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Seismic damage estimation through measurable dynamic characteristics

N. Lakshmanan†

Structural Engineering Research Centre, CSIR Campus, Taramani, Chennai 600113, India

B. K. Raghuprasad‡

Department of Civil Engineering, Indian Institute of Science, Bangalore 560012, India

K. Muthumani‡†, N. Gopalakrishnan‡‡ and R. Sreekala‡†

Structural Engineering Research Centre, CSIR Campus, Taramani, Chennai 600113, India

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Abstract. Ductility based design of reinforced concrete structures implicitly assumes certain damage under the action of a design basis earthquake. The damage undergone by a structure needs to be quantified, so as to assess the post-seismic reparability and functionality of the structure. The paper presents an analytical method of quantification and location of seismic damage, through system identification methods. It may be noted that soft ground storied buildings are the major casualties in any earthquake and hence the example structure is a soft or weak first storied one, whose seismic response and temporal variation of damage are computed using a non-linear dynamic analysis program (IDARC) and compared with a normal structure. Time period based damage identification model is used and suitably calibrated with classic damage models. Regenerated stiffness of the three degrees of freedom model (for the three storied frame) is used to locate the damage, both on-line as well as after the seismic event. Multi resolution analysis using wavelets is also used for localized damage identification for soft storey columns.

Keywords: seismic damage indicators; wavelet analysis; soft/weak storey structure.

1. Introduction

Damage caused to a civil structure during a seismic event needs to be quantified, so that future worthiness of the structure for performing its functions can be estimated. This shall also help in the estimation of repair and rehabilitation cost, as compared to replacement. The damage is stated in the form of an index, termed as damage index (DI). The conventional damage index require the hysteretic loop of the structure and its components and may not be readily available. Hence recourse can be taken to certain post-earthquake measurements like time period (this should be available in

† Director
‡ Professor
‡† Scientist
‡‡ Scientist, Corresponding Author, E-mail: ng@sercm.org

the pre-earthquake scenario also), plastic drift and crack-width, so that the damage index can be predicted. Hence a suitable calibration exercise has to be done to bench-mark conventional DI with those derived using measured change in time-periods of a structure in its various modes. The paper makes an attempt in the direction of DI-calibration with post-earthquake measurable parameter like time period. Damage sensitive open first storied, three storied reinforced concrete frame structure is subjected to non-linear dynamic analysis and compared to a normal and weak first storied structure. Available Di-Pasquale-Cakmak model, which defines damage based on initial and final time periods is modified through introduction of an exponent for the time period ratio (other than one) and fitted with the famous Park-Ang model. Future study shall involve suitable experimental validation conducted by authors as well as available in open literature.

2. Causative factors for seismic damage

Contemporary structures are designed in such a way that when the design earthquake occurs, they should respond in-elastically and are expected to sustain a controllable amount of damage. Codal provisions which stipulate the aseismic design procedures of structures are constantly re-written based on the knowledge gained from the most recent earthquakes. Thus structures designed using an older version of the code is unlikely to be safe with the current provisions.

Factors that affect the degree of damage to structures during an earthquake are wide and varied. Inaccurate determination of input acceleration and the frequency content of the response spectrum is the foremost reason, because of which a structure may suffer seismic damage. (Mexico city earthquake-1985 and the Kalamata earthquake-1986). There are a number of structural characteristics, which contribute towards their vulnerability (Penelis and Kappos 1997). Failures of RC buildings are invariably caused due to the failure of structural columns, which are the primary load carrying members, under actions of combined flexure, axial forces and shear forces. Column failure could be either due to the high strength and stiffness degradation, accentuated by the lack of proper ties at the critical regions of the columns. Short columns are known to fail by explosive shear failure, though such a phenomenon is rare. Plan wise un-symmetric arrangement of the primary load carrying members causing different seismic demands for different frames is another major reason for the seismic damage. Vertical irregularity of the buildings with an un-symmetry in elevation is also a major cause for damage.

Sudden reduction of stiffness at a particular floor level of a building causes dis-continuity in force and displacement flow and causes stress concentration in floor slabs. Open ground floor kept for the purpose of parking of vehicles is a typical example of a soft ground storied structure. Upper floors have more stiffness due to in-filled masonry, whose stiffness and strength are uncertain and degrading. The damage of such structures is concentrated in the ground floor and none of the other elements show any sign of damage. This is in sharp contrast to the capacity design concept of structures, wherein ductility and energy dissipation is primarily contributed by flexural elements and not through the structural actions of compression dominated columns or other shear dominated elements. Sudden reduction of strength also could be a cause for concern and these weak storied structures, though open throughout can exhibit a failure closer to a soft ground storied structure. Structures with weak beam and strong column, weakness and strength defined in terms of the moment capacity of the beam and column meeting at a particular joint, may also show a weak first storey failure.

3. Damage quantification and damage indices

It is to be noted that the idea of damage quantification is complex and subjective. However it is required to quantify the damage so as to take a suitable decision on the improvement of the structure commensurate with the available funds for rehabilitation, vis-a-vis the cost of re-building. Authors are part of the team, constituted after the Bhuj earthquake of 2001 for certifying structures for immediate occupancy, occupancy after minor repair, occupancy after major repair and demolition and re-building of the structure. In all these circumstances, it is required to define the state of damage undergone by a structure through a suitable non-dimensional index, whose value is zero if un-damaged and 1.0 if fully damaged. Such an index called as damage index (DI) is an attempt towards quantification of seismic damage. The damage indices are differentiated depending upon the level at which they are evaluated, namely local or global. Following reasons are attributed to the importance of seismic damage indices, in addition to retrofit decision making (Williams and Sexsmith 1995, CEB 1996).

