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Maximum concrete stress developed in unconfined flexural RC members

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Abstract. In flexural strength design of unconfined reinforced concrete (RC) members, the concrete compressive stress-strain curve is scaled down from the uni-axial stress-strain curve such that the maximum concrete stress adopted in design is less than the uni-axial strength to account for the strain gradient effect. It has been found that the use of this smaller maximum concrete stress will underestimate the flexural strength of unconfined RC members although the safety factors for materials are taken as unity. Herein, in order to investigate the effect of strain gradient on the maximum concrete stress that can be developed in unconfined flexural RC members, several pairs of plain concrete (PC) and RC inverted T-shaped specimens were fabricated and tested under concentric and eccentric loads. From the test results, the maximum concrete stress developed in the eccentric specimens under strain gradient is determined by the modified concrete stress-strain curve obtained from the counterpart concentric specimens based on axial load and moment equilibriums. Based on that, a pair of equivalent rectangular concrete stress block parameters for the purpose of flexural strength design of unconfined RC members is determined.

Keywords: rectangular stress block parameters; reinforced concrete; strain gradient; uni-axial concrete stress.

1. Introduction

In the flexural strength computation of reinforced concrete (RC) members, it is essential to determine the distribution of concrete stress in the compression zone at ultimate state. Assuming that plane sections remain plane before and after bending, which has been commonly adopted in evaluating the flexural capacity of typical RC sections (See Fig. 1a) (Park and Paulay 1975, Pam and Ho 2001, Pam *et al.* 2001, Wu *et al.* 2004, Au and Kwan 2004, Au *et al.* 2005, Kim 2005, Au and Bai 2006, Maghsoudi and Bengar 2006, Lin *et al.* 2006, Esmaeily and Peterman 2007, Kim 2007, Lu and Zhou 2007, Narayan and Venkataramana 2007, Bechtoula *et al.* 2009, Lam *et al.* 2009, Wang and Liu 2009), the strain distribution in the section would be linear as shown in Fig. 1(b). The stress distribution of concrete in the compression zone, which can be obtained from its uni-axial stress-strain curve, is shown in Fig. 1(c). Because of the nonlinearity of the actual stress-strain curve of concrete, the evaluations of the magnitude and location of resultant concrete compressive force are usually simplified using three parameters, i.e. k_1, k_2 and k_3, as shown in Fig. 1(d) (Hognestad *et al.* 1955, Soliman *et al.* 1967, Kaar *et al.* 1978, Sheikh and Uzumeri 1980, Ibrahim and MacGregor 1996, 1997, Attard and Stewart 1998, Bae and Bayrak 2003, Ozbakkaloglu and

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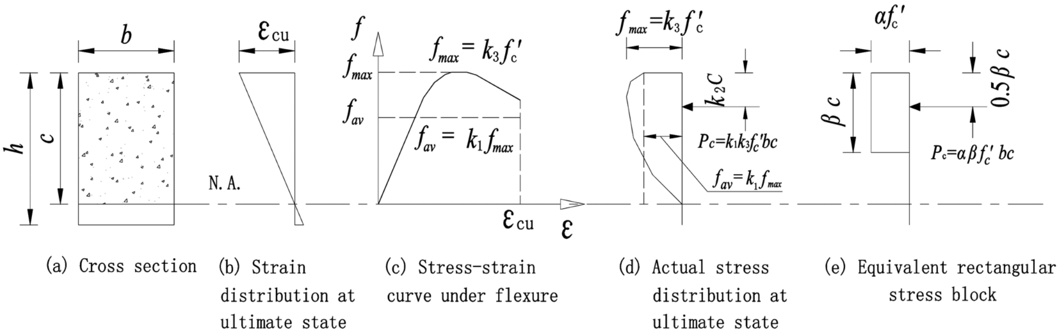


Fig. 1 Concrete stress block parameters

Saatcioglu 2004, Tan and Nguyen 2004, 2005). Amongst these parameters, k_1 is the ratio of average stress of concrete (f_{av}) over the compression area to maximum concrete stress developed under flexure (f_{max}); k_2 is the ratio of distance between the extreme compressive fibre and the resultant concrete compressive force (P_c) to that between the same fibre to the neutral axis (c); k_3 is the ratio of f_{max} to uni-axial concrete strength.

In practical flexural strength design of RC members, where the magnitude of resultant compressive force and its location are the most important parameters, the nonlinear concrete stress-strain curve is simplified into an equivalent rectangular stress block as shown in Fig. 1(e) (Hognestad *et al.* 1955, Ibrahim and MacGregor 1996, 1997, Ozbakkaloglu and Saatcioglu 2004, Tan and Nguyen 2005, Mertol *et al.* 2008), which has the same area and first moment of area as the actual nonlinear concrete stress-strain curve. The equivalent rectangular stress block is defined by two parameters, α and β, where α is the ratio of equivalent concrete compressive stress developed under flexure to concrete cylinder (f_c') or cube (f_{cu}) strength, and β is the ratio of the height of equivalent rectangular concrete compressive stress block to neutral axis depth (c). Since the area and the first moment of area of the equivalent concrere stress block is identical to those under the concrete stress-strain curve, α and β can be expressed equivalently in terms of k_1, k_2 and k_3 as follows

$$k_1 k_3 = \alpha\beta \tag{1}$$

$$k_2 = 0.5\beta \tag{2}$$

The equivalent rectangular concrete stress block has been widely adopted in various RC design codes for flexural strength design of RC members. The values of α and β in some of these codes, e.g. ACI Code (ACI Committee 318 2008), Australian Code (Standard Australia 2001), Eurocode 2 (European Committee for Standardization 2004) and New Zealand Code (Standard New Zealand 2005), for the flexural strength design of RC members are summarised in Table 1. From the table, it can be observed that: (1) The value of α and β are dependent only on the concrete strength, being larger for normal-strength concrete and smaller for high-strength concrete. (2) The values of a are smaller than 1.0, which indicates that the equivalent concrete stress that can be developed under flexure is smaller than the uni-axial strength. However, when the above values of α and β were adopted to evaluate the flexural strength of RC beams, it has been found consistently from previous experimental tests that the theoretical flexural strengths were significantly less than the measured flexural strengths. A comparison of flexural strengths predicted by ACI Code (ACI Committee 318

Table 1 Values of α and β stipulated in various current RC design codes

Design code	α	β
ACI 318[a] and AS 3600[b]	0.85 (for all f_c')	0.85 for $f_c' \leq$ 28MPa $0.85 - 0.007(f_c' - 28) \geq 0.65$ for $f_c' >$ 28MPa
Eurocode 2[c]	0.85 for $f_c \leq$ 50MPa $0.85-0.85\left(\dfrac{f_c'-50}{200}\right)$ for $50 < f_c' \leq$ 90 MPa	0.80 for $f_c' \leq$ 50MPa $0.8-\left(\dfrac{f_c'-50}{400}\right)$ for $50 < f_c' \leq$ 90MPa
New Zealand Code[d]	0.85 for $0 < f_c' \leq$ 55MPa $0.85 - 0.004(f_c' - 55)$ for $55 < f_c' \leq$ 80MPa 0.75 for $f_c' >$ 80MPa	0.85 for $0 < f_c' \leq$ 30MPa $0.85 - 0.008(f_c' - 30)$ for $30 < f_c' \leq$ 55MPa 0.65 for $f_c' >$ 55MPa

