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Blast analysis of concrete arch structures for FRP retrofitting design

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Abstract. Fiber Reinforced Polymer (FRP) is widely used for retrofitting concrete structures for various purposes. Especially, for the retrofitting of concrete structures subjected to blast loads, FRP is proven to be a very effective retrofitting material. However, a systematic design procedure to implement FRP for concrete structure retrofitting against blast loads does not exist currently. In addition, in case of concrete structures with inarticulate geometrical boundary conditions such as arch structures, an effective analysis technique is needed to obtain reliable results based on minimal analytical assumptions. Therefore, in this study, a systematic and efficient blast analysis procedure for FRP retrofitting design of concrete arch structure is suggested. The procedure is composed of three sequential parts of preliminary analysis, breach and debris analysis, and retrofit-material analysis. Based on the suggested procedure, blast analyses are carried out by using explicit code, LS-DYNA. The study results are discussed in detail.

Keywords: blast analysis procedure; concrete arch structure; preliminary analysis; breach/debris analysis; retrofit material analysis; FRP retrofitting; blast retrofit design.

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1. Introduction

Since the end of World War II, numerous researches on the behaviors of structures subjected to blast loads have been actively conducted (ASCE 1999, Winget *et al*. 2005). Single-Degree-of-Freedom (SDOF) and Multi-Degree-of-Freedom (MDOF) analysis methods proposed by Biggs (1964) have been widely used in blast analysis and design due to its convenience. In these methods, the global behavior of structures can be evaluated by simplification of structural system with lumped mass and spring constant. However, since the simplified analysis methods have a limitation in predicting nonlinear and localized behaviors of concrete structures, the development of a design practice based on sophisticated numerical analysis methods has been actively pursued (Byun *et al*. 2006, Choi and Krauthammer 2003, Lee *et al.* 2009, Marchand and Alfawakhiri 2005).

To overcome limitations of simplified analysis methods, extensive studies based on finite element method (FEM) with proper material models and element discretizations have been conducted. In order to implement blast load characteristics (i.e. infinitely short time duration, extreme localized pressure, fragmentations, etc) into FEM based blast analysis, element discretizations are needed (Li and Meng 2002, Nam *et al.* 2008a).

For most cases of blast analyses using FEM, explicit method is implemented because it can produce stable results. However, explicit FEM requires very small time steps to obtain reasonable results, which results in immense computational time, especially when a full 3D model is analyzed (Nam *et al*. 2008b, Tavárez 2001). For this reason, a simplified and localized modeling technique with partly simplified boundary and geometry conditions of the structures is still unavoidable for practical blast design and analysis even though current blast analysis software and computational hardware are rapidly advancing. However, the drawback of a simplified analysis is that improper assumptions can lead to significant errors throughout analysis.

Another urgently needed improvement in blast analysis is a systematic analytical and design procedure that can accurately and efficiently consider real structural behaviors. However, unfortunately, a lack of systematic blast analysis procedures based on physical and structural behaviors of concrete structures is limiting the actual applications and growth of the technology. Especially, when highly sophisticated structures of geometrically inarticulate structural types such as arches and domes need to be considered, a systematic design procedure and precise analytical tool are essentials in conducting proper blast evaluation and design.

The objective of this study is to suggest an effective and systematic blast analysis procedure for concrete structure retrofitting design based on a structural constitutive rationale. In order to achieve this objective, a concrete arch structure is selected as a target structure for FRP retrofitting and blast analyses. The suggested analysis technique introduces the methodology that can efficiently and accurately predict not only the structural behaviors but also retrofit effectiveness of concrete arch structure under blast loading by using simplified 3D models. For blast analysis, the explicit FEM code, LS-DYNA is used. The overall analysis results are used to confirm the applicability and feasibility of the suggested procedure. Also, the results of this study can be extended to the retrofit design of concrete shelter structures.

2. Blast analysis procedure for FRP retrofitting design

Concrete structures subjected to blast loads can be strengthened by various retrofitting materials and methods. Blast retrofitting materials and methods must be able to provide sufficient structural resistant capacity against very rapid and huge blast pressure wave. The resistance of structures under blast loads is dependent on failure strength and ductility of the retrofitting materials. Fiber

