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# Integration of the microplane constitutive model into the EPIC code

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**Abstract.** In this work the implementation of a production-level port of the Microplane constitutive model for concrete into the EPIC code is described. The port follows guidelines outlined in the Material Model Module (MMM) standard used in EPIC to insure a seamless interface with the existing code. Certain features of the model were not implemented using the MMM interface due to compatibility reasons; for example, a separate module was developed to initialize, store and update internal state variables. Objective strain and deformation measures for use in the material model were also implemented into the code. Example calculations were performed and illustrate the veracity of this new implementation.

**Keywords:** microplane model; constitutive model framework; EPIC code.

## 1. Introduction

Current world events have generated a renewed interest in modeling the material behavior of concrete. A number of important military applications depend on accurate material models for concrete under large strain, high strain rate conditions, such as the development of breaching criteria for reinforced concrete walls, design and engineering of blast resistant structures, and prediction of damage and residual load-carrying capacity of civilian structures. The importance of this problem was brought to the forefront in a recent workshop entitled "Modeling Concrete under High Impulsive Loadings", sponsored in part by US Army Corps of Engineers Engineer Research and Development Center (ERDC) and held in Austin TX on 20-21 March 2007. One of the conclusions of that workshop was that in spite of commonality and ubiquity of concrete in civilian structures, it is an extremely complex material that is very difficult to model under dynamic conditions.

Over the last three decades, a large number of material models have been proposed for concrete. Most models adhere to the classical approach wherein the material model is formulated directly in terms of stress and strain tensors and their invariants. One model which deviates from this approach is the Microplane model (Bazant *et al.* 2000), developed by Z. P. Bazant and his associates. This particular model is used extensively at ERDC and has demonstrated its veracity in accurate predictions for a wide range of applications in ballistic penetration and blast resistance. Others have effectively employed the microplane approach for different applications (e.g., see (Ožbolt *et al.*

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2008 and Ožbolt *et al.* 2005).

Conceptually, the Microplane model is simple. Instead of formulating constitutive equations in terms of stress and strain tensors and their invariants, material models are developed by relating stress vectors (or tractions) and strain vectors on a plane, called a *microplane*, characterized by the unit normal $n_i$. The strain vector $\varepsilon_{Nj}$, for example, is obtained as $\varepsilon_{Nj} = n_i \varepsilon_{ij}$, where $\varepsilon_{ij}$ is the strain tensor. Static equivalence between the macro and micro levels is enforced in an approximate sense by applying the principle of virtual work. Then, by applying very simple phenomenological constitutive rules, stress-strain boundaries applicable to concrete are developed. Four parameters are required to calibrate the model and represent the effects of friction in compression, tensile cracking, confinement and spreading cracks, pore collapse and isotropic cracking. Since stress states evolve independently on each microplane, sophisticated material behavior such as anisotropic cracking and damage evolution, dilatancy, Bauschinger effect, and hysteresis loops during cyclic loading can be captured. Numerical experiments have demonstrated that an implementation with 21 microplanes yields an acceptable tradeoff between accuracy (meaning that adding additional planes does not significantly alter the macro-stresses) and computational efficiency.

Different versions of the Microplane model have been implemented into the EPIC code previously as a means of demonstrating the utility of the model for predicting blast and ballistic penetration response for concrete. In the work by Bazant *et al.* (2000) the utility of various measures of stress and deformation were contemplated for use in the model. A set of appropriate measures (the same ones adopted in this work) arrived at were the unrotated or co-rotational Cauchy stress and the Green-Lagrange strain tensor. These measures are not conjugate in energy, but for concrete where the elastic and volumetric strains are always small, it was shown that the energy dissipation by large inelastic strains is always nonnegative. Calculations were run for a typical penetration and ground shock application and validated the use of the model for these problems. However, the code development was performed for demonstration purposes only and did not include many of the features required for production-run use, such as support for material libraries, dynamic memory allocation of variables, and restart capability.

In this technical effort a production-level port of the Microplane model has been developed for the EPIC finite element code (Biessel *et al.* 2006). This work was motivated primarily by the desire to develop a general framework within the EPIC code for easy integration of constitutive models, with the Microplane model implemented as a practical illustration of the framework. The port follows the material model programming guidelines in EPIC rigorously to insure a seamless interface with the existing code. EPIC uses a material model interface called Material Model Module (MMM) (Johnson *et al.* 2001) and the development conforms to the implementation guidelines in MMM to the extent that was practical. Additional modules were developed to address the unique requirements of the model not covered in the MMM interface. For example, the Microplane model requires frame indifferent measures of stress and deformation not available in EPIC, so additional routines were developed to store and update these measures. The large number of internal state variables required for the Microplane model also required special routines to be developed to initialize, store and update these variables. The restart package in EPIC was also modified to provide support for the model. Example problems are shown for code validation and to illustrate the utility of this model in the simulation of deformation, damage and failure to concrete structures.

