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Minimum deformability design of high-strength concrete beams in non-seismic regions

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Abstract. In the design of reinforced concrete (RC) beams, apart from providing adequate strength, it is also necessary to provide a minimum deformability even for beams not located in seismic regions. In most RC design codes, this is achieved by restricting the maximum tension steel ratio or neutral axis depth. However, this empirical deemed-to-satisfy method, which was developed based on beams made of normal-strength concrete (NSC) and normal-strength steel (NSS), would not provide a consistent deformability to beams made of high-strength concrete (HSC) and/or high-strength steel (HSS). More critically, HSC beams would have much lower deformability than that provided previously to NSC beams. To ensure that a consistent deformability is provided to all RC beams, it is proposed herein to set an absolute minimum rotation capacity to all RC beams in the design. Based on this requirement, the respective maximum limits of tension steel ratio and neutral axis depth for different concrete and steel yield strengths are derived based on a formula developed by the authors. Finally for incorporation into design codes, simplified guidelines for designing RC beams having the proposed minimum deformability are developed.

Keywords: beams; curvature; deformability; design aids; high-strength concrete; high-strength steel; reinforced concrete; rotation capacity.

1. Introduction

In the traditional design of reinforced concrete (RC) beams, much attention has been put on the design of sufficient flexural strength. Only a certain minimum level of flexural ductility and deformability are provided by some empirical deemed-to-satisfy rules to control the maximum tension steel and neutral axis depth. Except in Eurocode 2 (ECS 2004) that a set of more stringent requirements are imposed for beams with higher concrete strength, these empirical rules are not dependent on concrete and steel yield strength in most RC design codes. However, in a series of theoretical studies conducted by the authors on the ductility (Pam *et al*. 2001a, Ho *et al*. 2003, Lam *et al*. 2009) and deformability (measured in terms of normalised rotation capacity) (Ho *et al*. 2010a, Ho and Zhou 2010, Zhou *et al*. 2010) of RC beams using nonlinear moment-curvature analysis, it has been shown that at a given tension steel ratio or neutral axis depth, the ductility and deformability of RC beams depends significantly on concrete and steel yield strength. More importantly, the ductility and deformability provided to beams made of high strength concrete (HSC) and/or high-strength steel (HSS) would decrease to an unacceptably low level if the same deemed-to-satisfy rules are applied. This is because those rules were derived many years ago based on beams made of normal-strength concrete (NSC) and normal-strength steel (NSS) (Park and Ruitong 1988, Pam *et*

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al. 2001b). Considering nowadays that the adoption of HSC and HSS, which reduces the amount of construction materials under the same design load and hence lower the embodied carbon level in the structures, are getting more popular in tall buildings construction (Bilodeau and Malhotra 2000, Scrivener and Kirkpatrick 2008, Xu *et al*. 2008), the existing empirical rules for deformability design of RC beams should be revised to incorporate the adoption of HSC and/or HSS.

From performance-based design point of view, adequate flexural ductility and deformability design would prevent the beams from immediate collapse under earthquake attack (Lew 2007, Englekirk 2008, Park *et al*. 2008, Ho and Pam 2010). During an earthquake, RC beams with sufficient ductility and deformability would dissipate the enormous energy by redistributing moment to other parts of the beams through formation of plastic hinges at maximum moment regions (Wu *et al*. 2004, Chen *et al*. 2009, Li *et al*. 2009, Lu 2009, Nam *et al*. 2009, Weerheijm *et al*. 2009). In this connection, the reinforcement in the maximum moment regions should be designed carefully such that a minimum ductility and deformability would be provided to ensure that plastic hinges can be formed in the beam successfully (Ho and Pam 2003, Maghsoudi and Bengar 2006, Bechtoula *et al*. 2009, Pam and Ho 2009, Zhang *et al*. 2009, Sadjadi and Kianoush 2010). This can also be achieved by installing concrete-filled steel tube (Ellobody and YoWang *et al*. 2007, Cho *et al*. 2008, Feng and Young 2008, 2009), external steel plates (Altin *et al*. 2008, Su *et al*. 2009, Zhu and Su 2010) and FRP wraps (Lin *et al*. 2006, Hashemi *et al*. 2008, Wu and Wei 2010) at the maximum moment regions to enhance the confinement effect. For very tall building structures, the huge amount of energy induced by earthquake can also be dissipated more efficiently by installing dampers (Hwang *et al*. 2006, Chen and Ding 2008, Li and Xiong 2008, Chung *et al*. 2009) and adopting base isolation (Hino *et al*. 2008, Kim *et al*. 2008, Ates *et al*. 2009). However, even for beams not expected to resist seismic load, it is recommended that a certain minimum level of ductility and deformability should still be provided for the sake of resisting accidental load (Ho *et al*. 2004, Kwan *et al*. 2006, Lam *et al*. 2009), e.g. blasting and impact loads, and to provide extra safety to the occupants.

The authors have previously proposed a minimum flexural ductility in terms of curvature ductility factor (Ho *et al*. 2004) for RC beams design even they are not subjected to seismic risk. This level of minimum ductility is set at the ductility level provided previously to beams made of NSC and NSS using the empirical deemed-to-satisfy rules stipulated in the existing RC design codes (Ho *et al*. 2004, Kwan *et al*. 2006, Lam *et al*. 2009). Based on the proposed minimum ductility factors, the maximum limit of tension steel ratio and neutral axis depth were evaluated. In this study, the authors would similarly derive the minimum level of deformability for RC beams design even they are not subjected to seismic risk. Likewise, the minimum level of deformability would be set at the deformability level provided previously to beams made of NSC and NSS. The flexural design of RC beams possessing this minimum level of deformability is named by the authors as the "Minimum Deformability Design".

