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Modified heat of hydration and strength models for concrete containing fly ash and slag

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Abstract. This paper describes the development of modified heat of hydration and maturity-strength models for concrete containing fly ash and slag. The modified models are developed based on laboratory and literature test results, which include different types of cement, fly ash, and slag. The new models consider cement type, water-to-cementitious material ratio (w/cm), mineral admixture, air content, and curing conditions. The results show that the modified models well predict heat evolution and compressive strength development of concrete made with different cementitious materials. Using the newly developed models, the sensitivity analysis was also performed to study the effect of each parameter on the hydration and strength development. The results illustrate that comparing with other parameters studied, w/cm, air content, fly ash, and slag replacement level have more significantly influence on concrete strength at both early and later age.

Keywords: heat of hydration; strength; modeling; maturity; sensitivity.

1. Introduction

Concrete's compressive strength is generally used as a measure of overall concrete quality, because it is related to many other properties of concrete, such as elastic modulus and durability, and it is easy to test. Concrete strength at early age is also essential for construction operations, such as joint cutting times, formwork removal times, and pavement opening time. Guo (1989) states, "Knowledge of the early-stage strength of concrete is of special importance when concreting has to be carried out during cold weather." Since concrete strength development depends on its temperature history, which is determined by hydration process and environment, it is essential to have an accurate hydration model. Without accurate estimation of concrete temperature, it is hard to predict field concrete strength development accurately.

Both concrete compressive strength and hydration are influenced by many factors, such as concrete materials, environmental conditions, and the age of the concrete. Currently, several models

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have been developed to predict the strength (Popovics 1987, Alexander 1972, Tsivilis *et al.* 1995, Zelic *et al.* 2004, Pann *et al.* 2003, Hwang *et al.* 2004) and hydration (Maekawa, *et al.* 1999, De Schutter 1995, Schindler 2002).

However, most of the models only consider a few factors. Also, seldom models consider the effect of using different types of fly ash and slag, and the concrete temperature history.

This paper describes the development of the modified heat of hydration and maturity-strength models, which are able to predict strength and heat development of ternary concrete under different curing conditions. The developed models consider cement compositions and fineness, water-to-cementitious material ratio (w/cm), type and dosage of fly ash and slag, and curing conditions. The air content is also considered in the strength model. The work is based on a large quantity of experimental and literature data covering a wide range of cementitious materials, including different types of cement, fly ash, and slag.

2. Experimental work

The objective of the experiment is to characterize the hydration process and strength development of concrete containing different types and dosages of supplemental cementitious materials (SCMs). During the laboratory test phase, the quality control (slump, air content, and unit weight), calorimetry, and strength-maturity tests were performed.

Type I/II cement was selected for all of the laboratory tests conducted. The Type I/II cement has a specific surface area of 373 m^2/kg and a specific density of 3.20. The cement contains 53.2% C_3S, 19.4% C_2S, 6.1% C_3A, and 10.5% C_4AF. Four different types of fly ash, including Class C and

Table 1 Chemical composition of cement and SCMs

Chemical Compound	Type I/II Cement	Fly Ash				GGBF Slag		
		1 (Class C)	2 (Class C)	3 (Class C)	4 (Class F)	1	2.	3
CaO	62.32	27.11	16.77	28.89	1.51	37.13	36.61	36.74
SiO_2	20.75	31.83	46.92	32.62	45.33	37.18	35.68	37.32
Al_2O_3	4.49	19.02	15.11	19.32	23.02	9.17	11.24	9.00
Fe_2O_3	3.45	5.99	7.06	6.46	23.52	0.91	0.70	0.70
MgO	2.88	4.47	4.94	4.56	0.64	10.17	10.11	10.34
K_2O	0.67	0.27	2.17	0.35	1.76	0.43	0.41	0.37
Na_2O	0.09	2.12	3.30	1.86	0.36	0.32	0.34	0.31
SO_3	2.74	3.51	1.31	2.48	0.31	-	-	-
TiO_2	0.34	1.67	0.58	1.59	1.07	0.45	1.14	0.36
P_2O_5	0.10	1.03	0.20	1.11	0.17	0.02	0.02	0.02
SrO	0.05	0.52	0.44	0.48	0.05	0.04	0.08	0.04
Mn_2O_3	0.53	0.14	0.04	0.10	0.03	0.95	0.60	0.90
BaO		0.82	0.74	0.80	0.05	-	-	-
S		-	-	-	-	1.06	1.71	0.97
LOI	0.1	0.2	0.1	0.1	1.6	-	-	-

Table 2 Concrete mix proportion

Materials	Source	Weight/Amount
Coarse Aggregate	Fort Dodge	63.20 lb/ft^3
Fine Aggregate	Cordova	50.6 lb/ft^3
Cementitious Materials	Type I/II cement + SCMs	20.83 lb/ft^3
Water	Tap water	8.33 lb/ft^3
Water Reducer	WRDA-82	21.60 ml
Air Entraining	Daravair 1400	2.80 ml

Table 3 SCM replacement level by weight

Mix No.	Cement (%)	FA (%)	Slag (%)
1	100	0	0
2	100	0	0
3	85	15	0
4	70	30	0
5	55	45	0
6	40	60	0
7	85	0	15
8	70	0	30
9	55	0	45
10	40	0	60
11	85	3.75	11.25
12	70	7.50	22.50
13	55	11.25	33.75
14	40	15	45
15	85	11.25	3.75
16	70	22.50	7.50
17	55	33.75	11.25
18	40	45	15
19	70	30 (FA-2)	0
20	70	30 (FA-3)	0
21	70	30 (FA-4)	0
22	70	0	30 (Slag-2)
23	70	0	30 (Slag-3)

Class F, and three types of slag from different sources were used to make various blended cementitious materials together with the Type I/II cement. The chemical compositions of all cementitious materials are listed in Table 1. These four types of fly ash were selected because the CaO content covers a large range from 1.51 to 28.89%.

The limestone with a specific gravity of 2.67 was used as concrete coarse aggregate. It was sieved and recombined to obtain a desired gradation. The coarse aggregate was also saturated for 24 hours and dried to the surface saturated dry (SSD) condition before mixing. The fine aggregate used for the research is nature river sand and has a specific gravity of 2.67.

Twenty-two concrete mixes were prepared. All of the mixes have the same mix proportion (Table 2), but different SCM replacement levels. The replacement level of SCMs ranges from 0 to 60% (Table 3). In Table 3, the fly ash 1 and slag 1 were used for mix No. 3 to18. The w/cm ranges from 0.40 to 0.42, where "cm" means cement or cement plus fly ash and/or slag as cementitious material. The WRDA-82 water reducer and Daravair 1400 air entraining agent from Grace Construction Products were used in all mixes. The air content ranges from 3.8% to 8.4%.

The concrete specimen was prepared according to ASTM C 192, *Standard Practice for making and Curing Concrete Test Specimens in the Laboratory.* Immediately following the mixing, the concrete was placed into the 6×12″(150×300 mm) cylinder. Then, the sample was weighed and placed into the semi-adiabatic calorimeter (IQ drum) manufactured by Digital Site Systems, Inc. The data were recorded every 15 minutes. The entire test for concrete hydration took about 7 days under room condition.

For the maturity and strength tests, seventeen 4×8″ (100×200 mm) cylinder samples were prepared for each mix. To monitor the sample's thermal history, the thermocouples were inserted into the centers of two randomly selected samples right after the casting. The temperature was recorded every 15 minutes by the ilogger manufactured by Digital Site Systems, Inc. The samples were stored in the curing room with a constant temperature around 75°F (24°C) and 100% humidity. After 1 day of curing, the samples were demolded and stored in the curing room again until the test date. The strength tests were performed at 1, 3, 7, 14, and 28 days according to ASTM C39*, Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens*. The average value of three samples was used in the maturity strength modeling.

