Computers and Concrete, Vol. 9, No. 5 (2012) 389-402

DOI: http://dx.doi.org/10.12989/cac.2012.9.5.389

Prediction of elastic modulus of steel-fiber reinforced concrete (SFRC) using fuzzy logic

Mustafa Gençoğlu[1], Tayfun Uygunoğlu*[2], Fuat Demİr[3] and Kadir Güler[4]

[1]*Istanbul Technical University, Faculty of Civil Engineering, Division of Structural Engineering, 34469 Maslak, Istanbul, Turkey*
[2]*Afyon Kocatepe University, Engineering Faculty, Civil Engineering Department, 03200 Afyonkarahisar, Turkey*
[3]*Süleyman Demirel University, Engineering Faculty, Civil Engineering Department, 32260 Isparta, Turkey*
[4]*Istanbul Technical University, Faculty of Civil Engineering, Division of Structural Engineering, 34469 Maslak, Istanbul, Turkey*

(Received December 7, 2009, Revised July 27, 2011, Accepted August 20, 2011)

Abstract. In this study, the modulus of elasticity of low, normal and high strength steel fiber reinforced concrete has been predicted by developing a fuzzy logic model. The fuzzy models were formed as simple rules using only linguistic variables. A fuzzy logic algorithm was devised for estimating the elastic modulus of SFRC from compressive strength. Fibers used in all of the mixes were made of steel, and they were in different volume fractions and aspect ratios. Fiber volume fractions of the concrete mixtures have changed between 0.25%-6%. The results of the proposed approach in this study were compared with the results of equations in standards and codes for elastic modulus of SFRC. Error estimation was also carried out for each approach. In the study, the lowest error deviation was obtained in proposed fuzzy logic approach. The fuzzy logic approach was rather useful to quickly and easily predict the elastic modulus of SFRC.

Keywords: steel-fiber reinforced concrete; compressive strength; elastic modulus; fuzzy logic; modelling.

1. Introduction

The use of steel-fiber reinforced concrete (SFRC) in the world has been rapidly increasing for the last two decades. Many studies showed that steel fibers improve the mechanical properties of concrete such as flexural strength, compressive strength, tensile strength, creep behavior, impact resistance and toughness (Song and Hwang 2004, Sorooshian and Bayasi 1991, Uygunoğlu 2008). Addition of fibers makes the concrete more homogeneous and isotropic, and therefore it is transformed from brittle to a more ductile material (Ding and Kusterle 2000). SFRC offers more alternatives for the design of long-span beams and bridges, highway and airfield pavements, hydraulic structures, tunnel linings and more (Lim and Oh 1999, Kayali *et al.* 2003). In structural engineering, serviceability limit states, which may be critical for slender and thin sections of SFRC structures, are absolutely related with elastic modulus of concrete. Therefore, the modulus of elasticity of concrete is a

* Corresponding author, Ph D., E-mail: uygunoglu@aku.edu.tr

primary parameter required in structural analysis to determine the strain distribution and displacements of the structure (Rao and Prasad 2000, Baalbaki *et al.* 1992, Gesoğlu *et al.* 2002, Jang *et al.* 1996).

In the codes related with the design of concrete structures, the modulus of elasticity of concrete is usually proposed by empirical equations depending on a function of the compressive strength of concrete. According to these functions, the modulus of elasticity of concrete increases with its compressive strength (Topçu 2005, İsmail 2011, Williams 2010). In this case, it can be expected that the elastic modulus and compressive strength of concrete is affected by the mix design of the concrete. Therefore, it is difficult to state a certain relationship that is generally valid for the elastic modulus of concrete (Baalbaki *et al.* 1992, Jang *et al.* 1996, Demir 2005).

It is known that the contents and physical-mechanical properties of fibers in the concrete mixture have an effect on the compressive strength and elastic modulus of SFRC. Hence, it can be easily seen that the numbers of the effective parameters on the behavior of SFRC also increase. Because of that, it is fairly difficult to obtain the non-empirical equations which determine the elastic modulus and other mechanical properties of concrete.

Artificial intelligence applications have recently been used widely to model some of human interesting activities in many areas of science and engineering. Some of them were used to predict the steel fiber concrete properties. Many researchers have used the fuzzy logic approach to solve some of the problems related to structural and material engineering. These researchers have explained that the fuzzy logic systems are very effective on the realistic determination of mechanical properties of concrete and other materials. Demir (2005) developed a fuzzy logic approach to predict the elastic modulus of high strength concrete depending on the compressive strength of plain concrete, and compared the results of this approach with those of the equations proposed by the codes and literature. Ünal *et al.* (2007) developed a sensitive model for stress-strain curve of SFRC under compression by using the fuzzy logic approach. They reported that fuzzy sets can be used for non-linear curve estimation, too. Also, Uygunoğlu *et al.* (2006) applied a fuzzy model to determine the effect of fly ash content on compressive strength of concrete with the aim of optimization, and the results of this model were very close to experimental data. The highest error ratio was 15.45% between predicted and experimental results.

Eswari *et al.* (2008) carried out an experimental study on ductility performance of hybrid steel fiber reinforced concrete. The specimens incorporated from 0% to 2% volume fraction of polyolefin and steel fibers in different ratios. After experimental study, an Artificial Neural Network (ANN) model were developed for hybrid steel fiber concretes by authors, and a close agreement was obtained between the experimental and predicted results. Hodhod and Abdeen (2009) investigated the application of metallic steel fibers and natural (Linen) fibers in concrete industry. Following the experimental studies, the ANN technique was applied for predicting the performance of concrete with different mix proportions. The results of study showed that the ANN method with less effort was very efficiently capable of simulating and predicting the performance of concrete with different mix proportions and different types of fibers. Patodi and Purani (1998) generalized delta rule algorithm of ANN to predict the flexural behavior of SFRC *T*-beams using a computer program. Altun *et al.* (2008) used ANN to predict the compressive strength of lightweight concrete by adding steel fiber in dosage of 0, 10, 20, 30, 40, 50 and 60 kg/m^3. The ANN predicted the compressive strength of steel fiber added lightweight concrete with a relative absolute mean error of 6.75%. Adhikary and Mutsuyoshi (2006) developed an ANN models for predicting the ultimate shear strength of SFRC beams. It is found that the neural network model, with five input parameters, predicts the shear strength of beams more closely than the network with four input parameters.

