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Modeling of nonlinear cyclic response of shear-deficient RC T-beams strengthened with side bonded CFRP fabric strips

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Abstract. The use of Carbon Fiber Reinforced Polymers (CFRP) to strengthen reinforced concrete beams under bending and shear has gained rapid growth in recent years. The performance of shear strengthened beams with externally bonded CFRP laminate or fabric strips is raising many concerns when the beam is loaded under cyclic loading. Such concerns warrant experimental, analytical and numerical investigation of such beams under cyclic loading. To date, limited investigations have been carried out to address this concern. This paper presents a numerical investigation by developing a nonlinear finite element (FE) model to study the response of a cantilever reinforced concrete T-beam strengthened in shear with side bonded CFRP fabric strips and subjected to cyclic loading. A detailed 3D nonlinear finite element model that takes into account the orthotropic nature of the polymer's fibers is developed. In order to simulate the bond between the CFRP sheets and concrete, a layer having the material properties of the adhesive epoxy resin is introduced in the model as an interface between the CFRP sheets and concrete surface. Appropriate numerical modeling strategies were used and the response envelope and the load-displacement hysteresis loops of the FE model were compared with the experimental response at all stages of the cyclic loading. It is observed that the responses of the FE beam model are in good agreement with those of the experimental test. A parametric study was conducted using the validated FE model to investigate the effect of spacing between CFRP sheets, number of CFRP layers, and fiber orientation on the overall performance of the T-beam. It is concluded that successful FE modeling provides a practical and economical tool to investigate the behavior of such strengthened beams when subjected to cyclic loading.

Keywords: cyclic loading; nonlinear analysis; CFRP; reinforced concrete; side bonded.

1. Introduction

Externally bonded Carbon Fiber Reinforced Polymer (CFRP) composite plate has been extensively used recently as a strengthening member to reinforced concrete (RC) structural members in both shear and bending. Shear failure in reinforced concrete beams is brittle and undesirable. It is a complex phenomenon and it has been an area of continuous research, even for regular RC beams under monotonic loading. Limited research has been conducted on RC beams strengthened with CFRP when subjected to cyclic loading. In recent years, several experimental studies were carried out to investigate the effectiveness of CFRP in strengthening, rehabilitating and

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retrofitting of structural concrete members (ACI 440.2R-08 2008, Carolin 2001, Ritchie *et al.* 1991, Chajes *et al.* 1995, Norris *et al*. 1997, Khalifa *et al*. 2002). Most of this work does not address the behavior of CFRP strengthened beams under cyclic load. An overview of the state-of-the-art of existing research on beams strengthened in shear with Fiber Reinforced Polymer (FRP) has been reported by Alzoubi *et al*. (2007).

To better understand the complex shear behavior of reinforced concrete beams strengthened with CFRP under cyclic loading, numerical modeling as well as experimental testing is needed. Both tools examine the behavior of the structure and measure the load-deformation response of the member under consideration. Experimental testing is sometimes time consuming and expensive. On the other hand, reliable and validated numerical modeling could be used as a valid tool to predict the response and behavior of the structure with tremendous savings in time and money. The finite element method has been used widely to model reinforced concrete structures, especially when the concrete material behaves nonlinearly and when the tensile stress of concrete reaches the rupture value (Hawileh *et al*. 2009a, 2009b). In addition, 3-D finite element modeling gives full field of deformation, stress, and strain results.

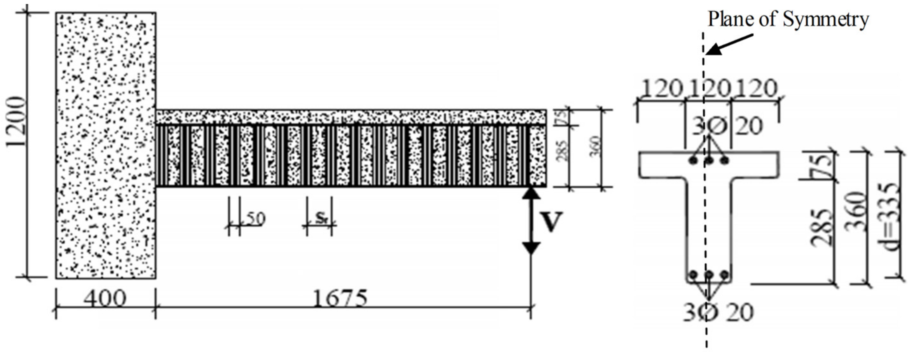
Considerable work has been reported in the literature related to the development and application of finite element procedures and models to simulate the response of strengthening RC beams with externally bonded FRP plates and sheets under static monotonic loading (Arduini *et al.* 1997, Tedesco *et al*. 1999, Kachlakev *et al.* 2002, Santhakumar *et al.* 2004, Kim and Aboutaha 2004, Hoque *et al*. 2007, Lee *et al.* 2008, Gamino *et al.* 2009). However, limited research (Tanarslan *et al.* 2008) has been conducted on the performance of CFRP strips under cyclic loading.