Notes:
[a]ACI Committee 318 (2008)
[b]Standard Australia (2001)
[c]European Committee for Standardization (2004) following UK National Annex
[d]Standard New Zealand (2005)

Table 2 Comparison of flexural strengths obtained from codes and previous tests

Specimen code	f_c' (MPa)	M_{ACI} (kNm) (1)	M_{EC} (kNm) (2)	M_{NZS} (kNm) (3)	M_t (kNm) (4)	$\frac{(1)}{(4)}$	$\frac{(2)}{(4)}$	$\frac{(3)}{(4)}$
Pecce and Fabbrocino (1999)								
A	41.3	97.0	97.0	97.0	104.0	0.93	0.93	0.93
B	41.3	45.0	45.0	45.0	49.6	0.91	0.91	0.91
C	42.3	636.7	636.7	636.7	712.5	0.89	0.89	0.89
Debernardi and Taliano (2002)								
T1	27.7	10.8	10.8	10.8	13.6	0.79	0.79	0.79
T2	27.7	20.5	20.6	20.5	23.6	0.87	0.87	0.87
T3	27.7	29.0	29.2	29.0	32.5	0.89	0.90	0.89
T4	27.7	46.9	46.8	46.9	59.8	0.78	0.78	0.78
T5	27.7	93.1	93.1	93.1	107.5	0.87	0.87	0.87
Rashid and Mansur (2005)								
A111	42.8	185.4	185.6	185.4	205.7	0.90	0.90	0.90
Lam *et al.* (2008)								
L-D	29.8	9.8	9.8	9.8	11.6	0.84	0.84	0.84
L-E	29.8	25.9	26.0	25.9	29.4	0.88	0.88	0.88
L-1	30.5	9.6	9.6	9.6	11.4	0.84	0.84	0.84
L-2	30.5	9.6	9.6	9.6	11.3	0.85	0.85	0.85
L-3	30.5	9.6	9.6	9.6	10.0	0.96	0.96	0.96

2008) M_{ACI}, Eurocode 2 (European Committee for Standardization 2004) M_{EC} and New Zealand Code (Standard New Zealand 2005) M_{NZS} with their corresponding measured flexural strengths M_t

obtained by the researchers (Pecce and Fabbrocino 1999, Ashour 2000, Debernardi and Taliano 2002, Rashid and Mansur 2005, Lam *et al.* 2008) is shown in Table 2. It is evident from the table that the measured flexural strengths can be underestimated by up to 22% in the worst scenario and the average underestimation is about 13%.

In fact, the maximum concrete compressive stress developed in flexure can be studied by k_3, while the equivalent stress developed in flexure can be studied by α. In the past, a plenty of experimental research has been conducted to investigate the range of k_3 and/or α in normal-strength RC members (Hognestad *et al.* 1955, Soliman *et al.* 1967, Kaar *et al.* 1978, Sheikh and Uzumeri 1980, Tan and Nguyen 2004, 2005). From the test results, it was found that: (1) The range of k_3 varied mostly from 0.8 to 1.0 for concentrically loaded columns and from 0.9 to 1.0 for eccentrically loaded columns, whereas α varied from 0.8 to 1.0 for both columns. These indicate that the maximum concrete stress that can be developed in concrete under flexure could be larger than that stipulated in the existing RC design codes. (2) The test results on k_3 and α are fairly scattered. This implies that the values of k_3 and α should be dependent on other factors apart from the uni-axial concrete (cylinder) strength (Kumar 2006, Tabsh 2006).

In this paper, the authors will investigate experimentally the maximum concrete compressive stress that can be developed under flexure as compared with the uni-axial strength. Several pairs of unconfined PC and RC column specimens were fabricated and tested. The 28-day concrete cylinder strengths ranged from 22 to 31MPa. Each pair of specimens contained identical properties. One of them was subjected to concentric load and the other to eccentric load. The purpose of the experiment is to determine the ratio of the maximum concrete compressive stress (f_{max}) that can be developed in the specimens subjected to eccentric load to the maximum compressive stress (σ_c) that can be developed in the counterpart specimens subjected to concentric load. The parameters of α and β for the equivalent concrete stress block are subsequently derived based on the obtained ratio. The applicability of the parameters obtained in this study for flexural strength design of unconfined RC members was verified by comparing the predicted flexural strengths (both by the values obtained in this study and those from existing RC design codes) with the measured flexural strength of RC beams and columns tested by different researchers. The proposed parameters α and β, after taking into account the strain gradient effect, should be able to predict accurately the flexural strength of unconfined RC beams and columns subjected to various axial load levels and strain gradients. For confined RC members, the same method can be adopted to assess the effects of strain gradient on the stress block parameters and equivalent rectangular stress block of confined concrete. However, the values of the stress block parameters will then be dependent on the content of confinement.

2. Experimental programme

2.1 Details of test specimens

In total four pairs of inverted T-shaped specimens were fabricated in this study, one pair was PC and three pairs were RC. Each pair of the specimens contained identical properties, and all the specimens had identical dimensions with the column 400 × 400 (cross section) × 1400 mm (height) and the base 400 × 400 (cross section) × 1500 mm (length). Fig. 2 describes the steel reinforcement details of the specimens. The test area in the specimens was in the middle 800 mm length of the column, while the rest was much more heavily reinforced than the test area in order to force failure to occur