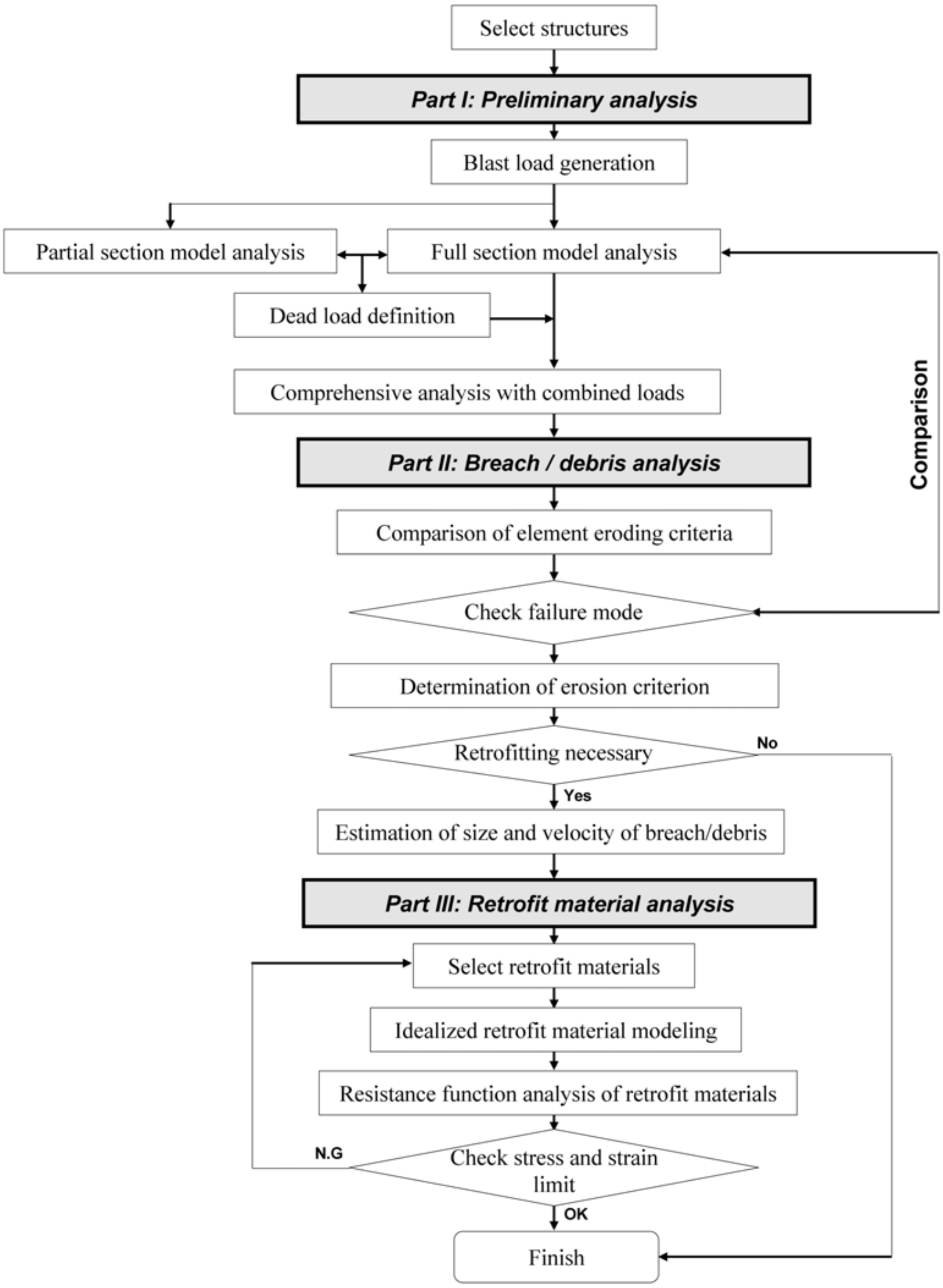


Fig. 1 Schematic procedure for blast retrofitting design analysis

composites such as Glass Fiber Reinforced Polymer (GFRP) and Carbon Fiber Reinforced Polymer (CFRP) are well acknowledged to have very high strength and ductility, which are the reasons for their wide usages in retrofitting concrete structures (Buchan and Chen 2007, Patoary and Tan 2003, Silva and Lu 2007, Tolba 2001).

Concrete structure retrofittings using FRP composites have distinct advantages over other retrofit materials because of the easiness of installation and natural architectural blending to original structures. FRP's retrofitting effectiveness is proven by their stress and strain responses (Byun *et al.* 2006, Mosalam and Mosallam 2001). In order to properly design FRP retrofitted structures against blast load, a systematic blast analysis procedure, which can reasonably evaluate the damage behavior, breach/debris possibility, and retrofitting capacity is required. However, presently, there is no real systematic design procedure for FRP retrofitted concrete structures under blast loading.

In this study, a blast analysis procedure considering FRP retrofit applications to concrete arch structures is suggested. The suggested blast analysis procedure is composed of three sequential parts of preliminary analysis, breach and debris analysis, and retrofit-material analysis as shown in Fig. 1. The preliminary analysis of concrete arch structures is conducted to understand structural damage distribution trends. Then, the size and velocity of breach and debris are calculated by erosion technique. Finally, the retrofitting material analysis is carried out to evaluate blast resistant effectiveness of FRP. The step-by-step considerations of the suggested blast analysis procedure are described in the following sections.

2.1. Step I: Preliminary analysis

To start off, the blast loading scenario is determined considering the geometry and boundary conditions of a target structure. After generating blast loads, the partial and full section models of arch structures with minimal empirical assumptions are defined. Then, analyses for the proposed partial and full section models are conducted to obtain information on the structural behaviors, damage distributions, and possibility of breach/debris of the target structure. The partial section model analysis is applied to reduce computation time. Throughout the analysis, the rate dependent constitutive material models considering damage and dynamic increasing effect are applied.

In the partial section model analysis, a portion of the structure simply modeled in 2D is used to predict the trend of structural behavior. The partially modeled section based on constitutive rationale should be able to realistically represent the global structural characteristics. Based on the partial section model analysis under several specific loading scenarios, the next step of preliminary analysis with fully modeled section is carried out.