## 2. The microplane model

The microplane model deviates significantly from classical approaches to constitutive modeling, where the material model is formulated directly in terms of stress and strain tensors and their invariants. Instead, the constitutive laws are formulated in terms of vectors rather than tensors, as a relation between the stress and strain components on a plane of arbitrary orientation in the material microstructure, called a *microplane* (Bazant 1986). The idea has its origins in the pioneering work of Taylor (1938) in formulating an approach to model plasticity in polycrystalline metals and became the basis for the so-called *slip line theory of plasticity*. In that theory it is assumed that only inelastic shear strains (the *slips*), with no normal strain, take place on the slip planes – the slip planes are analogous to the microplanes. It is assumed the planes of plastic slip are constrained statically to the stress tensor $\sigma_{ij}$, and that the elastic strain was not included on the slip planes, but instead was added to the inelastic strain tensor on the continuum level.

The microplane model is analogous but distinct to this approach in that stress and strain relationships are cast directly and individually on the microplanes. The basic hypothesis, which insures post-peak strain softening (Bazant 1986) is that the strain vector $\varepsilon_N$ on the microplane is the projection of the strain tensor $\varepsilon_{ij}$, i.e.

$$\varepsilon_{N_i} = \varepsilon_{ij} n_j \tag{1}$$

where **n** is the unit normal to the microplane with components $n_j$. The normal strain on the microplane is $\varepsilon_N = n_i \varepsilon_{N_i}$, that is

$$\varepsilon_n = N_{ij} \varepsilon_{ij} \tag{2}$$

where $N_{ij} = n_i n_j$. Likewise, the shear strains on each microplane are characterized by their components in the directions $M$ and $L$ and are given by the two orthogonal vectors **m** and **l** lying within the microplane, and the shear strain components $\varepsilon_M$ and $\varepsilon_L$ are given by

$$\varepsilon_M = M_{ij} \varepsilon_{ij}; \qquad \varepsilon_L = L_{ij} \varepsilon_{ij} \tag{3}$$

where $M_{ij} = m_i n_j$ and $L_{ij} = l_i n_j$. Because of these kinematic constraints relating strains on the microlevel (the microplane) to those on the macrolevel (continuum), static equivalence between the macro and micro levels can be enforced only approximately; this is done by applying the principle of virtual work. Written for a surface $\Omega$ of a unit hemisphere this gives

$$\sigma_{ij} = \frac{3}{2\pi} \int_{\Omega} (\sigma_N N_{ij} + \sigma_L L_{ij} + \sigma_M M_{ij}) d\Omega \tag{4}$$

where $\sigma_N$, $\sigma_L$ and $\sigma_M$ are the components of the stress vector on the microplanes in the normal and two tangential directions, respectively. The most general constitutive relation on the microplane level may be written as

$$\sigma_N(t) = \mathcal{F}[\varepsilon_N(t),\ \varepsilon_L(t),\ \varepsilon_M(t)] \tag{5a}$$

$$\sigma_L(t) = \mathcal{H}[\varepsilon_N(t),\ \varepsilon_L(t),\ \varepsilon_M(t)] \tag{5b}$$

$$\sigma_N(t) = \mathcal{M}[\varepsilon_N(t),\ \varepsilon_L(t),\ \varepsilon_M(t)] \tag{5c}$$

where $\mathcal{F}$, $\mathcal{H}$ and $\mathcal{M}$ are functionals that depend on the history of the microplane strains at time $t$. An intuitive understanding of the material behavior involved is necessary to formulate appropriate forms for these functionals, which in this case is concrete.

The evolution of appropriate microplane models for concrete has taken place over a period of several years. The model integrated in this study is referred to as *Model M3* (Bazant *et al.* 2000) and makes use of stress-strain boundaries (i.e., softening strain-dependent yield limits) on the microplane level. In this concept the microplane response remains elastic until any of the boundaries is reached. An advantage of this approach is that is allows different stress components to be defined as functions of different strain components. This is useful for simultaneous modeling of tensile, compressive and shear softening. The model was also extended to moderately large strains by using a method of scaling the parameters in the model.

### 2.1. EPIC

EPIC (*Elastic-Plastic Impact Computations*) is a general purpose three dimensional Lagrangian finite element code developed specifically for impact and penetration problems. The first documented version of EPIC occurred in Johnson (1977), and it has been under continuous development since then. A significant enhancement that occurred during the early part of this decade was the incorporation of an algorithm for automatically converting distorted finite elements into meshless particles. The latest version of EPIC released in Johnson *et al.* (2006) was the first parallel version of the code.

### 2.2. Integration

Incorporation of the microplane model into EPIC required extensive additions and modifications to the existing code. Many of these additions were performed to insure a seamless integration of the model into the production version. The integration included the following tasks:

(1) In order to conform to the material model interface in EPIC, a new material model type was added, and subroutines that query material type were modified to recognize this material. In addition, the EPIC material library was modified to include a fit to the model available for a particular concrete mix.