Karahan *et al*. (2008) developed an ANN model with levenberg-marquardt (LM), scaled conjugate gradient (SCG) and fletcher-powell conjugate gradient (CGF) algorithms for studying the strength properties of SFRC containing fly ash. The results obtained from the model and the experiments were compared, and it was found that the suitable algorithm is the LM algorithm. Jo *et al.* (2001) performed a study on modulus of elasticity of SFRC, having steel fiber with different aspect ratios, and compressive strength values of 30, 50 and 70 MPa. They reported that the current formulas used for *E*-modulus of SFRC are also valid for those of high strength SFRC. Teng *et al*. (2004) proposed an empirical formula to determine the relationship between effective elastic modulus of SFRC materials and variation of fiber volume fraction and aspect ratio. The empirical formula, based on the equivalent inclusion method proposed in the study, has represented an alternative method to calculating the effective elastic modulus of SFRC.

2. Currently used formulas for *E*-moduli

In ACI 544 code (1996), it is reported that the modulus of elasticity of SFRC is generally taken as equal to those of a similar plain concrete, when the volume percentage of fibers is less than 2%. However, in some of the codes and literature, the proposed equations for elastic modulus of SFRC have been derived from linear or power functions depending on the compressive strength. These equations are presented as follows

ACI 318 (2002) $$E_c = \gamma^{1.5} 4270 (f_c)^{1/2} \tag{1}$$

ACI 363 (1997) $$E_c = \left(\frac{\gamma}{2.346}\right)^{1.5} (10500 (f_c)^{1/2} + 70000) \tag{2}$$

CEB-FIB (1993) $$E_c = 10^{-4} ((f_c/10) + 8)^{1/3} \tag{3}$$

New RC Project (Jo *et al*. 2001) $$E_c = k_1 k_2 40250 \left(\frac{\gamma}{2.4}\right)^2 (f_c)^{1/3} \tag{4}$$

Jang (1996) $$E_c = k_1 k_2 \left(\frac{\gamma}{2.4}\right)^{1.5} (10000 (f_c)^{1/2} + 73000) \tag{5}$$

Jo (2001) $$E_c = 16191 \sqrt{f_c} - 13540 - 2.4 A_s \tag{6}$$

Where f_c, E_c, γ and A_s represent compressive strength (kgf/cm^2) elastic modulus of concrete (kgf/cm^2), unit weight (kg/dm^3) and aspect ratio of steel fiber, respectively; k_1 is coarse aggregate type (0.9-1.2), and k_2 is a coefficient related to the use of admixture (0.95-1.1) (Jo *et al.* 2001).

In this study, a fuzzy logic model was developed to predict the realistic *E*-moduli of SRFC as an alternative model to the former power functions. Furthermore, this newly developed model is independent from properties of SFRC components such as fiber volume fraction, aspect ratio and unit weight.

3. Fuzzy sets and logic

The fuzzy logic concept provides a natural way of dealing with problems in which the source of imprecision is an absence of sharply defined criteria rather than the presence of random variables. The main idea in fuzzy logic is the allowance of partial belonging of any object to different subsets of the universal set instead of belonging to a single set completely. Partial belonging to a set can be described numerically by a membership function (MF) which assumes values between 0 and 1 inclusive. Even if the measurements are carefully carried out as crisp quantities they can be fuzzified. Furthermore, if the form of uncertainty happens to arise because of imprecision, ambiguity or vagueness, then the variable is fuzzy and can be represented by a MF (Demir and Korkmaz 2007). In order to simplify the calculations, usually the MF is adopted as linear in practical applications. The fuzzy methodology application can be achieved through the fuzzy rules as outlined below (Sen 1998, Kiszka *et al.* 1985)

The input and output variables are divided into a number of subsets with simple triangular fuzzy MFs. Many fuzzy subsets such as triangular, Gaussian and exponential may possibly be used in fuzzification step. The triangular one is very common due to its simplicity (Demir and Korkmaz 2007). Generally, there are n^m fuzzy rules where n and m are the numbers of subsets and input variables, respectively. A fuzzy rule base can be achieved step-by-step from sets of input and output data as follows:

1. Try to model the problem with a minimum number of input variables.
2. Divide the range of each input variable into a number (4-8 in practice or usually m) of parts to give fuzzy subsets each with a triangular MF. Theoretically, the optimum number of fuzzy subsets can be found by minimizing the total squared error between the observations and predictions.
3. For each data point, corresponding membership values are computed in each fuzzy rule.
4. Compute the weight of each rule for data point m by multiplying the membership values that correspond to that rule and by squaring the result.
5. Store the output along with the complete set of rule weights.
6. Repeat the same steps for all the other data points.
7. Compute the weighted average.

4. Developing of fuzzy algorithm to prediction of elastic modulus of SFRC

It is a fact that the modulus of elasticity increases as the compressive strength of concrete increases, but the elastic modulus of concrete calculated by using the equations above is not consistent with the elastic modulus values of concrete calculated by the tests. The modulus of elasticity of SFRC is affected by the modulus of elasticity of the aggregate and by the volumetric proportion of steel-fibers in the concrete mixture (Yazıcı *et al.* 2007). However, properties of SFRC components affect the compressive strength of SFRC directly. Consequently, many parameters (e.g., modulus of elasticity of aggregate, volumetric proportion of steel-fiber in concrete) which affect the elastic modulus E_c and compressive strength f_c of SFRC can be taken into account in the developed fuzzy logic approach. This can be achieved by adding the related rules to the corresponding fuzzy approach which can lead to a more accurate elastic modulus of SFRC (Ünal *et al.* 2007).