(a) Indices can help the city planners, to predict the likely cost of earthquake, number of casualties and the amount of temporary accommodation needed.
(b) To assess the vulnerability of the structure to after-shocks.

A comprehensive and excellent coverage on the seismic damage indices for concrete structures is the state-of-the art paper by Williams and Sexsmith (1995).

Earliest damage index is based on ductility and is defined as the ratio of actual ductility reached to the maximum ductility capacity of the member. Similarly inter storey drift ratio could also be a damage index. Banon, *et al.* (1981) have suggested a flexural damage ratio (FDR) in lieu of damage index, which is the ratio of secant stiffness at some load level to the secant stiffness at virgin state. Roufaiel and Meyer (1987a, 1987b) have modified the ratio as,

$$FDR = \frac{\dfrac{1}{k_m} - \dfrac{1}{k_0}}{\dfrac{1}{k_f} - \dfrac{1}{k_0}} \tag{1}$$

Where, k_0 , k_m and k_f are the initial, intermediate and failure stage secant stiffness of the member. All the above definitions are good for static loading but may not represent the true damage under cyclic load effects. Accumulated effects of the damage could be best modeled using low-cycle fatigue formulations with the accumulated plastic deformation or the energy dissipated per cycle as the primary state variable.

Many of the models based on low-cycle fatigue formulation are based on Miner's rule which is stated as,

$$DI = \sum \frac{n_i}{n_{f,i}} \tag{2}$$

Where, n_i is the number of cycles at load i , and n_{fi} is the number of cycles to failure for the same load.

Chung, *et al.* (1987, 1989, 2006) have used a fatigue based formulation, wherein the moment strength reduction at the 'i-th' load level is defined as,

$$\Delta M_i = \Delta M_f \left[\frac{\phi_i - \phi_y}{\phi_f - \phi_y} \right]^{1.5} \tag{3}$$

ϕ_y, ϕ_i, ϕ_f are the yield, instantaneous and failure curvatures. The power of 1.5 means that large plastic deformations are much more damaging than small ones. Moment curvature failure envelope is also defined as,

$$M_{f,i} = M_f \frac{2\Phi_i}{\Phi_i + 1}; \Phi_i = \frac{\phi_i}{\phi_f} \tag{4}$$

Failure is deemed to have occurred, when the degraded moment−curvature curve intersects the failure envelope. For cycling at constant amplitude, number of cycles to failure is simply,

$$n_{f,i} = \frac{(M_i - M_{f,i})}{\Delta M_i} \tag{5}$$

The widely used and a popular damage index definition is due to Park and Ang (1985a), Park, *et al.* (1985b, 1987). This consists of a linear combination of a normalized deformation and energy absorption.

$$DI = \frac{\delta_m}{\delta_u} + \beta_e \frac{\int dE}{F_y \delta_u} \tag{6}$$

Where δ_m and δ_u are the instantaneous and ultimate displacements. dE is the incremental irrecoverable energy, F_y is the yield force and β is a constant to account for different types of structural elements.

In the above expression, the first term is due to the damage incurred due to pseudo-static displacement and the second term is due to cumulative energy loss. New version of IDARC (Kunnath and Reinhorn 1990, Valles, *et al.* 1996) modifies this expression and uses it as follows,

$$DI = \frac{\phi_m - \phi_y}{\phi_u - \phi_y} + \beta_e \frac{\int dE}{M_y \phi_u}. \tag{7}$$

Where, ϕ_y, ϕ_m and ϕ_u are the yield. Intermediate and ultimate curvatures, M_y is the yield moment and β_e is a constant. Park, *et al.* (1985) gives D=0.4 as a threshold value between repairable and irrepairable damages.

Park, *et al.* (1987) suggest the following detailed classification (Table 1),

Damage indices, mentioned earlier, are local ones and a global damage index is a weighted addition of the individual element index. The weights could be energy absorbed by each element. Rao, *et al.* (1998) have proposed a modified Park-Ang model based on the cyclic tests on normal and laced RC beams. The damage index has been defined as a simple linear relation in terms of monotonic and cyclic ductility indices and dissipated energy under both static and cyclic loading.

It is seen that nearly all the previously mentioned indexes are ideal in analytical computation, whereas in a practical building some of the parameters (like moment and curvatures) generated in

Table 1 Damage index vs damage description

Damage Index (D)	Damage Description
$D < 0.1$	No Damage or Localised Minor Cracking
$0.1 \le D < 0.25$	Minor Damage −Light Cracks throughout
$0.25 \le D < 0.4$	Moderate Damage, Severe Cracking, Localised Spalling
$0.4 \le D < 1.0$	Severe Damage, Crushing of Concrete, Reinforcement Exposed
$D \ge 1.0$	Collapsed

individual elements may not be readily calculated. There are tell-tale evidences of damage in a structure, due to the action of seismic forces and few of these are actually measurable. These include

(a) Presence of widespread cracking (flexure, shear, axial tension and bond) with a measurable crack widths
(b) Permanent out of plumb deflection.
(c) Decrease in stiffness
(d) Decrease in the frequency of the structure

Hence global damage indices based on these few measurable parameters are ideal tools for calculating the damage, particularly during a post-earthquake scenario. The damage index proposed by Di Pasquale and Cakmak (1987), Gomez and Cakmak (1990), is seemingly a handy tool, as it relates the damage to the change in the fundamental time periods of the structure. The index is given as,

$$DI_m = 1 - \frac{T_{und}}{T_d} \tag{8}$$

The present paper generally deals with the damage quantification using the above expression.