Tanarslan *et al.* (2008), tested in their experimental program, seven identical shear deficient RC cantilever T-beams under cyclic loading. The first beam was un-strengthened to serve as a control beam and the remaining six beams were strengthened in shear with different arrangements and anchorage of CFRP strips (side bonded, U-wrap, L-shaped, U jacketed, and double L-shaped jacketed). The authors of this paper developed a finite element model in a previous study (Hawileh *et al.* 2009a, 2009b) that correlated the experimental results of the control beam with reasonable accuracy. The finite element and the experimental results were in close agreement, especially, for the maximum deflection at failure.

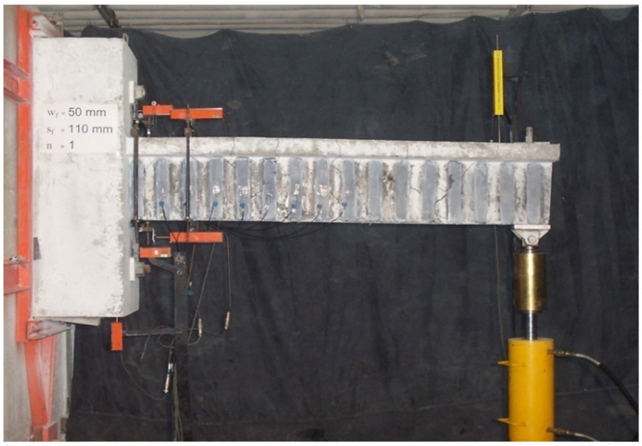
In this study a finite element model was developed to simulate the response and predict the behavior of a cantilever T-beam strengthened with side bonded CFRP strips under cyclic loading. The model was validated by comparing the load-deflection response envelopes and load-displacement hysteresis loops with the test results of Tanarslan *et al.* (2008) and the FE results. Crack patterns evolutions associated with the corresponding vertical displacement of the cantilever beam at different selected loading stages and at failure were also taken into account for comparison. In addition, a parametric study is conducted to investigate the effect of different spacing between the CFRP sheets, number of FRP layers as well as their orientation on the overall performance of the T-beam. The results of this study showed the viability and validity of the finite element method in modeling shear deficient RC beams strengthened with externally side bonded CFRP strips with reasonable accuracy when subjected to cyclic loading.

2. Details of the tested specimen

Fig. 1(a) shows the dimensions and reinforcement details of the tested beam (Tanarslan *et al.* 2008). The tested specimen is a cantilever T-beam with 1675 mm clear span, a total depth of 360 mm, web width of 120 mm, flange thickness of 75 mm and flange width of 360 mm, as shown in Fig. 1(a). The T-Beam is reinforced with top and bottom steel reinforcement as shown in Fig. 1(a), consisting of three bars, 20 mm in diameter at each level. No internal stirrups were provided as shear reinforcement. The beam is strengthened with 50 mm wide CFRP strips (Sikawrap 160c CFRP sheets bonded with Sikadur-330 epoxy resin). The clear spacing between the strips is 60 mm (110 mm center-to-center). Fig. 1(b) shows the actual T-beam specimen instrumented with LVDT and load cell. It is clear from Fig. 1(b) that the tested cantilever T-beam is connected to a column that is attached to a strong wall from one end and free from the other. The cyclic load is applied at the beam's free end by a loading column as shown in Fig. 1(b) consisting of a loading cell and hydraulic jack with a capacity of 600 KN and 1000 KN, respectively. A complete description of the experimental program, test setup and instrumentation are provided in (Tanarslan *et al.* 2008).



(a) Schematic sketch of tested specimen (all dimensions are in mm)



(b) Tested Specimen

Fig. 1 Tested specimen with side-bonded CFRP strips (Tanarslan *et al.* 2008)

3. Finite element model development

The developed finite model (FEM) has the same geometry, dimensions, materials, and boundary conditions of the tested beam. The FEM was analyzed using ANSYS program (ANSAY 2007). Making full use of symmetry of geometry, materials, and loading, only half of the tested beam was modeled resulting in tremendous saving of computational time due to the reduction of the number of elements by half. The developed finite element model is shown in Fig. 2.

ANSYS SOLID65 (ANSYS 2007) concrete element is used to simulate the concrete material. The element is capable of modeling the nonlinear behavior of concrete and cracking in tension. Each node of the solid element has three translational degrees of freedom in the x, y, and z directions. The reinforcement steel bars were modeled using LINK8 (ANSYS 2007) element. LINK8 is defined by two nodes with three translational degrees of freedom at each node. The element is capable of elastic-plastic deformation, stress stiffening, and large deflection. The CFRP (50 mm wide by 0.12 mm thick) strips were modeled using SHELL99 (ANSYS 2007) element with elastic orthotropic material properties [$E_x = 231$ GPa, $E_{y,z} = 12$ GPa, $\nu_{xy,yz} = 0.28$, $\nu_{yz} = 0.42$, $G_{xy,\,xz} = 5$ GPa, and $G_{yz} = 4$ GPa], where the x-axis is in the direction of the carbon fibers. The SHELL99 element is used to model layered applications of structural composites. The element allows up to 250 layers and composed of eight nodes (corner and midside nodes) with six degrees of freedom at each node: translations in the *x*, *y*, and *z* directions and rotations about the *x*, *y*, and z-axes. The advantage of using SHELL99 over SHELL63 (ANSYS 2007) is in the flexibility of specifying the nodes to be either at the top, middle, or bottom surface of the shell element. In this model, since the CFRP strips were externally bonded to the concrete, the nodes of the SHELL99 elements were specified at the bottom of the element. The SHELL99 element used in this study is composed of two layers. The first layer is 0.5 mm thick representing the adhesive epoxy material to simulate the bond between the CFRP and concrete surfaces. The second layer is 0.12 mm thick layer representing the CFRP sheet. The adhesive used in the experimental investigation is Sikadur-330 epoxy resin at the interface between the CFRP sheets and concrete surfaces. The epoxy resin had a modulus of elasticity of 4.5 GPa, poison ratio of