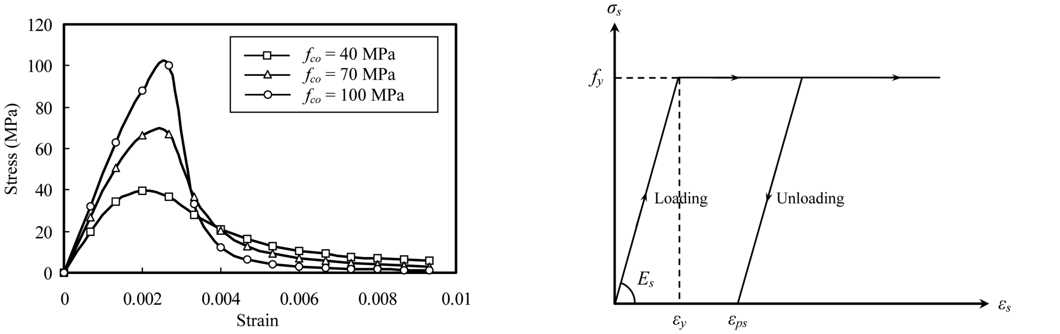
To evaluate the deformability of RC beams, the authors have carried out a series of parametric study to investigate the effects of some critical parameters on the deformability of RC beams (Ho *et al*. 2010a, Zhou *et al*. 2010). The deformability of RC beams was studied by "normalised rotation capacity" which is defined as the product of ultimate beam curvature and effective depth. From the results obtained, it was found that the deformability of RC beams increases as the degree of reinforcement decreases and confining pressure increases. On the other hand, the effects of concrete strength and steel yield strength on deformability are dependent on the degree of reinforcement and steel ratio. A formula for direct evaluation of deformability of RC beams based on the above parameters was then developed. In a separate study (Ho and Zhou 2010), the authors have also investigated the effects of these parameters on the interrelation between flexural strength and deformability. From the results, it was found that for a given concrete strength and confining pressure, there is a maximum limit of flexural strength and deformability that can be achieved simultaneously. Moreover, for a given pair of flexural strength and deformability, there is a maximum allowable limit of the degree of reinforcement λ or tension steel ratio ρ_t, beyond which the deformability can never be achieved apart from increasing the confining pressure or beam dimensions.

In this paper, the minimum deformability expressed in terms of normalised rotation capacity for designing RC beams located in non-seismic regions will be derived using the deformability equation previously developed by the authors. The minimum deformability derived will be comparable to that provided to RC beams made of NSC and NSS designed as per the empirical deemed-to-satisfy rules stipulated in the existing RC design codes. Having fixed the minimum deformability, the maximum degree of reinforcement λ and tension steel ratio ρ_t for designing RC beams with the proposed minimum deformability would be derived for different concrete and steel yield strength using the authors' developed formula (Zhou *et al.* 2010). Lastly, for practical design application, a set of simplified design guidelines that depend on concrete and steel yield strength are developed for minimum deformability design.

2. Nonlinear moment-curvature analysis

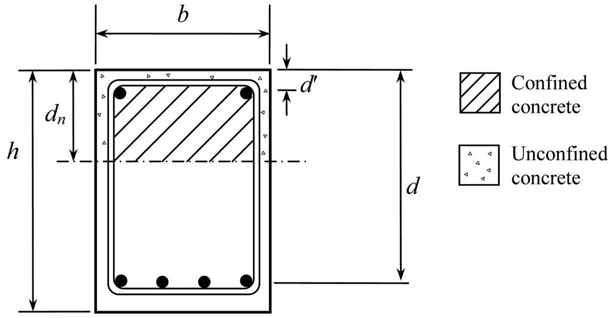
The method of nonlinear moment-curvature analysis developed previously by the authors (Pam *et al.* 2001a, Ho *et al.* 2003) has been adopted for a parametric study on the deformability analysis of RC beams. The stress-strain curves of concrete and steel reinforcement as per Attard and Setunge (1996) and Eurocode 2 (ECS 2004) were adopted in this study, the latter of which is stress-path dependence taking into account the unloading properties of steel in the post-peak stage of moment-curvature curves. The unloading path is having the same initial elastic modulus until it reaches zero steel stress. The stress-strain curves of concrete and steel are shown in Fig. 1.

Five assumptions are made in the analysis: (1) Plane sections before bending remain plane after bending. (2) The tensile strength of the concrete may be neglected. (3) There is no relative slip between concrete and steel reinforcement. (4) The concrete core is confined while the concrete



(a) Stress-strain curves of concrete (b) Stress-strain curve of steel with stress-path dependence considered

Fig. 1 Stress-strain curves of concrete and steel reinforcement



Section properties

- $b = 300$ mm
- $h = 600$ mm
- $d' = 50$ mm
- $d = 550$ mm
- $f_{co} = 40$ to 100 MPa
- $f_y = 400$ to 800 MPa
- $f_r = 0$ to 4 MPa
- $\rho_t = A_{st}/bd = 0.4$ to $2\rho_b$
- $\rho_c = A_{sc}/bd = 0$ to 2%

Fig. 2 Beam sections analysed

cover is unconfined. (5) The confining pressure provided to the concrete core by confinement is assumed to be constant throughout the concrete compression zone. Assumptions (1) to (4) are commonly accepted and have been adopted by various researchers (Au and Bai 2006, Kim 2007, Au *et al.* 2009, Bai and Au 2009, Wang and Liu 2009). Assumption (5) is not exact but nevertheless a fairly reasonable assumption (Ho *et al.* 2010b). In the analysis, the moment-curvature curve of the beam section is analysed by applying prescribed curvatures incrementally starting from zero. At a prescribed curvature, the stresses developed in the concrete and the steel are determined from their stress-strain curves. Then, the neutral axis depth and resisting moment are evaluated from equilibrium conditions, respectively. The above procedure is repeated until the resisting moment has increased to the peak and then decreased to 50% of the peak moment. Fig. 2 describes a typical beam sections adopted in the nonlinear moment-curvature analysis.

3. Parametric study for deformability

3.1 Flexural deformability analysis

In this study, the flexural deformability of beam sections are expressed in terms of normalised rotation capacity θ_{pl} defined as follows (Ho *et al.* 2010a)

$$\theta_{pl} = \phi_u d \tag{1}$$

where ϕ_u is the ultimate curvature, d is the effective depth of the beam section. The ultimate curvature ϕ_u is taken as the curvature when the resisting moment has dropped to $0.8M_p$ after reaching M_p, where M_p is the peak moment. The value of θ_{pl} represents the rotation capacity of beam with plastic hinge length l_p equal to its effective depth. For concrete beams subjected to pure flexure, the plastic hinge length remains relatively constant between $0.4d$ (Mendis 2001) and $0.5h$

(Standards New Zealand 2006) with concrete strength and tension steel ratio, where h is the overall beam depth. Therefore, it is fairly reasonable to use the proposed normalised rotation capacity to compare the deformability of different RC beams. For RC members subjected to flexure and axial load, the plastic hinge length will be dependent on other factors apart from the section dimension (Bae and Bayark 2008, Haskett *et al*. 2009, Pam and Ho 2009).

Based on the above definition, a comprehensive parametric study on the effects of various factors on the normalised rotation capacity has been conducted previously (Ho *et al*. 2010a, Zhou *et al*. 2010). The studied factors are: (1) Degree of reinforcement; (2) Concrete strength; (3) Steel yield strength; and (4) Confining pressure. The beam sections analysed has been shown in Fig. 2. The concrete strength f_{co} was varied from 40 to 100 MPa, the confining pressure f_r was varied from 0 to 4 MPa, the tension steel ratio ρ_t was varied from 0.4 to 2 times the balanced steel ratio, the compression steel ratio ρ_c was varied from 0 to 2%, and the tension f_{yt} and compression f_{yc} steel yield strength f_y were varied from 400 to 800 MPa.

