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Predicting shear strength of RC exterior beam-column joints by modified rotating-angle softened-truss model

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Abstract. A theoretical model known as the modified rotating-angle softened-truss model (MRA-STM), which is a modification of Rotating-Angle Softened-Truss Model and Modified Compression Field Theory, is presented for the analysis of reinforced concrete membranes in shear. As an application, shear strength and behaviour of reinforced concrete exterior beam-column joints are analysed using the MRA-STM combining with the deep beam analogy. The joints are considered as RC panels and subjected to vertical and horizontal shear stresses from adjacent columns and beams. The strut and truss actions in a beam-column joint are represented by the effective transverse compression stresses and a softened concrete truss in the proposed model. The theoretical predictions of shear strength of reinforced concrete exterior beam-column joints from the proposed model show good agreement with the experimental results.

Keywords: beam-column joints; shear strength; softened-truss model; reinforced concrete.

1. Introduction

When a reinforced concrete rigid frame is subjected to seismic loading, the beam-column joints play a very important role in transferring bending moments and shear forces among adjacent beams and columns. Both beams and columns will then deform in a double curvature. Inflection points generally occur near the mid-height of columns and mid-span of beams. The change of moments in the beams and columns across a joint will induce both vertical and horizontal shear forces, whose magnitude are typically many times higher that those of the adjacent beams or the column. It has been widely recognised that properly detailed beam-column joints are essential in maintaining the integrity and stability of ductile frame structures under earthquake attacks.

Currently, there is still little consensus about the design of beam-column joints. To have a common conclusion, the trilateral cooperative research by the US, New Zealand and Japan was conducted. The resisting mechanisms of joint shear are still in dispute. The uncertainties regarding to the behaviour of RC beam-column joints may be attributed to the numerous parameters involved and the interaction among different internal forces. Different approaches are, thus, proposed in different seismic codes of practices.

In the recent years, great progress has been achieved in the theoretical development of predicting the ultimate shear strength as well as the shear deformation of reinforced concrete membrane

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elements throughout their loading histories. The theories are mainly based on the truss model concept and cracked concrete is treated as a continuous material, so that Navier's principles of the mechanics of materials, namely the stress equilibrium, Mohr compatibility conditions and constitutive laws of materials of the cracked concrete and reinforcement, can be satisfied. The typical and rational models, including the Modified Compression Field Theory (MCFT) (Vecchio and Collins 1986), the Rotating Angle Softened Truss Model (RA-STM) (Hsu 1993) and the Fixed Angle Softened Truss Model (FA-STM) (Pang and Hsu 1995), have been developed for predicting the nonlinear shear behaviour of cracked reinforced concrete membrane elements. Some of these theories have shown to be applicable to analyse low-rise shear walls, framed shear walls and deep beams (Hsu 1998).

In this paper, a modified rotating-angle softened-truss model (MRA-STM) is presented for the analysis of reinforced concrete membranes in shear. The proposed model is then applied combining with the deep beam analogy for analysing and predicting the shear behaviour of reinforced concrete exterior beam-column joints. With the deep beam analogy, where the effective transverse compression stresses of a softened concrete truss in the shear element of a joint are introduced, the MRA-STM is shown to provide an effective and accurate means of predicting the shear strength of RC beam-column joints.

2. Modified rotating-angle softened-truss model (MRA-STM)

A modified rotating-angle softened-truss model (MRA-STM) for predicting the shear behaviour and strength of reinforced concrete membrane elements is derived based on the concepts of the RA-STM and MCFT, where "the concrete contribution" in cracked concrete membrane elements is adequately considered in the proposed model. The detailed development of the model is described elsewhere (Wong and Kuang 2009) and is summarised as follows.

2.1 Formulation of stresses and strains

Fig. 1 shows the stresses of reinforced concrete membrane elements subjected to shear. It is assumed that the angle of cracks in the concrete, α, is non-coincident with the angle of the concrete principal compressive stress, θ, and is kept rotating in correspondence with the level of stresses.