(2) The internal state variable package in EPIC was not sufficient to accommodate the arrangement or number of state variables required by the Microplane model. Rather than modify the existing internal state variable package, it was decided to develop a new set of subroutines to accommodate the internal state variable data array allocation, initialization and update and storage for the model.

(3) Frame indifferent measures of deformation are required by the Microplane model. Likewise, an objective measure of the stress is returned by the model. Standard measures of stress and deformation used in the EPIC code are the Cauchy stress and rate of deformation, respectively, and neither of these are frame indifferent. Transformation of these measures into objective measures is easily accomplished if the state of deformation is known. In this work, the unrotated or co-rotational Cauchy stress was assigned as the stress measure, and Green-Lagrange strain the deformation measure. Both of these measures are frame indifferent since they do not change value for a body subjected to rigid body motion.

In what follows, a summary of the work required to accomplish these tasks is given.

#### 2.2.1. Material model interface

Existing subroutines in EPIC were modified to enable support for the Microplane model.

Modifications were performed conforming to the Material Model Module (MMM) interface used in the code to insure a seamless integration of the model into the production code. Microplane materials were assigned to Material Type 16 and the various subroutines in the code that query material type were modified to recognize this material. Presently only one fit is available for the model – a fit to a 5000 psi concrete referred to as WES5000. This fit was inserted into the EPIC material library and assigned to Material 178.

### *2.2.2. Internal state variables*

A series of subroutines were developed to allocate, initialize, store and update the internal state variables required by the Microplane model. In this implementation, a total of 99 state variables were required. This includes three variables for each microplane (for the 28 planes used in the ERDC fit to WES5000 this is a total of 84 variables), one variable for the volumetric law (assumed to be the same for all planes), one variable to monitor the virgin/damaged state, nine variables to store the deformation gradient, and four variables to store additional post-processing quantities such as damage and equivalent inelastic strain. Due to the large number of state variables required by the model, subroutines were written so that allocation of the data arrays occurred only when this material model is used in a calculation. Initialization of these arrays takes place right after allocation in the calling sequence. Update and storage of the variables conforms to the standard used in EPIC wherein element data is retrieved and stored in 128-element blocks; this legacy feature was implemented in the code to support vectorization, and it remains for backward compatibility reasons.

### *2.2.3. Stress and deformation measures*

As was previously stated, the EPIC code uses rate of deformation and Cauchy stress as its measures of stress and deformation. In this implementation of the Microplane model, the Green-Lagrange strain and co-rotational Cauchy stress were used. These measures of stress and deformation are easily determined if the state of deformation is known. The Green-Lagrange strain, for example, can be recovered from time integration of the deformation rates. In the EPIC code, the rate of deformation $D_{ij}$ and the spin tensor $W_{ij}$ are stored; these are the symmetric and skew-symmetric parts of the velocity gradient $L_{ij}$. From this, the velocity gradient is obtained as

$$L_{ij} = D_{ij} + W_{ij} \tag{6}$$

The deformation gradient $F_{ij}$ is then recovered by integrating the velocity gradient forward in time using the relation

$$\frac{dF_{ij}}{dt} - L_{ik}F_{kj} = 0 \tag{7}$$

If there is no deformation in the initial state, then the components of $F_{ij}$ are initialized to the identity matrix, that is

$$F_{ij} = \delta_{ij} \tag{8}$$

where $\delta_{ij}$ is the Kronecker delta function. With the deformation gradient available, the Green-Lagrange strain is easily calculated as

$$E_{ij} = \frac{1}{2}(F_{ki}F_{kj} - \delta_{ij}) \tag{9}$$

The co-rotational Cauchy stress $\hat{\sigma}_{ij}$ on the other hand is recovered from the Cauchy stress as

$$\hat{\sigma}_{kl} = \sigma_{ij} R_{ik} R_{jl} \tag{10}$$

where $R_{ij}$ is the rotation tensor. The rotation tensor is calculated from a multiplicative decomposition called the polar decomposition of the deformation gradient given as

$$F_{ij} = R_{ik} U_{kj} \tag{11}$$

where $U_{ij}$ is called the right stretch tensor and is symmetric. There are a number of methods available for recovering $R_{ij}$ from $F_{ij}$; for example the incremental method proposed by Dienes (1979) or by applying the polar decomposition theorem (Chandrasekharaiah and Debnath 1994). The method implemented here is one developed by Danielson *et al.* (2007) which is not incremental; that is $R_{ij}$ is updated without the use of its previous computed values. Danielson's method begins by multiplying Eq. (11) by $F_{li}^T$ which gives

$$F_{li}^T F_{ij} = F_{il} F_{ij} = (R_{lm} U_{mi})^T R_{ik} U_{kj} = R_{ik} R_{im} U_{ml} U_{kj} = \delta_{km} U_{ml} U_{kj} = U_{kl} U_{kj} \tag{12}$$