In recent times there is a trend towards developing a unified damage and hysteretic model that shall evaluate the damage of different materials. A generalized damage index has been defined as the ratio between the initial and reduced resistance capacity of a structure, evaluated by using an evolution equation for the yield strength. (Colombo and Negro 2005). A hysteretic model proposed recently incorporates basic and post-capping strength, unloading and accelerated re-loading stiffness with experimental calibration from sub-assemblage tests on steel, concrete and plywood (Ibarra, *et al*. 2005). Kim, *et al.* (2005) have proposed a numerical method for the seismic damage assessment of reinforced concrete bridge columns and compare the results of the model with experimental work. A comprehensive comparison of almost all the existing damage indices is done. Recent work correlating the effects of structural damage to the change in the fundamental frequencies and dynamic parameters is due to Brun, *et al*. (2003) for a shear wall structure and Zembaty and Kowalski, *et al.* (2006), for an RC frame. In the latter work, dynamic identification of a damaged RC model frame is carried out using transfer function relationships, after subjecting it to various seismic levels in the shaking table.

Recent spurt in the development of wavelets have helped engineers to employ wavelet as a damage identification tool. The application of wavelets in the study of structural response to non-stationery force inputs is highlighted by Gurley and Kareem (1999). Wavelet application to ASCE health monitoring bench mark study for structural damage detection is due to Hera and Hou (2004). An excellent review paper on the existing literature on damage detection using wavelets is due to Kim and Melhem (2004). Recently Lakshmanan (2006), *et al.* have proposed a damage localization scheme using wavelets through rotation and curvature mode shapes.

4. Details of structure used in seismic damage analysis

The structure taken to illustrate the methodology is a three storied reinforced concrete structure with three bays (Fig. 1). The structure is further classified into three categories depending on its seismic performance, based on varying values of its column stiffness and strength in comparison to its beam stiffness and strength and whether there is an in-fill used or not.

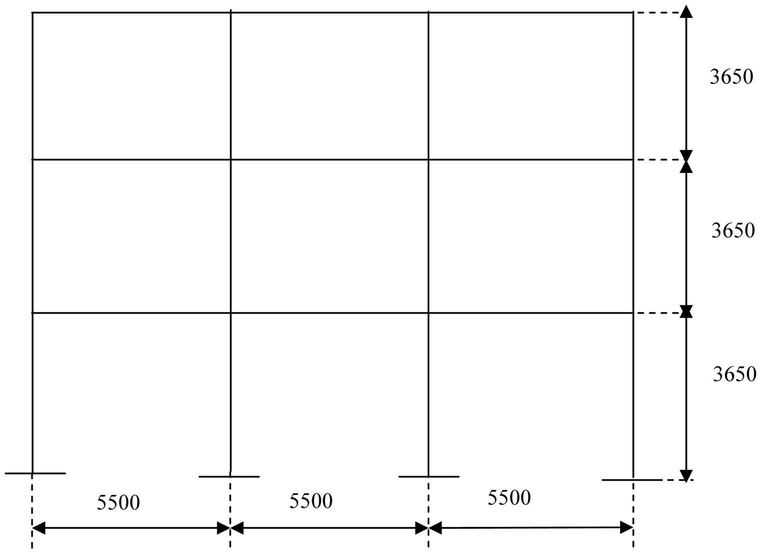

Fig. 1 Frame model used in the analysis

They are,

(a) A normal framed structure, characterized by an approximate linear first mode shape and with a collapse state similar to a beam side-sway mechanism

(b) A soft first storied structure characterized by a soft first storey fundamental mode shape and with a collapse state similar to a column side-sway mechanism.

(c) A weak first storied structure characterized by an approximate linear first mode shape and with a collapse state similar to a column side-sway mechanism. The seismic behavior is in between a normal frame and a soft first story frame.

Table 2 gives the summary of the structures taken for analysis.

A soft first storied structure is the most vulnerable structure, when subjected to seismic lateral forces. In a soft first storied structure, the stiffness of the first storey bay is smaller compared to the upper stories (*Soft storey is the one in which the lateral stiffness is less than 70 percent of that in*

Table 2 Summary of the structures taken for the non-linear dynamic analysis

Type of Structure	Stiffness	Strength	Mode Shape (Fig. 2)	Collapse Mechanism (Fig. 3)
Normal Frame (or Ordinary Frame)	Distributed	Distributed	Linear or Parabolic	Beam Side sway
Soft First Storied Frame	Stiffness of Upper stories more	Strength of Upper stories more	Typical Soft Storey mode shape	Column Side sway
Weak First Storied Frame	Distributed	Upper stories have higher strength (Collapse Moment) or Strong beam and weak column at first floor	Linear or Parabolic	Column Side sway