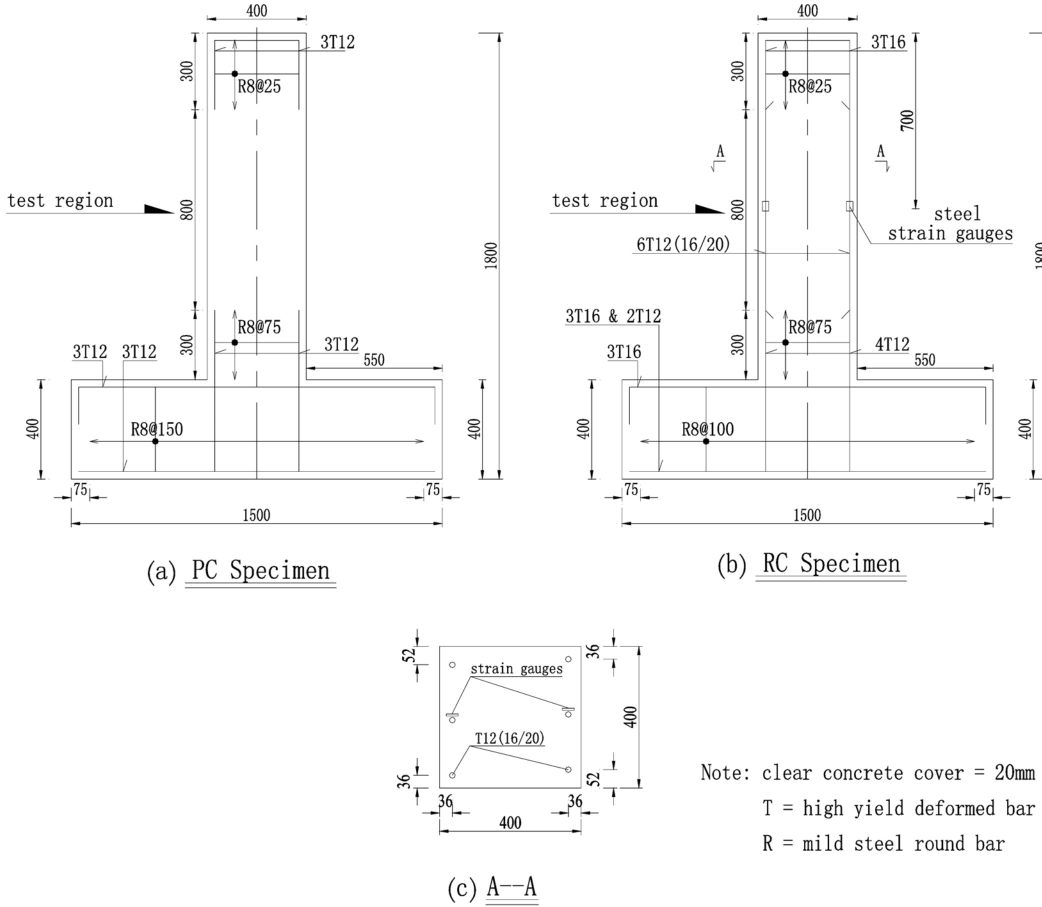


Fig. 2 Details of steel reinforcement

Table 3 Section properties and applied eccentricities

Specimen code	Loading mode	Longitudinal steel				f_c' (MPa)		Eccentricity (mm)
		ρ_s (%)	Detail	f_y (MPa)	E_s (GPa)	28[th] day	Testing day	
Plain concrete specimens:								
PC30-0-CON	concentric	0	---	---	---	29.6	30.0	0
PC30-0-ECC	eccentric	0	---	---	---	29.6	29.3	120
Reinforced concrete specimens:								
RC22-0.42-CON	concentric	0.42	6T12	538	203	22.2	28.7	0
RC22-0.42-ECC	eccentric	0.42	6T12	538	203	22.2	28.7	140
RC22-0.75-CON	concentric	0.75	6T16	533	203	21.9	27.4	0
RC22-0.75-ECC	eccentric	0.75	6T16	533	203	21.9	26.8	140
RC31-1.18-CON	concentric	1.18	6T20	536	200	30.7	34.5	0
RC31-1.18-ECC	eccentric	1.18	6T20	536	200	30.7	34.3	110

within the test region. The PC specimens did not contain any longitudinal reinforcement steel within the test area, while the RC specimens contained different amounts of longitudinal steel ratios (with respect to gross concrete area) of 0.42%, 0.75% and 1.18% within the test area. All the column specimens were unconfined. Table 3 lists out the section properties of the test specimens.

For each pair of column specimen, one was subjected to concentric compressive axial load and the counterpart to eccentric load with different eccentricities. The eccentricities applied varied from 110 to 140 mm for different columns, which are summarised in Table 3. The applied eccentricity has implication on the specimen's failure mode. When the column is subjected to small eccentricity, it would fail in compression where the tension steel would not yield. However, when the column is subjected to large eccentricity, it would fail in tension where the tension steel would yield. The two failure modes are located in two different regions on the column interaction curve as shown in Fig. 3.

The column specimens subjected to concentric load serve as reference specimens which were not subjected to flexure, while the counterpart eccentrically loaded specimens were subjected to flexure.

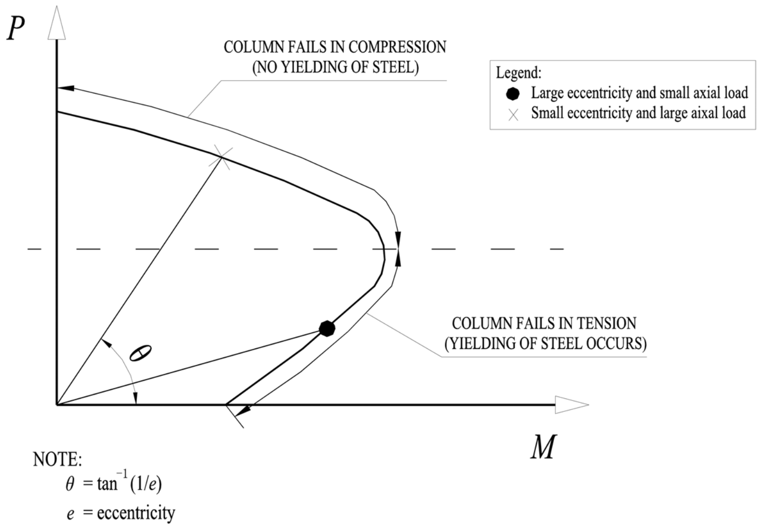
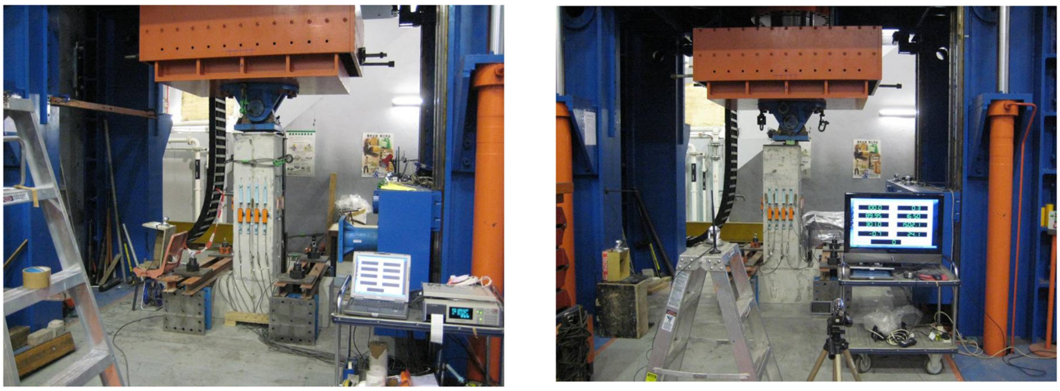


Fig. 3 Failure modes in column interaction diagram



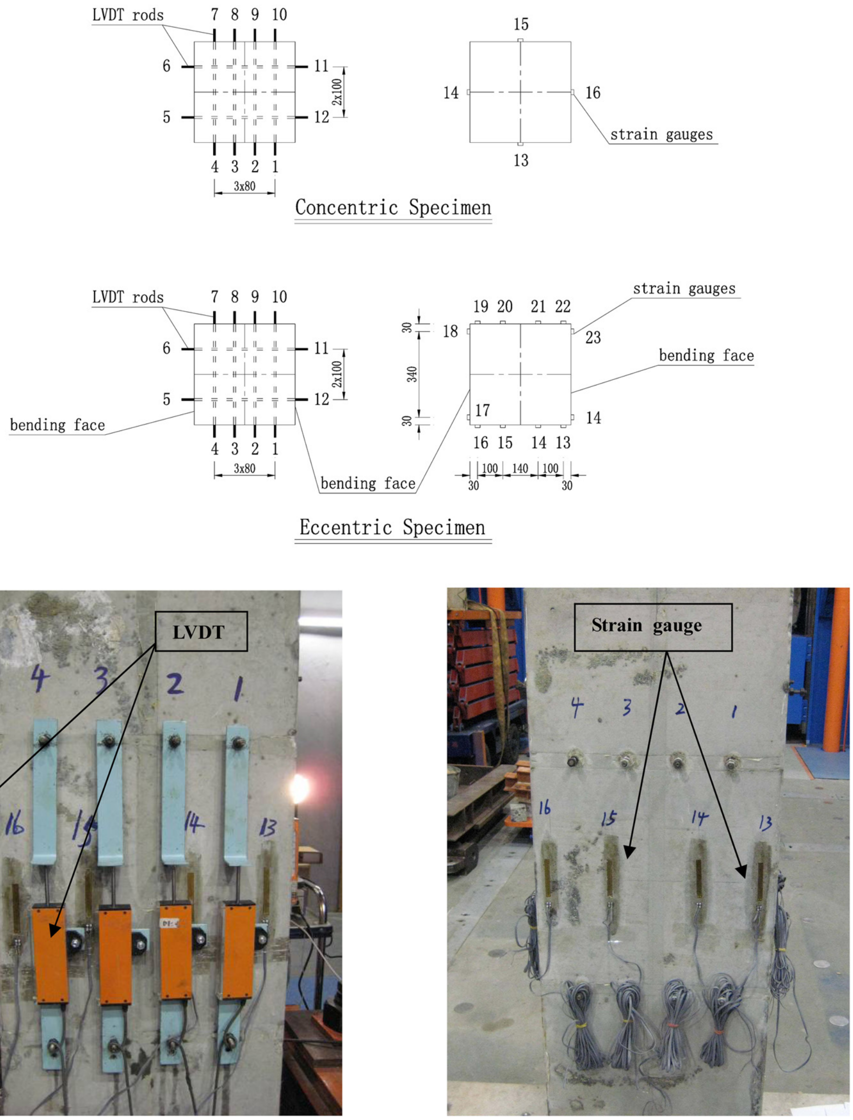
(a) Under concentric loading (b) Under eccentric loading