With $F_{ij}$ known, Eq. (12) is six nonlinear equations for the unknown components $U_{ij}$. These equations are easily solved by the Newton-Raphson method and, in practice, the solution converges to 8-9 significant figures of accuracy in usually less than 10 iterations. Once the solution for $U_{ij}$ is obtained, it can be inverted to get the solution for $R_{ij}$

$$R_{ik} = F_{ij} U_{kj}^{-1} \tag{13}$$

Finally, once the co-rotational Cauchy stress is determined from the Microplane model, it must be transformed back to obtain the Cauchy stress for use in EPIC. This is accomplished by applying the relation

$$\sigma_{kl} = \hat{\sigma}_{ij} R_{ki} R_{lj} \tag{14}$$

### 2.2.4. Subroutines and code

A series of modules and subroutines were developed to implement the Microplane model using the interface outlined in the previous section. Modules were developed to perform utility operations such as data array allocation and initialization, math operations such as matrix inversion and iterative equation solvers, and model operations to update the stress of an element given the deformation. The module microplane_mod contains the subroutine mpallocate which allocates data arrays for the model, including the internal state variable arrays, and the subroutine mp_assign_data which initializes material model parameters, direction cosines for the microplanes, and internal state variables. The module microplane_math_mod contains the subroutines right_polar_decomposition_nr which computes the rotation tensor given the deformation gradient, and the routine simeq6 which solves six linear algebraic equations of a special form obtained from the polar decomposition. The module microplane_calcs_mod includes subroutines for updating the stress and includes the subroutine sac5_m2_s3d for calculating the stress, and the subroutines c_vol and c_shear called by sac5_m2_s3d for applying the volumetric and shear laws, respectively. The subroutine mp3d_sac5_m2 is the main interface between the EPIC code and the model; in this routine the deformation gradient is updated, then the Green-Lagrange strain is computed, then the old value of the co-rotational stress is determined. The material model is called to get the new value of the co-rotational stress which is then rotated back to obtain the Cauchy stress. Using new values for the stress and deformation, several ancillary variables are then updated such as the damage and equivalent inelastic strain.

In addition to the new modules and subroutines, several existing modules and subroutines were modified to accommodate the model. The subroutine MATL was modified to recognize Material type 16 ("microplane" material) and to perform the appropriate initializations if this material type is present in a calculation. Likewise, the subroutine STRESS was modified to update the stress if this material is present. The module MATERIAL_MOD was modified to provide input parameters for material number 178-a fit of the Microplane model to a 5,000 psi concrete referred to as WES5000 by researchers at ERDC. Small changes were also required to several other routines, primarily to allow them to recognize Material type 16.

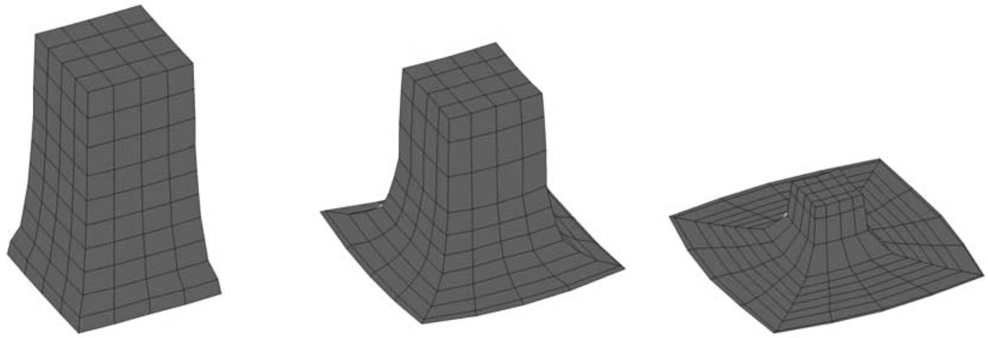
Subroutines and code were also modified to support restart capability of the model. The EPIC subroutine SAVE.f was modified to write all data associated with the Microplane model if Material type 16 was used in a calculation. The subroutines RREV62.f90 and RREV63.f90 were also modified to read restart files containing Material type 16. This included allocation of data arrays associated with internal state variables if a Microplane model material was present in the restart record.

## 3. Calculations

To verify the implementation is working properly and validate the model for a practical application, several calculations using the Microplane model were performed. In the first of these calculations, a 4-in square concrete rod 10 inches in length was impacted onto a rigid barrier at 4000 in/sec, commonly referred to as the *Taylor Test.* The finite element mesh consisted of 160 1-in cube hexahedrons. In Fig. 1 a plot of materials is shown at 1, 3 and 7 ms. As is evident, the impact face of the rod deforms and mushrooms as a result of the impact. No validation data was available for this problem; however, the deformation appears to be reasonable in the qualitative sense.