3.2 Balanced steel ratio and failure modes

The balanced steel ratio of a beam section without compression steel is defined as $\rho_{bo} = A_{sb}/bd$, where A_{sb} is the tension steel area that causes the most highly stressed tension steel just yield during failure. For beam section containing tension steel area less than the balanced steel area, the steel will yield during failure and the section is under-reinforced. Otherwise, the steel will not yield during failure and the section in over-reinforced. For beam sections with compression steel ratio ρ_c, the balanced steel ratio ρ_b is given by

$$\rho_b = \rho_{bo} + (f_{yc}/f_{yt})\rho_c \tag{2}$$

The values of ρ_{bo} for various concrete strengths and tension steel yield strength are listed in Table 1 for different tension steel yield strength (Ho *et al*. 2003). To facilitate practical design application, the following empirical equation was derived using regression analysis

$$\rho_{bo} = 0.005(f_{co})^{0.58}(1+1.2f_r)^{0.3}(f_{yt}/460)^{-1.35} \tag{3}$$

where f_r is the confining pressure provided by the confinement to the core concrete evaluated using the method proposed by Mander *et al*. (1988). All strengths are in MPa, 400 MPa $\leq f_{yt} \leq$ 800 MPa and $0 \leq f_r \leq$ 4 MPa.

Table 1 Balanced steel ratios

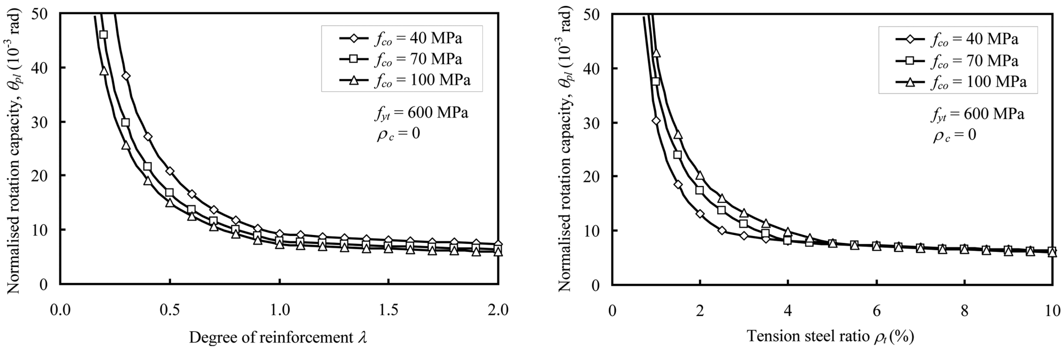
f_{co} (MPa)	Balanced steel ratios without compression reinforcement ρ_{bo}(%)		
	f_{yt} = 400 MPa	f_{yt} = 600 MPa	f_{yt} = 800 MPa
40	4.74	2.74	1.82
50	5.63	3.23	2.13
60	6.46	3.69	2.43
70	7.29	4.13	2.70
80	8.06	4.56	2.97
90	8.77	4.94	3.22
100	9.42	5.29	3.44

3.3 Effects of degree of reinforcement, concrete and steel yield strength

From previous theoretical studies on deformability of RC beams (Ho *et al*. 2010a, 2010b, Zhou *et al*. 2010), it is found that the most critical factors affecting the deformability of beam are the degree of reinforcement, concrete strength, tension and compression steel yield strength. The degree of reinforcement λ, which accounts for the degree of section being under- or over-reinforced, is expressed in Eq. (4)

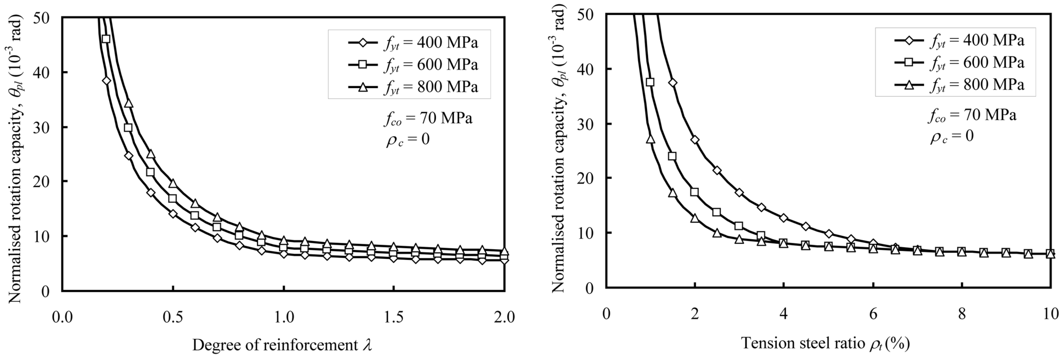
$$\lambda = \frac{f_{yt}\rho_t - f_{yc}\rho_c}{f_{yt}\rho_{bo}} \tag{4}$$

By this definition, the beam section is classified as under-reinforced, balanced and over-reinforced sections when λ is less than, equal to and larger than 1.0 respectively. To illustrate the effects of λ on the deformability of concrete beams, the normalised rotation capacity θ_{pl} is plotted against λ in Fig. 3(a) for different concrete strength. It could be seen that at a constant concrete strength, the deformability decreases as λ increases until reaching $\lambda = 1.0$, after which the deformability remains relatively constant. For the effect of concrete strength, it can be seen from Fig. 3(a) that at a constant λ, the deformability decreases as the concrete strength increases. However, if HSC is used at the same tension steel ratio ρ_t, it is evident from Fig. 3(b) that the deformability increases as concrete strength increases albeit that HSC is less deformable *per se*. This is because the balanced



(a) θ_{pl} against λ (b) θ_{pl} against ρ_t

Fig. 3 Variation of deformability with degree of reinforcement and tension steel ratio



(a) θ_{pl} against λ (b) θ_{pl} against ρ_t

Fig. 4 Effects of tension steel yield strength on deformability

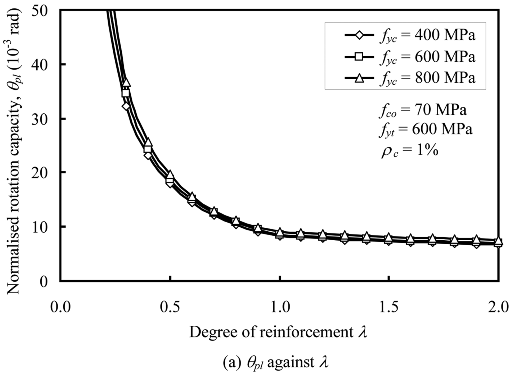


Fig. 5 Effects of compression steel yield strength on deformability

steel ratio increases as concrete strength increases. And for a given ρ_t, λ decreases and the deformability increases as concrete strength increases.