The relationship of the crack and principal angles is shown in Fig. 2. From the Mohr's circle of average stresses, the corresponding average stresses of cracked concrete in the proposed model are expressed by

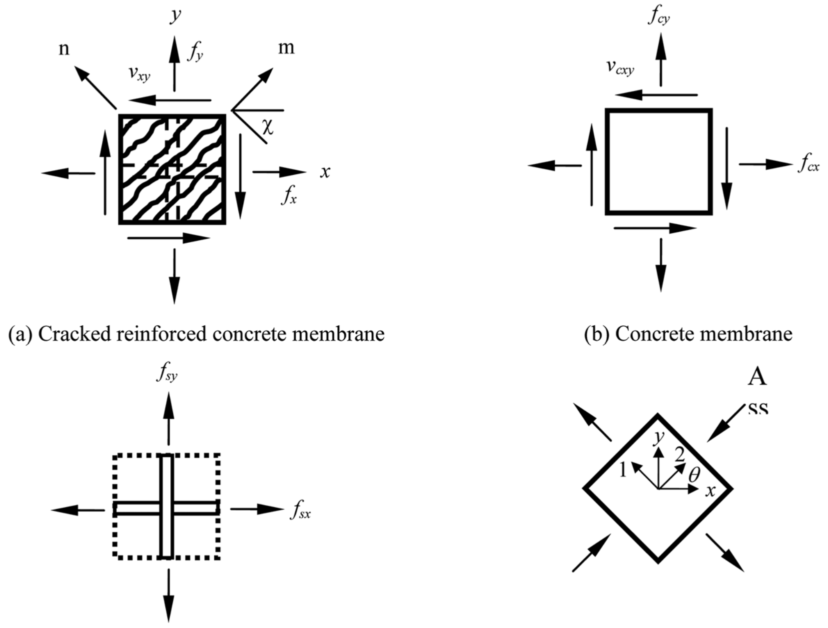
$$f_{cx} = f_{c1} - v_{cxy} \cot\theta \tag{1}$$

$$f_{cy} = f_{c1} - v_{cxy} \tan\theta \tag{2}$$

$$v_{cxy} = \frac{f_{c1} - f_{c2}}{2} \sin 2\theta \tag{3}$$

where

f_{cx}, f_{cy} : average concrete stresses in x and y directions, respectively,
f_{c1}, f_{c2} : average principal stresses of concrete in 1 and 2 directions, respectively,
v_{cxy} : average shear stress of concrete in the x-y coordinate.



(a) Cracked reinforced concrete membrane (b) Concrete membrane

(c) Steel reinforcement (d) Principal angle and stresses

Fig. 1 Stresses of cracked reinforced concrete membrane elements subjected to shear

Similarly, from the Mohr's circle of average strains, the corresponding average strains of cracked concrete in the proposed modified rotating-angle softened-truss model are expressed by

$$\varepsilon_{cx} = \frac{\varepsilon_{c1} - \varepsilon_{c2}}{2}(1 - \cos 2\theta) + \varepsilon_{c2} \tag{4}$$

$$\varepsilon_{cy} = \varepsilon_{cx} + (\varepsilon_{c1} - \varepsilon_{c2})\cos 2\theta \tag{5}$$

$$\gamma_{cxy} = 2(\varepsilon_{cy} - \varepsilon_{c2})\tan\theta \tag{6}$$

where

ε_{cx}, ε_{cy} : average concrete strains in *x*- and *y*-directions, respectively,
ε_{c1}, ε_{c2} : average principal strains of concrete in 1 and 2 directions, respectively,
γ_{cxy} : average shear strain of concrete in the *x*-y coordinate.

2.2 Constitutive laws for materials

In deriving the MRA-STM, the average compressive stress-strain relationships of concrete in compression proposed by Belarbi and Hsu (1995) are used

$$f_{c2} = \zeta f_c'\left[2\left(\frac{\varepsilon_{c2}}{\zeta\varepsilon_o}\right) - \left(\frac{\varepsilon_{c2}}{\zeta\varepsilon_o}\right)^2\right] \quad (\varepsilon_{c2}/(\zeta\varepsilon_o) \le 1) \tag{7}$$

$$f_{c2} = \zeta f_c'\left\{1 - \left[\left(\frac{\varepsilon_{c2}}{\zeta\varepsilon_o} - 1\right)\Big/\left(\frac{4}{\zeta} - 1\right)\right]^2\right\} \quad (\varepsilon_{c2}/\zeta\varepsilon_o > 1) \tag{8}$$