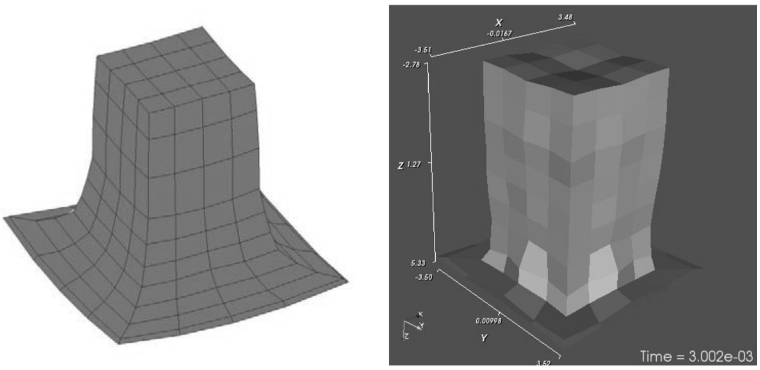
Figs. 2 and 3 show results from a calculation performed to compare results obtained from the EPIC implementation of the Microplane model to the same model in another Lagrangian code called PRONTO (Taylor and Flanagan 1989). In these calculations the conditions were the same as the Taylor test example presented in Fig. 1. In Fig. 2, the deformation is compared at 3 ms and in Fig. 3, the rod length history is compared. As is evident, the deformation and rod length history compare reasonably; however, they are not identical. Because there are significant differences in the implementations used by these two Lagrangian codes, an exact agreement cannot be expected. For example, the PRONTO calculation was run using element deletion (meaning that elements with large deformations were discarded from the calculation), but the EPIC calculation was not.

As part of the validation, the impact of a projectile onto a concrete target was also simulated. The problem setup is depicted in Fig. 4. The projectile was a steel penetrator and is 13.6 and 2 inches in length and diameter, respectively. The concrete target had a diameter of 72 inches on the face, and thicknesses of 5, 8.5 and 10 inches were considered. A mesh consisting of 230858, 399208 and 476130 tetrahedral elements was used in the calculation of the 5, 8.5 and 10 inch targets, respectively. Eroding slidelines were enabled to permit the removal of finite elements that accumulated excessive distortion, with the erosion strain set equal 1.5. A single plane of symmetry was used in the calculation so that half the problem was run. Results from one of these calculations are shown in Figs. 5(a)-(e), where material plots are shown at 0, 500, 1000, 1500 and 2000 $\mu$s. As is evident, impact of the projectile results in compete perforation of the target leaving a cavity approximately 7



(a) 1 ms (b) 3 ms (c) 7 ms

Fig. 1 Taylor anvil impact of a 10-in bar at 4000 in/s




(a) EPIC (b) PRONTO

Fig. 2 Comparison of deformations at 3 ms

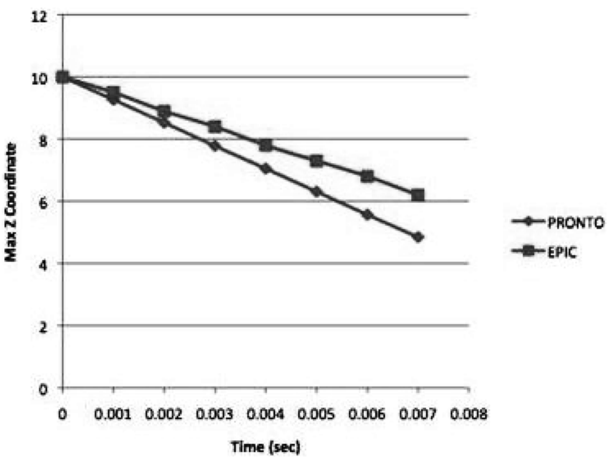



Fig. 3 Comparison of bar length history

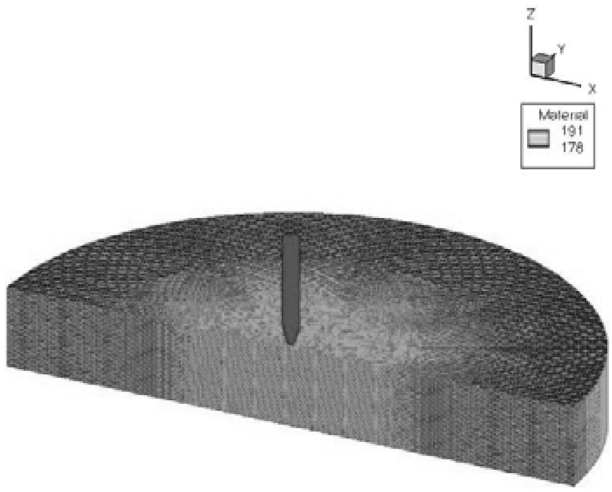



Fig. 4 Setup of penetration simulation

in diameter. The cavity represents the boundary above which the inelastic strain accumulated in the target material is above 1.5; however, a more realistic failure strain for the concrete is 0.07. In Figs. 6, 7 and 8 the cavities produced from the impact are shown for the 5, 8.5 and 10 inch targets, respectively, along with the profile obtained from experimental data for these targets (Cargile 1999). In light of the stochastic nature of failure in concrete, the 7% inelastic strain boundary compares reasonably with the boundaries of the cavity produced in the experiments.