To illustrate the effects of steel yield strengths on the deformability of concrete beams, θ_{pl} is plotted against λ and ρ_t in Figs. 4(a) and 4(b) respectively for different tension steel yield strength f_{yt}, whereas θ_{pl} is plotted against λ and ρ_t in Figs. 5(a) and 5(b) respectively for different compression steel yield strength f_{yc}. Generally, it is observed from Fig. 4 that at a constant λ, the deformability increases as the tension steel yield strength increases, notwithstanding that HSS is less deformable *per se*. However, it decreases as the tension steel yield strength increases at a given ρ_t. From Figs. 5(a) and 5(b), it is seen that the deformability increases only very slightly as the compression steel yield strength increases at constant λ. Nevertheless, the deformability increases significantly as the compression steel yield strength increases at constant ρ_t.

3.4 Effects of neutral axis depth

Alternatively, the degree of beam section being under- or over-reinforced may be expressed in terms of x_u/x_{ub}, where x_u and x_{ub} are the neutral axis depths of beam section and the respective balanced section, respectively. Since the neutral axis depth varies with the beam curvature, the stage

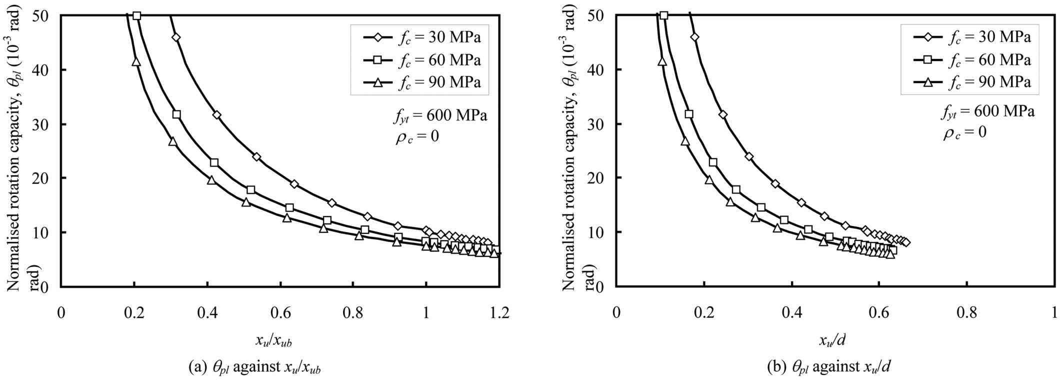


Fig. 6 Variation of deformability with neutral axis depth at different concrete strength

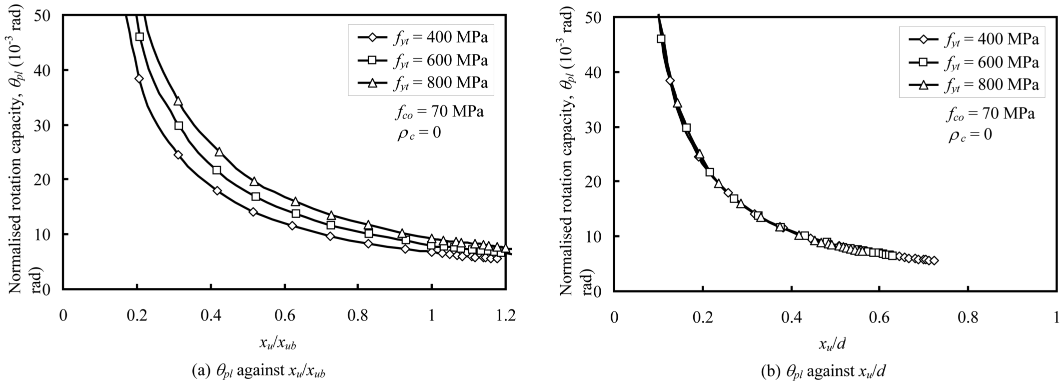


Fig. 7 Variation of deformability with neutral axis depth at different steel yield strength

at which the x_u or x_{ub} is measured should be defined. In most of the existing RC design codes, the neutral axis depths are measured at the ultimate limit state when the section is reaching its maximum moment. For the sake of consistency, the neutral axis depths presented in this study are all referring to those at the maximum moment at ultimate limit state. To study the effect of x_u/x_{ub} and x_u/d on the deformability of beams, θ_{pl} is plotted against x_u/x_{ub} and x_u/d in Figs. 6 and 7 for different concrete strength and tension steel yield strength respectively. The variation of θ_{pl} with x_u/x_{ub} is similar to that with λ as shown in Figs. 3(a) and 4(a). On the contrary, the variation of θ_{pl} with x_u/d is different from that with ρ_t as shown in Figs. 3(b) and 4(b). It can be observed from Fig. 6 that the deformability decreases as x_u/x_{ub} or x_u/d increases until x_u/x_{ub} is equal to 1.0, after which the deformability remains relatively constant. And at a given ratio of x_u/x_{ub} or x_u/d, the deformability decreases as concrete strength increases. Therefore, the use of HSC would decrease the deformability of concrete beam at a specified ratio of x_u/x_{ub} or x_u/d. It is then evident that the empirical deemed-to-satisfy rules stipulated in the existing RC design codes would provide a smaller deformability to beam when HSC is adopted because they are concrete strength independent. For the effects of steel yield strength, it is apparent from Fig. 7(a) that at a given x_u/x_{ub}, the deformability increases as the tension steel yield strength increases. Nonetheless, at a given x_u/d, it can be seen from Fig. 7(b) that the deformability is insensitive to the tension steel yield strength.

4. Minimum deformability design of concrete beams

4.1 Empirical deemed-to-satisfy rules provided by existing codes

In the existing RC design codes, the deformability of concrete beams is provided by some sets of empirical deemed-to-satisfy rules that limit the maximum tension steel ratio or neutral axis depth. The respective rules of some existing RC design codes are extracted and highlighted as follows:

(1) American Code ACI 318 (ACI Committee 2008): Clause 10.3.5 of the code limits the maximum tension steel strain to not less than 0.004. This is equivalent to limit the tension steel ratio to not more than 0.75 of the balanced steel ratio.