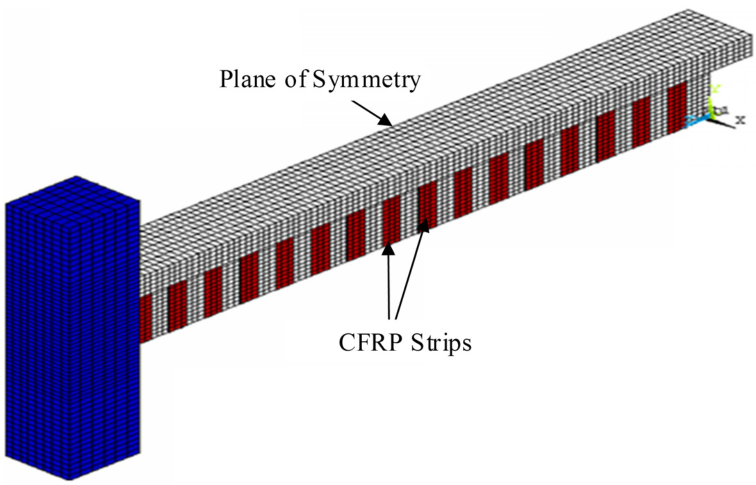


Fig. 2 Three-dimensional isometric view of FEM

ν_{xy} = 0.28, and a tensile strength and elongation at failure of 30 MPa and 0.009, respectively. The epoxy material was simulated as an isotropic elastic material until it reachs its ultimate tensile strength, after which the failure is brittle and this behavior is simulated by reducing the material's tensile strength linearly to zero at a strain of 0.012. In addition, perfect bond between the concrete and reinforcement steel was assumed in this study through sharing the same nodes. The total number of elements used in this model is 18000.

Failure of the reinforced concrete is simulated in this study according to William and Warnke (1975) along with the Von Misses failure criterion. The Compressive, tensile strength and modulus of elasticity of concrete were taken as 32.4 MPa, 3.54 MPa, and 26937 MPa respectively. To define the plastic behavior of concrete, a multi-linear compressive stress-strain curve was defined and shown in Fig. 3. A coefficient of 0.2 (typical range is between 0.0 and 1.0) for the open and closed shear transfer coefficients is used for the concrete constitutive material data table needed for the William and Warnke (1975) model. In addition, a tensile stiffness multiplier upon reaching the concrete rupture strength of 0.6 is used (ANSYS 2007). The rigid concrete column support used in the experiment didn't examine any cracking and hence was modeled using element type SOLID45 (ANSYS 2007) as an elastic concrete material having a modulus of elasticity and poison ratio of 26.39 GPa and 0.2, respectively. Finally, for the reinforcement steel bars, the Young's modulus of Elasticity and yield stress were taken as 205 GPa and 460 GPa, respectively. In addition, the nonlinear material property of steel is assumed to be elastic-perfectly plastic (Hawileh *et al.* 2009a, 2009b) and the Von Misses failure criteria were used to define the yielding of the steel reinforcement. The failure criteria adopted in the study is simulated and defined by the following assumptions:

- The beam will fail when the steel reinforcement reaches its yield stress.
- The CFRP fabric sheet will de-bond from the adjacent concrete surface when the strain in the direction of the CFRP fibers reaches 0.004, according ACI440.2R-08 (2008) provisions.
- Concrete fails when strain in the top compression fiber of the concrete exceeds 0.003.
- CFRP ruptures when the stress exceeds its rupture stress