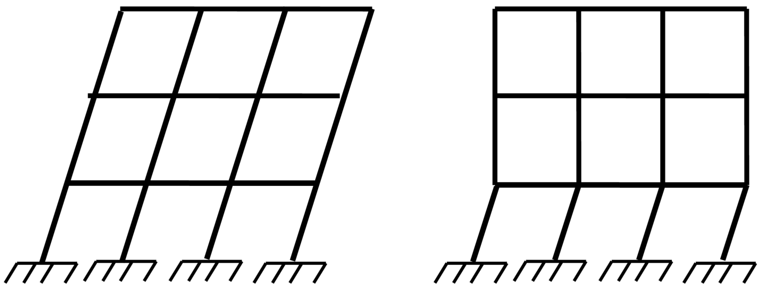

Fig. 2 Linear or parabolic mode shape & soft storey mode shape

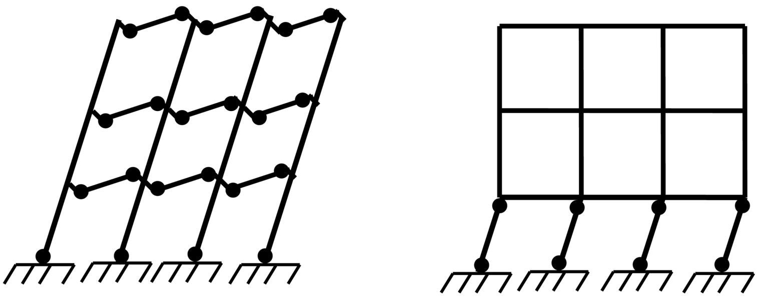

Fig. 3 Beam side-sway and column side-sway mechanisms

the storey above or less than 80 percent of the average lateral stiffness of the three storeys above. IS 1893 (Part 1): 2002). Because of such a configuration, more seismic force is attracted in the first storey level and the displacement demand has to be realized fully from the first storey bay.

A typical example is a structure, provided with infill in all stories other than the first storey. The poor performance of a soft first storied structure is due to the following reasons.

(a) A high level of ductility is expected from an axially loaded compressive member like a column.

(b) P-δ effects and

(c) The total seismic displacement demand is expected only from the first storey bay, whereas in a conventional structure, the seismic displacement demand is distributed over all stories

Because of these reasons, the damage is sudden, catastrophic and without warning.

A weak first storied structure is similar to a soft storied one but the initial un-damaged state does not show a mode shape similar to a soft-storied one (*Weak storey is the one in which the storey lateral strength is less than 80 percent of that in the storey above*-IS 1893 (Part 1): 2002). Hence it may not attract as much of seismic force as a soft first storied structure, but during a seismic event it may start acting like a soft first storey. A typical example is a strong column and weak beam structure, but without infill effects in all floors.

5. Non-linear dynamic analysis

The above three categories of structure are subjected to an El-Centro time history magnified uniformly such that the PGA is 1.0 g. IDARC 4.1 is used to perform the non-linear time history dynamic analysis of the structure. The stiffness-degrading factor for columns is such that the category is 'severe'. Natural frequencies and mode shapes of the structure are obtained both prior to damage and during damage at every one second interval. The changed frequencies for the first three mode of the structure are correlated with the damage index of the structure. System identification of the three different structures (normal or ordinary, weak storey and soft storey) are performed using the information of the natural frequencies and mode shapes as obtained from the IDARC output at every one second. A program is written in Fortran, which shall regenerate the flexibility, stiffness and mass matrices from the modal parameters, given as input. Diagonal elements of the stiffness and flexibility matrices are compared at every stage. Mass matrix is assumed as not undergoing any change.

In real life structure, undergoing a base excitation from an earthquake, even the information of a time wise variation of modal parameters is not available explicitly. Only available information could be the variation of the acceleration or displacement responses of the structure at various floor levels. Hence an additional exercise has to be done to evaluate the system parameters from the response histories collected at various floor levels.

The special signal processing tools are required to achieve the capture of system characteristics from the response time histories. Had the structure been linear throughout the time duration of earthquake, the transfer function of the response acceleration with reference to the base acceleration could be taken and system characteristics could be established.

Transfer function of the seismically excited structure is defined as

$$TRF(f) = \frac{Response\_accln \cdot (f)}{base\_accln \cdot (f)} \tag{9}$$

In the above equation, the response in the time domain is converted to frequency domain using an FFT technique.

This response is normalized with reference to the base acceleration in the frequency domain, and the transfer function is established. The peaks in the transfer function give the natural frequencies (poles) and the imaginary part of the transfer function gives a better estimation of the mode shapes of the structure (residues). There are time stepping techniques like Ibrahim's time domain techniques for establishing the modal parameters using a singular value decomposition scheme (SVD).

However, the damage changes the characteristics of the structure to step-wise linear and an FFT of the whole response is not correct as the system itself is not linear. Time-frequency analysis tools are to be resorted to obtain the instantaneous elastic characteristics of the system. In this context, there are techniques using short term Fourier Transform and wavelet analysis. The short term Fourier transform is a windowed Fourier transform, with the central value of the over-lapped window moves over the time duration. Wavelet technique is an additional tool that has come in handy for scientists and engineers towards localizing the events. Wavelet technique is also used to capture the instantaneous linear transfer function of the response signal.