In the fuzzy logic approach, there are three independent steps. The first step is fuzzification. In this step, events related with concrete strength and elastic modulus is considered as having ambiguous characteristics, and their domain is divided into many successive fuzzy subsets. Hence domain change is fuzzified.

For application of the fuzzy approach, the compressive strength and elastic modulus of SFRC are

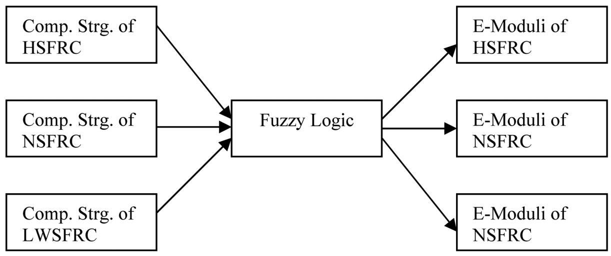

Fig. 1 Schematic representation of developed fuzzy logic model

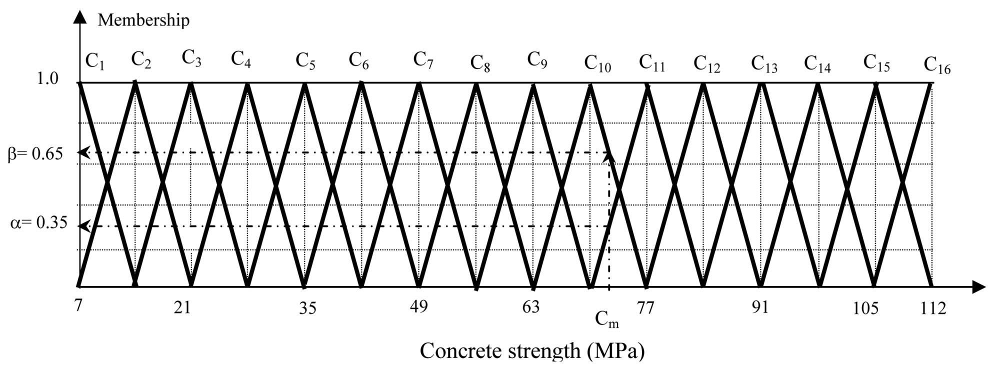

Fig. 2 Fuzzy subset membership functions for compressive strength class

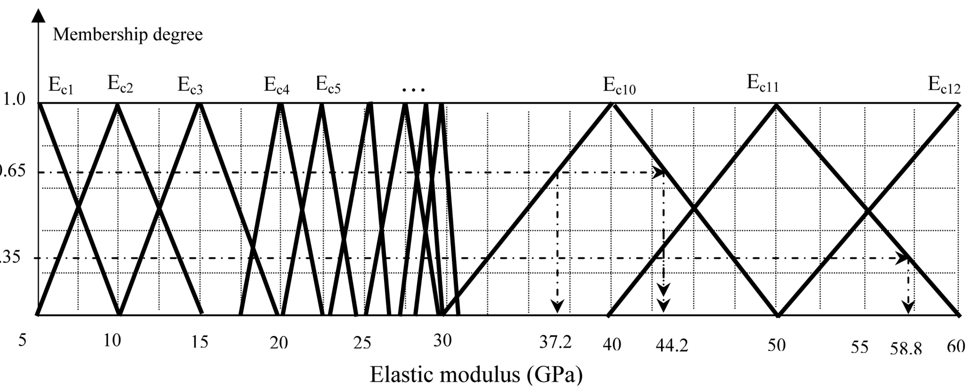

Fig. 3 Fuzzy subset membership functions for elastic modulus

derived from different references (Kayali *et al.* 2003, Rao and Prasad 2000, Düzgün *et al.* 2005, Kurihara *et al.* 2000, Haktanır *et al.* 2007, Jo *et al.* 2001). The schematic representation of developed fuzzy logic model was presented in Fig. 1. First of all, the compressive strength domain was divided into sixteen triangular subsets which are normal and complementary. Here, normality implies that fuzzy subset has membership value equal to 1 at least for one of the members. On the other hand, the compressive strength fuzzy subsets as C_1, C_2, C_3, …, C_{16} were considered to have triangular MFs represented in Fig. 2. The value of compressive strength f_c in Fig. 2, which includes low, normal and high strength concrete increases from 7 to approximately 112 MPa. Elastic moduli's subdivision to twelve subsets were labeled as E_{c1}, E_{c2}, E_{c3}, …, E_{c12} in increasing magnitude (Fig. 3).

This subdivision was considered as valid for the whole range of experimental results.

The second step is the conclusion in which the effective factors are matched according to the solution of the problem. Many fuzzy conditional statements were used to describe a certain situation. In this study, the compressive strength and elastic modulus of concrete werer combined with fuzzy subsets in accordance with Eq. (7) through the following fuzzy rule base.

R_i : IF f_c is $C_i + C_{i+1}$ THEN E_c' is $E_{ci} + E_{ci+1}$ (i = 1, 2, 3, …, 11)

$$R_i : \text{IF } f_c \text{ is } C_i + C_{i+1} \text{ THEN } E_c' \text{ is } E_{c11} + E_{c12}\ (i \geq 11) \tag{7}$$

Final step was the defuzzification. In this step, fuzzy outputs were converted to the crisp values. There are different defuzzification methods that can be used according to the overall purposes (Demir and Korkmaz 2007). In this study, weighted average method was used for defuzzification.