3. Literature data collected

The quantitative effect of cement type on heat of hydration and concrete strength development were studied using literature data. Lerch *et al.* (1948) conducted the heat evolution test on a series of cement, including eight Type I cement, five Type II cement, three Type III cement, three type IV cement, and one Type V cement. The range and average values of the main compound compositions and Blain fineness are listed in Table 4. The heat of hydration up to 72 hours was obtained by

Table 4 Cement chemical and physical properties (Lerch *et al.*, 1948)

Cement Type		C_3S (%)	C_2S (%)	C_3A (%)	C_4AF (%)	SO_3 (%)	Blain Fineness (m^2/kg)
I	Range	42.5-64.5	10.1-32.2	7.5-13.2	6.5-10.7	1.6-1.9	322.9-398.5
	Average	50.3	23.8	10.8	8.1	1.7	341.3
II	Range	34.0-51.0	24.0-41.0	3.7-6.6	9.7-16.6	1.2-1.9	289.1-369.7
	Average	41.5	33.3	5.4	13.5	1.6	321.0
III	Range	56.0-60.0	13.0-17.0	10.4-10.8	6.4-7.7	1.7-2.3	527.2-579.5
	Average	58.0	15.0	10.6	7.1	2.1	553.4
IV	Range	20.0-27.0	48.0-55.5	3.5-6.2	8.2-15.2	1.5-2.1	350.1-384.6
	Average	25.3	51.5	4.9	11.6	1.9	366.5
V		41.0	39.0	3.7	10.0	1.4	348.3

conduction calorimeter method with sample cured under 75°F. Heat of solution method was conducted on paste samples cured under 70°F up to one year. Wood (1992) documented the compressive strength for concrete made with different types of cement. The cements have the same chemical and physical properties as the cement listed in Table 4. All concrete samples were cured under 73°F and 100% relative humidity for up to 10 years. The w/cm ranges from 0.40 to 0.61, where "cm" means cement as cementitious material.

4. Test results

4.1. Heat of Hydration

Fig. 1-3 show the effect of the type and replacement level of fly ash and slag on the heat evolution process. As shown in Fig. 1(a), the replacement with slag can significantly reduce the heat generation at the early age. The reduction increases as the slag replacement increases. Note, that the reduction is not proportional to the slag replacement. As the time increases, the difference is reduced due to the Pozzolanic reaction. Fig. 1(b) indicates that blended cement with different slag has different heat of hydration characteristics. At an early age, the generated heat is similar for all three cements containing different slag. After about 20 hours, the cement with slag 2 increases much faster than the other two cements.

Fig. 2 shows that the fly ash has a similar effect on heat evolution process as the slag. However, at the same replacement level, the heat reduction caused by fly ash replacement is smaller than the heat reduction caused by slag. At 7 days, the heat generated by the cements containing 15-45% fly ash is similar to heat generated by the Type I/II cement. Similar to slag, different type of fly ash have different heat curves. The Class F fly ash is less reactive than the Class C fly ash. Even for Class C fly ash, the fly ash with higher CaO content will generate more heat than those with lower CaO. This is consistent with the approach by Schindler and Folliard (2005) that CaO content of fly ash appears to be an indicator of fly ash effect on cement hydration.

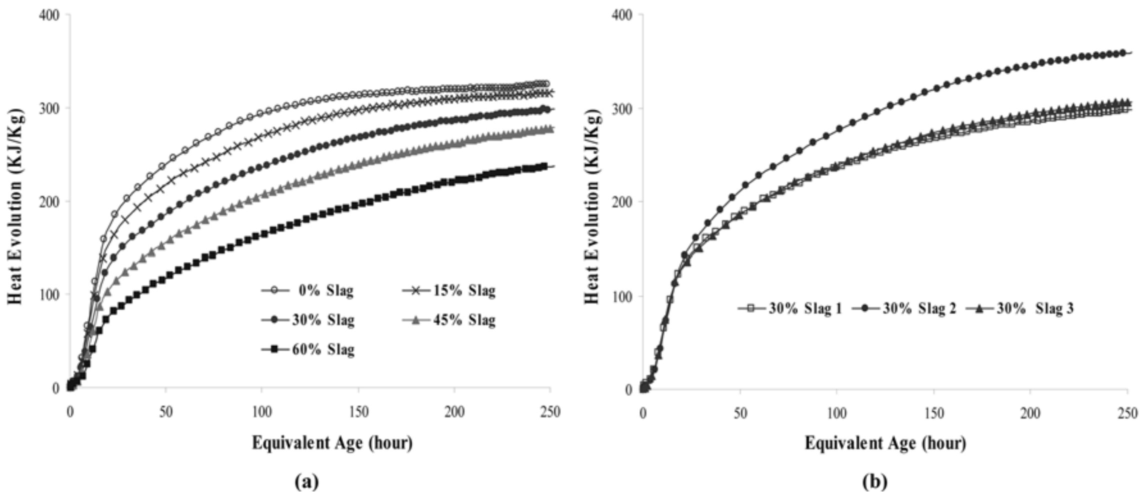


Fig. 1 Effect of slag replacement on heat of hydration. (a) slag replacement level and (b) slag type

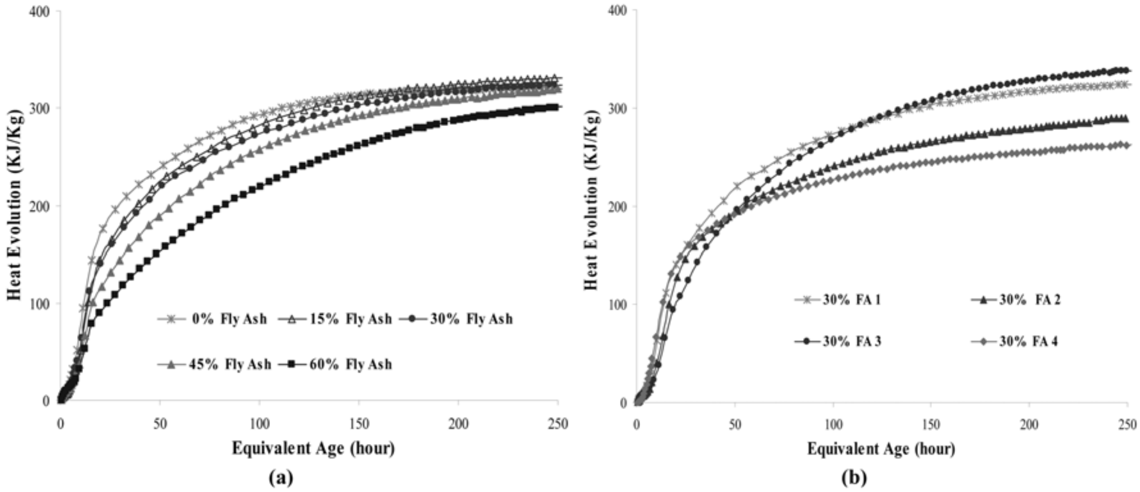


Fig. 2 Effect of fly ash replacement on heat of hydration. (a) fly ash replacement level and (b) fly ash type

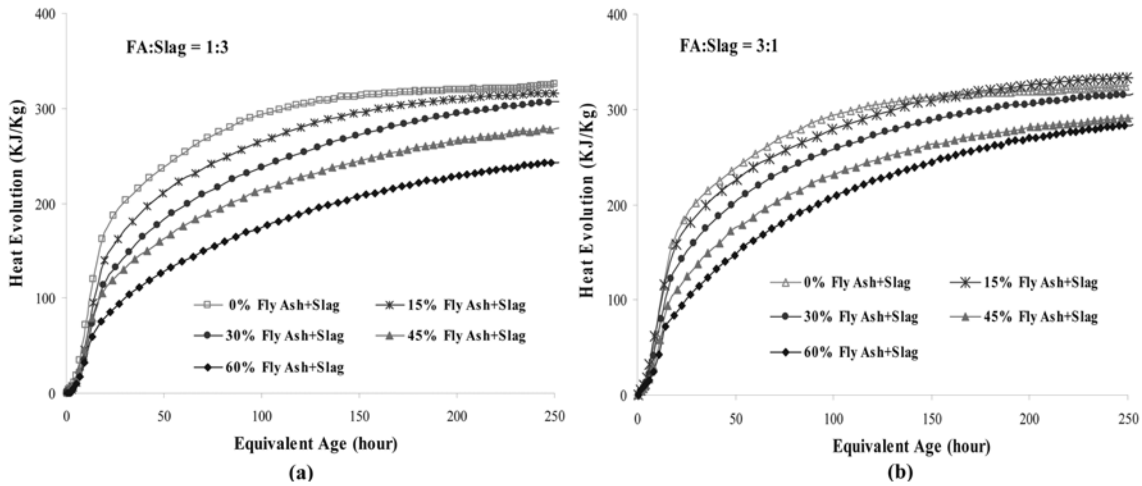


Fig. 3 Effect of fly ash and slag replacement on heat of hydration. (a) fly ash:slag=1:3 and (b) fly ash: slag=3:1

The ternary mix is used to check the interaction between the fly ash and slag. The effect of the slag and fly ash mixture has a similar effect as the fly ash and slag (Fig. 3). When the fly ash dominates, it reacts like the fly ash and vice versa. The generated heat is reduced at early age regardless of fly ash-to-slag ratio. At a later age, the heat of evolution is different for different fly ash-to-slag ratio.