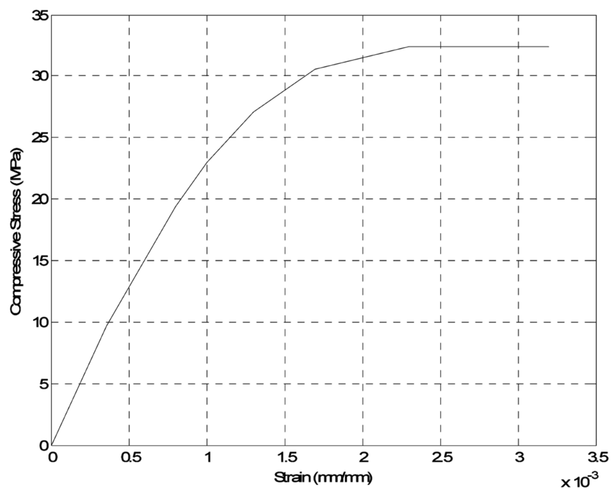


Fig. 3 Compressive stress-strain curve for concrete

4. Loading, boundary conditions, and nonlinear solution

The tested T-beam is free at one end and fixed to a column at the other end. Similar boundary conditions in the FEM were assigned to simulate the experimental setup as shown in Fig. 4. The beam has one plane of symmetry and is simulated by restraining the beam with rollers along the axis of symmetry (i.e. displacement was restrained in the plane perpendicular to the plane of symmetry). The beam is loaded with a cyclic loading history shown in Fig. 5 at its free end as shown in Fig. 4. This is simulated in the FEM by applying cycles of loading and unloading (tension and compression) in several load steps and sub-steps.

The applied cyclic loads are divided into a series load steps and sub-steps. At the end of each incremental solution, the stiffness matrix of the model is adjusted before proceeding to the next incremental load step in order to reflect the nonlinear changes in the model's structural stiffness. The automatic time stepping option is turned on in this model to predict and control load step size increments. The model stiffness is updated by Newton-Raphson equilibrium iterations which default to 30 equilibrium equations.

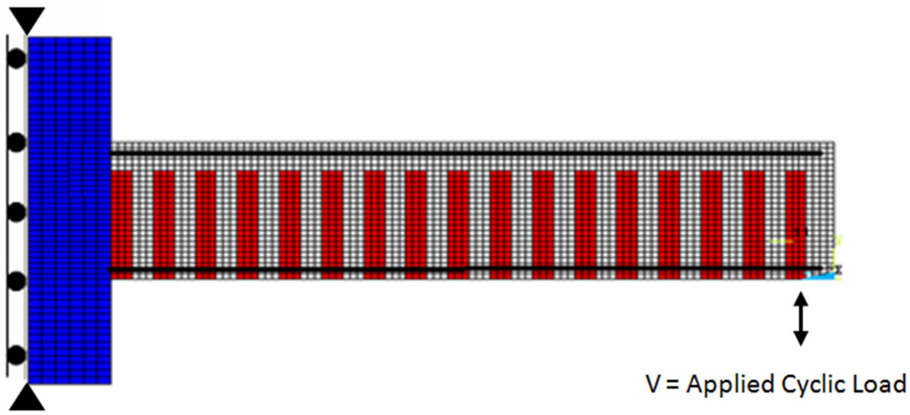


Fig. 4 Meshed FEM with applied loading and boundary conditions

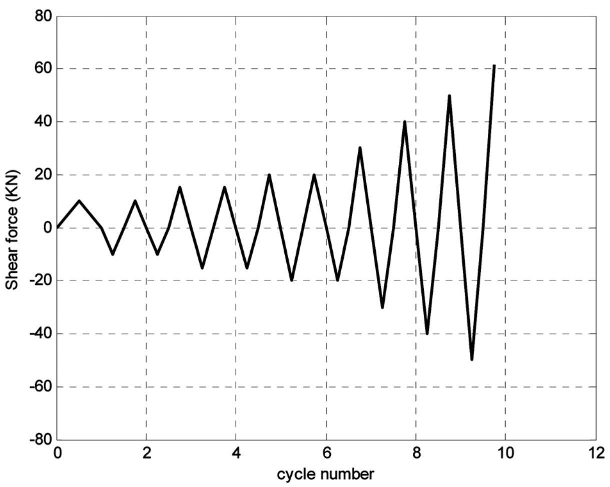


Fig. 5 Cyclic load history

Convergence of the developed finite element model depends on the highly nonlinear response of the reinforced concrete elements due to crack generation and propagation, and the associated loss of stiffness in such elements after reaching their tensile rupture strength. This might lead to divergence and failure and thus termination of the analysis. In this study, failure of the FE model is defined when the solution for a 0.005 N load increment does not converge.

The convergence criteria are based on force, displacement, and the tolerance limits of the convergence criteria (ANSYS 2007). In this study, the force convergence criterion controls the convergence and results of the developed model. A default force convergence tolerance value of 0.5% is initially selected. It has been found that convergence was difficult to achieve using such tight default value due to the nonlinear response and brittleness of the concrete elements with the associated large deflections. Thus, in order to obtain convergence of the equilibrium iterations, the force convergence tolerance limit value is increased by one order of magnitude. Thus a convergence tolerance value of 0.1 is used in this study for force checking.

5. Results and discussions

5.1 Model validation

The tested beam failed due to debonding of the CFRP strips at a load of 57.17 kN upon the completion of the previous 50 kN load cycle. The predicted FE results were compared with the experimental ones by using response envelopes of applied cyclic load versus measured deflection at the free end of the cantilever beam. Response envelopes were developed by connecting peak deflection points of loading cycles. Comparison between the test and predicted finite element results is presented in Fig. 6. A comparison between the predicted load and deflection at failure and the associated results obtained from experimental test are shown in Table 1. In addition, Fig. 7 shows the load-displacement response history (hysteresis loops) predicted by the FE model and that of the experimental test.