Fig. 4 Test set-up

By comparing the test results of both specimens, the effect of strain gradient can be assessed. The axial load acting on the column was produced by a computerised electro-hydraulic servo controlled multi-purpose testing machine having a maximum axial loading capacity of 10,000 kN. Fig. 4 shows the test set-up of column specimens subjected to concentric and eccentric loads.

2.2 Instrumentation

The types of instrumentation used for monitoring the behaviour of column specimens are described as follows:



LVDT Strain gauge

Fig. 5 Details of instrumentation

(a) Strain gauges - A strain gauge was attached on each longitudinal steel bar in the middle layer within the testing region to measure the axial and bending strains for both specimens subjected to concentric and eccentric loads. The details of steel strain gauges are shown in Fig. 2(c). On the other hand, a concrete strain gauge was attached on each face of every concentrically loaded specimen, while there were in total twelve gauges (two on each bending face and four on each face perpendicular to the bending face) attached on every eccentrically loaded specimen to measure the strain distribution. The details of concrete strain gauges are shown in Fig. 5.

(b) Linear variable differential transducer (LVDT) - For every specimen, a total of twelve LVDTs were installed on four sides of the specimen test area to measure the deformation due to axial load and/or bending moment. Two LVDTs were installed on each of the bending faces and four LVDTs were installed each on the side that is perpendicular to the bending face. The complete installation of LVDTs is shown in Fig. 5.

2.3 Test procedure

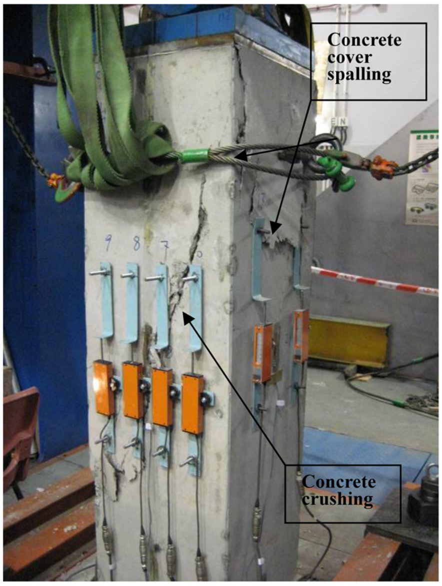
For column specimens subjected to concentric load, a 20 mm steel plate was installed on top of the column. This steel plate was installed to ensure that a smooth contact surface was provided between the specimen and loading platen such that the stress in every part of concrete can be taken as the average stress of the gross concrete area. On the contrary, for column specimens subjected to eccentric load, a guided steel roller was installed at prescribed eccentricity on top of the 20 mm steel plate. In all specimens, the loading was applied in a displacement-controlled manner at a rate of 0.36 mm per minute. All the data from the above instrumentation were recorded by a data logger. The loading application stops after the applied load has reached the maximum value and dropped slightly after the maximum value.

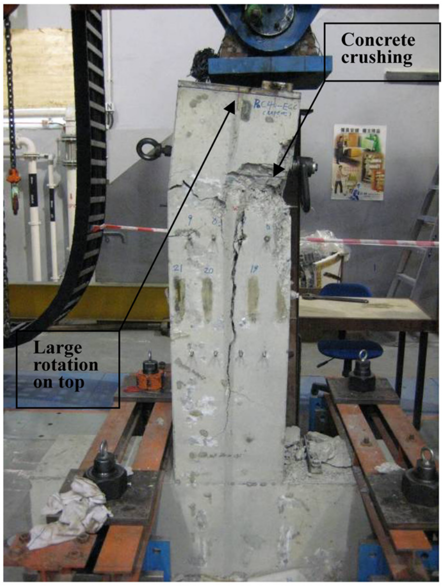
3. Test results and discussion

3.1 Test observations

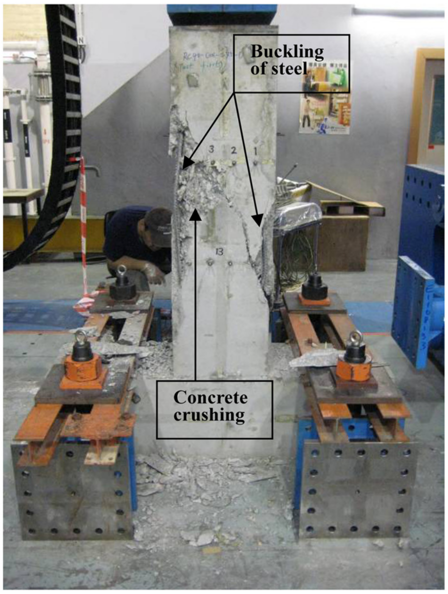
In all specimens, initially the applied axial load increased fairly linearly as the axial displacement of the column increased. No significant damage was observed in the concentrically and eccentrically loaded specimens. As the load increased beyond 70% of the maximum axial load, the axial displacement of the column increased more rapidly as the stiffness of the columns reduced. After the applied load reached the maximum value, cover crushing and spalling were observed. Although it was seen that minor spalling of cover had occurred at the top of RC22-0.42-ECC due to the large eccentricity, however, it was believed that there would not be any adverse effect happen on the evaluation of the maximum concrete stress because the concrete stress had been evaluated based on the measured maximum axial load before the local spalling of cover took place.