In the full section model analysis, the modeling for the breach/debris analysis is performed considering structural behaviors such as damage state, displacement distribution, and local velocity. The critical scenario of the highest possibility of breach and debris is assessed to determine the retrofitting strategy and to prepare for the next step of breach/debris analysis. The results of full and partial section model analyses are compared to confirm the consistency of the defined models.

2.2. Step II: Breach/debris analysis

In the second analysis step of breach/debris analysis, breach/debris effect is investigated based on the results of the preliminary analysis. The pre-defined full section model from the preliminary analysis is used to evaluate structural retrofit necessity. The breach/debris effect is evaluated by checking the erosion of element for the pre-estimated assumptions from the preliminary analysis results. The mass and velocity of breach/debris are calculated on the eroded elements, where the maximum values of the size and velocity are selected to suggest a conservative retrofit design.

2.3. Step III: Retrofit material analysis

If the retrofit is needed, the retrofitting effect can be evaluated by two different approaches in the third analysis step. One approach is to perform an additional structural analysis including finite element modeling of a whole retrofitted structure. The other approach is to perform retrofit material analysis focusing on the capacity and failure of retrofit materials obtained from the previous full section model in the second analysis step. The former approach has an advantage of being able to directly show the retrofit effect such as energy absorption, catching effect, etc. However, it requires immense modeling effort and computation time due to an increased number of elements. On the other hand, the latter approach has advantages of being able to directly confirm the capacity and failure of retrofit materials with less modeling effort and computation time. In this study, the latter approach of performing the retrofit material analysis is selected for the third analysis step. Breach or debris considered as a rigid body with its mass and initial velocity estimated from the previous breach/debris analysis is applied as a load to selected retrofitting materials. The capacity and failure of the retrofit materials can be evaluated by the displacement and the stress/strain states of the retrofit materials.

3. Preliminary analysis

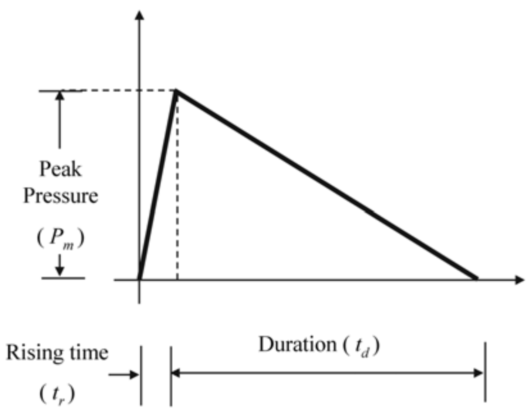
The main objective of the preliminary analysis is to obtain the information about structural behaviors before the breach/debris analysis. As stated before, the preliminary analysis is composed of partial and full section model analyses. For the analyses, nonlinear dynamic material models are used in the explicit code, LS-DYNA. Damage model considering the dynamic increasing effect of materials and accumulative damages is used for concrete material (LSTC 2003, Malvar *et al*. 1997, Tavárez 2001). This model properly predicts the compressive and tensile behaviors of concrete under blast pressure by using three failure surfaces of maximum, residual and yield failure surface derived from experimental data. For the reinforcement, piecewise plastic material model is used (Jones 1983, LSTC 2003).

3.1. Blast load generation

The blast load is generated by using a computer program called CONWEP (Hyde 1992, TM5-1300 1990). The idealized blast pressure and time curve as shown in Fig. 2 is used for the analysis (Kingery and Bulmash 1982). Based on the idealized blast pressure curve, two different scenarios of detonations, namely on-ground-detonation and above-ground-detonation, are applied as explosive loading conditions as shown in Fig. 3.

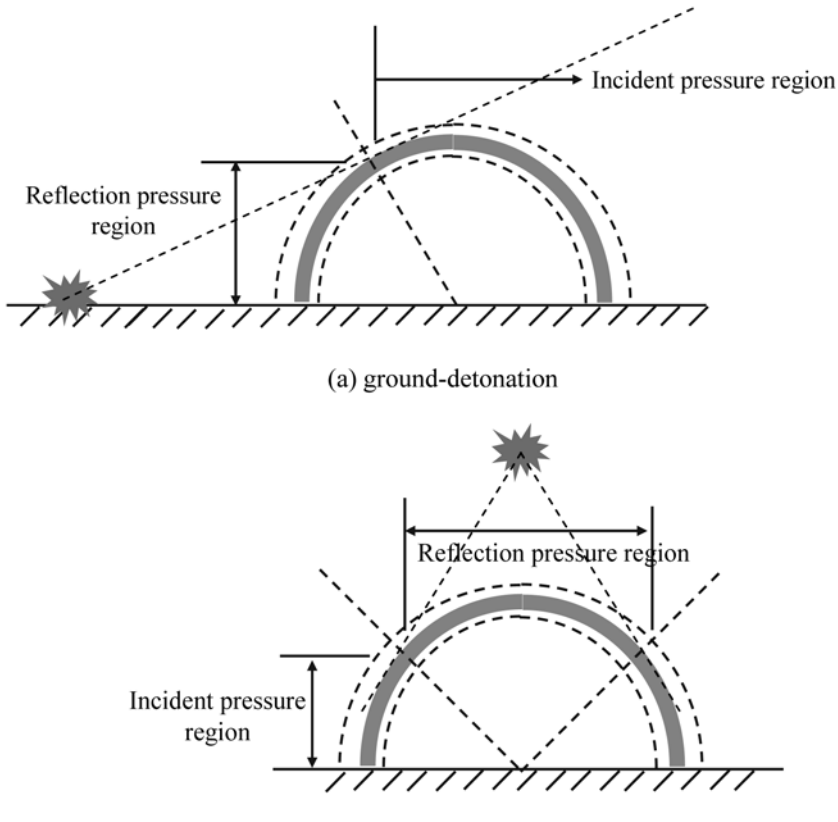
3.2. Partial section model analysis

In the partial section model analysis, behaviors of roof of arch structure are analyzed. The objectives