4.2. Compressive strength

Fig. 4-6 show the effect of the type and replacement level of fly ash and slag on compressive strength. The replacement of slag significantly reduces concrete strength at early age (Fig. 4(a)). The strength reduction increases as the slag content increases. At the equivalent age of about10

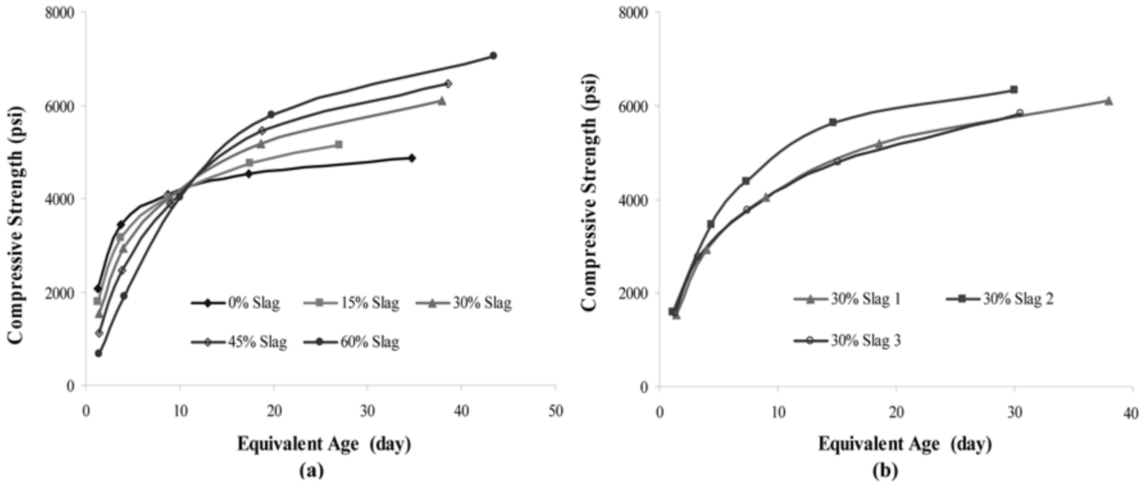


Fig. 4 Effect of slag replacement on strength development. (a) slag replacement level and (b) slag type

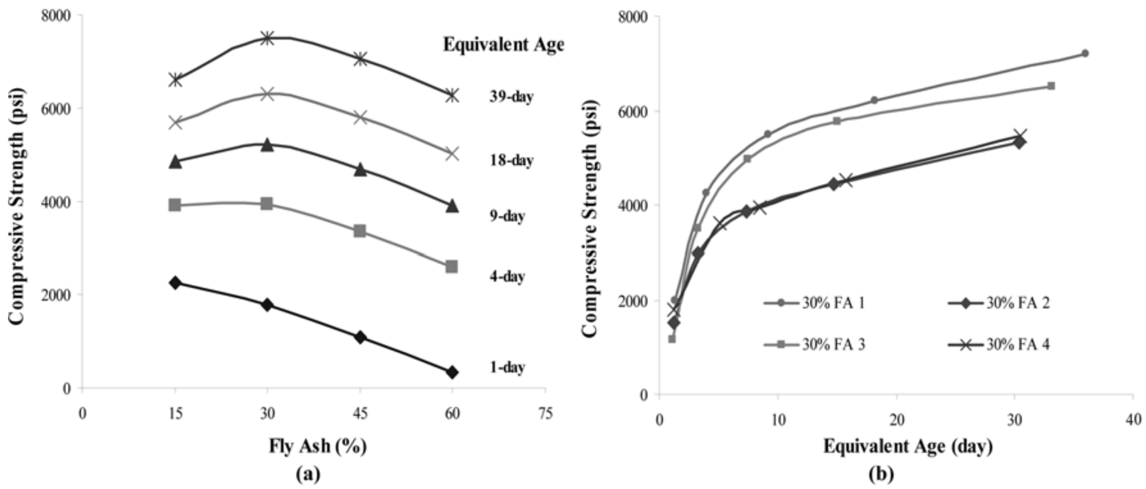


Fig. 5 Effect of fly ash replacement level on strength development. (a) fly ash replacement level and (b) fly ash type

days, all concretes reach a similar compressive strength value. After that, the concrete with slag starts to exceed the concrete without slag. The concrete with 60% slag replacement has the highest compressive strength at the equivalent age of about 40 days. The strength enhancement caused by the slag replacement is due to the Pozzolanic reaction. Fig. 4(b) shows the strength development characteristics of concrete with different types of slag replacement. For the first few days, the compressive strength is similar for all different concrete mixtures. After about 3 days, the concrete containing slag 2 starts to gain strength much faster than concrete containing the other two types of slag. At later age, the strength difference among concrete containing these three types of slag is reduced. Concretes containing slag 2 and 3 have similar compressive strength.

Fig. 5(a) shows the influence of fly ash replacement on concrete strength development. The compressive strength decreases as the replacement of the fly ash increases at 1 day. After 4 days,

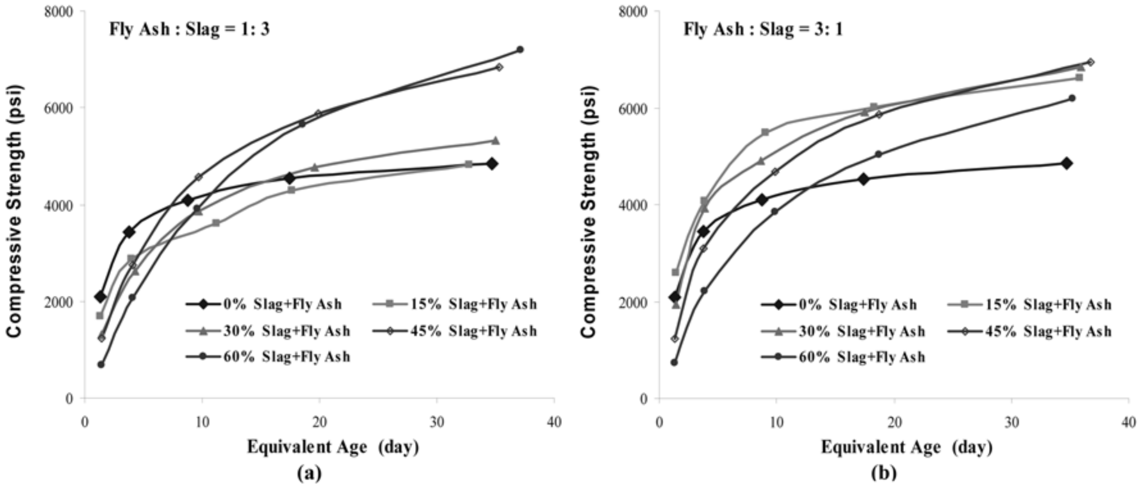


Fig. 6 Effect of fly ash and slag replacement on strength development. (a) fly ash:slag=1:3 and (b) fly ash:slag=3:1

the concrete with 30% fly ash replacement has the highest compressive strength. Compressive strength noticeably decreases as the fly ash content increases beyond 30%. The concrete with 60% fly ash replacement has the lowest compressive strength. Fig. 5(b) indicates that concretes with different types of fly ash have different compressive strength. The difference is mainly caused by their chemical composition, especially CaO content, which is listed in Table 1. Since fly ash 1 and 3 have similar CaO content, they have similar compressive strength. Concrete with fly ash 2 and 4 have similar compressive strengths despite the different chemical composition. This is due to the difference of the air content, which is only 3% for concrete with fly ash 4 and 6.4% for concrete containing fly ash 2. The reduced air content increases the compressive strength. If both of them had the same air content, the concrete with fly ash 4 (Class F) would have lower compressive strength.