Once fuzzy subsets of elastic modulus of SFRC and compressive strength were determined, it is possible to estimate the elastic modulus from a given concrete strength through the following steps where the numerical calculations were based on given experimental data.

Step 1. Locate the measured concrete strength C_m, say as 75 MPa, on the horizontal axis of Fig. 2. It is possible to find two successive fuzzy subsets, for instance, C_{10} and C_{11}, for which the membership does not vanish.

Step 2. Find the membership degrees, namely, $\alpha = 0.35$ and $\beta = 0.65$ corresponding to these two successive concrete strength fuzzy subsets C_{11} and C_{10} respectively. It is important to notice at this stage that by definition, $\alpha + \beta = 1.0$

Step 3. Enter the compressive strength fuzzy subset domain from the fuzzy rule base by considering α and β membership degrees as shown, for instance, in Fig. 2.

Step 4. In Fig. 3 for $\alpha = 0.35$, compressive strength should yield two values, namely $\alpha_1 = 44200$ MPa and $\alpha_2 = 58800$ MPa from the corresponding E_{c11} fuzzy subset in the elastic modulus of concrete domain. Likewise, for $\beta = 0.65$, $\beta_1 = 37200$ MPa and $\beta_2 = 44200$ MPa are obtained from E_{c10} as shown in the same figure. In fact, E_{c10} and E_{c11} together present the fuzzy consequent, i.e., answer to the elastic modulus estimation in the form of the fuzzy subset.

Step 5. For the defuzzification of fuzzy set, the arithmetical average values from each fuzzy subset were calculated as follows

$$\bar{\alpha} = \frac{\alpha_1 + \alpha_2}{2} = \frac{44200 + 58800}{2} = 51500 \text{ MPa and}$$

$$\bar{\beta} = \frac{\beta_1 + \beta_2}{2} = \frac{37200 + 44200}{2} = 40700 \text{ MPa} \tag{8}$$

Step 6. The elastic modulus of steel-fiber concrete value, E_e, was calculated as the weighted average of these two simple arithmetic averages as

$$E_e = \alpha\bar{\alpha} + \beta\bar{\beta} = 0.35 \times 51500 + 0.65 \times 40700 = 44480 \text{ MPa} \tag{9}$$

It is possible to execute these steps for each of concrete strength which leads to either fuzzy subset estimation in a vague form or after its defuzzification by Eq. (9) for a single elastic modulus estimation value.

The fuzzy model was tested on the data provided by the formulas that were proposed by the codes. Since elastic modulus of concrete data changes slightly and smoothly over the range, the fuzzy portion of elastic modulus of concrete is expected to be slightly different for different test results. Also, error (E) was estimated by using Eq. (10).

$$E = \left(\frac{R_1 - R_2}{R_1}\right) \times 100 \tag{10}$$

Where, R_1 and R_2 are results of E_c from FL approach and from experimental study, respectively.

5. Results and discussion

Fuzzy logic is a successful common modeling technique. The elastic modulus of SFRC has been modeled by fuzzy logic. The elastic modulus values of SFRC determined by fuzzy logic approach are evaluated and compared with the values obtained from equations in the codes and literature and test results to check the compatibility of the proposed fuzzy logic model equation. The elastic modulus of SFRC calculated by the fuzzy logic approach developed in this study was compared with current elastic modulus assumption formula proposed by ACI 318 (2002), ACI 363 (1997),

Table 1 Experimental results for modulus of elasticity of concretes

Authors	Steel fiber type	Vf (%)	Aspect ratio	Compr. Str. (MPa)	Modulus of elasticity (GPa)
Kayali *et al.* (2003) (HSFRC)	Straight	0	37.5	65	24
		0.56	37.5	61	21
		1.13	37.5	62	21
		1.7	37.5	61	21
Düzgün *et al.* (2000) (LWSFRC)	Hooked end	0	75	19.47	10.33
		0.5	75	20.05	10.74
		1	75	20.85	10.96
		1.5	75	21.5	11.38
		0	75	15.99	9.03
		0.5	75	16.63	9.23
		1	75	17.4	9.84
		1.5	75	18.1	10.58
		0	75	13.6	7.63
		0.5	75	14.24	7.91
		1	75	14.92	8.74
		1.5	75	15.44	9.26
		0	75	10.87	6.71
		0.5	75	11.88	6.94
		1	75	12.58	7.33
		1.5	75	13.16	8.64
Rao and Prasad (2000) (HSFRC)	Straight	0	70	72.5	38.39
		0.31	70	70	37.56
		0.47	70	69	37.86
		0.62	70	76.7	38.97

Table 1 Continued

Authors	Steel fiber type	Vf (%)	Aspect ratio	Compr. Str. (MPa)	Modulus of Elasticity (GPa)
Haktanır *et al*. (2007) (NSFRC)	Hooked end	0	40/80	37.6	32
		0.32	40	40.2	33.3
		0.5	40	35.6	29.6
		0.32	80	41.6	30.8
		0.5	80	35.3	25.8
Kurihara *et al.* (2000) (HSFRC)	Intended	1	50	52.2	30.3
		2	50	51.9	30.1
		4	50	97.5	37.8
		6	50	87.6	37.6
	Intended	1	50	27.6	22.9
		1	50	48.6	30.9
		1	50	60.5	28.8
	Straight	1	50	35.3	27.9
		1	50	52.2	30.3
		1	50	66.5	30.9
	Deformed	1	50	33.5	25.2
		1	50	49.2	29.8
		1	50	62.7	30.1
Jo *et al.* (2001) (HSFRC)	Hooked end	0	75	42.3	23.29
		0.5	75	42.7	26.34
		0.75	75	38.1	28.24
		1	75	35.1	24.64
		1.5	75	33.7	22.47
		0	75	61.8	33.60
		0.5	75	61.1	35.42
		0.75	75	61.9	35.24
		1	75	53.9	32.11
		1.5	75	61.6	36.98
		0	75	65.9	38.51
		0.5	75	72.4	39.16
		0.75	75	72.8	38.58
		1	75	72.4	41.14
		1.5	75	62.9	29.10