For this purpose, a band limited white noise, with a frequency content of 0.5-50 Hz is generated for time duration of 5.12 seconds (for 1024 points). This white noise is appended to the existing El-Centro time history prior and after a critical damaging event at 7 to 8 seconds. The response of the three storied structure is obtained both in pre-damage and post-damage states. System characteristics are also obtained using this narrow band base acceleration random time history before and after application of actual earthquake motion.

The transfer function of the response acceleration normalized with reference to the base acceleration, both in the frequency domain is computed and plotted. Natural frequencies are identified from the peaks of the transfer function and mode shapes are obtained from the imaginary part of this function. Natural frequencies are also obtained as a snap shot output from IDARC, which calculates the dynamic characteristics using the instantaneous stiffness values.

6. Re-generation of stiffness matrix through system identification techniques in frequency domain

System identification of a structural system means deriving the properties of a system using the values of its response or output, also referred to as an inverse problem. For example if the natural frequencies and mode shapes of a structure are experimentally evaluated, using an ambient vibration or through any other excitation, then it is possible to regenerate the system matrices like stiffness, flexibility or mass. The flexibility matrix of a structure could be derived using the mode shapes (ortho-normalized) and frequencies of a structure and the derivation is as follows:

The steady state dynamic displacement response vector for a structure, acted upon by a force 'F_d' (at the degree of freedom number 'j'), with an excitation frequency, 'ω', and having a damping of 'ξ' can be given as,

$$\{u\}_j = \sum_{i=1}^{N} \frac{F_d \phi_{i,j}}{M_i(\omega_{n,i}^2 - \omega^2 + 2i\xi\omega_{n,i}\omega)} \{\phi\}_i \tag{10}$$

Where $\phi_{i,j}$ is the mode shape coefficient for the j-th DOF and i-th mode. ω, $\omega_{n,i}$ are the forcing and natural frequencies.

The column of a dynamic flexibility matrix for the degree of freedom number 'j', when a unit load is applied at 'j' is obtained from the above equation, by substituting $F_d = 1.0$. Also, the modal mass M_i for an ortho-normalized mode shape is, $M_i = 1.0$. Substituting in the above equation, the dynamic flexibility matrix could be written as,

$$[F]_{dyn} = \sum_{i=1}^{N} \frac{\{\phi\}_i \{\phi\}_i^T}{(\omega_{n,i}^2 - \omega^2 + 2i\xi\omega_{n,i}\omega)} \tag{11}$$

Static flexibility can be obtained from the above equation by substituting, excitation frequency, $\omega = 0.0$. This is given as,

$$[F]_{stat} = \sum_{i=1}^{N} \frac{\{\phi\}_i \{\phi\}_i^T}{\omega_{n,i}^2} \tag{12}$$

Similarly it is possible to derive an expression for the mass matrix of the system as,

$$[M]^{-1} = \sum_{i=1}^{N} \{\phi\}_i \{\phi\}_i^T \tag{13}$$

Stiffness matrix can be obtained by inverting the flexibility matrix, if information on all significant modes are present. For example in the case of the three-storied structure taken in the analysis, there are three significant lateral modes corresponding to the translational degree of freedom for each floor level. This information on the frequencies and mode shapes could then be used to generate the flexibility matrix and then the stiffness matrix is obtained by inverting the flexibility matrix. However in cases, where the information is not full, i.e., information of mode shapes and frequencies are not available for as many modes as possible, a partial flexibility matrix could be built. The matrix may not be well conditioned and stiffness matrix can not be obtained by inverting the flexibility matrix.

However, a partial build-up of stiffness matrix could be obtained using the following equation,

$$[K]_{stat} = \sum_{i=1}^{N} \omega_{n,i}^{2}[M]\{\phi\}_i\{\phi\}_i^T[M] \tag{14}$$

The above equation requires information on mass matrix, which could be obtained relatively easy. Also, damage alters only the stiffness or flexibility matrices and mass matrix can be assumed as un-affected.

More than just monitoring the frequencies and mode shapes and correlating them with the damage state, the system matrices could be re-generated from the modal parameters (like frequencies and mode shapes) and these are compared with the damages. Advantages of the correlation of damage with system matrices are,

(a) Magnitude of damage could be estimated by the change in the magnitudes of the system flexibility and stiffness when they are compared at each stage of damage with reference to the un-damaged state.

(b) Observing the changes at each of the DOF and relating them with the members, contributing the stiffness of that DOF, position of damage could estimate position of damage.

7. Discussion of results

There is a considerable drop in the natural frequencies of the system, after the major damaging event. In the case of the normal structure (ordinary open storey frame), a considerable warning is obtained in the form of appreciable frequency changes for all the three modes. The change in the frequencies shows a general down ward trend, which is obtained by averaging adjacent terms of the frequency values, written as snap-shot option from IDARC output. Fig. 4 to Fig. 6 show the variation of frequencies in the first three flexural modes, for normal (ordinary), soft and weak first storey structures. There is a fluctuating trend, over and above this averaged line. At any point of time, structure is essentially at the initial stiff portion or at the softened portion. Both the force-