The effect of the slag and fly ash mixture on strength is similar to its effect on heat evolution. When the fly ash dominates, it reacts more like the fly ash and vice versa (Fig. 6).

5. Modle development

5.1. General concept and approach

The influence of time and environmental temperature on the development of concrete properties has been studied for several decades. Saul (1951) introduced the term “maturity” that approximates the complex effect of time and temperature on concrete strength development. Later Rastrup (1954) and McIntosh (1956) introduced the concept of equivalent age, which represents the time at a specified temperature that is required to produce maturity equal to the maturity achieved by a curing period at temperature different from the specified temperature. Today, the maturity method is widely used because of its simplicity and low cost. Freiesleben Hansen, and Pedersen (1985), Schindler *et al.* (2005), and Schindler (2002) suggested that the degree of hydration and strength development can be expressed in an exponential form.

$$S = S_u \cdot \exp\left(-\left(\frac{\tau}{t_{eq}}\right)^{\beta}\right) \tag{1}$$

Where, S_u is ultimate limiting strength or ultimate degree of hydration. τ is time parameter. β is slope parameter. t_{eq} is the equivalent age.

In Eq. 1, three parameters, S_u, τ, β, control the process of hydration and strength development. In this study, these three parameters will be modeled according to the test and literature data. The method of model development is following the approaches documented in Schindler and Folliard (2005) and Schindler (2002). The process is divided into three steps:

Step 1: Calculate the equivalent age from the semi-adiabatic heat evolution test, maturity-strength test, and literature data. 70°F (21.1°C) was selected as the reference temperature. The activation energy is an important factor for equivalent age calculation. In this study, the activation model developed by Schindler (2002) is adopted.

Step 2: Performing nonlinear regression analysis to fit each set of the data obtained from Step 1 with Eq. (1) to obtain parameters for each set of data.

Step 3: Establishing the relationship between the parameters in Eq. 1 and the factors that affect concrete hydration and strength development through statistical analysis, which is the model for strength development.

5.2. Heat of hydration model

The term "degree of hydration" is used to describe the extent of the hydration process and defined as the ratio between the amount of cementitious material that has reacted at time t and the original cementitious material amount. It was found that there is an excellent linear relationship between the degree of hydration and heat liberation (Parrot *et al.* 1990, Schindler *et al.* 2005, and Schindler 2002). Therefore, Eq.(1) can be expressed as follows:

$$H(t) = H_{total} \cdot \alpha_u \cdot \exp\left(-\left[\frac{\tau}{t_{eq}}\right]^{\beta}\right) \tag{2}$$

Where, $H(t)$ is the heat generated at time t (kJ/kg). H_{total} is the total hydration heat of the cementitious materials (kJ/kg). α_u is the ultimate degree of hydration.

To determine the total hydration heat of cementitious materials, it is necessary to know the total hydration heat of ordinary Portland cement (OPC), fly ash, and slag. The total heat of OPC is the sum of the specific heat of its chemical compounds. The specific heat of the individual compounds proposed by Lerch and Bogue (1934) are used. These values were also adopted and verified by Schindler *et al.* (2005).

$$H_{cem} = 500P_{C_3S} + 260P_{C_2S} + 866P_{C_3A} + 420P_{C_4AF} + 1186P_{freeCaO} + 850P_{MgO} + 624P_{SO_3} \tag{3}$$

Where, H_{cem} is the total heat of hydration of OPC. P_{C_3S}, P_{C_2S}, P_{C_3A}, P_{C_4AF}, $P_{free\ Cao}$, P_{MgO}, and P_{SO_3} are the weight percentage of each chemical compound of Portland cement.

For the fly ash and slag, only limited data are available. Kishi *et al.* (1994) recommended a total heat generation value of 461 kJ/kg for slag. Schindler *et al.* (2005) demonstrated that this value is appropriate for slag. Therefore, it is adopted in this paper. For fly ash, Schindler *et al.* (2005) define the total heat of hydration of fly ash as $1800 \cdot P_{FA\text{-}CaO}$ (J/g) based on the engineering judgment. Kishi *et al.* (1994) suggested a value of 209 kJ/kg for fly ash with 8.8% CaO. In this study, it is assumed that the ultimate degree of hydration is the same for different types of fly ash. Based on this assumption and the semi-adiabatic test data, the equation by Schindler *et al.* (2005) could be modified as following.

$$H_{total,FA} = 15.9 \cdot (P_{FA-CaO} \times 100) + 74.3 \tag{4}$$

Where, $P_{FA\text{-}CaO}$ is the CaO content of the fly ash. Based on the assumed total heat of fly ash and slag hydration and OPC, the total heat of cementitious materials is now calculated as follows.

$$H_{total} = P_{cem} \cdot H_{cem} + 461 P_{Slag} + P_{FA} \cdot (15.9 \cdot (P_{FA-CaO} \times 100) + 74.3) \tag{5}$$

Where, P_{cem} is the cement content in the cementitious material. P_{slag} is the slag content. P_{FA} is the fly content.

Schindler *et al.* (2005) first had proposed the model for the parameters α_u, τ, and β. However, during the development of that model, no ternary mix had been tested and the effect of type of slag was not considered in the model. The Fig. 1(b) shows that the type of slag has a significant effect on the heat evolution process. In order the account for the effect of slag type, the hydraulic index (HI), proposed by Mantel (1994), is calculated considered.

$$HI = \frac{P_{CaO} + P_{MgO} + P_{Al_2O_3}}{P_{SiO_2}} \tag{6}$$

Where, P_{Cao}, $P_{Al_2O_3}$, and P_{SiO_2} are the weight percentage of CaO, Al_2O_3, and SiO_2 of slag.

By adding the effect of the ternary mix and slag type, the model proposed by Schindler *et al.* (2005) is modified as followings:

$$\tau = 2.649 \cdot P_{C_3S}{}^{-0.541} \cdot P_{C_3A}{}^{-0.122} \cdot P_{SO_3}{}^{-1.191} \cdot Blaine^{-0.567} \cdot \exp(3.018 \cdot P_{slag} + 8.365 \cdot P_{FA} \cdot P_{FA-CaO}) \tag{7}$$

$$\beta = P_{C_3S}{}^{0.280} \cdot P_{C_3A}{}^{0.143} \cdot P_{SO_3}{}^{1.378} \cdot Blaine^{-0.994} \cdot \exp(-9.210 \cdot P_{slag} \cdot (1 - 0.568 \cdot HI) + 11.272) \cdot (1 - 0.519 \cdot P_{FA}) \tag{8}$$

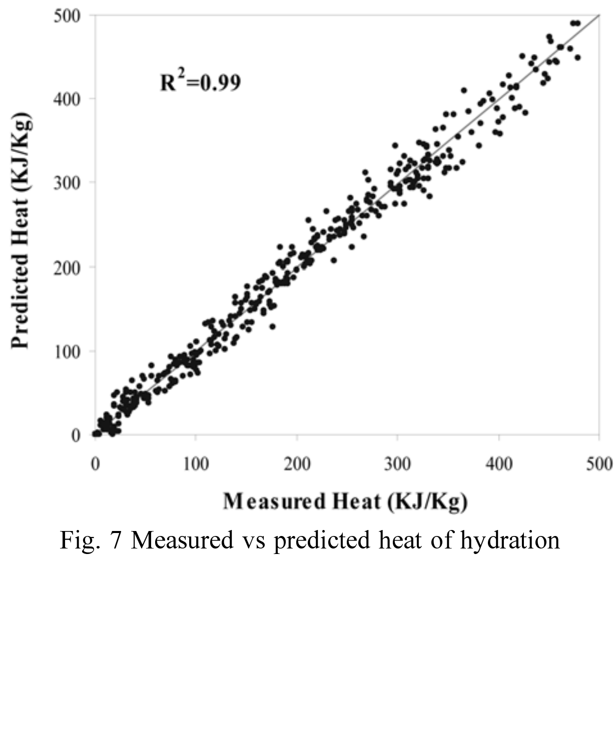


Fig. 7 Measured vs predicted heat of hydration

$$\alpha_u = \frac{1.031 \cdot w/cm}{0.194 + w/cm} + 0.361 \cdot P_{FA} + 4.285 \cdot P_{slag} \cdot (0.730 \cdot HI - 1) \leq 1 \quad (9)$$

Where, *w/cm* is water-to-cementitious material ratio.