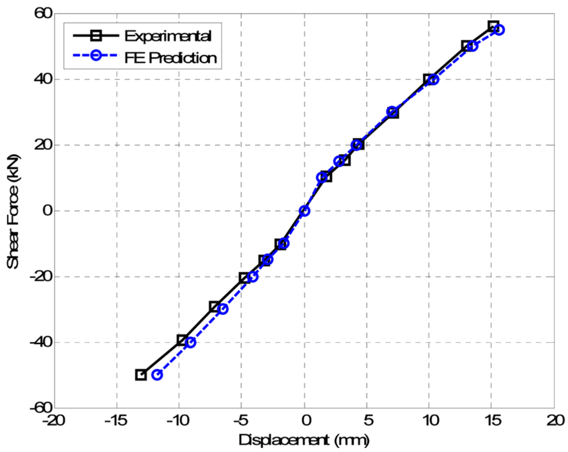


Fig. 6 Cyclic response load-deflection envelope

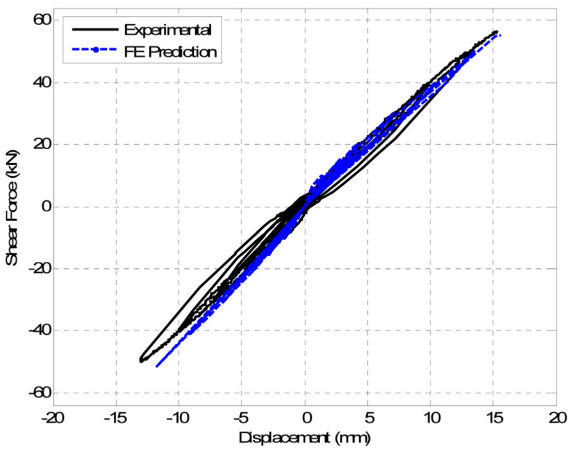


Fig. 7 Response hysteresis envelopes of the T-beam

Table 1 Comparison between the predicted FE and experimental results

	Experimental	FE prediction	% Difference (Pred. – Exp) / Exp.
Ultimate load (kN)	57.17	55.07	-3.8
Maximum deflection (mm)	15.60	15.62	0.14