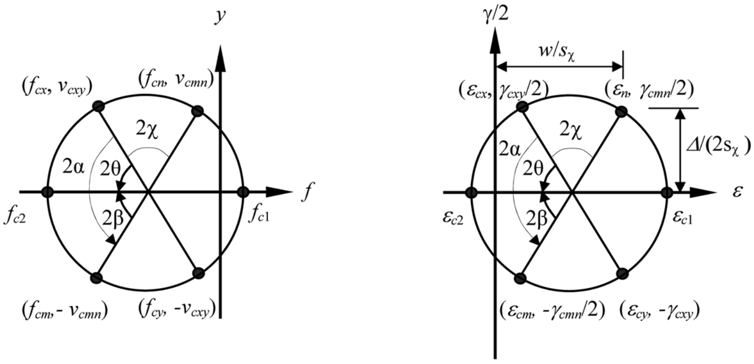


Fig. 2 Mohr circles of average stress and average strain for cracked concrete

where
f_c' : cylinder strength of concrete,
ε_o : concrete strain at peak stress,
ε_{c2} : average principal compressive strain of concrete,
ζ : softening coefficient, given by

$$\zeta = \frac{5.8}{\sqrt{f_c'}}\frac{1}{\sqrt{1+400\varepsilon_{c1}}} \le \frac{0.9}{\sqrt{1+400\varepsilon_{c1}}} \tag{9}$$

where
ε_{c2} : average principal compressive strain of concrete.
The average tensile stress-strain relationship of concrete is given as follows

$$f_{c1} = E_c\varepsilon_{c1} \quad (\varepsilon_{c1} \le \varepsilon_{cr}) \tag{10}$$

$$f_{c1} = f_{cr}\left(\frac{0.00008}{\varepsilon_{c1}}\right)^{0.4} \quad (\varepsilon_{c1} > \varepsilon_{cr}) \tag{11}$$

where
$E_c = 3875\sqrt{f_c'}$ (MPa): Young's modulus of concrete,
$\varepsilon_{cr} = 0.00008$: cracking strain of concrete,
$f_{cr} = 0.31\sqrt{f_c'}$ (MPa) : cracking strength of concrete.
The bilinear stress-strain relationship is used for steel reinforcement

$$f_c = E_s\varepsilon_s \quad (\varepsilon_s \le \varepsilon_y) \tag{12}$$

$$f_s = f_y \quad (\varepsilon_s > \varepsilon_y) \tag{13}$$

where
E_s : Young's Modulus of steel reinforcement (N/mm^2),
f_s : steel stress along x or y axis, expressed in f_{sx} and f_{sy} (N/mm^2),
f_y : steel strength along x or y axis (N/mm^2),
ε_s : strain of reinforcement along x or y axis.

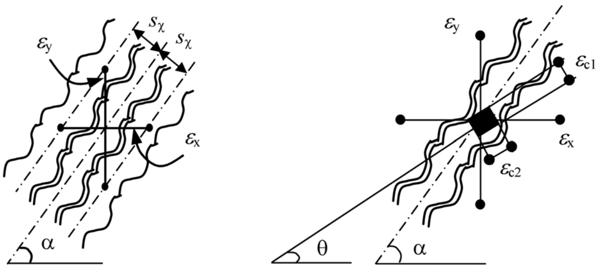


Fig. 3 Strain state of concrete strut

2.3 Solution scheme

To determine the steel stresses and check the stress equilibrium of reinforced concrete membrane elements, the total strains are needed to be determined. Fig. 3 shows the strain state of a concrete strut, where ε_x and ε_y are the total average smeared-strains of cracked concrete in x and y directions, respectively.

Before obtaining the total average strains of the reinforced concrete membrane elements, local strains at cracks are calculated, where kinematic conditions at the crack interface should be considered owing to crack opening and slip deformation. The crack width w and slip displacement Δ are directly proportional to the crack spacing s_χ, the normal strain ε_{cn} and shear strain γ_{cmn} across cracks, given by

$$w = s_\chi \varepsilon_{cn} \tag{14}$$

$$\Delta = s_\chi \gamma_{cmn} \tag{15}$$

$$\varepsilon_{cn} = \varepsilon_{c2} \sin^2\beta + \varepsilon_{c1}\cos^2\beta \tag{16}$$

$$\gamma_{cmn} = (\varepsilon_{c1} - \varepsilon_{c2})\sin 2\beta \tag{17}$$

$$s_\chi = 1/(\sin\alpha/s_{mx} + \cos\alpha/s_{my}) \tag{18}$$

where

α : crack angle,
β : angle between α and θ, equal to $(\alpha - \theta)$,
s_{mx}, s_{my} : mean values of crack spacing in x- and y-directions, respectively.