TNT (kN)	4.5
Standoff (m)	7.6
Peak pressure (MPa)	5.2
Rising time (msec)	0.01
Duration (msec)	1.71

Fig. 2 Blast load generation



(a) ground-detonation

(b) above-ground-detonation

Fig. 3 Blast pressure application ranges with different scenarios

of the partial sectional analysis are to evaluate concrete damage, possible breach/debris, displacement trends, velocity trends, and further analysis details. A 2D partial section model with roller boundary conditions at both ends is modeled as shown in Fig. 4. The peak pressure of 5.2 MPa from the blast wave induced by 454 kg charge weight and 7.6m stand-off distance is applied on the arch-roof.

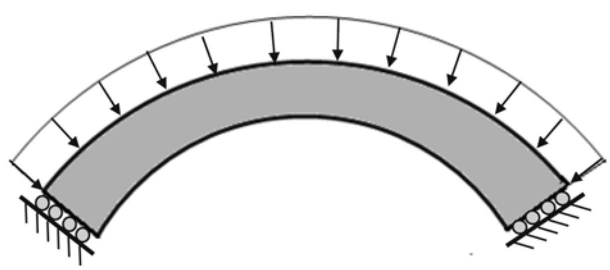


Fig. 4 Loading and boundary conditions of partial section model

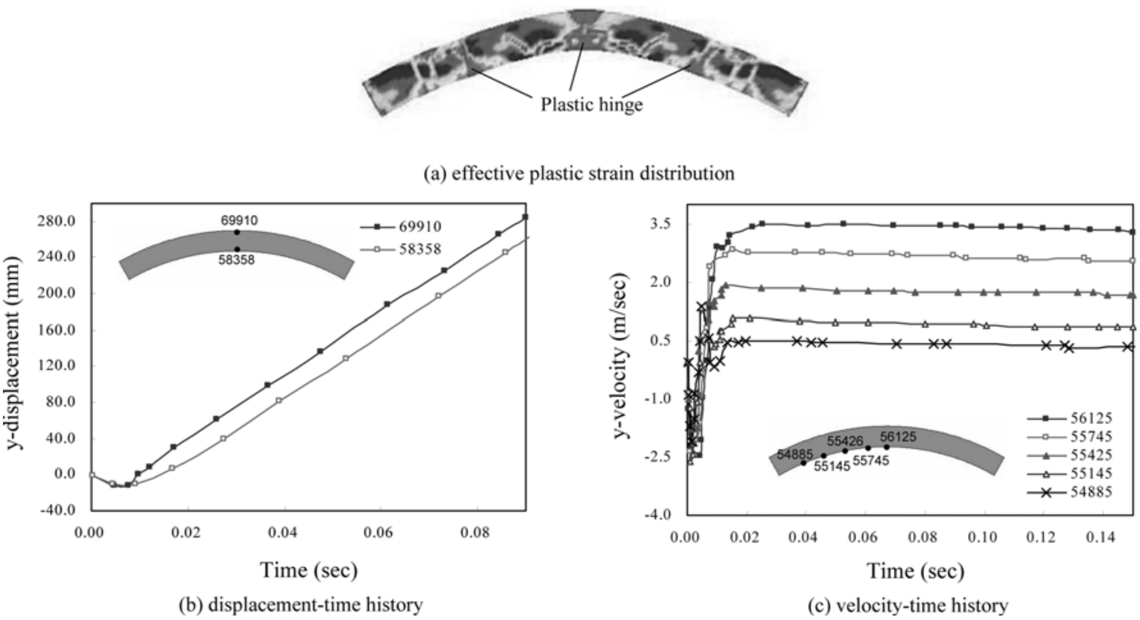


Fig. 5 Partial section model analysis results

The results of the partial section analysis for the arch-roof are shown in Fig. 5. The maximum displacement and velocity of 12.4 mm and 2.9 m/sec, respectively, are calculated. Three major plastic hinges are formed and significant damages are observed throughout the arch-roof. The possibility of breach/debris occurrence is evaluated as very high from the large response differences between top and bottom of the arch-roof.

3.3. Full section model analysis

In the full section model analysis, a full cross section with 1 m thickness is modeled as shown in Fig. 6. A plane-strain boundary condition is applied in the thickness direction and fixed boundary conditions are applied at the both bottom-ends of the arch structure. The main objectives of the global analysis are to validate the full section model by comparing with the results from the partial section model analysis and to evaluate the damage behavior of the full section model under more realistic loading condition.