The scatter plot of the predicted heat of hydration from the proposed model versus the experimentally measured values is shown in Fig. 7. The figure shows that the predicted values are very close to the measure value. A high R^2 value, 0.99, is achieved for the overall prediction, indicating that 99% of the variation in the measured values can be explained by the proposed model. Therefore, the model is fairly accurate.

5.3. Maturity-strength model

5.3.1. Factors considered in strength development

Concrete compressive strength increases exponentially at early age. After a few days, the rate of the strength gain becomes slower but the strength gain continues as long as water can reach the unhydrated cement particles. Numerous factors can affect the way that concrete strength develops. The factors considered in the developed model include cement chemical and physical properties, w/

Table 5 Estimated parameters for laboratory test

Fly Ash (%)	Slag (%)	w/cm	S_u (psi)	τ (hour)	β
0	0	0.42	6158	32.262	0.483
15	0	0.42	9081	56.981	0.403
30	0	0.41	10452	74.277	0.439
45	0	0.41	8629	79.868	0.599
60	0	0.41	7377	99.254	0.720
0	15	0.42	7344	59.919	0.446
0	30	0.42	8336	91.680	0.502
0	45	0.42	8672	121.894	0.592
0	60	0.42	9290	164.482	0.717
3.75	11.25	0.42	6578	56.833	0.426
7.50	22.50	0.42	7973	106.680	0.457
11.25	33.75	0.42	10792	161.275	0.492
15	45	0.42	11191	232.000	0.589
11.25	3.75	0.41	8311	42.698	0.512
22.50	7.50	0.41	9890	83.334	0.420
33.75	11.25	0.41	10490	137.498	0.488
45	15	0.40	10370	215.578	0.496
30 (FA-2)	0	0.41	11352	221.413	0.318
30 (FA-3)	0	0.41	9540	138.056	0.341
30 (FA-4)	0	0.41	9372	82.297	0.516
0	30 (Slag-2)	0.42	8512	96.174	0.447
0	30 (Slag-3)	0.42	8599	74.967	0.538

cm, type and dosage of fly ash and slag, curing conditions, and air content.

5.3.2. Nonlinear regression analysis

For each set of the maturity-strength result, the nonlinear regression analysis was performed to obtain S_u, τ, and β in Eq. (1). The estimated values through the nonlinear regression for each set of data are summarized in Table 5 and 6 for the laboratory test and literature data, respectively. Table 5 shows that the ultimate strength, S_u, increases continuously as the slag replacement level increases until 60% replacement. However, for concrete with fly ash, the S_u first increases and then decreases as the replacement level increases beyond 30%. The strength increase is mainly caused by the Pozzolanic reaction. The reactive silicate of the fly ash and slag reacts with the calcium hydroxide (CH), which is formed by cement hydration, and forms calcium silicate hydrate (CSH) gel, which is every efficient in filling up large capillary spaces. Both the CH consumption and CSH gel production can increase concrete strength. Maekawa *et al.* (1999) stated that the CH consumption for a unit weight of fly ash and slag is different. Sufficient supply of CH can apply to slag reaction up to 60-70%, but only 30% for fly ash reaction. Therefore, the ultimate strength starts to decrease for cement containing more than 30% fly ash. Unlike the replacement level trend, the S_u is almost the same for different types of fly ash and slag. The time parameter τ increases when SCM was added. The larger the τ, the slower the early age strength development is. The slope parameter β decreases slightly and then increases as the SCM levels increase. Both τ and β depend on the type of SCMs.

Table 6 summarizes the estimated parameter values for the literature data set (wood 1992). Types II, IV, and V cement have higher S_u values than Types I and III cement for both w/cm. Type III cement has the lowest τ values, meaning that it hydrates fast at an early age. On average, Type IV cement has the highest τ values, which are 394 for a w/cm of 0.55 and 237 for a w/cm of 0.42. The β values are not significantly influenced by w/cm for all types of cement. Table 6 also shows that w/cm can affect S_u and τ. The S_u values decrease as the w/cm increases. The τ values, on the other hand, increase as the w/cm increases.

5.4. Modeling of concrete strength parameters

Similar to the heat of hydration model, strength parameters, S_u, τ, and β can also be expressed as functions of the chemical compositions of cement, fly ash, and slag and also the replacement level of fly ash and slag.

The multivariate analysis was performed to identify the factors that are related to the strength parameters, S_u, τ, and β. After the initial analysis of literature data, it was found that ultimate strength is related to weight percentage of C_2S (P_{C2S}), w/cm, and air content; time parameter τ is related to w/cm, P_{C3S}, P_{C3A}, and P_{SO3}, and the Blaine value; and slope parameter β is related to $P_{C3S+C2S}$ and P_{C3A}, and the Blaine value. The results in Table 5 reveal that the strength parameters, S_u, τ, and β, are related to fly ash and slag replacement level. The time and slope parameters, τ and β, are also affected by the type of the fly ash and slag. Thus, strength parameters can be expressed as follows:

Ultimate Strength S_u

$S_u = f\ (w/cm, Air, P_{C2S}, P_{FA}, P_{slag})$

Table 6 Estimated strength parameters for literature data (Wood, 1992)

Cement Type	Cement No.	w/cm	S_u (psi)	τ (hour)	β	w/cm	S_u (psi)	τ (hour)	β
Type I	11	0.56	6952	79.853	0.609	0.41	9075	70.084	0.609
	12	0.55	7163	104.572	0.677	0.41	9448	75.014	0.681
	13	0.57	6968	151.420	0.501	0.42	8950	98.837	0.509
	14	0.55	6946	108.629	0.531	0.41	9112	89.313	0.533
	15	0.59	6306	48.139	0.751	0.45	7974	39.955	0.757
	16	0.56	6791	95.146	0.553	0.41	8824	63.677	0.535
	17	0.61	6176	97.579	0.587	0.46	8506	81.529	0.575
	18	0.58	6692	94.817	0.664	0.49	8501	76.389	0.629
	Average		6749	97.519	0.609		8799	74.350	0.604
Type II	21	0.56	7966	263.110	0.489	0.40	9833	168.053	0.508
	22	0.54	7140	158.599	0.578	0.41	9665	100.840	0.585
	23	0.54	7345	138.479	0.456	0.42	9167	120.460	0.461
	24	0.56	6492	144.700	0.497	0.42	8320	102.699	0.502
	25	0.57	7185	210.285	0.491	0.42	9514	179.800	0.46
	Average		7226	183.035	0.502		9300	134.370	0.503
Type III	31	0.6	6360	43.074	0.479	0.49	7836	40.170	0.484
	33	0.55	6798	31.517	0.48	0.43	7853	23.981	0.446
	Average		6579.1	37.300	0.480		7844	32.076	0.465
Type IV	41	0.53	7928	482.917	0.472	0.40	10436	227.054	0.479
	42	0.55	7963	426.764	0.432	0.42	10914	289.766	0.427
	43	0.54	7550	273.460	0.498	0.42	9506	193.475	0.48
	Average		7814	394.380	0.467		10285	236.765	0.462
Type V	51	0.54	8024	268.634	0.429	0.41	10239	138.489	0.426

Time Parameter τ
τ = f (w/cm, P_{C3S}, P_{C3A}, P_{SO_3}, Blaine, P_{FA}, $P_{FA\text{-}CaO}$, P_{Slag}, HI)
Slope Parameter β
β = f ($P_{C3S+C2S}$, P_{C3A}, Blaine, P_{FA}, $P_{FA\text{-}CaO}$, P_{Slag}, HI)

Where, *Air* is air content (%). w/cm is water-to-cementitious material ratio, where “cm” means cement or cement plus fly ash and/or slag as cementitious material.