CEB-FIB model (1993), New RC Project (Jo *et al*. 2001), Jang *et al.* (1996) and Jo *et al.* (2001). As seen from the experimental results (Table 1), the modulus of elasticity of SFRC increases as compressive strength of SFRC increases depending on the aspect ratio and volume fraction of steel fibers (Yazıcı *et al*. 2007). However, the orientation of fibers in concrete mix is extremely effective on mechanical behavior of SFRC. Therefore, it is not possible to say that the compressive strength and elastic modulus of SFRC can always be increased by adding steel fibers into concrete mixtures.

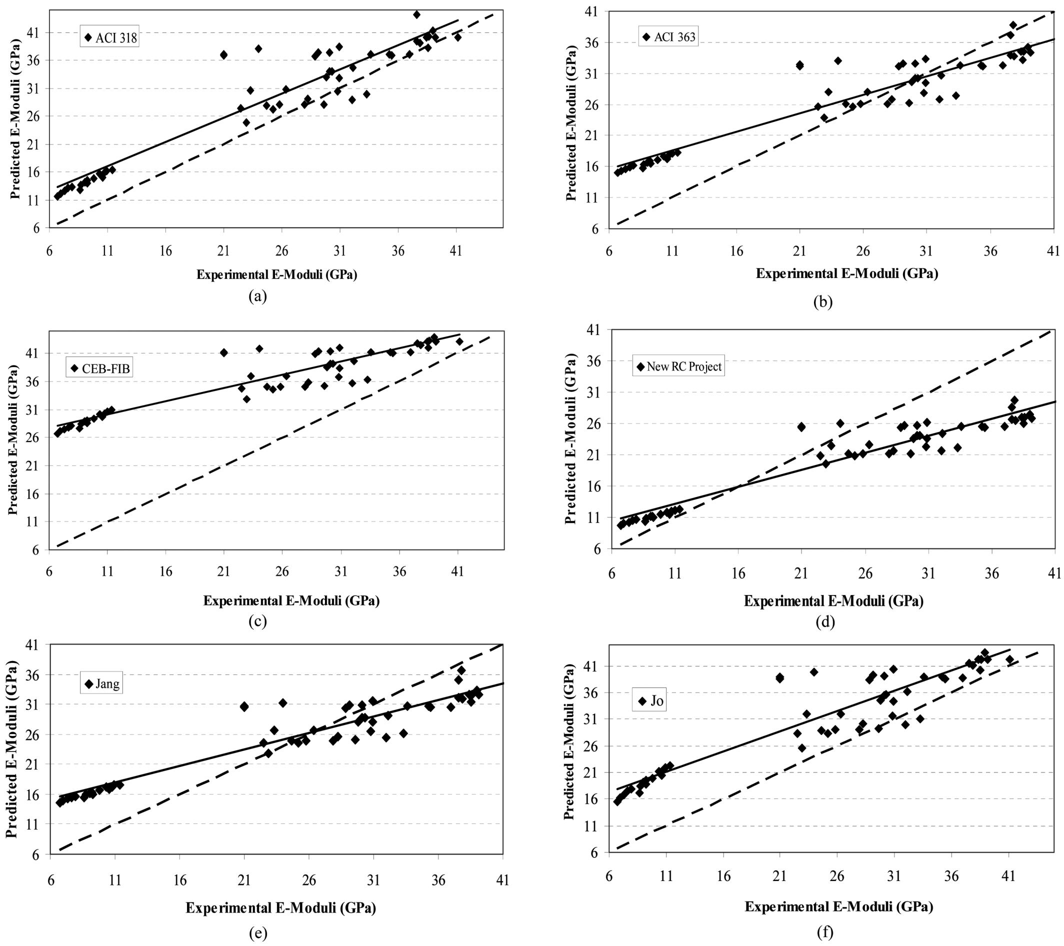

Fig. 4 Comparison of power functions and experimental results for E-moduli of SFRC

Sometimes, compressive strength of SFRC can be decreased due to orientation or distribution of fibers in concrete mixture. This case can be seen in SFRC which has especially high fiber content. In other words, some experimental results have shown that the compressive strength and elastic modulus of SFRC could be lower than the expected values. These unexpected results can be eliminated (minimized) by the fuzzy logic approach developed in this study.

All of the power functions and proposed models for elastic modulus of SFRC are given respectively and comparatively in Figs. 4(a), (b), (c), (d), (e), (f) and Fig. 5 together with experimental results. It is seen that the elastic modulus values calculated from the equations in the codes and literature are very close to those obtained from experiments for low, normal (NSFRC) and high (HSFRC) strength steel fiber reinforced concrete.

The closest elastic modulus value to experimental results among the current power functions proposed for E-modulus of SFRC is obtained from the power equation proposed by ACI 318 (2002). The highest deviation has been obtained by CEB-FIB model (1993), and the deviation was seen on the elastic modulus of low strength concrete. The results of the calculation model proposed by New RC Project are the closest results to experimental results among proposed equations in the

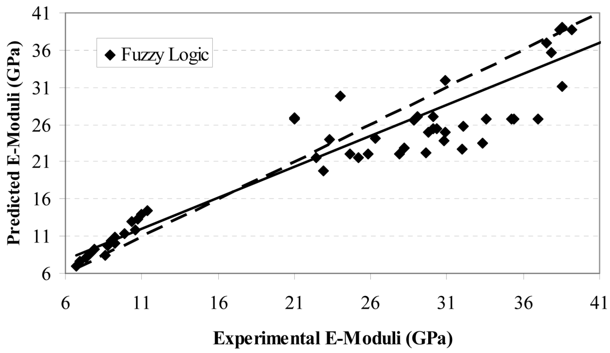

Fig. 5 Comparison of fuzzy logic model and experimental results for *E*-moduli of SFRC

codes and literature for light SFRC. Because of that, the calculation model of New RC Project can be proposed as a reliable model to determine elastic modulus of light SFRC.