The third validation problem performed in this study was the simulation of an embedded detonation problem. The problem setup is depicted in Fig. 9. A concrete target 36 inches square on the face and 12 inches in thickness is reinforced with two layers of a grid of 0.5 inch square steel bars. The grid is arranged in a 5 inch square pattern. In the center of the target is a cylindrical cavity 1 inch in diameter and 8.5 inches in length. The cavity is filled with 5 inches of Composition C-4 explosive with a 3.5 inch unfilled opening above it. A mesh consisting of 995272 hexahedral elements was used to simulate the problem. Two planes of symmetry were used in the calculation, so that a quarter of the problem was run. Concrete elements were discarded at an inelastic strain of 2.0 so that finite elements with a large accumulated distortion could be removed from the calculation. Although this strain this is much larger than the failure strain of the concrete, it is necessary for the failed material to remain in the calculation, particularly for an embedded detonation simulation, since the concrete can still support compression. Results of the calculation are shown in Figs. 10(a) -(b), where materials and inelastic strain are shown at 1.0 ms after detonation. In Fig. 10(a), materials with an inelastic strain below 0.07 are shown. As is evident, the removed concrete exposes the first square of the rebar grid around the detonation and a portion of the second and third layers of the rebar grid. There is also evidence of strain accumulated at the boundaries of the target caused by reflected waves. In Fig. 10(b), the inelastic strain contours are shown. In Figs. 11(a)-(b), results obtained from an experiment of this setup are shown (Danielson *et al*. 2008). As is evident, the calculation produces the main features seen in the test, including the extent of the cavity produced in the concrete was well as the failure seen near the lateral boundaries of the target.