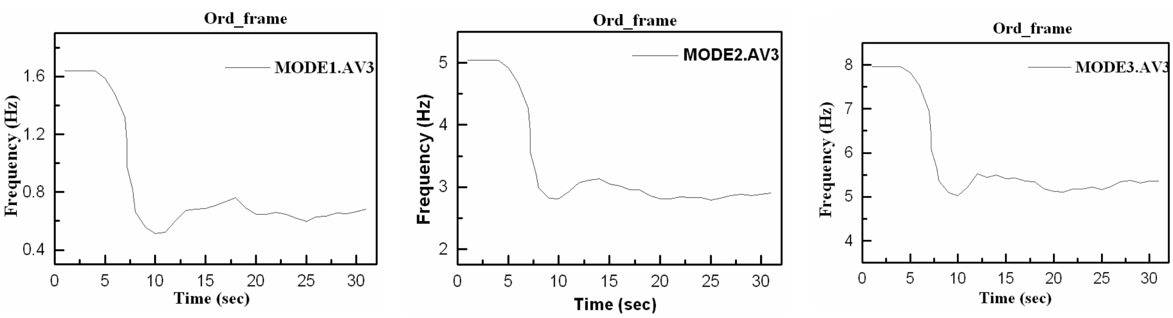

Fig. 4 Frequency variation in three modes for the normal frame

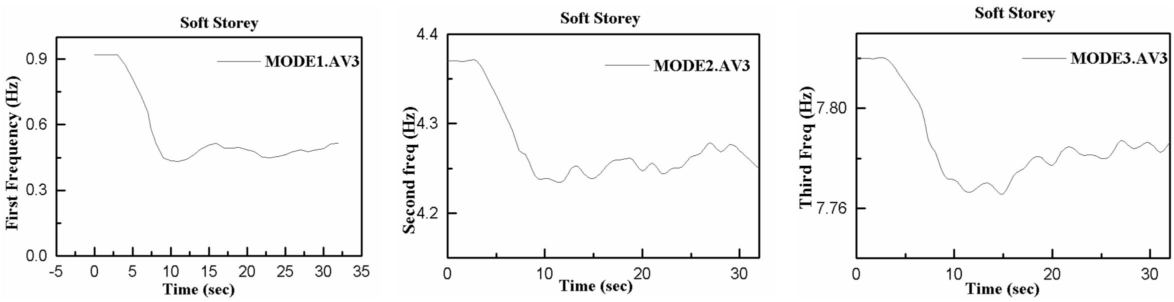

Fig. 5 Frequency variation in three modes for the soft storey frame

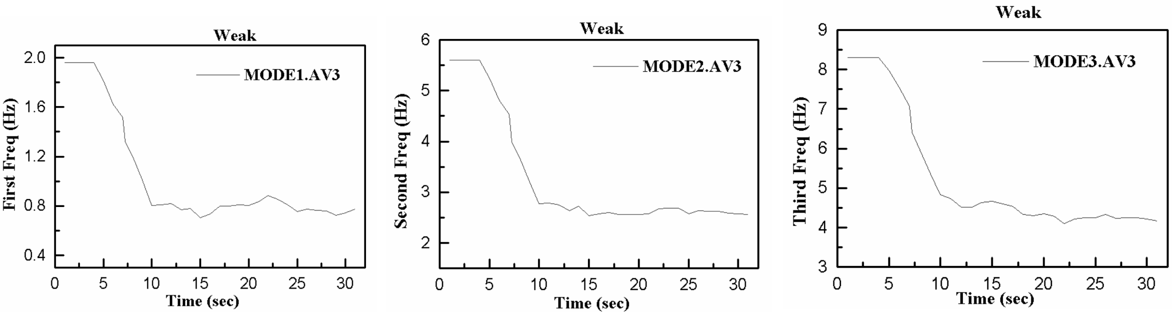

Fig. 6 Frequency variation in three modes for the weak storey frame

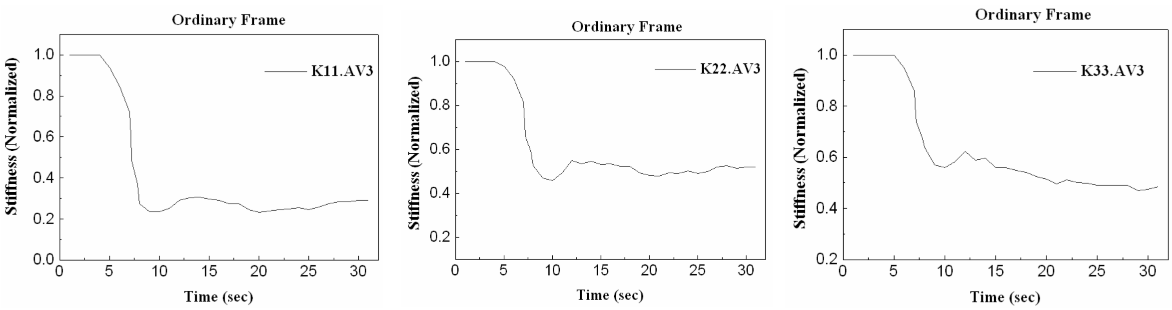

Fig. 7 Stiffness variation in three floors for the normal frame

displacement (and Moment-Curvature) lines are tri-linear but undergo change in the form of a stiffness drop. The averaged line shows the irretrievable general damage and the associated stiffness drop, whereas the fluctuating line is due to the current state of the system (whether the member is between the cracked and yield state or between the yield and ultimate states).