The longitudinal, transverse and shear strains caused by crack opening, ε_{wx}, ε_{wy} and γ_{wxy} and the longitudinal, transverse and shear strains caused by crack slipping, $\varepsilon_{\Delta x}$, $\varepsilon_{\Delta y}$ and $\gamma_{\Delta xy}$ are calculated by

$$\varepsilon_{wx} = \frac{w}{s_\chi}\cos^2\chi,\ \varepsilon_{wy} = \frac{w}{s_\chi}\sin^2\chi,\ \gamma_{wxy} = \frac{w}{s_\chi}\sin 2\chi \tag{19}$$

$$\varepsilon_{\Delta x} = -\frac{\Delta}{2s_\chi}\sin 2\chi,\ \varepsilon_{\Delta y} = \frac{\Delta}{2s_\chi}\sin 2\chi,\ \gamma_{\Delta xy} = \frac{\Delta}{s_\chi}\cos 2\chi \tag{20}$$

where

χ : crack orientation, equal to (90°-α).

The crack spacing s_χ may be calculated following the procedure recommended by Vecchio and

Collins (1986). Combining the average strains given by Eq. (14) to Eq. (16) and local strains caused by crack opening and slipping calculated by Eqs. (19) and (20), respectively, gives the total strains of the concrete element membrane

$$\varepsilon_x = \varepsilon_{cx} + \varepsilon_{wx} + \varepsilon_{\Delta x} \tag{21}$$

$$\varepsilon_y = \varepsilon_{cy} + \varepsilon_{wy} + \varepsilon_{\Delta y} \tag{22}$$

$$\gamma_{xy} = \gamma_{cxy} + \gamma_{wxy} + \gamma_{\Delta xy} \tag{23}$$

The total applied stress and shear capacity can be determined by

$$f_x = f_{cx} + \rho_{sx} f_{sx} = -p_1 \tag{24}$$

$$f_y = f_{cy} + \rho_{sy} f_{sy} = -p_2 \tag{25}$$

$$v_{xy} = v_{cxy} \tag{26}$$

where

ρ_{sx}, ρ_{sy} : reinforcement ratios in x and y directions respectively,

f_{sx}, f_{sy} : reinforcement stresses in x and y directions respectively.

Note that when a reinforced concrete membrane element is subjected to shear as well as transverse compressive stresses in x and y directions, $f_x = -p_1$ and $f_y = -p_2$.

When a reinforced concrete membrane element is subjected to increasing shear, the first set of cracks will form in the major principal concrete stress direction when the principal concrete stress reaches tensile strength of the concrete. The orientation of the crack angle remains constant until the principal concrete stress exceeds tensile strength of the concrete. New cracks will then form in the major principal stress direction. Hence it is assumed that the set of currently open cracks will close and disappear. The crack angle of the concrete changes from the initial crack to the point of failure.

3. Analysis of concrete beam-column joints by MRA-STM

3.1 Deep beam analogy

Consider a simply supported deep beam subjected to concentrated loading at the top shown in Fig. 4. The arch action between the applied load and the support reaction can be modelled by introducing the effective transverse compressive stress p (Mau and Hsu 1987). The transverse stress p is directly proportional to the shear stress v and the shear span-to-depth ratio and given by

$$p = K v \tag{27}$$

where

$$K = \frac{2d_v}{h} \quad (0 < a_v/h \le 0.5) \tag{28a}$$

$$K = \frac{4d_v}{3a_v}\left(1 - \frac{a_v}{2h}\right) \quad (0.5 < a_v/h \le 2) \tag{28b}$$