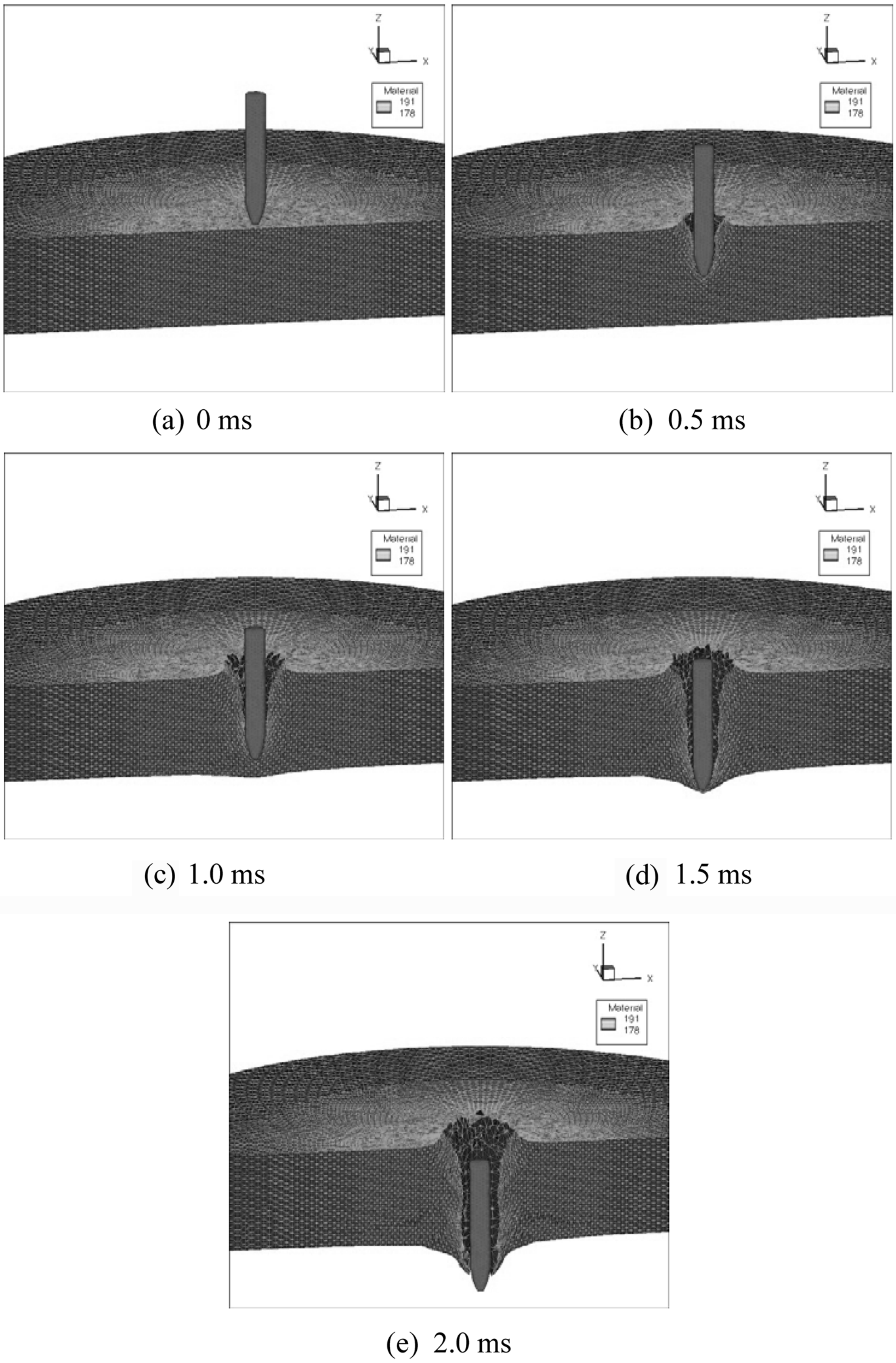



Fig. 5 Time history of rod penetration

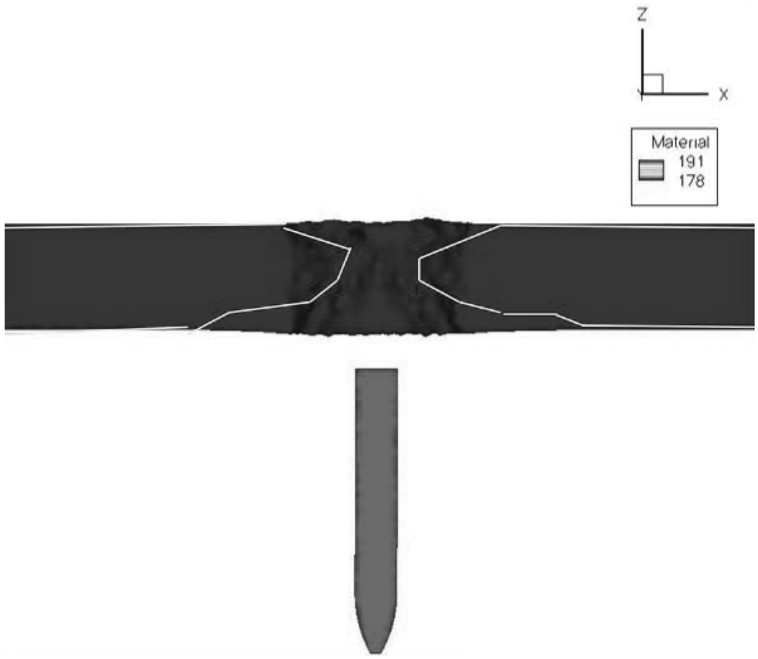



Fig. 6 Comparison of cavities produced in 5-in concrete target

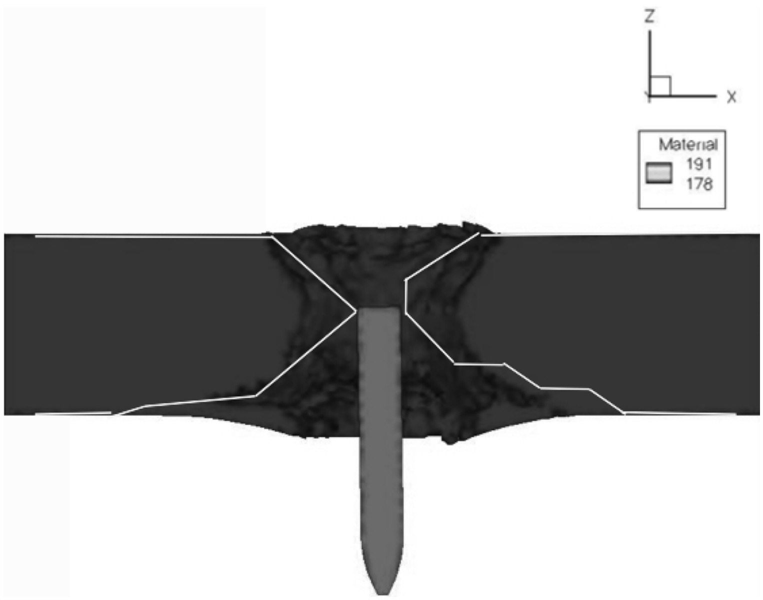



Fig. 7 Comparison of cavities produced in 8.5-in concrete target

## 4. Conclusions

In this work the implementation of a production-level port of the Microplane constitutive model for concrete into the EPIC code is described. The overall goal of this work was to develop a general