For validation of the model, two types of blast loads of on-ground- and above-ground-detonations

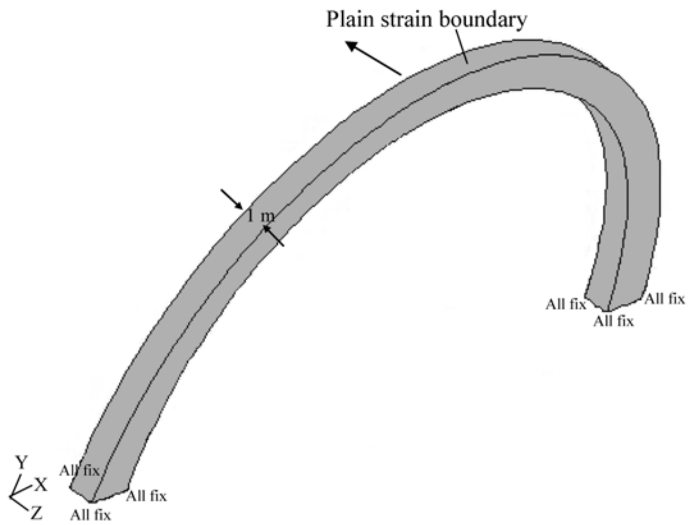


Fig. 6 Full section model

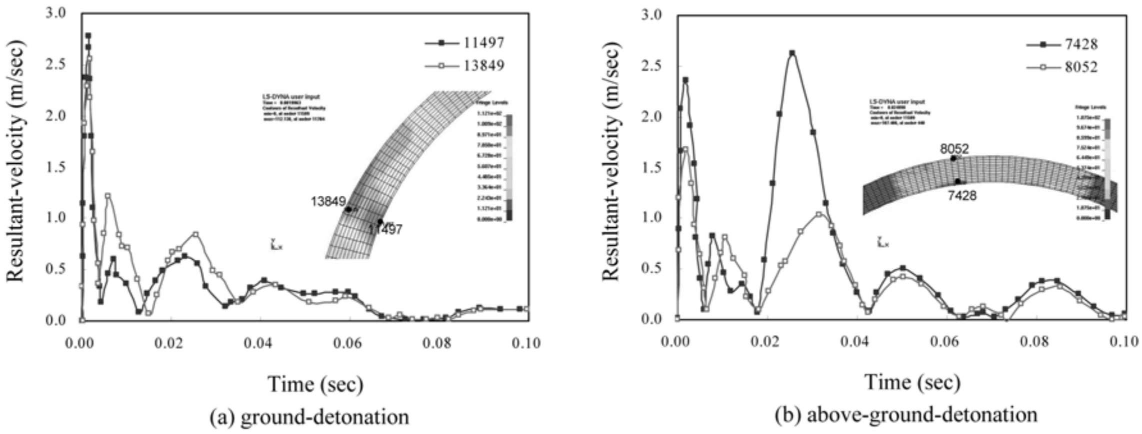
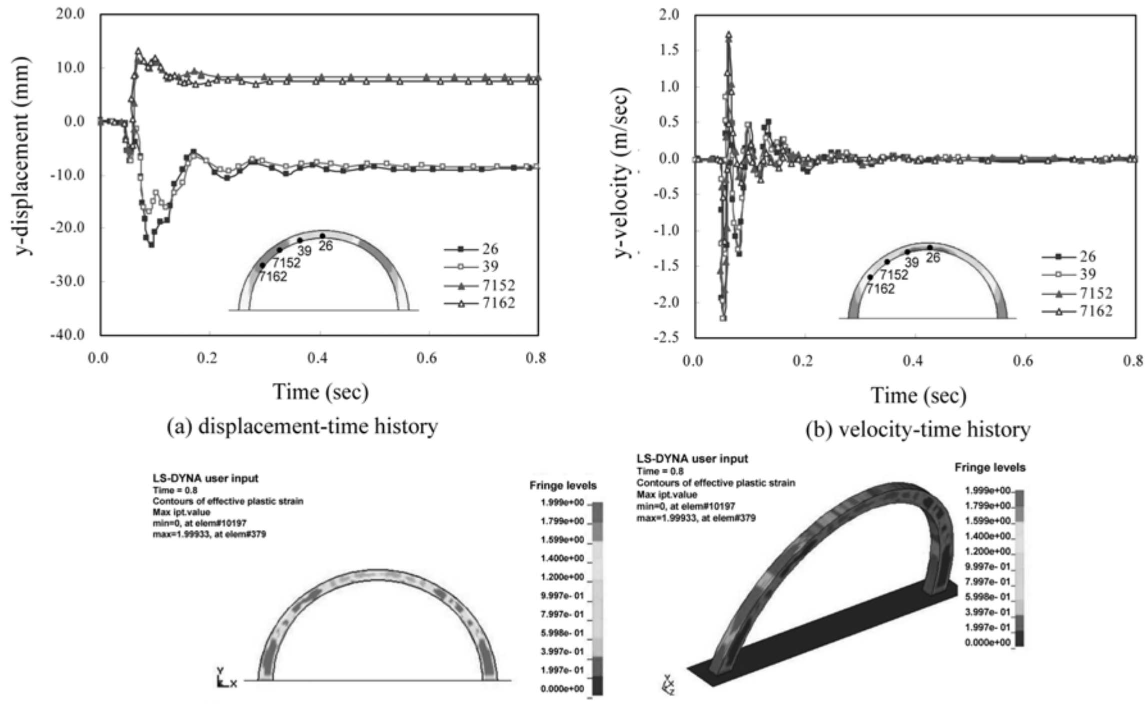