$$K = 0 \quad (a_v/h > 2) \tag{28c}$$

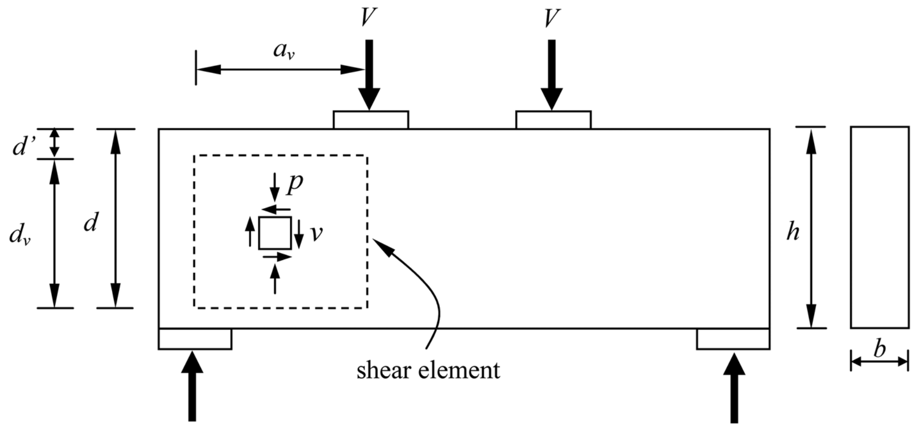


Fig. 4 Concrete stresses within shear span of a deep beam

Similar to the shear element of a deep beam, the shear element in the joint core of an exterior beam-column joint, as shown in Fig. 5, is subjected to horizontal and vertical shears induced by the adjacent beam and columns. In addition, two effective transverse compressive stresses in both horizontal and vertical directions, p_1 and p_2, are introduced. In general, the shear strength of a RC beam-column joint may involve two sources: truss mechanism and strut mechanism (Paulay and Priestley 1992). The truss mechanism is a function of the shear strength of the cracked concrete, while the strut mechanism is a function of the span-to-depth ratio of the joint, which is similar to the arch action of a deep beam shown in Fig. 4. The effective transverse stresses p_1 and p_2 and shear stress v can be directly related by a factor that is dependent upon the shear span-to-depth ratio of the beam and column.

The shear element in a joint core can be considered to be bounded by the main reinforcing bars in the beam and column, as shown in Fig. 6; where d_{sh} and d_{sv} are the horizontal and vertical dimensions of the shear element, l_h and l_v are the horizontal and vertical spans of the joint, and h_b

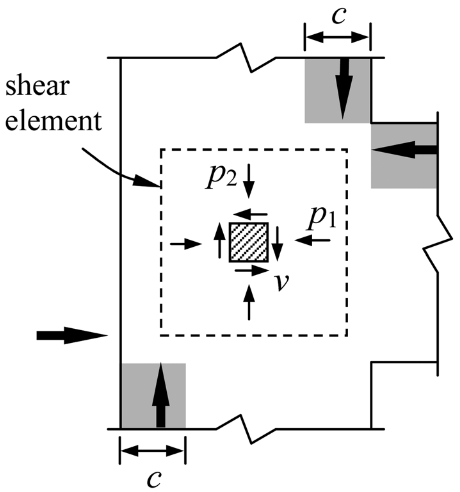


Fig. 5 Stresses in an exterior beam-column joint

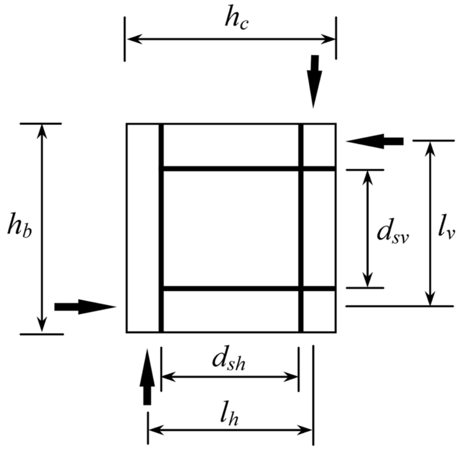


Fig. 6 Dimensions of a beam-column joint

and h_c are the beam and column depths, respectively. Analogous to the effective transverse stress, p, of the deep beam shown in Fig. 4, which is expressed by Eq. (27), the effective transverse stress, p_1, of the joint shown in Fig. 5 is given by

$$p_1 = K_1 v \tag{29}$$

where

$$K_1 = \frac{2d_{sh}}{h_c} \quad (l_v/h_c \leq 0.5) \tag{30a}$$

$$K_1 = \frac{4}{3}\frac{d_{sh}}{l_v}\left(1 - \frac{l_v}{2h_c}\right) \quad (0.5 < l_v/h_c \leq 2.0) \tag{30b}$$