Moreover, it can be seen that the elastic modulus values of SFRC calculated by fuzzy logic model are extremely convergent to the results obtained from the tests (from Fig. 5). It is obvious that these results calculated by fuzzy logic model remain within the central parts of the scatter diagram for

Table 2 Error estimations between experimental and predicted results for modulus of elasticity of SFRC

Concrete type	Comp. Str. (MPa)	Results of error estimation (%)						
		ACI-318	ACI-363	CEB-FIB	New RC Project	Jang	Jo	Fuzzy logic
HSFRC	65	58.22	37.44	74.14	8.07	30.08	66.32	24.58
	61	75.17	53.21	95.31	20.92	45.10	83.93	27.14
	62	76.60	54.19	96.25	21.58	46.00	85.49	28.10
	61	75.17	53.21	95.31	20.92	45.10	83.93	27.14
LWSFRC	19.47	51.06	71.14	192.09	14.65	65.82	105.42	24.88
	20.05	47.44	66.07	182.90	11.35	60.84	100.69	23.84
	20.85	47.33	64.69	179.84	10.55	59.41	100.79	26.82
	21.5	44.09	60.12	171.52	7.57	54.91	96.56	26.54
	15.99	56.60	84.69	219.39	22.82	79.48	111.54	15.17
	16.63	56.24	82.76	215.22	21.74	77.50	111.35	18.09
	17.4	49.91	73.73	198.73	15.93	68.61	103.10	15.85
	18.1	42.20	63.48	180.37	9.25	58.57	92.92	12.48
	13.6	70.92	108.71	265.00	37.72	103.34	129.49	14.02
	14.24	68.71	103.95	255.52	34.89	98.55	126.91	16.31
	14.92	56.29	87.05	225.01	24.00	81.97	110.58	10.98
	15.44	50.06	78.29	209.06	18.38	73.35	102.45	7.99
	10.87	73.76	123.23	296.77	45.33	118.24	131.13	2.83
	11.88	75.63	121.06	290.34	44.74	115.81	134.52	9.51
	12.58	71.12	112.60	273.86	39.68	107.37	129.03	10.50
	13.16	48.48	82.63	220.13	20.29	78.02	99.09	-1.62

Table 2 Continued

Concrete Type	Comp. Str. (MPa)	Results of error estimation (%)						
		ACI-318	ACI-363	CEB-FIB	New RC pRoject	Jang	Jo	Fuzzy logic
HSFRC	72.5	4.46	-10.28	12.47	-29.93	-15.18	9.99	1.07
	70	4.92	-9.57	13.76	-29.22	-14.48	10.40	-1.49
	69	3.34	-10.79	12.37	-30.11	-15.63	8.71	-5.97
	76.7	5.85	-9.60	12.69	-29.67	-14.60	11.55	7.26
NSFRC	37.6	-9.75	-16.36	11.64	-32.46	-20.34	-6.12	-29.06
	40.2	-10.32	-17.61	9.29	-33.64	-21.59	-6.61	-29.43
	35.6	-5.06	-11.38	18.91	-28.31	-15.54	-1.40	-25.00
	41.6	-1.37	-9.78	19.29	-27.43	-14.17	2.76	-22.40
	35.3	8.46	1.36	36.11	-17.98	-3.39	12.58	-14.34
HSFRC	52.2	12.31	-0.04	29.35	-20.43	-5.17	17.58	-15.84
	51.9	12.73	0.40	29.99	-20.06	-4.74	18.01	-15.61
	97.5	23.04	2.72	25.00	-21.45	-3.19	30.13	26.72
	87.6	17.24	-1.15	21.61	-23.80	-6.74	23.82	22.34
	27.6	8.05	4.51	43.65	-14.87	-0.06	11.50	-13.97
	48.6	6.26	-4.63	24.25	-23.81	-9.44	11.09	-19.42
	60.5	27.21	11.36	42.07	-12.07	5.47	33.54	-7.64
	35.3	0.30	-6.27	25.86	-24.15	-10.66	4.14	-20.79
	52.2	12.31	-0.04	29.35	-20.43	-5.17	17.58	-15.84
	66.5	24.30	7.72	36.17	-15.42	1.92	30.70	3.56
	33.5	8.18	1.81	37.39	-17.48	-2.89	12.18	-14.29
	49.2	10.86	-0.64	29.30	-20.68	-5.67	15.93	-16.11
	62.7	23.90	8.05	37.37	-14.86	2.30	30.15	-10.30
	42.3	31.51	20.04	58.47	-3.51	14.17	37.06	3.03
	42.7	16.84	6.53	40.50	-14.41	1.31	21.80	-8.51
	38.1	2.97	-4.74	26.99	-23.12	-9.29	7.07	-18.89
	35.1	13.24	5.90	42.28	-14.29	0.95	17.53	-10.72
	33.7	21.68	14.42	54.32	-7.27	9.13	26.17	-3.88
	61.8	10.19	-3.76	22.53	-24.10	-8.87	15.70	-20.24
	61.1	3.95	-9.10	15.87	-28.26	-13.91	9.13	-24.61
	61.9	5.15	-8.18	16.88	-27.59	-13.06	10.41	-23.96
	53.9	7.70	-4.50	23.21	-24.10	-9.43	12.80	-19.96
	61.6	-0.02	-12.65	11.25	-31.10	-17.28	4.97	-27.52
	65.9	-0.72	-13.88	8.96	-32.34	-18.51	4.36	-19.25
	72.4	2.35	-12.08	10.23	-31.34	-16.89	7.76	-1.16
	72.8	4.15	-10.58	12.05	-30.19	-15.48	9.67	1.08
	72.4	-2.59	-16.32	4.91	-34.65	-20.90	2.56	-5.93
	62.9	28.36	11.89	42.22	-11.85	5.93	34.82	-7.22