The regenerated flexibility and stiffness matrices are also plotted and all the diagonal stiffness terms (K_{11}, K_{22} and K_{33}) undergo changes (Fig. 7 to Fig. 9). K_{11}, K_{22} and K_{33} are the ground, second and top floor diagonal stiffness values. For normal frame, changes in the values of the frequencies are 58%, 40% and 34% with reference to the un-damaged frequencies in the first, second and third modes respectively. The changes in the values of the stiffness are 70%, 50% and 46% with reference to the un-damaged stiffness in the first, second and third floors respectively. In the case of the soft first storied structure, the changes in the values of the frequencies are 50%, 2% and 0.5% with reference to the un-damaged frequencies in the first, second and third modes respectively. The

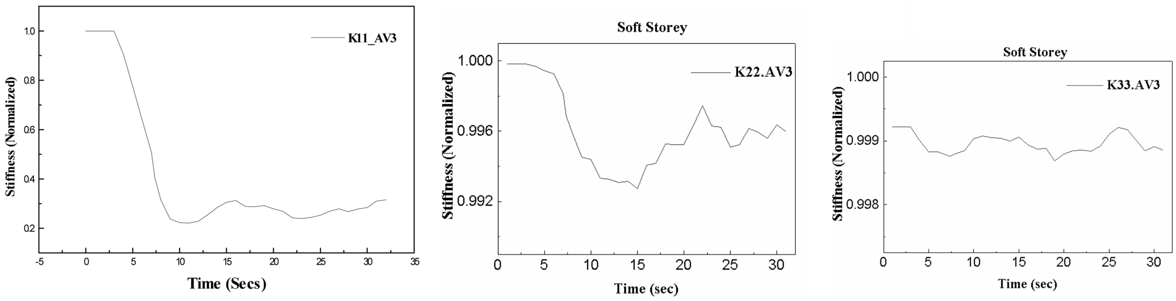

Fig. 8 Stiffness variation in three floors for the soft storey frame

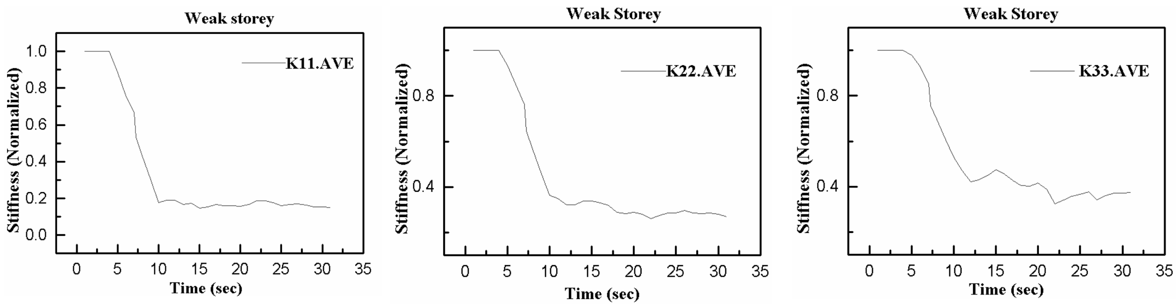

Fig. 9 Stiffness variation in three floors for the weak storey frame

changes in the values of the stiffness are 70%, 0.4% and 0% with reference to the un-damaged stiffness in the first, second and third floors respectively. It is seen that, the major change in the frequency occurs only in the first mode and the majority of the stiffness change occurs in the ground storey bay. This means that damage in the upper storey beams and columns are negligible. In the case of the weak first storied structure, the changes in the values of the frequencies are 59%, 51% and 51% with reference to the un-damaged frequencies in the first, second and third modes respectively. The changes in the values of the stiffness are 75%, 70% and 60% with reference to the un-damaged stiffness in the first, second and third floors respectively. It is seen that, the change in the frequencies occur uniformly in all modes. It is worthwhile noting that the stiffness of the ground storey bay is less than 70% of the stiffness of the storey above this level after occurrence of damage. However the final failure is due to the damage in the ground storey columns. Typical transfer function variation (ratio of top response acceleration and ground acceleration in frequency domain)

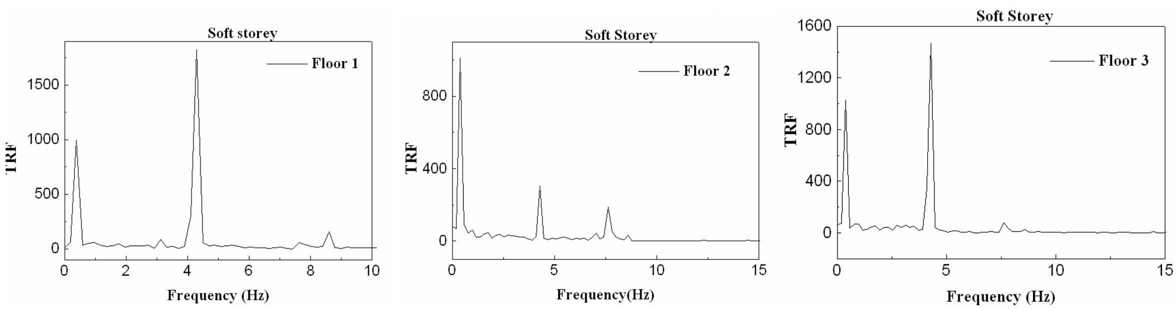

Fig. 10 Transfer function of response acceleration (X 100) for soft storey structure