Similarly,

$$p_2 = K_2 v \tag{31}$$

where

$$K_2 = \frac{2d_{sv}}{h_b} \quad (l_h/h_b \leq 0.5) \tag{32a}$$

$$K_2 = \frac{4}{3}\frac{d_{sv}}{l_h}\left(1 - \frac{l_h}{2h_b}\right) \quad (0.5 < l_h/h_b \leq 2.0) \tag{32b}$$

In Fig. 5, the depth of the flexural compression zone, c, of columns connected to the joint may be estimated by (Paulay *et al.* 1992)

$$c = \left(0.25 + 0.85\frac{N}{f_c' A_g}\right) h_c \tag{33}$$

where
N : axial compressive load on column (N),
f'_c : compressive strength of standard concrete cylinder (N/mm^2),
A_g : gross area of column section (mm^2),
h_c : column depth (mm).

Thus the vertical span of the joint l_h shown in Fig. 6 is given by $(h_c - c)$. Whereas for beam sections, the moment arm l_v of the horizontal forces, as shown in Fig. 6, may be taken as $0.9d$ (Parker and Bullman 1997), where d is the effective depth of the beam. The effective transverse stresses in the joint, p_1 and p_2, can then be determined using Eqs. (29) and (31).

3.2 Solution procedure

A flow chart of solution procedure for predicting the shear strength of exterior RC beam-column joints by MRA-STM is shown in Fig. 7. The proposed model can capture not only the influence of vertical and transverse reinforcement, but also the effect of the span-to-depth ratio of a joint, which is reflected by the values of p_1 and p_2, on the shear behaviour and shear strength of the joints.

It has been shown that the smaller the span-to-depth ratio of a joint, the larger the values of effective transverse compressive stress which results in the higher shear strength. On the other hand, axial load on columns is also considered as one of the main factors affecting the shear strength of joints. It has no further effect on the joint shear strength when the axial load reaches to an extent