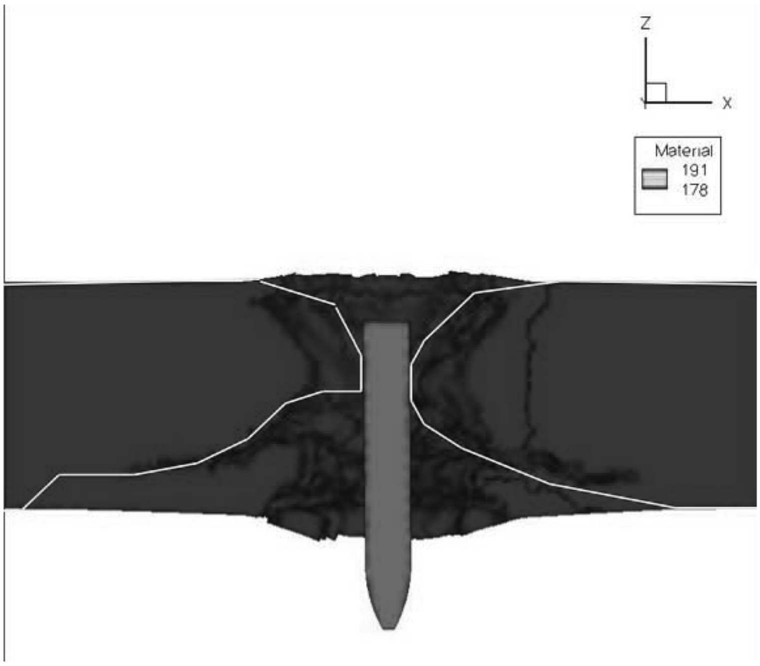



Fig. 8 Comparison of cavities produced in 10-in concrete target

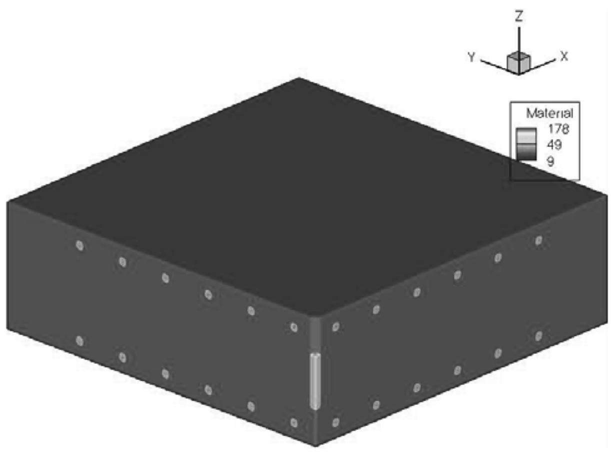
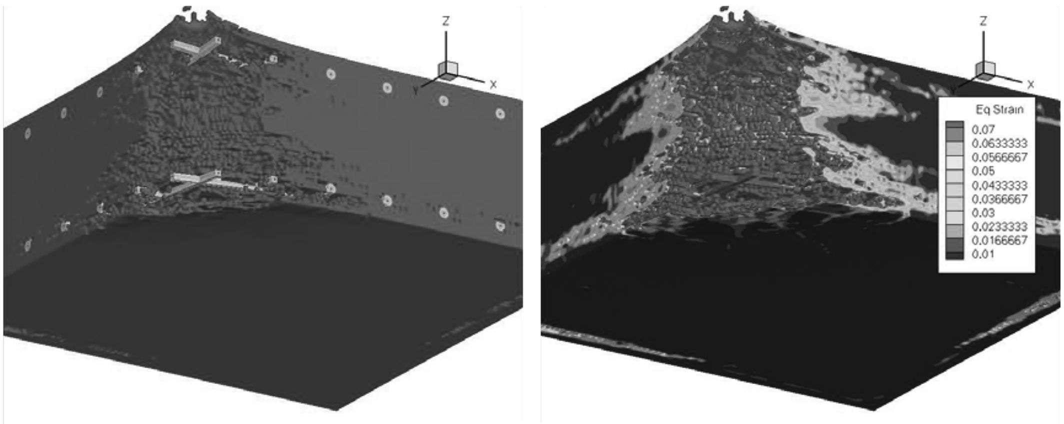



Fig. 9 Setup of embedded detonation simulation

framework within the EPIC code for easy integration of constitutive models, with the Microplane model implemented as a practical illustration of the framework. To the extent possible, the port follows guidelines outlined in the Material Model Module (MMM) standard used in EPIC to insure a seamless interface with the existing code. Objective stress and deformation measures have also been implemented as part of the material model installation. Validation problems run include Taylor anvil impact, projectile penetration and embedded detonation and illustrate the veracity of this new capability.




(a) Materials (b) Inelastic strain

Fig. 10 Materials and plastic strain 1.0 ms after detonation

(a) View 1 (b) View 2

Fig. 11 Cavity produced from embedded detonation

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