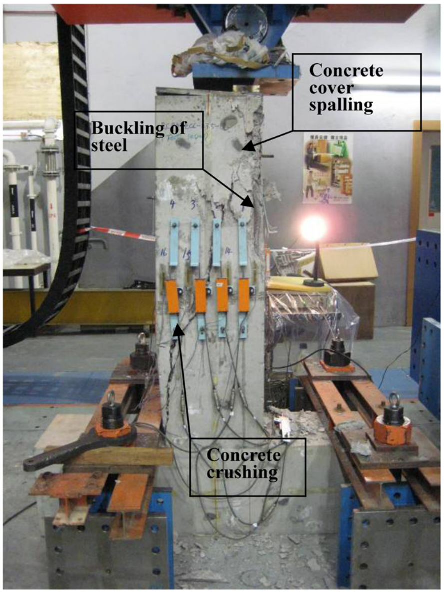
Apart from RC22-0.42-ECC, failure occurred within the test region of the other specimens. In all concentrically loaded specimens, compression crushes occurred around the mid-height of the columns together with the inelastic buckling of longitudinal steel (for RC specimens). For eccentrically loaded PC specimen, severe concrete crushing on the compression side and flexural cracking on the





(a) PC30-0-CON (b) PC30-0-ECC





(a) RC22-0.75-CON (b) RC22-0.75-ECC

Fig. 6 Some observed behaviour of test specimens after test

tension side accompanied by large column rotation were observed. In other eccentrically loaded RC specimens, inelastic buckling of longitudinal steel bars on the compression side was observed in addition to concrete crushing and flexural cracks. Fig. 6 shows the condition of two selected pairs of specimens after failure has occurred.

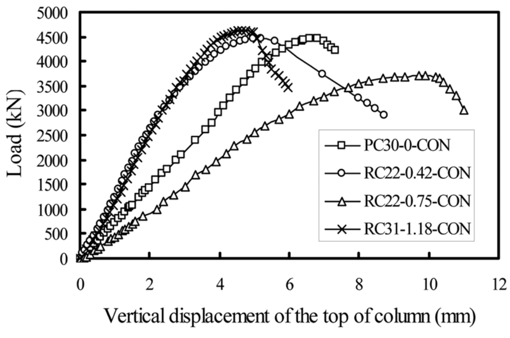
3.2 Test results of concentrically loaded specimens

The measured concrete compressive force of the concentrically loaded column specimens is plotted against the axial displacement of the column in Fig. 7(a). For the PC specimens, the total concrete compressive force is equal to the compressive axial load applied by the actuator. However for RC specimens, the total concrete compressive force was determined by subtracting the compressive force contributed by the longitudinal steels from the applied axial load.

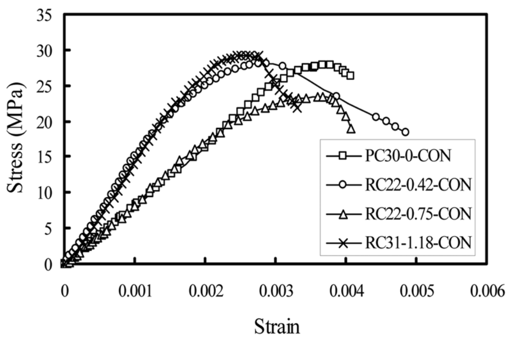
Fig. 7(b) shows the graph of the concrete compressive stress plotted against the concrete strain for all concentrically loaded specimens. The concrete stress in the RC specimens is evaluated by dividing the total concrete force (obtained in Fig. 7(a)) by the gross area of concrete (with area of steel subtracted). The concrete strain is taken as the average readings of the twelve LVDTs installed on all column faces divided by the gauge length. These concrete stress-strain curves will be used later to evaluate the maximum concrete compressive stress that can be developed in the eccentrically loaded specimens.

3.3 Test results of eccentrically loaded specimens

The measured concrete compressive force of the eccentrically loaded specimens is plotted against the axial displacement of the column in Fig. 8. Similarly, the concrete compressive forces of the RC



(a) Load-displacement curves



(b) Stress-strain curves

Fig. 7 Load-displacement and stress-strain curves of concrete of concentrically loaded specimens

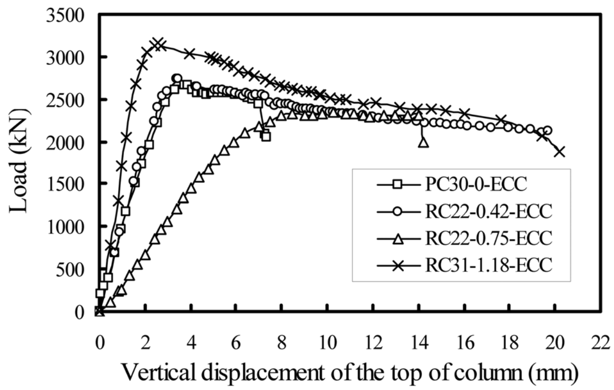


Fig. 8 Load-displacement curves of concrete of eccentrically loaded specimens

specimens were obtained by subtracting the steel force from the total load. The moment capacities of the specimens at any stage were evaluated by multiplying the obtained axial load at that stage with the prescribed eccentricity. These axial loads and moments will be used at a later stage to back calculate the maximum concrete stress that can be developed in the eccentrically loaded specimens.

4. Derivation of concrete stress block parameters

4.1 Derivation of k_1, k_2 and k_3

In this study, the maximum concrete compressive stress that can be developed in flexure is investigated by determining the ratio of maximum concrete compressive stress developed in the eccentrically loaded specimens (f_{max}) to that in the concentrically loaded counterpart specimens (σ_c), which is equivalent to k_3. The value of k_3 can be evaluated by equalising the theoretical and the measured axial force and moment of the eccentrically loaded specimens. The theoretical axial load and moment were computed based on the stress-strain curve obtained from the concentrically loaded specimens multiplied by k_3. A numerical analysis that iterates the value of k_3 for each specimen to match the theoretical with measured axial loads and moments is thus required.

In the numerical method, the stress-strain curve of each concentrically loaded specimen was obtained by fitting the measured stress and strain data using parabolic function as shown in Eq. (3)

$$\sigma = A_1\varepsilon^2 + A_2\varepsilon + A_3 \tag{3}$$

where σ and ε are respectively the concrete stress and strain developed in specimens subjected to concentric load, while A_1, A_2 and A_3 are coefficients obtained by regression analysis.

From the definition of k_3, the concrete stress-strain curve developed under flexure (σ') can be obtained by multiplying both sides of Eq. (3) by k_3, which is expressed in Eq. (4)

$$\sigma' = k_3\sigma = k_3(A_1\varepsilon^2 + A_2\varepsilon + A_3) \tag{4}$$

The value of k_3 for each eccentrically loaded specimen can be determined by matching the theoretical axial force (P) and moment (M) calculated using the concrete stress-strain curve as shown in Eq. (4) with the measured maximum axial load and moment. The formulas for evaluating P and M are expressed in Eqs. (5a) and (5b) respectively (compression is taken as positive)

$$P = \int_A k_3(A_1\varepsilon^2 + A_2\varepsilon + A_3)dA + \sum_{i=1}^{n} f_{si}A_{si} \tag{5a}$$

$$M = \int_A k_3(A_1\varepsilon^2 + A_2\varepsilon + A_3)\left(\frac{h}{2} - c + x\right)dA + \sum_{i=1}^{n} f_{si}A_{si}\left(\frac{h}{2} - d_1\right) \tag{5b}$$

$$\varepsilon = \frac{x}{c}\varepsilon_{cu} \tag{5c}$$

where A is the area of compression zone, x is the distance of a very small strip dA from the neutral axis; n is the total number of steel bars, f_{si} and A_{si} are respectively the stress and area of the i^{th} steel bar, d_i is the distance of the i^{th} steel bar from the extreme compressive concrete fibre and ε_{cu} is the ultimate concrete strain. The ultimate concrete strain is the concrete strain at extreme compressive concrete fibre when the eccentrically loaded specimens reached the maximum moment (Park and Paulay 1975, Wu *et al.* 2004).