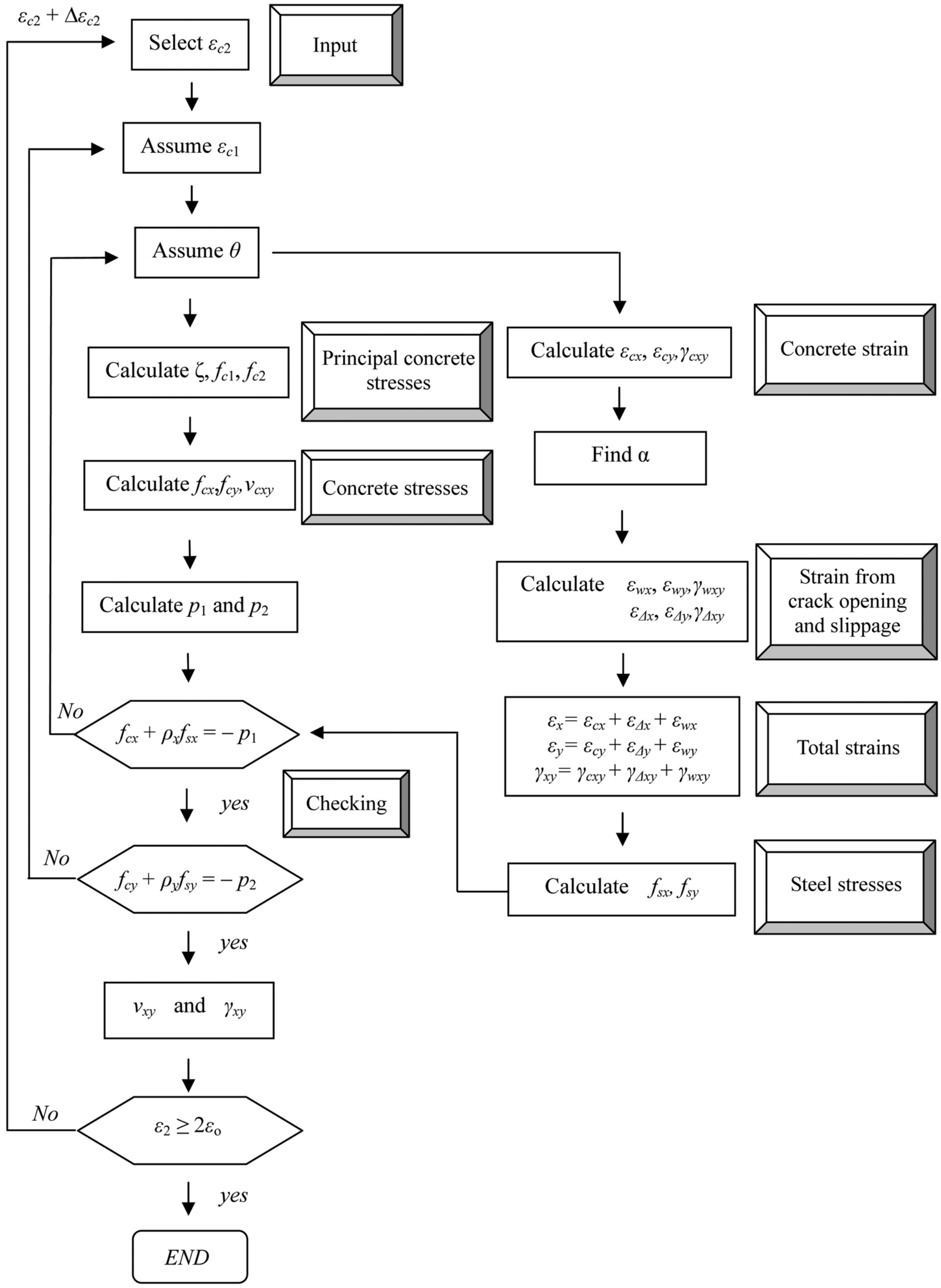


Fig. 7 Flow chart of solution procedure for determining shear stress-strain relationship of beam-column joints by MRA-STM

that the ratio l_h / h_b is smaller than 0.5, which may explain why high axial load does not exhibit beneficial effect on the RC beam-column joint shear strength (Kitayama *et al*. 1991).

Table 1 Experimental and predicted shear strengths of exterior beam-column joints

Tests	Specimen	f_c' (MPa)	V_{exp} (kN)	V_{pred} (kN)	V_{exp}/V_{pred}
Ortiz	BCJ 1	34.0	304.3	303.3	1.00
	BCJ 2	38.0	338.2	410.9	0.82
	BCJ 3	33.0	319.2	310.3	1.03
	BCJ 4	34.0	351.7	517.2	0.68
	BCJ 5	38.0	311.1	342.1	0.91
	BCJ 6	35.0	311.1	329.0	0.95
Parker	6a	44.0	782.0	705.6	1.11
	6b	44.8	839.3	780.6	1.08
	6c	45.6	759.3	841.2	0.90
	6d	40.4	977.1	855.6	1.14
	6e	44.0	963.5	879.6	1.10
	6f	42.4	951.6	834.0	1.14
Sarsam	EX2	52.5	176.3	144.7	1.22
Scott	C1AL	33.4	91.7	104.6	0.88
	C4	41.4	125.4	125.3	1.00
	C4A	44.3	134.4	129.3	1.04
	C4AL	35.8	119.5	117.7	1.02
	C7	35.2	84.3	82.8	1.02
Scott and Hamil	C4ALN0	42.4	110.5	88.2	1.25
	C4ALN1	45.6	141.5	128.5	1.10
	C4ALN3	41.6	148.6	148.8	1.00
	C4ALN5	50.4	166.8	159.7	1.04
Taylor	P1/41/24	33.0	92.9	90.0	1.03
	P2/41/24	29.0	86.2	86.7	0.99
	P2/41/24A	46.5	114.4	104.5	1.09
	A3/41/24	27.0	89.3	83.1	1.07
	D3/41/24	53.0	125.7	109.5	1.15
	B3/41/24	22.0	76.2	93.2	0.82
Kuang and Wong, Wong and Kuang	BS-L	30.9	315.5	330.8	0.95
	BS-OL	30.9	219.1	220.0	1.00
	BS-LL	42.1	398.8	391.5	1.02
	BS-U	31.0	341.2	330.8	1.03
	BS-L-LS	31.6	344.9	336.8	1.02
	BS-L-V2T10	32.6	398.8	353.0	1.13
	BS-L-V4T10	28.3	402.9	334.8	1.20
	BS-L-H1T10	33.3	389.3	477.2	0.82
	BS-L-H2T10	42.1	479.3	567.0	0.85
	BS-L-300	34.1	505.0	623.5	0.81
	BS-L-600	36.4	283.9	283.5	1.00