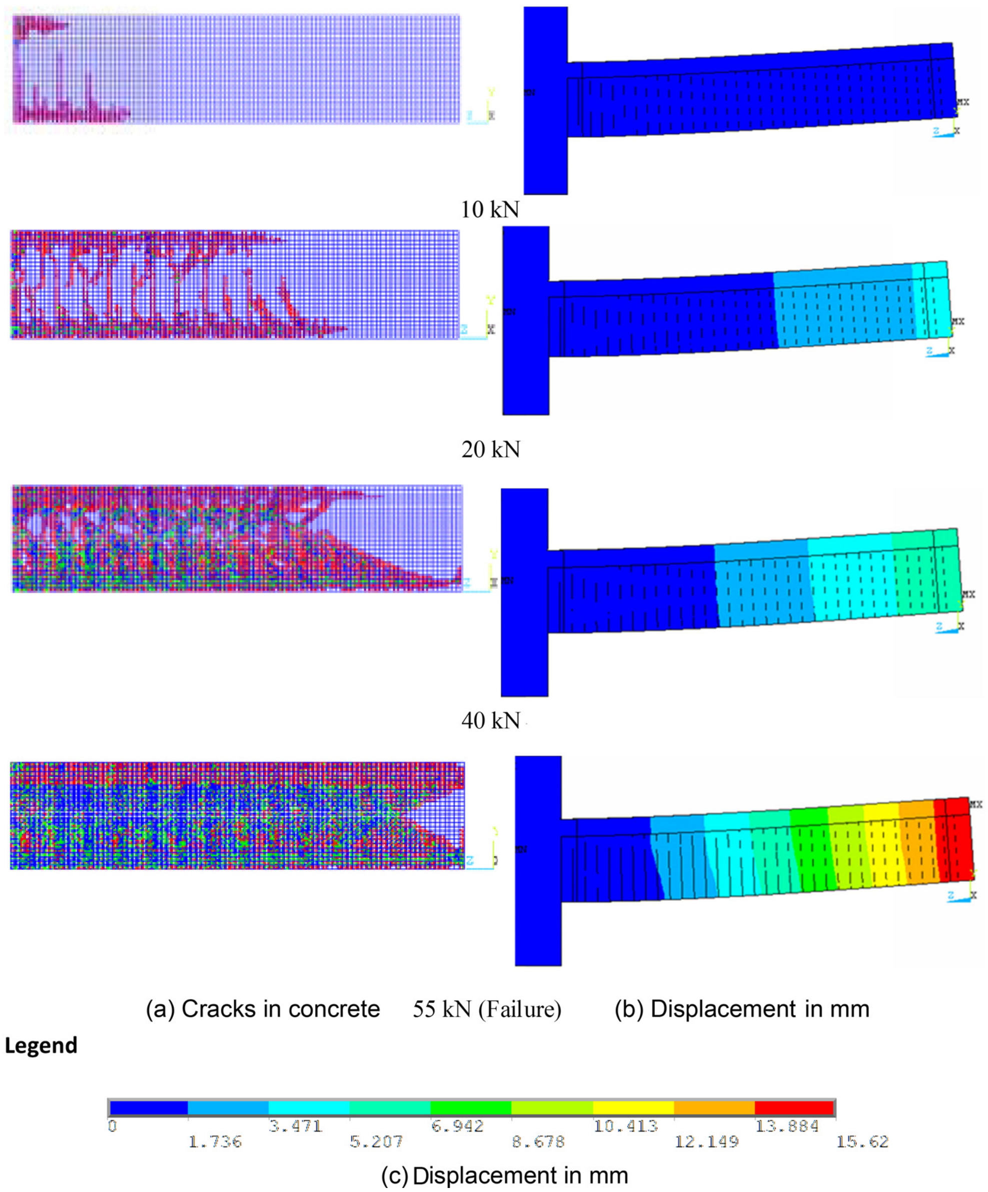


Fig. 8 Sample evolution of crack patterns and vertical deflection of the T-beam

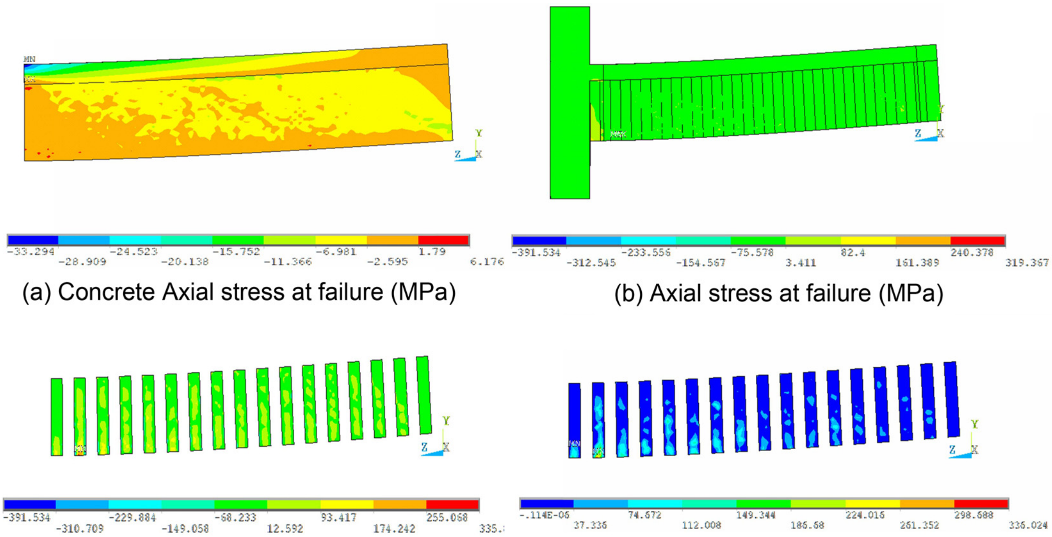
It is clear from Table 1 that the difference between the predicted and experimentally measured ultimate load and ultimate deflection of the T-beam is less than 5%. In addition, the FE response envelope agreed reasonably well with the experimental results as indicated in Fig. 6. The response of the developed FE model is slightly stiffer than that of the tested beam at large negative amplitude (downward deflection) of the applied cyclic load. This deviation is reasonable and could be related to the complex nature of the problem with uncertainties in the concrete mix which might be a little bit different than the developed FE model with the assigned isotropic material properties. The complete hysteresis responses presented in Fig. 7 show reasonable agreement between the predicted and the experimentally measured ones, however, the FE predictions show slightly stiffer behavior for the negative load at ultimate values.

5.2 Model behavior

Having such reliable and validated finite element model is advantageous over the experimentally measured data in many aspects. The output of the experimental test are usually limited to that recorded by discrete number of strain gauges and LVDT at few points within the beam at specified time or load while the finite element model provides full deformation and stress results throughout the beam for the entire cyclic loading history. Fig. 8 shows samples of evolutions of crack patterns that developed in the FE model of the T-beam during cyclic loading and the corresponding displacement of the T-beam at selected load steps (end of positive displacement cycles).

It was observed that the first crack was appeared as minor flexural cracks close to the column support face (region of high bending moments) and then propagated to major shear cracks in the entire shear span. As expected, as the amplitude of the cyclic load increases the deflection in the cantilever increases and more cracks developed progressively as shown in Fig. 8.

Fig. 9(a) shows the axial stress distribution in concrete at the failure stage and Fig. 9(b) shows the axial stress of the entire model. In addition, Fig. 9(c) and (d) show the predicted axial and principle stress in the CFRP fabric strips at failure, respectively. It is observed that the strips in middle span of the cantilever experienced the most axial stress at failure.