(2) Australian Code AS3600 (Standard Australia 2001): Clause 8.1.3 of the code limits the neutral axis depth to not more than 0.4d for all concrete strength.

(3) Chinese Code GB50011 (Ministry of Construction 2001): Clause 6.3.3 of the code requires the neutral axis depth to be smaller than $0.35d$, where d is the effective depth of the beam section.
(4) European Code EC2 (ECS 2004): Clause 5.6.3.2 of the code limits the neutral axis depth to not more than $0.45d$ when f_{ck} 50 ≤ MPa or $0.35d$ when $f_{ck} > 50$ MPa, in which f_{ck} is the characteristic concrete cylinder strength.
(5) New Zealand Code NZS3101 (Standards New Zealand 2006): Clause 9.3.8.1 of the code restricts the neutral axis depth to not more than $0.75x_{ub}$.

From the above, the minimum deformability expressed in terms of normalised rotation capacity θ_{pl} provided by various existing design codes may be evaluated by their respective values of $\theta_{pl,min}$ at different concrete strength and steel yield strength. In order to reflect the ranges of concrete and steel that are commonly adopted in practical construction, the minimum deformability at f_{co} = 30 and 100 MPa and f_{yt} = 400 and 800 MPa are calculated using nonlinear moment-curvature analysis and summarised in Table 2. Alternatively, the minimum deformability could be calculated using the following formulas previously developed by the authors (Zhou *et al.* 2010)

$$\theta_{pl} = 0.03m(f_{co})^{-0.3}(\lambda)^{-1.0n}\left(1+110(f_{co})^{-1.1}\left(\frac{f_{yc}\rho_c}{f_{yt}\rho_t}\right)^3\right)\left(\frac{f_{yt}}{460}\right)^{0.3} \tag{5a}$$

$$m = 1+4f_{co}^{0.4}(f_r/f_{co}) \tag{5b}$$

$$n = 1+3f_{co}^{0.2}(f_r/f_{co}) \tag{5c}$$

The validity of Eq. (5) has been verified by comparing with the measured deformability of beams tested by other researchers. The comparison is shown in Tables 3 and 4 for NSC and HSC beams respectively.

It can be seen from Table 2 that the actual deformability provided to concrete beams with different materials' strength is within a very wide range from an average value of 0.0168 rad when f_{co} = 30 MPa and f_{yt} = 800 MPa to an average value of 0.0097 rad when f_{co} = 100 MPa and f_{yt} = 400 MPa. Therefore, the existing empirical rules, which are material strength independent, are actually not able to provide a consistent level of deformability to concrete beams. Particularly for beams made of HSC and HSS, the deformability could be as low as 65% of that provided previously to beams made of NSC and NSS. With a view to providing a consistent level of minimum deformability to concrete beams, it is proposed herein to set a fixed minimum normalised rotation capacity $\theta_{pl,min}$ for all beams instead of complying with the deemed-to-satisfy rules.

Table 2 Deformability of RC beams provided in existing design codes

Design codes	Normalised rotation capacity θ_{pl} (rad)			
	f_{co} = 30 MPa		f_{co} = 100 MPa	
	f_{yt} = 400 MPa	f_{yt} = 800 MPa	f_{yt} = 400 MPa	f_{yt} = 800 MPa
American Code ACI318	0.0119	0.0163	0.0083	0.0115
Australian Code AS3600	0.0166	0.0165	0.0096	0.0095
Chinese Code GB50011	0.0196	0.0196	0.0111	0.0111
Eurocode EC2	0.0141	0.0138	0.0111	0.0111
New Zealand Code NZS3101	0.0130	0.0177	0.0084	0.0117
Average	0.0150	0.0168	0.0097	0.0110

Table 3 Comparison with experimental results on rotation capacities of NSC beams

Code	f_c' (MPa)	f_r (Mpa)	f_{yt} (Mpa)	ρ_t (%)	ρ_c (%)	θ_{pl} by Eq. (5) (rad) [1]	θ_{pl} by others (rad) [2]	θ_{pl} by EC2 (rad) [3]	$\frac{[1]}{[2]}$	$\frac{[3]}{[2]}$
					Nawy *et al.* (1968)					
P9G1	33.6	0.00	328	1.73	0.71	0.0870	0.0650	0.0330	1.34	0.51
P11G3	35.1	0.50	328	1.73	0.71	0.1536	0.1110	0.0320	1.38	0.29
P3G4	37.5	1.30	452	1.73	0.71	0.1232	0.1340	0.0260	0.92	0.19
P4G5	39.1	1.30	452	1.73	0.71	0.1217	0.1360	0.0265	0.89	0.19
					Pecce and Fabbocino (1999)					
A	41.3	0.98	471	2.60	0.05	0.0255	0.0220	0.0100	1.16	0.45
B	41.3	0.94	454	1.10	0.05	0.0736	0.1220	0.0265	0.60	0.22
					Debernardi and Taliano (2002)					
T1A1	27.7	0.46	587	0.67	0.30	0.1433	0.1035	0.0310	1.38	0.30
T3A1	27.7	0.46	587	2.00	0.59	0.0270	0.0290	0.0080	0.93	0.28
T5A1	27.7	0.35	587	0.63	0.22	0.0978	0.1130	0.0300	0.87	0.27
T6A1	27.7	0.35	587	1.28	0.22	0.0311	0.0245	0.0160	1.27	0.65
					Haskett *et al.* (2009)					
A1	38.2	0.67	315	1.47	0.0	0.0313	0.0360	0.0269	0.87	0.75
A2	42.3	0.32	318	1.47	0.0	0.0226	0.0205	0.0280	1.10	1.37
A3	41.0	0.31	336	1.47	0.0	0.0209	0.0168	0.0270	1.24	1.61
A4	42.9	1.29	315	2.95	0.0	0.0222	0.0305	0.0172	0.73	0.56
A5	39.6	0.59	314	2.95	0.0	0.0136	0.0207	0.0154	0.66	0.74
A6	41.1	0.31	328	2.95	0.0	0.0103	0.0118	0.0153	0.87	1.30
B1	43.0	0.65	329	1.47	0.0	0.0293	0.0277	0.0278	1.06	1.00
B2	41.8	0.31	322	1.47	0.0	0.0222	0.0152	0.0277	1.46	1.82
B3	42.9	1.29	321	2.95	0.0	0.0217	0.0218	0.0168	1.00	0.77
B4	42.9	0.64	323	2.95	0.0	0.0138	0.0120	0.0166	1.15	1.38
C2	26.0	0.39	329	1.47	0.0	0.0219	0.0258	0.0203	0.85	0.79
C3	25.6	0.32	330	1.47	0.0	0.0201	0.0187	0.0200	1.07	1.07
C4	25.9	1.23	325	2.95	0.0	0.0205	0.0297	0.0080	0.69	0.27
C5	23.4	0.64	328	2.95	0.0	0.0126	0.0130	0.0080	0.97	0.62
C6	27.4	0.34	319	2.95	0.0	0.0102	0.0125	0.0080	0.82	0.64
					Average				1.01	0.72
					Standard deviation				0.24	0.47