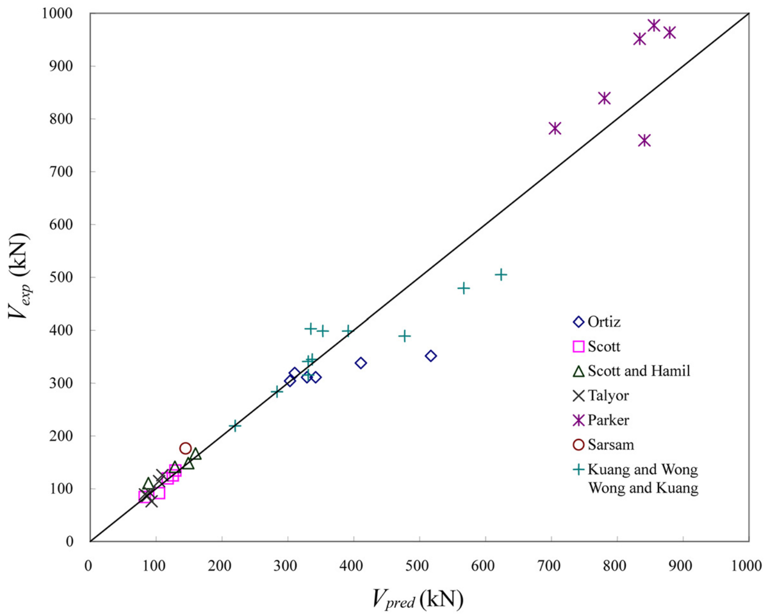


Fig. 8 Correlation of experimental and predicted joint shear strengths

3. Experimental comparison

Thirty-nine concrete, exterior beam-column joints are analysed using the proposed modified rotating-angle softened-truss model. Among these joint specimens, twenty-eight were tested under monotonic loading by Talylor (1974), Sarsam *et al.* (1985), Scott (1992), Ortiz (1993), Parker (1997), Scott and Hamil (1998), and eleven were tested under reversed cyclic loads by Kuang and Wong (2006) and Wong and Kuang (2008). The experimental shear strengths of the beam-column joint specimens, V_{exp}, and comparisons with the theoretical predictions, V_{pred}, are presented in Table 1.

It is shown from Table 1 that the overall mean ratio of the experimental joint shear strengths to the theoretical predictions, V_{exp}/V_{pred}, is 1.01, whereas the corresponding coefficient of variance is 12.2%. The correlation of the experimental and predicted joint shear strengths is shown in Fig. 8. Hence two sets of results show good agreement.

4. Conclusions

In this paper, a theoretical model known as the modified rotating angle softened truss model (MRA-STM), which is modified from Rotating-Angle Softened-Truss Model and Modified Compression Field Theory, is presented for the analysis of reinforced concrete membranes subjected to shear. The proposed theory is applied combining with the deep beam analogy for predicting the shear strength of reinforced concrete exterior beam-column joints. A beam-column joint can be considered as a RC shear panel which is subjected to horizontal and vertical shear stresses transferred from adjacent columns and beams. With the deep beam analogy, the strut and truss actions in a beam-column joint are represented by the effective transverse compression stresses in the corresponding softened concrete truss in the proposed model. Experimental results of shear strength of total 39 reinforced concrete exterior beam-column joints with various joint steel, beam and column depths and the axial load ratios are compared with the theoretical predictions from the proposed model. They show good agreement; hence the MRA-STM combining with the deep beam analogy is shown to provide an effective, yet accurate, theorectical means of predicting the shear strength of RC beam-column joints.

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