(a) Concrete Axial stress at failure (MPa)

(b) Axial stress at failure (MPa)

(c) Axial Stress on CFRP, MPa

(d) Principle stress on CFRP (σ1), MPa

Fig. 9 Stress distribution of concrete and CFRP

6. Parametric study and sensitivity analysis

The contribution of the FRP external reinforcement to the shear capacity of the beam is based on the fiber orientation and an assumed crack pattern, and could be determined according to the ACI440.2R-08 (2008) shear provisions as follows

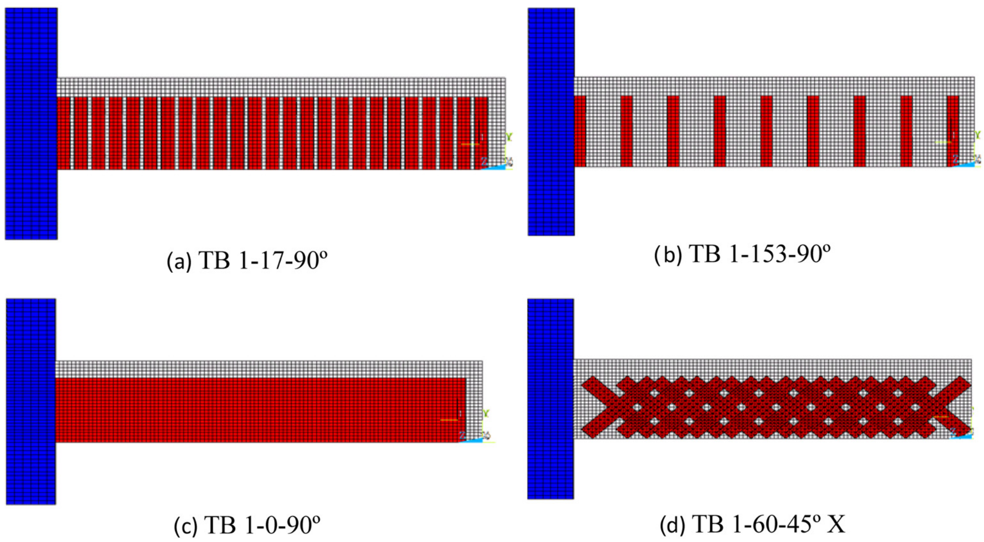
$$V_f = \frac{A_{fv} f_{fe}(\sin\alpha + \cos\alpha) d_{fv}}{s_f} \tag{1}$$

where,

$$A_{fv} = 2nt_f w_f \tag{2}$$

A_{fv} = area of shear reinforcement, mm^2
n = number of plies of FRP reinforcement
t_f = nominal thickness of one ply of FRP reinforcement, mm
w_f = width of FRP reinforcing plies, mm
s_f = center-to-center spacing between FRP strips, mm
d_{fv} = effective depth of FRP shear reinforcement, mm
f_{fe} = effective stress in the FRP at section failure, MPa
a = fiber orientation angle from the horizontal axis, in degrees

It is clear from Eq. (1) and Eq. (2) that the number of layers, areas of the FRP shear reinforcement, center-to-center spacing between FRP strips, and FRP fiber orientation are the major parameters that contributes to the member's shear capacity. In order to investigate the effect of these parameters on the shear resistance of the T-beam, a parametric study was carried out. The parameters varied were the spacing between FRP strips, the number of FRP strip layers, and their corresponding fiber orientation.



(a) TB 1-17-90º (b) TB 1-153-90º (c) TB 1-0-90º (d) TB 1-60-45º X

Fig. 10 Sample of the FE models used in the parametric study

A total of seven models were built and analyzed to investigate the different parameters associated with the external strengthening via side bonded CFRP fabric strips. Samples of these models are shown in Fig. 10. The models were designated by the beam title (TB) followed by the number of layers (n), clear spacing, and orientation of the fibers (α).The results of the parametric study will be discussed in the subsequent sections. For instance, "TB 1-60-90º" refers to the FE model that has one layers of CFRP fabric strips, clear spacing between CFRP fabric strips of 60 mm (110 mm center to center spacing), and fiber orientation of 90º. Similarly, "TB 1-0-90º" refers to the model that has one layer of CFRP fabric strips, clear spacing between CFRP fabric strips of zero mm (50 mm center to center spacing), i.e. with full side bonded continuous ply sheet of CFRP fabric, and fiber orientation of 90º. In addition, "TB 1-60-45º X" refers to the model that has a one layer of inclined attached CFRP fabric strips, clear spacing between CFRP strips of 60 mm (110 mm center to center spacing), and fiber orientation of +45º and -45º, respectively. The width of the CFRP fabric strips in all the developed models is 50 mm. The results of the parametric study will be discussed in the subsequent sections.

6.1 Effect of spacing between CFRP fabric strips

The first parameter studied was the spacing between the FRP strips. Four different clear spacing were studied, 0, 17, 60, and 153 mm; the 60 mm case was the validated model. As expected the more spacing between the CFRP strips, the more shear cracks and deflection observed in the FE model and leads to inefficient CFRP external strengthening. The zero mm clear spacing case, which corresponds to the use of continuous CFRP fabric sheet side-bonded all over the T beam, achieved 18.25% increase on the ultimate failure load and 312.7% increase on the deflection at failure while the 17 mm spacing achieved a 8.94% increase in the ultimate failure load and 85.1% increase in the deflection at failure over the validated model with 60 mm spacing. The 153 mm showed 14.67% decrease in the ultimate failure load and 12.8% decrease in the deflection at failure over the validated model with 60 mm spacing.