4.2 Derivation of minimum normalised rotation capacity

For beams located in non-seismic regions, even though they are not expected to resist earthquake load, they should still be designed with a minimum value of deformability $\theta_{pl,min}$ for the sake of providing extra safety to the occupants in the case of accidental loads. This minimum value can be derived by referring to the deformability that was being provided in the past to beams made of NSC

Table 4 Comparison with experimental results on rotation capacities of HSC beams

Code	f_c' (MPa)	f_r (Mpa)	f_{yt} (Mpa)	ρ_t (%)	ρ_c (%)	θ_{pl} by Eq. (5) (rad) [1]	θ_{pl} by others (rad) [2]	θ_{pl} by EC2 (rad) [3]	$\frac{[1]}{[2]}$	$\frac{[3]}{[2]}$
					Pecce and Fabbocino (1999)					
AH	93.8	0.98	471	2.60	0.05	0.0271	0.0220	0.0170	1.23	0.77
CH	95.4	1.11	534	2.20	0.04	0.0300	0.0380	0.0170	0.79	0.45
					Ko *et al.* (2001)					
6-65-1	66.6	2.26	415	3.59	0.79	0.0547	0.0472	0.0150	1.16	0.32
6-75-1	66.6	2.33	427	4.27	0.77	0.0399	0.0412	0.0100	0.97	0.24
8-50-1	82.1	2.42	443	3.35	0.80	0.0580	0.0482	0.0160	1.20	0.33
8-65-1	82.1	2.33	427	4.27	0.77	0.0398	0.0450	0.0100	0.88	0.22
8-75-1	82.1	2.15	394	4.97	0.79	0.0338	0.0484	0.0080	0.70	0.17
$7\text{-}62^{00}\text{-}1$	70.8	1.91	408	3.16	0.00	0.0403	0.0530	0.0135	0.76	0.25
$7\text{-}62^{15}\text{-}1$	70.8	1.91	408	3.16	0.79	0.0587	0.0510	0.0160	1.15	0.31
					Lopes and Bernardo (2003)					
A(64.9-2.04)	64.9	0.59	555	2.04	0.20	0.0248	0.0200	0.0210	1.24	1.05
A(63.2-2.86)	63.2	0.62	575	2.86	0.20	0.0161	0.0180	0.0110	0.89	0.61
A(65.1-2.86)	65.1	0.62	575	2.86	0.20	0.0161	0.0150	0.0110	1.07	0.73
B(82.9-2.11)	82.9	0.59	555	2.11	0.20	0.0243	0.0210	0.0180	1.16	0.86
B(83.9-2.16)	83.9	0.59	555	2.16	0.20	0.0237	0.0200	0.0180	1.19	0.90
B(83.6-2.69)	83.6	0.62	575	2.69	0.20	0.0178	0.0210	0.0150	0.85	0.71
B(83.4-2.70)	83.4	0.62	575	2.70	0.20	0.0177	0.0200	0.0150	0.89	0.75
					Average				1.01	0.54
					Standard deviation				0.18	0.28

and NSS as per the empirical deemed-to-satisfy rules stipulated in the existing RC design codes. In this study, the authors suggest to adopt the average value of the normalised rotation capacity obtained for NSC beams of $f_{co} = 30$ MPa and $f_{yt} = 400$ MPa designed in accordance with the empirical deemed-to-satisfy rules stipulated in various existing RC design codes for the benchmark of the minimum deformability, which is equal to 0.015 rad.

5. Methods of providing minimum deformability

5.1 By controlling the maximum degree of reinforcement

Based on this specified minimum normalised rotation capacity $\theta_{pl,min}$ = 0.015 rad, it is seen from Eq. (5a) that there is a corresponding maximum allowable value of λ, denoted by λ_{max}, for each chosen f_{co} and f_{yt}, which is shown in Eq. (6a).

$$\lambda_{max} = \left\{0.03m(f_{co})^{-0.3}\left(\frac{f_{yt}}{460}\right)^{0.3}\left(1+110(f_{co})^{-1.1}\left(\frac{f_{yc}\rho_c}{f_{yt}\rho_t}\right)^3\right)(\theta_{pl,min})^{-1.0}\right\}^{\frac{1}{n}} \tag{6a}$$

The value of λ_{max} with respect to the specified minimum deformability $\theta_{pl,min} = 0.015$ rad are evaluated rigorously by nonlinear moment-curvature analysis and the value of which are summarised in Tables 5 to 7 for different combination of concrete and steel yield strengths of f_{co} = 30, 60 and 90 MPa and f_{yt} = 400, 600 and 800 MPa. Alternatively, the value of λ_{max} can also be obtained by substituting $\theta_{pl,min} = 0.015$ rad into Eq. (6a) when there is no compression steel and confining pressure

$$\lambda_{max} = 2.0(f_{co})^{-0.3}\left(\frac{f_{yt}}{460}\right)^{0.3} \tag{6b}$$

It can be seen from the above tables that the value of λ_{max} decreases significantly as the concrete strength increases from 30 to 100 MPa at a given steel yield strength. On the other hand, the value of λ_{max} increases slightly as the steel yield strength increases from 400 to 800 MPa at a given concrete strength. Obviously, a lower value of λ_{max} should be set for the proposed minimum deformability design of concrete beams when HSC is adopted. And a slightly higher value of λ_{max} should be set for the design when HSS is adopted. However, the value of λ_{max} may or may not be decreased when both HSC and HSS are adopted in the beam design.

The corresponding maximum allowable tension steel ratio $\rho_{t,max}$ for singly-reinforced concrete beam section (i.e. $\rho_c = 0\%$) having different concrete and steel yield strength can be derived by multiplying the value of λ_{max} with the respective balanced steel ratio ρ_{bo}. These values of $\rho_{t,max}$ have been listed in Tables 5 to 7. It can be observed from these tables that although the value of λ_{max} decreases substantially as the concrete strength increases, the value of $\rho_{t,max}$ does increase as the concrete strength increases because the value of ρ_{bo} increases considerably with the concrete strength. Therefore, the advantage of using of HSC is that it can increase the maximum design limit of the flexural strength, while at the same time maintain the provision of minimum deformability to the beam section. On the other hand, it is seen that the value of $\rho_{t,max}$ decreases significantly as the steel yield strength increases because the balanced steel ratio ρ_{bo} decreases as the steel yield strength increases. Nevertheless, since higher strength steel is adopted, the provision of a lower tension steel ratio may or may not lead to a reduction in the flexural strength.