Fig. 7 Results of full section model analysis according to different detonations

are applied. The results for the two types of blast loads are shown in Fig. 7. The differences of velocity-histories between the critical outer and inner points are shown to be almost constant during the loading durations. These trends are similar to those of the partial section model analysis for the arch-roof, which confirm the validity of the full section model. From the direct comparison with the partial analysis, more significant difference of velocity-histories through thickness is observed after an application of loading is computed. This difference is due to the domineering displacement global structural mode. Generally, this significant response difference is directly related to breach/debris action and it is shown that the above-ground-detonation has a relatively higher possibility of breach/debris than on-ground-detonation.

After validation of the full section model, dead load considering dynamic relaxation is combined with blast load for more realistic loading condition. Blast loading scenario is considered as above-ground-detonation. The results of the full section model analysis for the combined loading condition are shown in Fig. 8. The vertical displacements are more significant than the horizontal displacements



(a) displacement-time history (b) velocity-time history

(c) effective plastic strain

Fig. 8 Full section model analysis for the combined load

along the surface with strong possibility of breach due to warping of concrete. The velocities are concentrated on the thin inner surface and the critical changes along the surface are not observed. Breach/debris is anticipated by the concrete failure at the inner surface of roof and the side-wall of the arch structure. Therefore, it is estimated that the surface retrofit is needed for the target structure to prevent breach and debris.

4. Breach/debris analysis

In the breach/debris analysis, kinetic energy based erosion technique is used to calculate breach/debris mass, volume, and velocity in the damaged elements. The erosion technique is more applicable in a short stand-off explosion and it can directly represent breach/debris of the damaged structures. So, this technique can be effectively used for the evaluation of retrofit required area under blast loading. For the erosion option in the analysis, the failure criteria of eroded elements are assumed based on the plastic strains (Hartmann 2005). For determining optimum erosion criteria, the convergences of volume, mass, and velocity of breach/debris between the adjacent elements are comprehensively investigated according to the several different levels of erosion criteria. These convergences can be utilized for the validation of applied erosion model and the simulation of breach/debris actions in analytical point of view. The plastic strains of 2%, 5%, 10%, and 20% are considered for determining the optimum erosion criterion and the failure modes according to the different erosion criteria are shown in Fig. 9. In case of 2%, most of surface elements are eliminated