With the identified parameters, regression analysis was performed for the literature and experimental test data. It is found that there is a linear relationship between the logarithm of the strength parameters and cement properties. However, the fly ash and slag percentage have a quadratic effect on the strength parameters. The small P-values, which are “the probability of observing data as or more extreme as the actual outcome when the null hypothesis is true” (Dallal 2008), obtained for each independent variable (less than 0.05) indicate that these relationships are statistically significant. After that, all the data are combined to develop the final model. The nonlinear model is fitted using the least square function. Iterative methods are used to search for the least-squares estimates. The Iterations stop when one of the three criteria--objective change, parameter change, and gradient--is met. The best fitted model is listed in Eq. (10) to (12).

$$S_u = \exp(-1.810 \cdot w/cm + 10.143) \cdot (1 - 0.051 \cdot Air) \cdot (P_{C_2S}{}^{0.143} + 2.155 \cdot P_{FA} - 3.544 \cdot P_{FA}{}^2 + 0.711 \cdot P_{Slag} - 0.367 \cdot P_{Slag}{}^2) \quad (10)$$

$$\tau = \exp[-4.462 \cdot (P_{FA-CaO} - 0.2711) - 2.147 \cdot (HI - 1.519)] \cdot [\exp(3.401 \cdot w/cm - 2.709) \cdot p_{C_3S}{}^{-1.659} \cdot P_{C_3A}{}^{-0.378} \cdot P_{SO_3}{}^{-1.154} \cdot Blaine^{-0.229} + 120.306 \cdot P_{FA} + 190.939 \cdot P_{Slag} + 1221.717 \cdot P_{FA} \cdot P_{Slag}] \quad (11)$$

$$\beta = \begin{bmatrix} \exp(3.039) \cdot (P_{C_3S} + P_{C_2S})^{-0.653} \cdot P_{C_3A}{}^{0.303} \cdot Blaine^{-0.525}) \\ -0.463 \cdot P_{FA} + 1.377 \cdot P_{FA}{}^2 - 0.427 \cdot P_{Slag} + 1.392 \cdot P_{Slag}{}^2 \end{bmatrix} \cdot \exp[1.993 \cdot (P_{FA-CaO} - 0.2711) + 0.924 \cdot (HI - 1.519)] \quad (12)$$

For Eq. (11) and (12), the $P_{FA\text{-}CaO}$ value is 0.2711 when there is no fly ash added. The *HI* value is 1.519 without slag.

The predicted values and experimentally determined values for each parameter are plotted in Fig. 8 to 10. These figures indicate that the proposed Eq. (10) to (12) accurately predict the strength parameters by considering the effect of the w/cm, air content, cement chemical composition, cement fineness, SCMs, chemical compositions of SCMs, and thermal activity of the cementitious material. The R^2 values for S_u, τ, and β are 0.955, 0.959, and 0.852 respectively, meaning that over 95% variation of the experimentally determined S_u, τ values can be explained by the proposed model, Eq. (10) and (11). The R^2 value of β is lower than the other two parameters, which may indicate that other variables not included in Eq. (12) may also influence the parameter β.

The scatter residual plots, which plot the residual versus the parameter, are shown in Fig. 11. These figures show that the residuals are randomly distributed. No apparent patterns for residuals

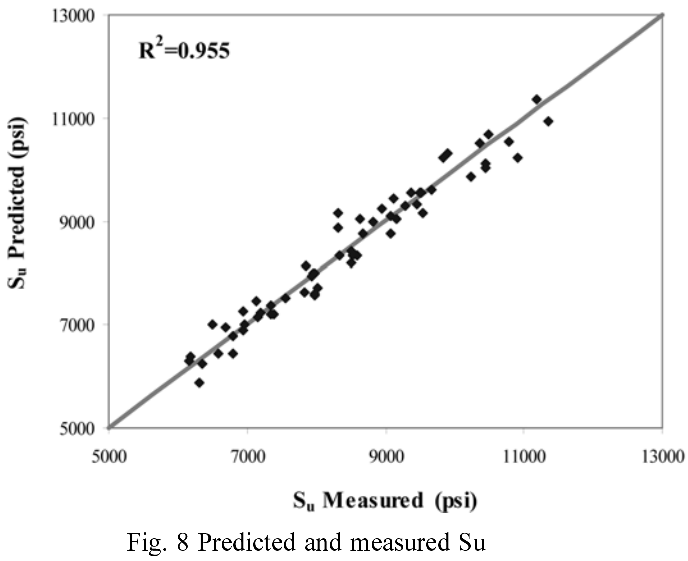


Fig. 8 Predicted and measured Su

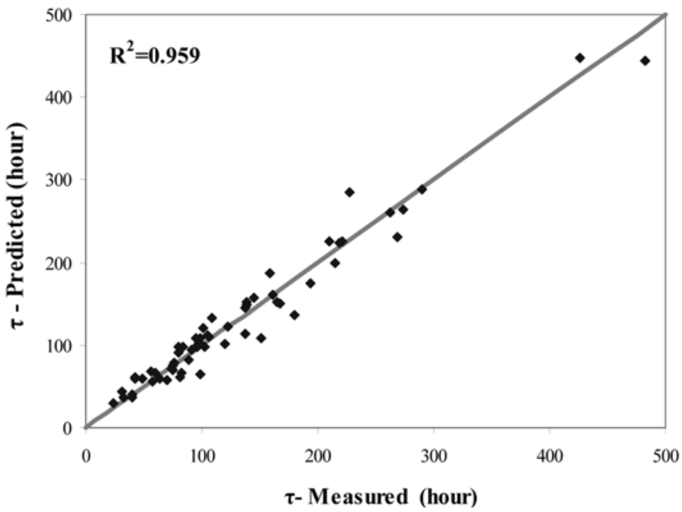


Fig. 9 Predicted and measured τ

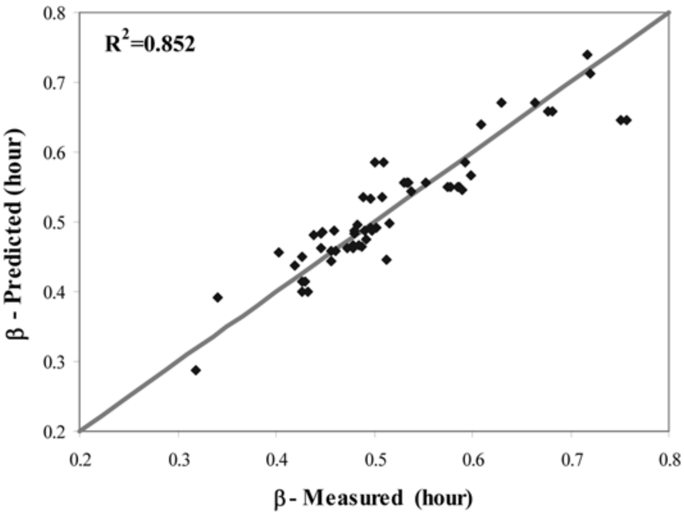


Fig. 10 Predicted and Measured β

with strength parameters are observed. There are no apparent outliers in the plot. The residues of the Su are within 1,000 psi and less than 10% of the test-estimated S_u values.

5.5. Modeling results

The strength prediction from this modified maturity-strength model is presented in Fig. 12 with an R^2 value of 0.975. Fig. 13 (a) and (b) show the comparison of the predicted and measured strength-tested maturity-strength results from Wood (1992) and model-fitted results for typical Types I, II, III, IV, and V cement concrete with different w/cm. Both figures show that the proposed model can predict strength development with maturity for various types of cement and different w/cm.