Table 3 The maximum deviations of proposed approaches

Proposed model	ACI-318	ACI-363	CEB-FIB	New RC Project	Jang	Jo	Fuzzy
Max. Deviation	(+) 76.6	(+) 123.23	(–) 73.81	(+) 45.33	(+) 118.24	(+) 134.52	(–) 29.43

any given concrete strength. Hence, the proposed method of fuzzy logic estimation leads to elastic modulus estimations in a vague form as either fuzzy subset or as a defuzzified value, E_e. On the other hand, it can be seen that the elastic modulus of SFRC computed by the developed fuzzy logic approach in this study is fairly compatible with the test results. Furthermore, the most convergent results can be found for elastic modulus of concretes with low compressive strength by the fuzzy logic model. Also, elastic modulus values determined by the fuzzy logic approach developed for the normal and high strength of SFRC are close to the results obtained from the experiments. According to this result, more realistic elastic modulus of SFRC can be obtained by the fuzzy logic model developed in this study.

The error estimations of the results obtained from the power function formulas given by different sources and fuzzy logic approach developed in this study were carried out and these are summarized in Table 2. The fuzzy logic approach developed in this study may be assumed to be a reliable model in comparison with the tested data, because this calculation model has the smallest error percent. In Table 3, maximum deviations for each assumption are presented to determine the best approach. In general, the highest deviations are obtained on low strength SFRC. When it is compared with the experimental results in the literature for the elastic modulus values of SFRC, the most realistic *E*-modulus results for low, normal and high strength SFRC are obtained from the fuzzy logic approach developed in this study. As seen from Table 2 and Table 3, the highest error estimation obtained from fuzzy logic approach was (–) 29.43%. However, the highest error ratio among all the other models is (+) 134.52% in the model proposed by Jo *et al*. (2001). The error ratios in other models are also fairly high. However, the highest error ratios are obtained from low SFRC for almost all the proposed power function formulas. Due to compression strength changing according to the distribution of the steel fiber in the concrete, a very sensitive model is developed by fuzzy logic approach.

6. Conclusions

Elastic modulus of concrete is very important in the non-linear design of structures. Currently used formulas for *E*-Moduli of SFRC depend on the compression strength concrete. The elastic modulus values of plain concrete and SFRC are estimated by using linear equations or original power functions depending on compressive strength. However, it can be seen that the results of two analyses are extremely different from each other. In this study, a new approach is presented for elastic modulus of low, normal and high strength steel fiber reinforced concretes. When the developed fuzzy model was compared with the experimental results in the literature for the elastic modulus values of SFRC, the most realistic *E*-modulus results for low, normal and high strength SFRC are obtained. The developed fuzzy algorithm does not provide an equation, but it can be adjusted itself to any type of linear or non-linear form through fuzzy subsets of linguistic elastic modulus and compressive strength variables. When some test results related with SFRC and the results of the empirical statements proposed in the codes and literature are compared with the results of the fuzzy logic approach developed in this study, it can be seen that the fuzzy logic approach method developed in this study has the smallest error ratios among all of the proposed equations for predictions of elasticity modulus of SFRC with low, normal and high strength. The highest error estimation obtained from fuzzy logic approach was (–) 29.43%. The adopted methodology in this paper has many advantages in comparison to the existing ones. Presented method is easily applicable and provides more realistic results. The methodology is rather simple in order to obtain more accurate results. Consequently, the developed and proposed fuzzy logic model can be used for determination of E-moduli of SFRC as an alternative to using power functions.

References

ACI 318.RM-02 (2002), Building code requirements for structural concrete. ACI Committee 318, American Concrete Institute.

ACI 363.R-92 (1997), State-of-art-report on high strength concrete. ACI Committee 363, American Concrete Institute.

ACI 544.1R-96 (1996), State-of-the-art report on fiber reinforced concrete. ACI Committee 544, American Concrete Institute, 1996.

Adhikary, B.B. and Mutsuyoshi, H. (2006), "Prediction of shear strength of steel fiber RC beams using neural networks", *Constr. Build. Mater.,* **20**(9), 801-811.

Altun, F., Kişi, Ö. and Aydın, K. (2008), "Predicting the compressive strength of steel fiber added lightweight concrete using neural network", *Comp. Mater. Sci.*, **42**(2), 259-265.

Baalbaki, W., Aitcin, P.C. and Ballivy, G. (1992), "On prediction modulus of elasticity in high-strength concrete", *ACI Mater. J.,* **89**(5), 517-520.

CEB-FIB Model Code (1993), Bull. D'information CEB. 213/214. Lausanne.

Demir, F. and Korkmaz, A. (2008), "Prediction of lower and upper bounds of elastic modulus of high strength concrete", *Constr. Build. Mater.,* **22**(7), 1385-1393.

Demir, F. (2005), A new way prediction of elastic modulus of normal and high strength concrete-fuzzy logic, *Cement Concrete Res.*, **35**(8), 1531-1538.

Demir, F., Uygunoğlu, T. and Ünal, O. (2006), "Fuzzy logic approach to predict to modulus of elasticity of steel fiber reinforced concrete in compression", Seventh International Congress on Advances in Civil Engineering, ACE06-513, Yıldız Technical University, İstanbul.