6.2 Effect of the number of layers of CFRP fabric strips

Two cases were studied in this section, the effect of using two layer and three layers of CFRP fabric strips as opposed to the experimental work that used one layer of CFRP fabric strips only. The two cases had the same CFRP fabric strip width and clear spacing and were taken as 50 mm and 60 mm, respectively. The use of three layers of CFRP fabric strips reinforcement increased the load at failure by 14.38% as well as the deflection at failure by 277.6% whereas the use of two layers of CFRP fabric strips reinforcement increased the load at failure by 5.31% and the deflection at failure by 52% – all with respect to the validated model with one layer of fabric strips that have 50 mm width and 60 mm clear spacing.

6.3 Effect of orientation of tension fibers of CFRP fabric strips

This case implemented the use of inclined CFRP fabric strips attached with X-cross shapes along with fibers orientation of +45º and -45º. This case achieved 18.02% increase in the ultimate failure load and 518.4% increase in the deflection at failure over the validated model with 50 mm fabric strips width and 60 mm clear spacing and vertical orientation angle of 90 degrees.

Table 2 Parametric study

	Load at failure (kN)	Increase in load at failure (%)	Deflection at failure (mm)	Increase in deflection at failure (%)
Validation model (TB 1-60-90°)	55.07	-	15.6	-
TB 1-0-90°	65.13	18.25	62.2	312.7
TB 1-17-90°	60	8.94	27.9	85.1
TB 1-153-90°	47	-14.67	13.14	-12.8
TB 2-60-90°	58	5.31	22.9	52.0
TB 3-60-90°	63	14.38	56.9	277.6
TB 1-60-45° X	65	18.02	93.2	518.4

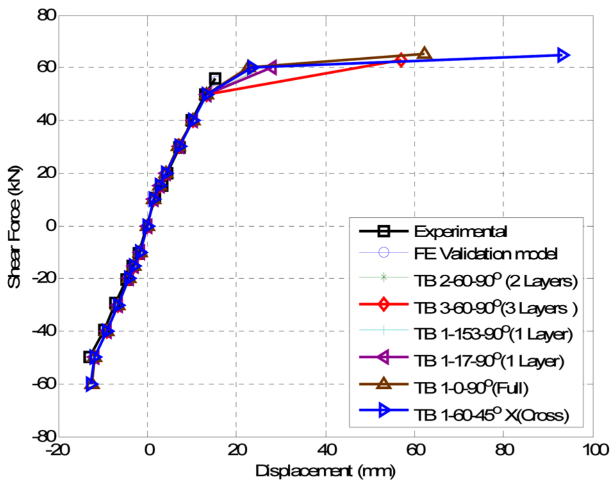


Fig. 11 Failure load-displacement relations of experimental and all FE cases

Table 2 presents a summary of the load at failure and deflection at failure for all the considered cases while Fig. 11 shows a summary of the response load-deflection relationships of all cases considered in the parametric studies.

It is observed from Fig. 11 that all cases have similar behavior at the elastic region, however, at the inelastic region the load and the corresponding displacement are different from one case to another.

7. Conclusions

A nonlinear finite element model is developed in this study to simulate the response and predict the behavior of a shear deficient RC cantilever T-beam strengthened with sided CFRP fabric strips. The model was validated by comparing the FE results with an experimental test. In addition, a parametric study was carried out to investigate the effect of different spacing between CFRP fabric strips, number of layers of CFRP fabric strips and fibers orientation of CFRP fabric strips. The following conclusions can be drawn from the results of this study:

- The load-deflection response of the finite element model is in a reasonable agreement with the measured experimental load-deflection at almost all stages of the cyclic loading.
- The crack patterns of the finite element model resemble that of the experimental crack patterns and they correlate well with the vertical deflection and the amplitude of the cyclic load.
- The developed model is effective in producing good prediction in the elastic and plastic ranges of the applied load history.
- A parametric study was conducted that simulates the effects of using multiple layers, different center to center spacing and fiber orientation of the CFRP fabric strips.
- The larger the spacing between the CFRP fabric strips, the larger the crack propagation and deflection.
- The use of two and three layers of the CFRP fabric strips yielded an increase in the failure load by 5.31% and 14.38%, respectively.
- The use of continuous CFRP fabric sheet as external reinforcement appears to achieve the highest load at failure while the use of inclined orientation (at angles of +45° and −45°) of CFRP fabric strips seems to achieve the highest deflection at failure.
- At failure the number of layers, center to center spacing and fiber orientation of the CFRP fabric strips considerably influence the magnitude of load at failure and the deflection at failure.
- The developed and verified finite element model in this study could be used as a tool for further investigation of CFRP strengthened reinforced concrete beams under cyclic loading and as a supplement or alternative to experimental testing, which is usually more expensive.

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