The neutral axis depth c is also determined from equalising the theoretical and measured axial load and moment. It should be noted that although the neutral axis depth could be assessed by linearly interpolating the concrete strains obtained by LVDTs at the extreme compression and tension fibres, the value of such is not adopted in the evaluation of k_3 in this study. It is because the measurement of average concrete strain over the gauge length of LVDTs will underestimate the actual tensile strain of concrete, which is a very small and localised value, and subsequently overestimate the neutral axis depth. Therefore, in lieu of using the neutral axis depth obtained from direct measurement, the respective value computed based on the axial force and moment equilibriums were adopted.

By substituting Eq. (5c) into Eqs. (5a) and (5b), k_3 and c can be solved simultaneously. Based on these values, k_1 and k_2 can be solved respectively from Eqs. (6a) and (6b) as follows

$$P = k_1 k_3 \sigma_c bc + \sum_{i=1}^{n} f_{si} A_{si} \tag{6a}$$

$$M = k_1 k_3 \sigma_c bc\left(\frac{h}{2} - k_2 c\right) + \sum_{i=1}^{n} f_{si} A_{si}\left(\frac{h}{2} - d_i\right) \tag{6b}$$

The values of k_1, k_2, k_3 and c evaluated for the eccentrically loaded specimens are listed in Table 4 together with their corresponding σ_c and strain gradient ϕ. From the table, it is apparent that all values of k_3 are larger than 1.0, which reveal that strain gradient does enhance the maximum concrete compressive stress that could be developed in flexural members. It is also seen that the values of k_3 increases as ϕ increases, while k_1 and k_2 remains relatively constant at 0.67 and 0.40 respectively.

The average values of k_1, k_2 and k_3 are compared with the respective values obtained by other researchers (Hognestad *et al.* 1955, Kaar *et al.* 1978, Swartz *et al.* 1985, Mansur *et al.* 1997, Tan and Nguyen 2004, 2005) in Table 5. From the table, it is obvious that the average values of k_1 and k_2 obtained in this study are slightly smaller than that obtained by other researchers. However, the average value of k_3 obtained in this study is larger than those obtained by other researchers, which

Table 4 Values of k_1, k_2 and k_3

Specimen code	f_c' (MPa) at testing day	σ_c (MPa)	c (mm)	k_1	k_2	k_3	k_1k_3	ϕ (rad/m)
Plain concrete specimen:								
PC30-0-ECC	29.3	28.1	199.2	0.683	0.401	1.675	1.144	0.0176
Reinforced concrete specimens:								
RC22-0.42-ECC	28.7	28.1	217.7	0.656	0.403	1.626	1.066	0.0142
RC22-0.75-ECC	26.8	23.5	229.0	0.691	0.392	1.537	1.062	0.0135
RC31-1.18-ECC	34.3	29.3	287.0	0.642	0.397	1.111	0.713	0.0101
Average	30.0	27.3	---	0.668	0.398	1.487	0.996	---

Table 5 Comparison of authors' obtained values of k_1, k_2 and k_3 with other researchers'

Researcher	f_c' (MPa)	k_1	k_2	k_3	k_1k_3
Hognestad *et al.* (1955)	27.6	0.790	0.450	0.940	0.743
	34.5	0.750	0.440	0.920	0.690
Kaar *et al.* (1978)	45.0	0.722	0.400	0.970	0.700
Swartz *et al.* (1985)	57.0	0.714	0.415	0.980	0.700
Mansur *et al.* (1997)	57.2	0.704	0.420	0.980	0.690
Tan and Nguyen (2004, 2005)	48.3	0.700	0.380	0.930	0.651
Authors' results	30.0	0.668	0.398	1.487	0.996

indicates that the maximum compressive stress developed in concrete under flexure should be larger than that predicted by previous researchers. Furthermore, it is also seen that the value of the product k_1k_3 obtained in this study is generally larger than those obtained by other researchers. It reveals that the total compressive force that could be developed by concrete under flexure should also be larger.

4.2 Derivation of equivalent rectangular concrete stress block parameters

The values of α and β of the eccentrically loaded specimens can be determined by Eqs. (7a) and (7b), which are derived based on axial load and moment equilibrium conditions as shown in Fig. 1(e).

$$P = \alpha\beta f_c' bc + \sum_{i=1}^{n} f_{si} A_{si} \tag{7a}$$

$$M = \alpha\beta f_c' bc\left(\frac{h}{2} - \frac{\beta}{2}c\right) + \sum_{i=1}^{n} f_{si} A_{si}\left(\frac{h}{2} - d_i\right) \tag{7b}$$

The neutral axis depth obtained from Eq. (5) is adopted in solving the above equations for α and

Table 6 Values of equivalent rectangular stress block parameters and strain gradient

Specimen code	α	β	ε_{cu}	ϕ (rad/m)
PC30-0-CON	0.937	---	---	0.0
RC22-0.42-CON	0.819	---	---	0.0
RC22-0.75-CON	0.858	---	---	0.0
RC31-1.18-CON	0.849	---	---	0.0
Average	0.866	---	---	0.0
PC30-0-ECC	1.426	0.802	0.0035	0.0176
RC22-0.42-ECC	1.323	0.806	0.0031	0.0142
RC22-0.75-ECC	1.357	0.783	0.0031	0.0135
RC31-1.18-ECC	0.898	0.794	0.0029	0.0101
Average	1.251	0.796	0.0031	---

β, which are listed in Table 6. To illustrate the effect of strain gradient on the equivalent concrete stress, the values of α obtained from eccentrically loaded specimens are compared with those of respective concentrically loaded specimens, which are also summarised in Table 6. From the comparison, it is clear that: (1) The value of α for the eccentrically loaded specimens is larger than that of the concentrically loaded specimens. It therefore implies that flexural RC members could develop larger equivalent concrete stress than purely axially loaded RC members. (2) α increases as the strain gradient increases, which implies that the equivalent concrete stress that can be developed in flexural RC members increases as strain gradient increases. (3) For concentrically loaded specimens, the average value of α is about 0.866, which is very close to the currently adopted value of α = 0.85 in ACI code (ACI Committee 318 2008), Australian Code (Standard Australia 2006), Eurocode 2 (European Committee for Standardization 2004) and New Zealand Code (Standard New Zealand 2006). From the above, it is convinced that the existing codes could predict accurately the equivalent concrete stress and thus the strength of RC columns subjected to pure axial load. However, it underestimates those of RC beams and columns subjected to flexure with or without axial load.

4.3 Verification against flexural strength of RC beams and columns

To validate the obtained equivalent rectangular concrete stress block parameters, the average values of α = 1.25 and β = 0.80 listed in the last row of Table 6, as well as ε_{cu} = 0.0031 which are the obtained average value of ε_{cu}, are used to evaluate the flexural strengths of RC members tested by other researchers, which include beams (Pecce and Fabbrocino 1999, Ashour 2000, Debernardi and Taliano 2002, Rashid and Mansur 2005, Lam *et al.* 2008), columns subjected to low axial load level, i.e. $0 < P/A_g f_c' \leq 0.2$ where A_g is the column cross-section area, (Watson and Park 1994, Mo and Wang 2000, Marefat *et al.* 2005) and columns subjected to medium axial load level, i.e. $0.2 < P/A_g f_c' \leq 0.5$, (Sheikh and Khoury 1993, Watson and Park 1994, Mo and Wang 2000, Lam *et al.* 2003, Marefat *et al.* 2005, Marefat *et al.* 2006). These predicted flexural strengths M_p are compared with their respective measured strengths M_t as well as with their respective theoretical strengths calculated based on the following RC design codes: M_{ACI} based on ACI Code (ACI Committee 318 2008), M_{EC} based on Eurocode 2 (European Committee for Standardization 2004) and M_{NZS} based on New Zealand Code (Standard New Zealand 2005). The comparison is summarised in Table 7 for beams and Tables 8 to 9 for columns.