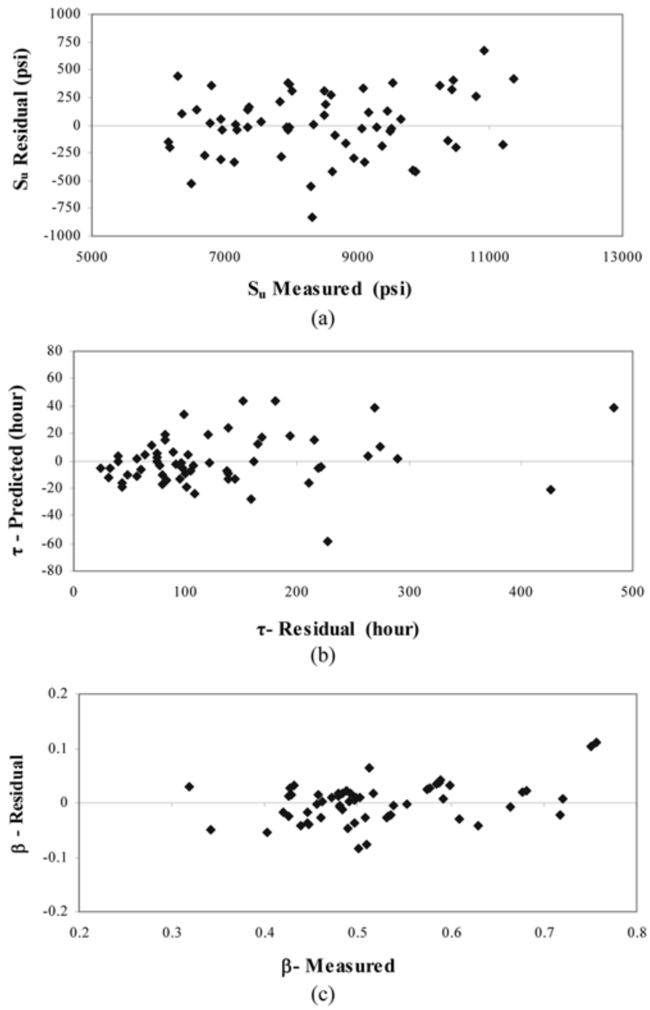


Fig. 11 Residual vs strength parameters

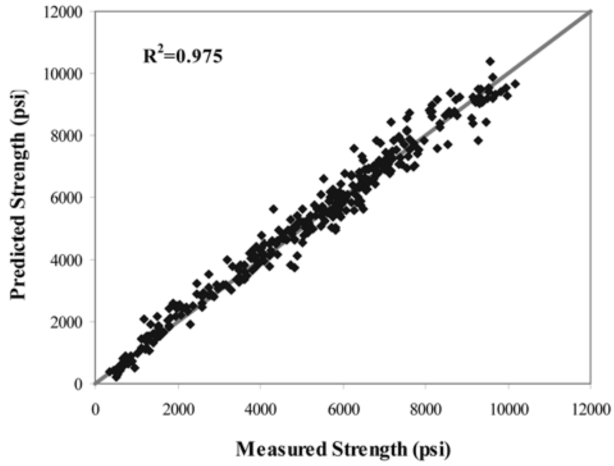
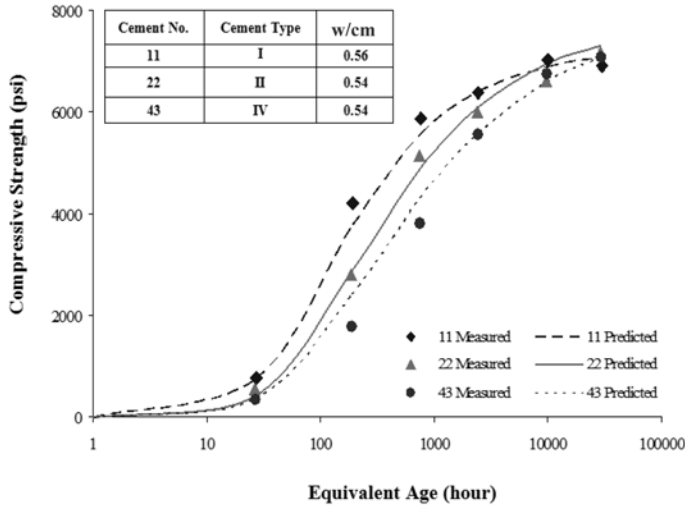
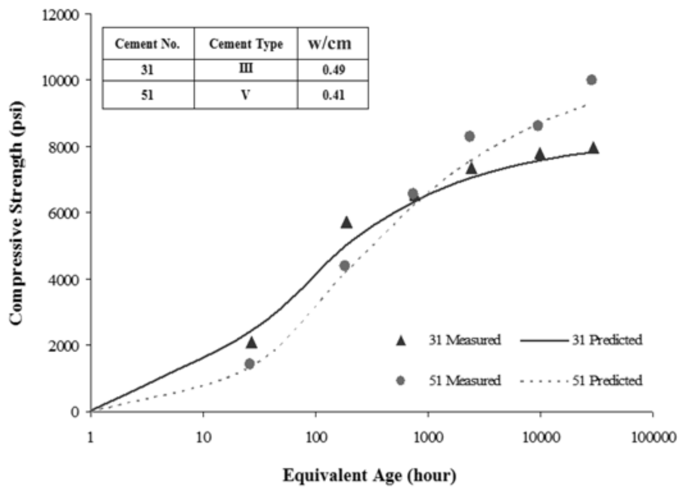


Fig. 12 Measured vs predicted strength



(a) Types I, II, and IV Cement



(b) Types III and V Cement

Fig. 13 Measured vs predicted strength, types III and V cement

Fig. 14 to 16 present the predicted results for blended cement concrete containing different amounts of fly ash and slag. The predicted results indicate that the proposed model is able to predict strength development for blended cement concrete containing different levels of fly ash, slag, and mixtures of fly ash and slag. The difference between the measured and predicted strength is fairly small.

5.6. Sensitivity analysis

In addition to predicting concrete strength development, a reliable strength model can be used for sensitivity analysis of the factors that affect concrete strength. The results from such sensitivity

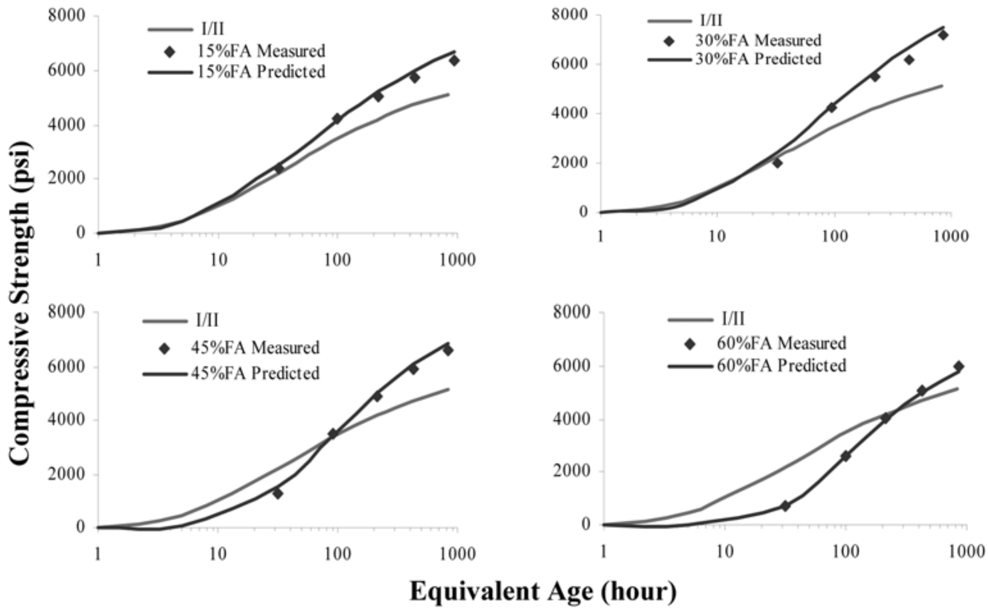


Fig. 14 Measured vs predicted strength for type I/II cement with different levels of fly ash replacement (w/cm = 0.42)