Ding, Y. and Kusterle, W. (2000), "Compressive stress-strain relationship of steel fibre-reinforced concrete at early age", *Cement Concrete Res.*, **30**(10), 1573-1579.

Düzgün, O.A., Gül, R. and Aydin, A.C. (2000), "Compressive stress-strain relationship of steel fibre-reinforced concrete at early age", *Cement Concrete Res.*, **30**(10), 1573-1579.

Eswari, S., Raghuanath, P.N. and Suguna, K. (2008), "Ductility performance of hybrid fibre reinforced concrete", *American J. Appl. Sci.*, **5**(9), 1257-1262.

Gesoğlu, M., Güneyisi, E. and Özturan, T. (2002), "Effects of end conditions on compressive strength and static elastic modulus of very high strength concrete", *Cement Concrete Res.*, **32**(10), 1545-1550.

Haktanir, T., Ari, K., Altun, F. and Karahan, O. (2007), "A comparative experimental investigation of concrete, reinforced-concrete and steel-fibre concrete pipes under three-edge-bearing test", *Constr. Build. Mater.*, **21**(8), 1702-1708.

Hodhod, H. and Abdeen, M.A.M. (2009), "Experimental investigation and numerical modeling of the effect of natural and steel fibers on the performance of concrete", *Int. J. Eng.* (IJE), **4**(5), 321-337.

Ismail, H. Cagatay and Riza Dincer (2011), "Modeling of concrete containing steel fibers: toughness and mechanical properties", *Comput. Concrete,* **8**(3), 357-369.

Jang, I.Y., Park, H.G. and Yoon, Y.S. (1996), "A proposal of elastic modulus equation for high-strength and ultra-high-strength", *Kor. Concrete Inst.*, **8**(6), 213-222.

Jo, B.W., Shon, Y.H. and Kim, Y.J. (2001), "The evalution of elastic modulus for steel fiber reinforced concrete", *Russ. J. Nondestruct.*, **37**(2), 152-161.

Karahan, O., Tanyıldızı, H. and Atıs, C.D. (2008), "An artificial neural network approach for prediction of long-term strength properties of steel fiber reinforced concrete containing fly ash", *J. Zhejiang Univ. Sci. A*, **9**(11), 1514-1523.

Kayali, O., Haque, M.N. and Zhu, B. (2003), "Some characteristics of high strength fiber reinforced lightweight aggregate concrete", *Cement Concrete Comp.*, **25**(2), 207-213.

Kiszka, J.B., Kochanskia, M.E. and Sliwinska, D.S. (1985), "The influence of some fuzzy implication operators on the accuracy of a fuzzy model Part II", *Fuzzy Set Syst.*, **15**(3), 223-240.

Kurihara, N., Kunieda, M., Kamada, T., Uchida, Y. and Rokugo, K. (2000), "Tension softening diagrams and evaluation of properties of steel fiber reinforced concrete", *Eng. Fract. Mech.*, **65**(2-3), 235-245.

Lim, D.H. and Oh, B.H. (1999), "Experimental and theoretical investigation on the shear of steel fibre reinforced concrete beams", *Eng. Struct.*, **21**(10), 937-944.

Patodi, S.C. and Purani, V.S. (1998), "Modeling flexural behavior of steel fibre reinforced concrete beams using neural networks", *J. Build. Mater. Construc.*, **4**, 28-35.

Rao, G.A. and Prasad, B.K.R. (2000), Fracture toughness of fiber reinforced high strength concrete. Fourteenth Engineering Mechanics Conference, Department of Civil Engineering, The University of Texas, USA., May 21-24.

Sen, Z. (1998), "Fuzzy algorithm for estimation of solar irradiation from sunshine duration", *Sol. Energy*, **63**(1), 39-49.

Song, P.S. and Hwang, S. (2004), "Mechanical properties of high-strength steel fiber-reinforced concrete", *Constr. Build. Mater.*, **18**(9), 669-673.

Sorooshian, P. and Bayasi, Z. (1991), "Fiber type effects on the performance of steel fiber reinforced concrete", *ACI Mater. J.*, **88**(2), 53-60.

Teng, T.L., Chu, Y.A., Chang, F.A. and Chin, H.S. (2004), "Calculating the elastic moduli of steel-fiber reinforced concrete using a dedicated empirical formula", *Comput. Mater. Sci.*, **31**(3-4), 337-346.

Topçu, İB. (2005), "Alternative estimation of the modulus of elasticity for dam concrete", *Cement Concrete Res.*, **35**(11), 2199-2202.

Ünal, O., Demir, F. and Uygunoğlu, T. (2007), "Fuzzy logic approach to predict stress-strain curves of steel fiber-reinforced concretes in compression", *Build. Environ.*, **42**(10), 3589-3595.

Uygunoğlu, T. and Ünal, O. (2006), "A new approach to determination of compressive strength of fly ash concrete using fuzzy logic", *J. Sci. Ind. Res.*, **65**(11), 894-899.

Uygunoğlu, T. (2008), "Investigation of microstructure and flexural behavior of steel-fiber reinforced concrete", *Mater. Struct.*, **41**(8), 1441-1449.

Williams, E.M., Graham, S.S., Akers, S.A., Reed, P.A. and Rushing, T.S. (2010), "Constitutive property behavior of an ultra-high-performance concrete with and without steel fibers", *Comput. Concrete,* **7**(2), 191-202.

Yazıcı, Ş., İnan, G. and Tabak, V. (2007), "Effect of aspect ratio and volume fraction of steel fiber on the mechanical properties of SFRC", *Constr. Build. Mater.*, **21**(6), 1250-1253.

NB