From Tables 7 to 9, it can be concluded that:

(1) The flexural strengths of RC beams and columns subjected to low, medium load levels predicted by the average values of α = 1.25, β = 0.80 and ε_{cu} = 0.0031 have the best agreement with their measured flexural strengths.

(2) For RC beams, the average ratio of the predicted to measured flexural strength is 0.94, whereas the respective ratios of the theoretical to measured flexural strength of ACI, EC2 and NZ are 0.91, 0.91 and 0.91 respectively. It is evident that the adoption of the average stress block parameters obtained in this study can increase the accuracy of flexural strength prediction by 3% in average.

(3) For RC columns subjected to low axial load level, the average ratio of the predicted to measured flexural strength is 0.95, whereas the respective ratios of the theoretical to measured flexural strength of ACI, EC2 and NZS are 0.84, 0.86 and 0.84 respectively. It is evident that the adoption of the average stress block parameters obtained in this study can increase the

Table 7 Comparison of proposed strengths of beams

Specimen code	f_c'	$^{\#}M_p$ (1)	$^{\#}M_{ACI}$ (2)	$^{\#}M_{EC}$ (3)	$^{\#}M_{NZS}$ (4)	$^{\#}M_t$ (5)	$\frac{(1)}{(5)}$	$\frac{(2)}{(5)}$	$\frac{(3)}{(5)}$	$\frac{(4)}{(5)}$
Pecce and Fabbrocino (1999)										
A	41.3	99.7	97.0	97.0	97.0	104.0	0.96	0.93	0.93	0.93
B	41.3	43.9	45.0	45.0	45.0	49.6	0.89	0.91	0.91	0.91
C	42.3	710.0	636.7	636.7	636.7	712.5	1.00	0.89	0.89	0.89
Ashour (2000)										
B-N2	48.6	56.1	53.6	53.6	53.6	58.2	0.96	0.92	0.92	0.92
B-N3	48.6	81.6	77.1	77.1	77.1	80.6	1.01	0.96	0.96	0.96
B-N4	48.6	104.9	98.4	98.4	98.4	99.6	1.05	0.99	0.99	0.99
Debernardi and Taliano (2002)										
T1	27.7	11.0	10.8	10.8	10.8	13.6	0.81	0.79	0.79	0.79
T2	27.7	21.0	20.5	20.6	20.5	23.6	0.89	0.87	0.87	0.87
T3	27.7	30.3	29.0	29.2	29.0	32.5	0.93	0.89	0.90	0.89
T4	27.7	47.8	46.8	46.9	46.8	59.8	0.80	0.78	0.78	0.78
T5	27.7	91.3	93.1	93.1	93.1	107.5	0.85	0.87	0.87	0.87
T6	27.7	170.5	170.5	171.2	170.5	192.4	0.89	0.89	0.89	0.89
T7	27.7	234.2	217.2	221.6	217.2	221.6	1.06	0.98	1.00	0.98
T8	27.7	81.4	81.1	81.2	81.1	93.9	0.87	0.86	0.86	0.86
T9	27.7	149.0	152.0	151.9	152.0	182.7	0.82	0.83	0.83	0.83
T10	27.7	323.4	324.6	324.7	324.6	330.4	0.98	0.98	0.98	0.98
Rashid and Mansur (2005)										
A111	42.8	190.3	185.4	185.6	186.2	205.7	0.93	0.90	0.90	0.90
A211	42.8	297.5	286.8	286.6	286.6	276.8	1.07	1.04	1.04	1.04
*Lam *et al.* (2008)										
L-A	29.8	14.8	14.6	14.6	14.6	14.0	1.06	1.05	1.05	1.05
L-C1	29.8	14.8	14.6	14.6	14.6	14.2	1.04	1.03	1.03	1.03
L-C2	29.8	14.8	14.6	14.6	14.6	14.4	1.03	1.01	1.01	1.01
L-D	29.8	9.9	9.8	9.8	9.8	11.6	0.85	0.84	0.84	0.84
L-E	29.8	26.7	25.9	26.0	25.9	29.4	0.91	0.88	0.88	0.88
L-1	30.5	9.8	9.6	9.6	9.6	11.4	0.86	0.84	0.84	0.84
L-2	30.5	9.8	9.6	9.6	9.6	11.3	0.87	0.85	0.85	0.85
L-3	30.5	9.8	9.6	9.6	9.6	10.0	0.98	0.96	0.96	0.96
						Average	0.94	0.91	0.91	0.91

Notes: #All moments in kNm; f_c' in MPa
*f_c' is taken as $0.8f_{cu}$

accuracy of flexural strength prediction by 10% in average.

(4) For RC columns subjected to medium axial load level, the average ratio of the predicted to measured flexural strength is 0.98 whereas the respective ratios of the theoretical to measured flexural strength of ACI, EC2 and NZS are 0.82 0.83 and 0.82 respectively. It is evident that

Table 8 Comparison of proposed strengths of columns subjected to low axial load level

Specimen code	f_c'	$\frac{P}{A_g f_c'}$	$^{\#}M_p$ (1)	$^{\#}M_{ACI}$ (2)	$^{\#}M_{EC}$ (3)	$^{\#}M_{NZS}$ (4)	$^{\#}M_t$ (5)	$\frac{(1)}{(5)}$	$\frac{(2)}{(5)}$	$\frac{(3)}{(5)}$	$\frac{(4)}{(5)}$
Watson and Park (1994)											
1	47.0	0.100	326.8	302.0	306.9	302.0	335.2	0.97	0.90	0.92	0.90
Mo and Wang (2000)											
C1-1	24.9	0.113	331.9	284.3	289.9	284.3	351.4	0.94	0.81	0.82	0.81
C2-1	25.3	0.111	333.5	285.4	290.8	285.4	347.2	0.96	0.82	0.84	0.82
C3-1	26.4	0.106	335.6	286.4	291.4	286.4	353.4	0.95	0.81	0.82	0.81
C1-2	26.7	0.158	360.3	305.7	311.7	305.7	374.6	0.96	0.82	0.83	0.82
C2-2	27.1	0.156	361.2	306.2	312.1	306.2	399.9	0.90	0.77	0.78	0.77
C3-2	27.5	0.153	362.1	306.7	312.4	306.7	395.5	0.92	0.78	0.79	0.78
Marefat *et al.* (2005)											
STCM-9	24.0	0.190	22.5	22.5	22.8	22.5	23.3	0.97	0.97	0.98	0.97
SBCC-7	27.0	0.160	45.3	41.6	41.7	41.6	45.1	1.00	0.92	0.93	0.92
							Average	0.95	0.84	0.86	0.84

Notes: $^{\#}$All moments in kNm; f_c' in MPa

the adoption of the average stress block parameters obtained in this study can increase the accuracy of flexural strength prediction by 16% in average.