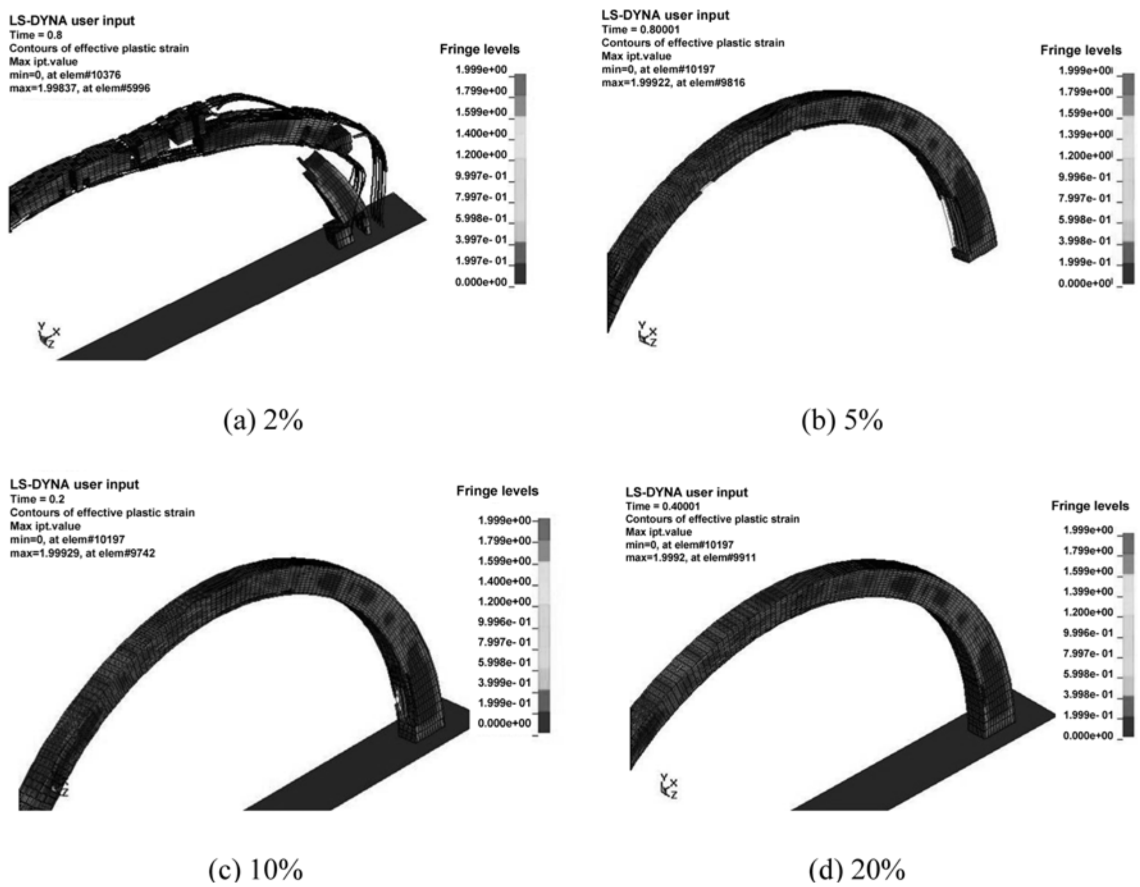


Fig. 9 Failure modes with different erosion criteria

Table 1 Summary of breach/debris analysis

Erosion Criteria		Volume (m^3)	Mass (kN sec^2/m)	Velocity (m/sec)
5%	Min	0.063	0.41	1.93
	Max	0.173	1.10	3.84
10%	Min	0.091	0.58	3.73
	Max	0.109	0.70	3.73
20%	Min	0.027	0.17	2.97
	Max	0.064	0.41	2.95

and the structure is fully collapsed. On the contrary, there is almost no eroded element in case of 20%. The breach/debris analysis results for the optimum erosion criterion are summarized in Table 1. 10% plastic strain is evaluated as the optimum erosion criterion, which represents the most realistic breach/debris action based on the convergence of volume, mass, and velocity of breach/debris. Based on the determined velocity, volume, and mass, the breach/debris size is estimated for the next analysis step of evaluation of retrofitting material evaluations.

5. Evaluation of retrofitting materials

For retrofitting materials which can prevent fragmentation, FRPs are considered to be the best retrofitting material due to its high strength and flexibility (Crawford *et al.* 1997, Muszynski and Purcell 2003). In this study, Carbon Fiber Reinforced Polymer (CFRP) and Glass Fiber Reinforced Polymer (GFRP) are considered as retrofit materials. For modeling of FRP, Belytschko-Lin-Tsay shell element is adopted (Belytschko and Tsay 1981, Kollar and Springer 2002, LSTC 2003). The material properties of FRP used in the analysis are listed in Table 2. Each FRP is subjected to be loaded by velocity of 3.73 m/sec and mass of 0.67 kNsec^2/m/m based on the breach/debris analysis results. The physical concept of an idealized breach/debris in the full section model is shown in Fig. 10. An ideally estimated breach/debris with mass and initial velocity is applied as a load to FRP retrofitting materials and a fixed boundary condition is assumed at both ends. Gaps at both ends are

Table 2 Material properties of retrofit materials

Properties	CFRP	GFRP
Failure Strength (MPa)	3,540	500
Fracture Strain (%)	1.5	2.0
Tensile Modulus (MPa)	235,000	25,300
Thickness (mm)	0.17	1.0