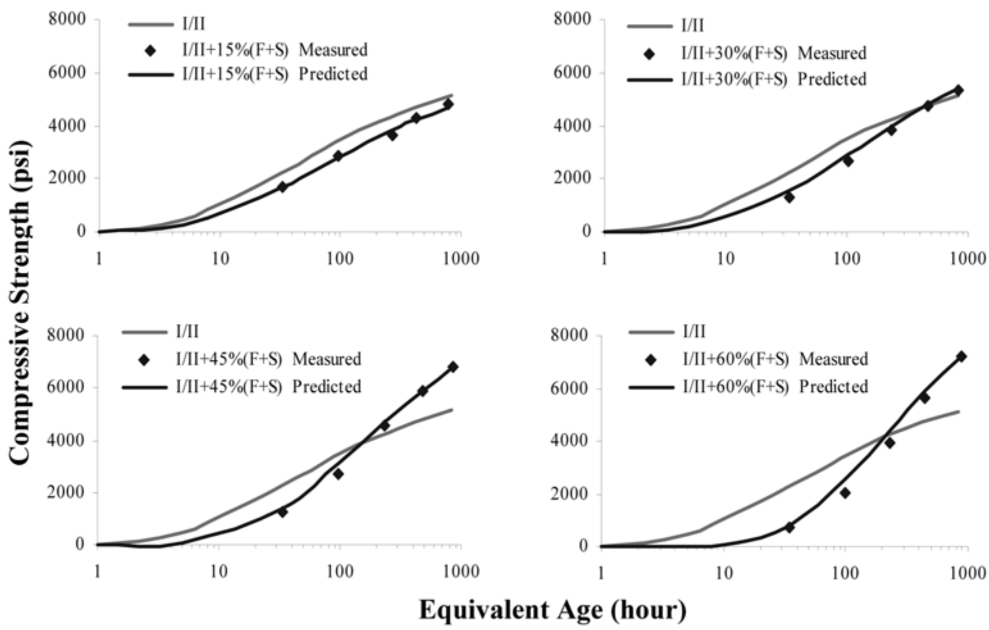


Fig. 15 Measured vs predicted strength for type I/II cement with different levels of slag and fly ash replacement (w/cm = 0.42; slag : FA = 3:1)

analysis can provide engineers with insight into concrete mix design modification and field temperature management. The sensitivity analysis is performed by varying only one parameter at a time. During the sensitivity analysis, three values, including typical, high, and low values for normal

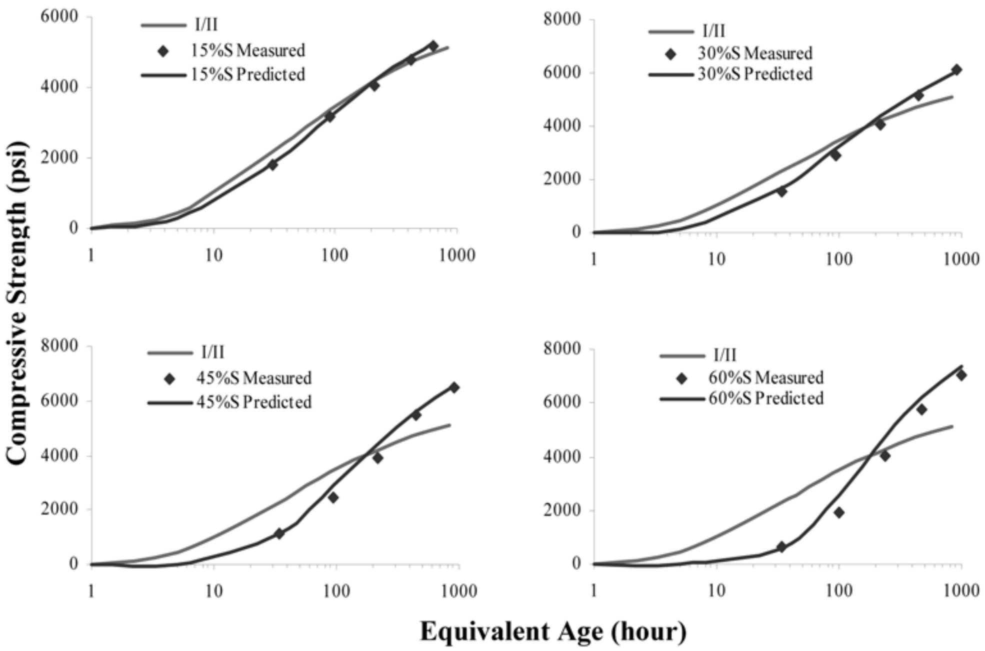


Fig. 16 Measured vs predicted strength for type I/II cement with different levels of slag replacement (w/cm = 0.42)

concrete, are selected. The difference between the typical and low/high values is calculated. The results of the sensitivity analysis are tabulated in Table 7.

The data show that the effect of each factor varies with time. On average, cement type has greater influence on strength at early age than on strength at later age. Type III cement has the highest strength at early age and lowest strength at later age.

The w/cm strongly influences strength. The lower the w/cm, the higher the strength is. When the w/cm decreases from 0.45 to 0.3, the strength increases about 49% at early age and 33% at 1 year. The strength increase is caused by the reduced porosity and increased IT zone strength (Mehta 1986).

The influence of air content is constant regardless of age. Table 7 shows that the lower the air content, the higher the compressive strength. According to the model, Eq. 10, the loss of concrete due to voids is about 5.1% for every 1% increase in air content. This is consistent with the value of 5.5% strength loss suggested by Mindess and Young (1981).

Cement with a high Blaine value gains higher strength at early age and lower strength at later age. When compared with other factors, the influence of the Blaine value is smaller.

The fly ash level and CaO content of fly ash significantly influences strength development. The influence of fly ash replacement level increases as time increases. The strength increases 15% at 7 days when fly ash increases from 0 to 30%. This value becomes 37% at 1 year. On the other hand, the influence of the CaO content in fly ash decreases as time increases. The strength values indicate that CaO is an effective means of differentiating the effect of different types of fly ash.

Similar to fly ash, the influence of slag replacement level increases as concrete age increases. Concrete containing slag has lower strength at early age and high strength at later age. The

Table 7 Summary of the sensitivity analysis

Factors	Range	Compressive Strength, S (psi)								
		7 Days			28 Days			1 Year		
		S	ΔS*	ΔS/S (%)	S	ΔS	ΔS/S (%)	S	ΔS	ΔS/S (%)
Cement Type	I	3939	0	0	5202	0	0	6135	0	0
	II	3399	-541	-14	4770	-433	-11	6042	-93	-2
	III	3705	307	8	4709	-61	-2	5587	-456	-12
	IV	2157	-1549	-39	3935	-774	-20	6035	448	11
	V	2935	-1004	-25	4268	334	8	5834	-201	-5
w/cm	0.45	3939	0	0	5202	0	0	6135	0	0
	0.30	5888	1949	49	7217	2015	39	8142	2008	33
	0.60	2512	-1427	-36	3674	-1528	-29	4603	-1532	-25
Air Content	6	3939	0	0	5202	0	0	6135	0	0
	4	4518	579	15	5967	765	15	7036	902	15
	9	3071	-868	-22	4055	-1147	-22	4782	-1352	-22
Blaine	350	3939	0	0	5202	0	0	6135	0	0
	289	4027	88	2	5342	140	3	6204	70	1
	580	3779	-160	-4	4934	-268	-5	5967	-168	-3
FA%	0	3939	0	0	5202	0	0	6135	0	0
	15	4541	602	15	6325	1122	22	7804	1669	27
	30	4547	607	15	6663	1461	28	8408	2273	37
CaO% of FA	10	3137	0	0	5005	0	0	7369	0	0
	1.5	2717	-420	-13	4336	-669	-13	6712	-657	-9
	28	4642	1505	48	6752	1747	35	8446	1077	16
Slag%	0	3939	0	0	5202	0	0	6135	0	0
	30	3624	-315	-8	5616	414	8	7292	1157	19
	60	3231	-708	-20	6436	1233	22	8419	2285	31
HI of Slag	1.5	3543	949	27	5534	838	15	7255	312	4
	1.7	4492			6372			7567		

*Strength difference

strength is reduced 20% at 7 days as 60% of the cement is replaced by slag. However, the strength for concrete containing 60% slag is 31% higher than the OPC concrete. Table 7 also shows that concrete containing slag with different HI has different strength development properties. Therefore, the HI of slag could be an effective means of differentiating the effect of different types of fly ash.

6. Conclusions

This paper presents the development of modified heat of hydration and maturity-strength models for concrete containing fly ash and slag. Twenty-two sets of test data of different concrete mixtures, including different types of fly ash and slag, and literature data containing different types of cement were used in the modeling. The modeling results show that the modified models are able to predict heat evolution and compressive strength development accurately for concrete made with different cementitious materials. The R^2 for the overall prediction are 0.99 and 0.975 for the hydration and strength model respectively.

Sensitivity analysis for concrete compressive strength was performed for the maturity-strength model to analyze the effect of very single variable in the modified maturity-strength model. The results show that the influence of each factor varies with time. The Blaine value has a smaller influence on concrete strength compared with other factors. The w/cm, air content, fly ash, and slag content significantly influence concrete strength at both early and later age.

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