(5) It is observed that the accuracy of flexural strength predicted by using the average values of α, β and ε_{cu} obtained in this study improves for RC beams and columns subjected to low and medium axial load levels. This indicates that the average values of the equivalent rectangular concrete stress block parameters (i.e. α and β) obtained in this study represent more precisely the equivalent concrete stress developed under flexure.

5. Conclusions

The maximum and equivalent concrete stress that can be developed in unconfined RC members under flexure were studied experimentally. In total four pairs of inverted T-shaped specimens were fabricated, one pair was PC and three pairs were RC. Each pair of them consisted of identical cross-section properties, and one of them was subjected to concentric axial load while another to eccentric axial load. All the column specimens were unconfined. The effects of strain gradient on the maximum concrete stress that can be developed under flexure was studied by the parameter k_3, which is the ratio of the maximum concrete stress developed under flexure to uni-axial concrete strength σ_c. The value of k_3 was determined by equalising the theoretical and the measured values of axial force and moment of the eccentrically loaded specimens. On the other hand, the effects of strain gradient on the equivalent concrete stress developed under flexure was studied by the parameter α, which is the ratio of the equivalent concrete stress to the uni-axial concrete cylinder strength.

From the obtained results, it can be seen that the values of k_3 and α were larger than those obtained by previous researchers. It is observed that the values of k_3 obtained is larger than 1.0,

Table 9 Comparison of proposed strengths of columns subjected to medium axial load level

Specimen code	f_c'	$\frac{P}{A_g f_c'}$	$^{\#}M_p$ (1)	$^{\#}M_{ACI}$ (2)	$^{\#}M_{EC}$ (3)	$^{\#}M_{NZS}$ (4)	$^{\#}M_t$ (5)	$\frac{(1)}{(5)}$	$\frac{(2)}{(5)}$	$\frac{(3)}{(5)}$	$\frac{(4)}{(5)}$
Sheikh and Khoury (1993)											
AS-19	32.3	0.470	218.1	178.5	183.9	179.2	219.7	0.99	0.81	0.84	0.82
Watson and Park (1994)											
2	44.0	0.300	472.8	405.9	410.2	406.0	486.0	0.97	0.84	0.84	0.84
3	44.0	0.300	473.6	405.9	410.2	406.0	479.1	0.99	0.85	0.86	0.85
4	40.0	0.300	443.3	382.1	385.3	382.3	448.1	0.99	0.85	0.86	0.85
5	41.0	0.500	524.0	372.9	383.2	373.4	525.8	1.00	0.71	0.73	0.71
6	40.0	0.500	516.0	367.2	376.7	367.8	526.4	0.98	0.70	0.72	0.70
Mo and Wang (2000)											
C1-3	26.1	0.216	382.1	322.8	329.9	322.8	427.7	0.89	0.75	0.77	0.75
C2-3	26.8	0.210	384.3	324.1	331.3	324.1	427.1	0.90	0.76	0.78	0.76
C3-3	26.9	0.209	383.4	323.5	330.5	323.5	423.8	0.90	0.76	0.78	0.76
Lam *et al.* (2003)											
X6	31.9	0.450	31.4	28.5	29.0	28.6	37.1	0.85	0.77	0.78	0.77
X7	35.7	0.450	35.2	29.7	30.5	29.8	37.1	0.95	0.80	0.82	0.80
Marefat *et al.* (2005)											
NTCM-14	20.1	0.310	18.6	16.0	16.0	16.0	16.8	1.11	0.95	0.95	0.95
NBCC-12	25.2	0.230	25.4	22.0	22.4	22.0	21.7	1.17	1.01	1.03	1.01
NBCM-11	24.5	0.250	45.1	38.6	38.5	38.6	44.6	1.01	0.87	0.86	0.87
SBCM-8	28.0	0.220	52.0	46.0	46.0	46.0	58.7	0.89	0.78	0.78	0.78
Marefat *et al.* (2006)											
NTMM13	21.0	0.310	18.6	16.0	16.0	16.0	17.3	1.08	0.93	0.93	0.93
							Average	0.98	0.82	0.83	0.82

Notes: $^{\#}$All moments in kNm; f_c' in MPa

which indicates that the maximum concrete stress developed under flexure could be larger than its uni-axial strength. For equivalent concrete stress, it is found that the values of α of specimens subjected to flexural were larger than the respective value stipulated in the existing RC design codes (i.e. $\alpha = 0.85$), whereas those of specimens subjected to pure axial load is almost identical to that stipulated in the existing design codes. It is also deduced that the values of k_3 and α were not only dependent on the concrete strength but also on the strain gradient.

The validity of using the average values of α, β and ε_{cu} in flexural strength evaluation of unconfined RC members was checked by comparing the theoretical strengths of beams and columns subjected to low as well as medium axial load levels with the their measured strengths obtained by previous researchers. The predicted flexural strengths were also compared with those predicted by various RC design codes: ACI (or AS), EC2 and NZ. From the comparisons, it was seen that the obtained values of α, β and ε_{cu} predict more accurately the flexural strengths of unconfined RC beams and columns subjected to low as well as medium axial load level than the existing RC design codes. The accuracy improvement was about 3% for RC beams and could reach about 16% for columns subjected to medium axial load level.

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CC

Notations

A_g	Column cross-section area
A_s	Area of steel bar
b	Width of cross section
c	Neutral axis depth
d	Distance of longitudinal steel bar to extreme compressive fibre
E_s	Young's modulus of steel bar
f_{av}	Average concrete compressive stress over compression area in flexural members
f_c'	Uni-axial concrete compressive strength represented by cylinder strength
f_{cu}	Uni-axial concrete compressive strength represented by cube strength
f_{max}	Maximum concrete compressive stress developed under flexure
f_s	Stress of steel bar
f_y	Yield strength of steel bar
h	Depth of cross section
k_1	Ratio of average stress (f_{av}) over compression area to maximum stress developed under flexure (f_{max})
k_2	Ratio of distance between extreme compressive fibre and resultant force of compressive stress block (P_c) to that between the same fibre to neutral axis (c)
k_3	Ratio of f_{max} to uni-axial concrete strength σ_c
LVDT	Linear variable displacement transducer
M	Moment or flexural strength of column section
M	Maximum measured moment (Eqs. 5, 6 and 7 only)
M_{ACI}	Moment calculated based on ACI Code
M_{EC}	Moment calculated based on Eurocode 2
M_{NZS}	Moment calculated based on New Zealand Code 3101
M_p	Moment calculated based on average values of equivalent rectangular concrete stress block parameters obtained in this study
M_t	Measured moment of specimen from tests
n	Number of longitudinal steel bars
P	Axial load subjected by column section
P	Axial load at maximum moment (Eqs. 5, 6 and 7 only)
PC	Plain concrete
P_c	Resultant force of concrete compressive stress block
RC	Reinforced concrete

x	Neutral axis depth
α	Ratio of equivalent concrete compressive stress developed under flexure to concrete cylinder (f_c') or cube (f_{cu}) strength
β	Ratio between height of equivalent rectangular concrete compressive stress block and neutral axis depth
ε	Concrete strain
ε_{cu}	Concrete strain at extreme compressive fibre measure at maximum load
ρ_s	Longitudinal reinforcement ratio
σ	Concrete stress developed in concentrically loaded specimens
σ_c	Maximum uni-axial concrete compressive stress developed in concentrically loaded specimens
ϕ	Strain gradient