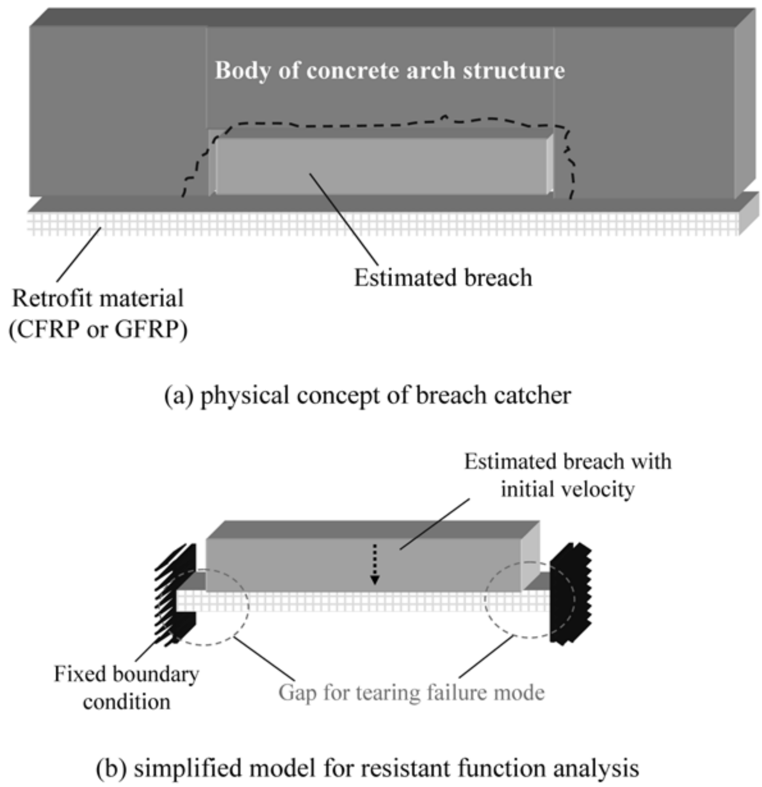


Fig. 10 Concept of simplified retrofit analysis model

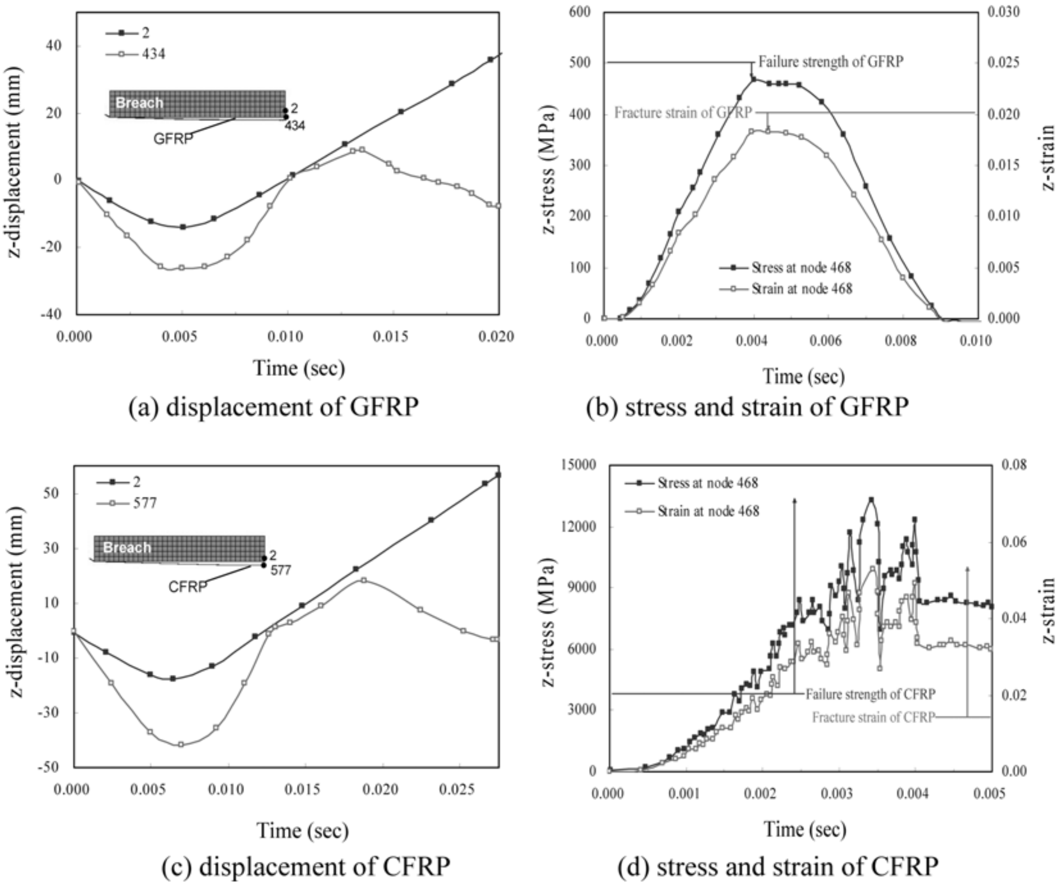


Fig. 11 Results of retrofit material analysis

modeled for the simulation of tearing failure mode.

The results of the retrofit material analysis are shown in Fig. 11. In case of GFRP application, the maximum stress corresponds to 94% of ultimate strength and the maximum strain corresponds to 90% of ultimate strain. The residual capacity of GFRP retrofitting is calculated to be an increase of approximately 6-10%. In case of CFRP application, the maximum stress corresponds to 336% of ultimate strength and the maximum strain corresponds to 353% of ultimate strain. For the CFRP retrofitting, more than four layers of CFRP sheets should be required. Even though the strength and stiffness of CFRP are much higher than those of GFRP, the retrofitting effect of GFRP appears to be superior than CFRP in the analysis results. This trend is caused by the difference of application thickness, which can alter the energy absorbing capacity of retrofitting materials.

6. Conclusions

In this study, a systematic blast analysis procedure for the FRP retrofitting design of concrete arch structures is presented as step-by-step process. The conclusions of this study are summarized as follows:

(1) A systematic blast analysis procedure composed of three steps of preliminary analysis, breach/debris analysis, and retrofit material analysis is suggested. Through the procedure, multi-level analytical investigations are conducted to minimize analytical assumptions, and partial and full section models are considered for efficient analysis calculations;

(2) Damage of structure and possibility of breach/debris are evaluated in the preliminary analysis. In the next analysis step, breach/debris action is assessed by the breach/debris analysis. 10 % of plastic strain was adopted for the erosion criterion of element by comparing the convergences of volume, mass, and velocity. The estimated size and velocity of breach/debris is used for the retrofit material analysis;

(3) The resistant capacity of retrofit materials is evaluated by the stress and strain states under the loading of breach/debris. By using the simplified retrofit material model, computing time can be substantially reduced and retrofit effectiveness can be directly evaluated;

(4) For retrofitting materials, GFRP and CFRP are adopted. In the case of CFRP retrofit, both stress and strain exceeded its ultimate strength and strain. In case of GFRP, the maximum stress and strain corresponded to approximately 92% of its ultimate strength and strain. GFRP application appeared to have a superior resistant capacity in the analysis results. However, since the different application thicknesses were used in the analysis, the results should be considered as relative. Blast resistant capacity and catching effect against breach/debris can be affected by application thickness dependent energy absorbing capacity;

(5) Based on the proposed design procedure for concrete arch structures, expanded applications can be performed on more sophisticated concrete structures with different inarticulate geometry. And further studies about 3D boundaries of breach/debris and accurate debonding failure modeling are needed for more precise and realistic design analysis.

Acknowledgements

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