In order to study the flexural strength that can be achieved by the beam sections designed for the proposed minimum deformability, the maximum moment capacity expressed in terms of $M_p/(bd^2)$ for the beam section having different concrete and steel yield strength were calculated for ρ_c = 0%, 0.5% and 1.0%. The results are tabulated in Tables 8 to 10. It can be observed from these tables that the maximum flexural strength achieved by the beam section designed with $\theta_{pl,min}$ = 0.015 rad increases significantly as the concrete strength increases and increases slightly as the tension steel yield strength increases. The advantages of using higher strength materials are now evident. The use of HSC and/or HSS would allow a higher flexural strength to be achieved at the proposed minimum deformability, albeit that HSC and HSS are less deformable *per se*. On the other hand, the addition of compression steel would always increase the $\rho_{t,max}$ for achieving the proposed minimum deformability. It therefore allows a higher flexural strength to be achieved in the minimum deformability design of RC beams.

5.2 By controlling the maximum neutral axis depth

The method of limiting the neutral axis depth for the provision of minimum flexural deformability has been adopted by most of the existing RC design codes. These maximum limits of neutral axis depth are expressed as a certain fraction of different parameters in different codes. For instance, NZS3101 limits the neutral axis depth (at maximum moment) to not more than 0.75 of the neutral axis depth of the balanced section. On the other hand, AS3600, EC2 and GB50011 limit the neutral axis depth to not more than a certain fraction of the beam's effective depth. The maximum limits set to the neutral axis depth in these codes are applicable to both singly- and doubly RC beam sections. However, these limits are not dependent on the concrete and steel yield strength (except for EC2 that depend on concrete strength only). Therefore, the deformability provided to beam section with HSC and HSS will be lower than that of beam section made of NSC and NSS.

As before, the maximum limits of neutral axis depth expressed in the ratio of neutral axis to effective depth x_u/d, which is a more commonly used ratio in the existing RC design codes, for minimum deformability design of RC beam section are derived rigorously using nonlinear moment-curvature analysis for different concrete and steel yield strength. The so obtained maximum value of x_u/d are summarised in Table 11. It is noted that the maximum value of x_u/d decreases significantly as the concrete strength increases from 30 to 100 MPa. Nevertheless, the maximum value of x_u/d remain fairly constant with tension steel yield strength within the range from $f_{yt} = f_{yc}$ = 400 to 800 MPa. Thus, a lower maximum limit of x_u/d should be set when HSC is used for the purpose of minimum deformability design.

6. Simplified design guidelines

The maximum limits of the degree of reinforcement λ_{max} and neutral axis to effective depths ratio x_u/d have been derived and presented in Tables 5 to 11. These tables can be used for concrete beam

Table 5 Values of λ_{max} and $\rho_{t,max}$ for $\theta_{pl,min}$ = 0.015 rad when f_{yt} = 400 MPa

f'_{co} (MPa)	ρ_{bo} (%)	λ_{max}	$\rho_{t,max}$ (%)
30	3.815	0.626	2.388
40	4.735	0.565	2.675
50	5.625	0.526	2.958
60	6.455	0.497	3.208
70	7.285	0.472	3.438
80	8.055	0.454	3.657
90	8.765	0.439	3.848
100	9.415	0.423	3.982

Table 6 Values of λ_{max} and $\rho_{t,max}$ for $\theta_{pl,min}$ = 0.015 rad when f_{yt} = 600 MPa

f_{co} (MPa)	ρ_{bo} (%)	λ_{max}	$\rho_{t,max}$ (%)
30	2.225	0.715	1.590
40	2.735	0.652	1.783
50	3.225	0.611	1.970
60	3.685	0.581	2.141
70	4.125	0.555	2.289
80	4.555	0.534	2.432
90	4.935	0.519	2.561
100	5.285	0.501	2.647

Table 7 Values of λ_{max} and $\rho_{t,max}$ for $\theta_{pl,min}$ = 0.015 rad when f_{yt} = 800 MPa

f_{co} (MPa)	ρ_{bo} (%)	λ_{max}	$\rho_{t,max}$ (%)
30	1.495	0.800	1.196
40	1.815	0.737	1.337
50	2.125	0.695	1.477
60	2.425	0.662	1.605
70	2.695	0.636	1.714
80	2.965	0.616	1.826
90	3.215	0.597	1.919
100	3.435	0.578	1.985

(Note f_r = 0 MPa and ρ_c = 0% in Tables 5 to 7)

Table 8 Values of $\rho_{t,max}$ and $M_p/(bd^2)$ for $\theta_{pl,min}$ = 0.015 rad when $f_{yt} = f_{yc}$ = 400 MPa

f'_{co} (MPa)	$\rho_{t,max}$ (%)			$M_p/(bd^2)$ (MPa)		
	ρ_c = 0%	ρ_c = 0.5%	ρ_c = 1.0%	ρ_c = 0%	ρ_c = 0.5%	ρ_c = 1.0%
30	2.388	3.000	3.625	7.929	10.045	12.173
40	2.675	3.279	3.888	9.152	11.263	13.384
50	2.958	3.548	4.144	10.301	12.384	14.481
60	3.208	3.798	4.382	11.310	13.401	15.476
70	3.438	4.011	4.613	12.240	14.278	16.410
80	3.657	4.221	4.802	13.115	15.131	17.198
90	3.848	4.418	4.988	13.886	15.919	17.954
100	3.982	4.576	5.180	14.463	16.576	18.727

Table 9 Values of $\rho_{t,max}$ and $M_p/(bd^2)$ for $\theta_{pl,min}$ = 0.015 rad when $f_{yt} = f_{yc}$ = 600 MPa

f'_{co} (MPa)	Maximum value of ρ_t (%)			$M_p/(bd^2)$ (MPa)		
	ρ_c = 0%	ρ_c = 0.5%	ρ_c = 1.0%	ρ_c = 0%	ρ_c = 0.5%	ρ_c = 1.0%
30	1.590	2.240	2.857	7.924	11.044	14.191
40	1.783	2.417	3.029	9.151	12.253	15.326
50	1.970	2.583	3.196	10.292	13.345	16.406
60	2.141	2.744	3.343	11.319	14.363	17.386
70	2.289	2.892	3.491	12.223	15.290	18.335
80	2.432	3.014	3.610	13.085	16.072	19.116
90	2.561	3.135	3.709	13.868	16.830	19.787
100	2.647	3.253	3.790	14.423	17.552	20.355

design with minimum deformability. However, in practical design application, it may not be practical to incorporate the above tables into a design code. For incorporation into RC design codes, simplified guidelines are preferred. Referring to the maximum allowable values of the degree of reinforcement λ_{max} summarised in Tables 5 to 7, it can be observed that the effect of the steel yield strength is relatively small when compared with that of the concrete strength on λ_{max}. Therefore, it is proposed herein that the effects of steel yield strength be ignored and the simplified guidelines

