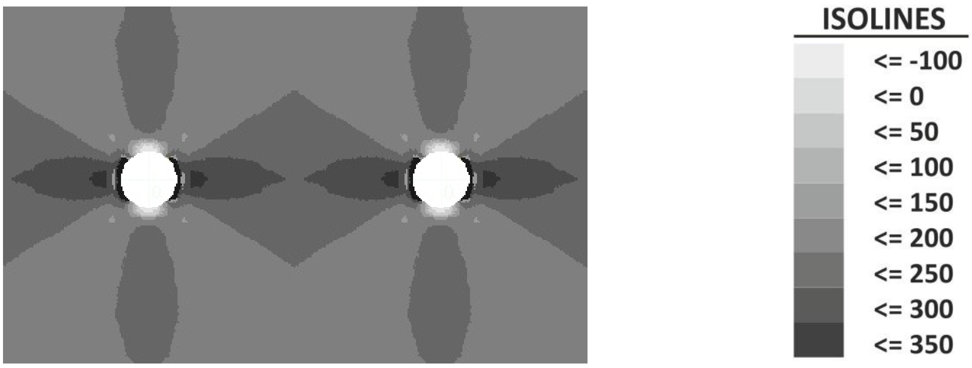


Fig. 14 Stress components values σ_{xx} distribution

Analysis of stress intensity values P_u illustrates the interference of the holes (Fig. 13). Holes vicinities are unloaded, and there are no plasticity zones. Entire strip is in a stressed state. Increased values are observed along the axis of the holes.

Distribution of stress components σ_{xx} values confirms the pattern view of stressed state (Fig. 14). Concentration tensile stresses values on hole system sides caused by their interference. Increased stressed state also observed in areas between the holes.

At the end this paper, stretching of infinite strip with the system of two vertically positioned holes is investigated. Distances between the holes are: vertically h = 0.2 cm, horizontally - l = 0.5 cm.

Distribution of stress intensity P_u values is shown in Fig. 15. Unloading of stress stated strip is the result of holes' vertically as well as horizontally directed interference. Stress intensity values of strip are slightly less than in the case of uniaxial hole system.

Stress values distribution of σ_{xx} component is shown in Fig. 16. Unlike the isolated plate (Fig. 11), there is no zone of compressive stress between vertical holes. Moreover, the center of the area is most susceptible to stretching stresses.

In conclusion, it should be noted that analysis of obtained results allows revealing of additional holes location, reducing the stress concentration and unloading the vicinity of concentrators.

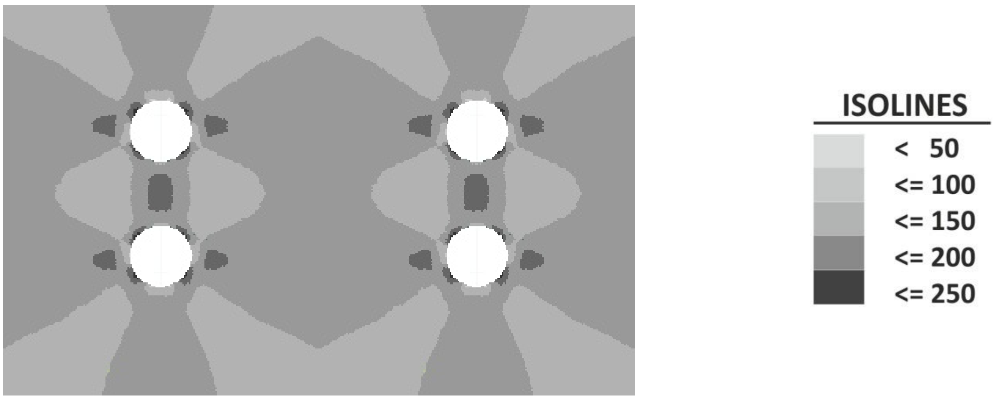


Fig. 15 Stress intensity values P_u distribution

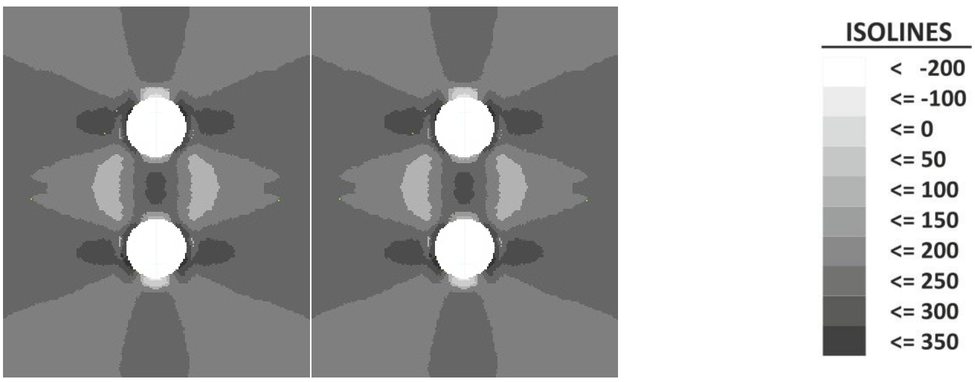


Fig. 16 Stress components values σ_{xx} distribution

7. Conclusions

A numerical modeling of fibrous composites strain process has been proposed in this study.

- Software package developed for new fibrous materials designing purposes with allows predicting pre-determined properties. It allows you to automate the mechanical parameters and ratios of fiber and matrix volume fractions determining processes.
- Stress-strain state of unidirectional composite materials is investigated and related fiber volume fraction influence specifications determined.
- It's revealed that in presence of horizontal isolated crack, plastic strain zone in the plane isotropy (p_u) is formed over the entire contour of the crack.
- The presence of lateral cracks causes curvature of areas in the vicinities of cracks. Plastic strain zone in the plane of isotropy (p_u) are formed in area crack tips.
- Fibrous composites have elastic-plastic characteristics when fibers volume fraction is in the range of 30% to 60%. Interaction of fiber and matrix is not provided when fiber volume fraction is less than 30% and when more than 60% - composite behaves as a brittle material.
- Two vertically positioned holes interference is studied. Hole system location is identified at which it works for the stressed state unloading.
- Uniaxial and two vertically positioned hole system interference in an infinite strip is studied. It's found that the presence of two vertical holes reduces concentrators influence and improves strength of construction.

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