Table 10 Values of $\rho_{t,max}$ and $M_p/(bd^2)$ for $\theta_{pl,min}$ = 0.015 rad when $f_{yt} = f_{yc}$ = 800 MPa

f'_{co} (MPa)	Maximum value of ρ_t (%)			$M_p/(bd^2)$ (MPa)		
	ρ_c = 0%	ρ_c = 0.5%	ρ_c = 1.0%	ρ_c = 0%	ρ_c = 0.5%	ρ_c = 1.0%
30	1.196	1.878	2.713	7.921	11.933	15.980
40	1.337	1.992	2.639	9.152	13.164	17.121
50	1.477	2.104	2.744	10.286	14.282	18.206
60	1.605	2.214	2.830	11.317	15.280	19.195
70	1.714	2.313	2.910	12.206	16.163	20.038
80	1.826	2.397	3.004	13.097	16.917	20.900
90	1.919	2.454	3.064	13.853	17.482	21.525
100	1.985	2.533	3.099	14.421	18.155	21.963

Table 11 Maximum value of x_u/d for $\theta_{pl,min}$ = 0.015 rad

f'_{co} (MPa)	Maximum value of x_u/d		
	f_{yt} = 400 MPa	f_{yt} = 600 MPa	f_{yt} = 800 MPa
30	0.426	0.425	0.425
40	0.372	0.372	0.372
50	0.339	0.339	0.338
60	0.322	0.322	0.322
70	0.301	0.300	0.300
80	0.284	0.283	0.283
90	0.275	0.275	0.275
100	0.259	0.259	0.259

(Note $f_y = f_{yt} = f_{yc}$ in Table 11)

are dependent only on the concrete strength. Based on this, the following guidelines for limiting the value of λ_{max} in order to ensure the provision of minimum deformability (i.e. $\theta_{pl,min} \geq 0.015$ rad) are developed:

In the case of $400 \leq f_{yc} = f_{yt} \leq 800$ MPa, λ_{max} should not exceed 0.60 when $f_{co} \leq 30$ MPa, should not exceed 0.50 when 30 MPa $< f_{co} \leq 60$ MPa, and should not exceed 0.40 when 60 MPa $< f_{co} \leq 100$ MPa.

For incorporating the maximum allowable limits of neutral axis to effective depths presented in Table 11 into RC design codes for practical design application, the following guidelines are developed:

In the case of $400 \leq f_{yc} = f_{yt} \leq 800$ MPa, x_u/d should not exceed 0.40 when $f_{co} \leq 30$ MPa, should not exceed 0.30 when 30 MPa $< f_{co} \leq 60$ MPa, and should not exceed 0.25 when 60 MPa $< f_{co} \leq 100$ MPa.

7. Conclusions

The flexural deformability of RC beams were studied by nonlinear moment-curvature analysis in terms of normalised rotation capacity. From the study, it was found that the major factors determining the deformability of a RC beam section is the degree of reinforcement λ or the neutral axis depth at maximum moment (expressed in dimensionless ratio of x_u/x_{ub} or x_u/d). However, the variations of deformability with λ and x_u/x_{ub} or x_u/d are not unique and dependent on the concrete and steel yield strength. Because of such dependence, the current empirical deemed-to-satisfy rules, which are concrete and steel yield strength independent, are not able to provide a consistent level of minimum deformability to concrete beams. Of greater concern is that the level of deformability provided by the existing rules to RC beams made of HSC and/or HSS is much lower than that provided in the past to beams made of NSC and NSS. The average difference is about 35% lower for HSC beams.

In order to provide a consistent level of deformability to RC beams, it is proposed to set a minimum normalised rotation capacity for deformability design. This proposed deformability is comparable to that provided in the past to RC beams made of NSC and NSS, which should be provided to all beams even though they are not located in seismic regions. In this study, the provision of minimum deformability to RC beams is achieved by either limiting the maximum allowable degree of reinforcement λ_{max} or the neutral axis to effective depth ratio x_u/d. These maximum allowable values were derived using the nonlinear moment-curvature analysis for different combination of concrete strength from 30 to 100 MPa and steel yield strength from 400 to 800 MPa. From the results, it is evident that maximum allowable values of λ_{max} and x_u/d decrease significantly as the concrete strength increases. Hence, it is inappropriate to set any fixed maximum limit to the value of λ_{max} and x_u/d. Moreover, it was also found that at the specified minimum deformability, the flexural strength of beam section increases as the concrete strength and/or steel yield strength increases. Therefore, the use of HSC and HSS would improve the maximum design limit of flexural strength while at the same time provide minimum deformability to the RC beams. Lastly, simplified guidelines for incorporation into RC design codes that limit the value of λ_{max} and x_u/d to ensure provision of the proposed minimum deformability have been developed.

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CC

Notations

A_{sb}	Balanced steel area
A_{sc}	Area of compression steel
A_{st}	Area of tension steel
b	Breadth of beam or column section
d	Effective depth of beam or column section
E_s	Elastic modulus of steel reinforcement
f_{co}	Peak stress on stress-strain curve of unconfined concrete
f_r	Confining pressure
f_y	Yield strength of steel reinforcement
f_{yc}	Yield strength of compression steel
f_{yt}	Yield strength of tension steel
h	Total depth of the beam section
l_p	Plastic hinge length
M_p	Peak moment
x_u	Neutral axis depth of beam section at maximum moment
x_{ub}	Neutral axis depth of balanced section at maximum moment
ε_{ps}	Residual plastic strain in steel reinforcement
ε_s	Strain in steel
θ_{pl}	Normalised rotation capacity of beam
$\theta_{pl,min}$	Minimum required normalised rotation capacity of beam
λ	Degree of reinforcement
λ_{max}	Maximum allowable degree of reinforcement
ϕ_u	Ultimate curvature
ρ_b	Balanced steel ratio (= A_{sb}/bd)
ρ_{bo}	Balanced steel ratio for beam section with no compression steel
ρ_c	Compression steel ratio (= A_{sc}/bd)
ρ_t	Tension steel ratio (= A_{st}/bd)
σ_s	Stress in steel reinforcement