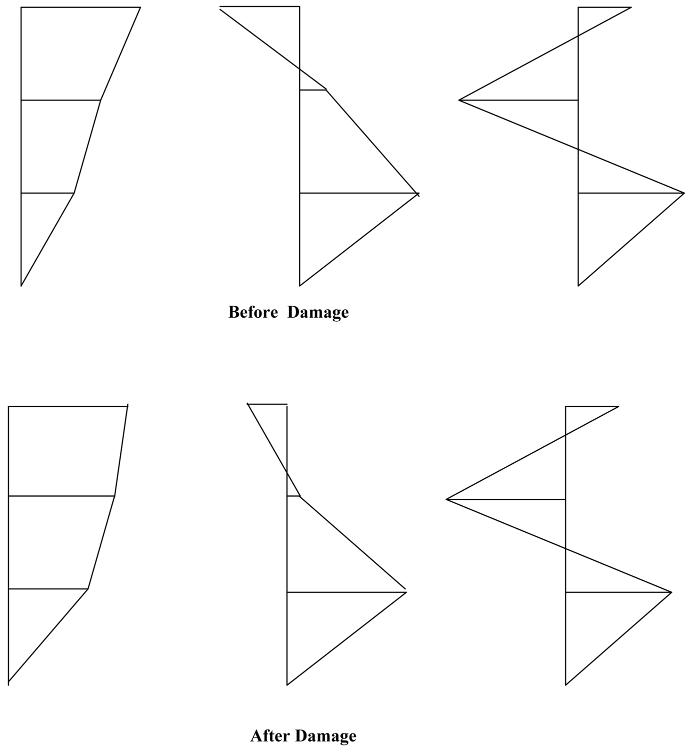

Fig. 11 Mode shape of the weak frame computed from transfer function

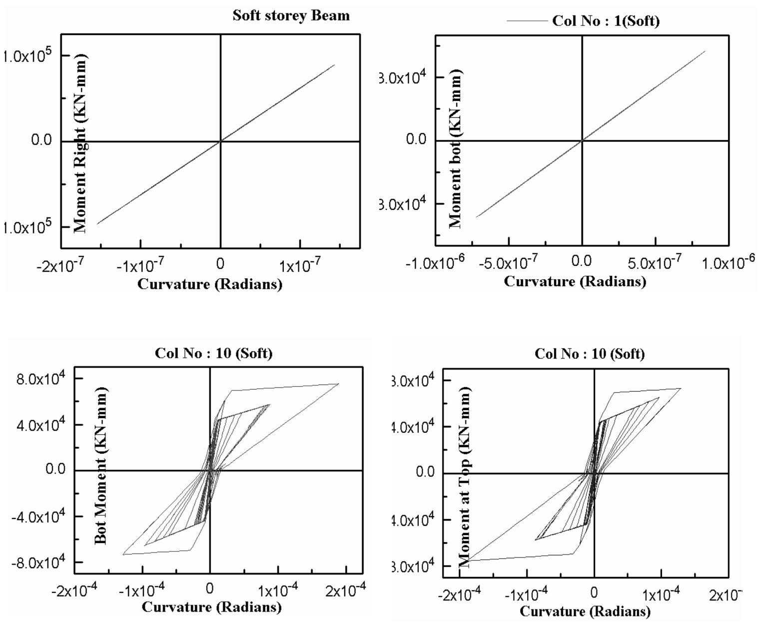

Fig. 12 Moment curvature variation of beams, upper storey column and ground storey columns for the soft storey frame

for a soft storey frame is shown in Fig. 10. Mode shapes of the weak storey frame before and after damage are shown in Fig. 11. This clearly shows that the first mode shape switches from the normal frame shape to the soft storey type as damage progresses.

The cyclic deteriorating moment–curvature relationship of typical columns and beams are plotted. It is seen that in the case of a normal frame, considerable energy dissipation is obtained through inelastic excursions of the beams in addition to columns. Such a trend is not seen in the case of a soft storied structure, where beams at all levels behave elastically, thus imposing severe ductility requirements on the column. Also the top columns of the soft storied structure act elastically. Fig. 12 shows

```
Normal Frame                              Soft Storey Frame

+-----------+-----------+-----------+     +-----------+-----------+-----------+
x           O           O           x     !           !           !           !
!           !           !           !     !           !           !           !
!           !           !           !     !           !           !           !
!           !           !           !     !           !           !           !
x           O           O           x     !           !           !           !
+x---------x+O---------O+x---------x+     +-----------+-----------+-----------+
O           O           O           x     !           x           x           !
!           !           !           !     !           !           !           !
!           !           !           !     !           !           !           !
!           !           !           !     !           !           !           !
O           O           O           x     !           x           x           !
+O---------O+-----------+O---------O+     +-----------+-----------+-----------+
O           O           O           O     O           O           O           O
!           !           !           !     !           !           !           !
!           !           !           !     !           !           !           !
!           !           !           !     !           !           !           !
O           O           O           O     O           O           O           O
```

Fig. 13a Hinge and damage location for normal and soft storied frame

```
Weak Storey Frame

+-----------+-----------+-----------+
!           !           !           !
!           !           !           !
!           !           !           !
!           !           !           !
!           !           !           !
+-----------+-----------+-----------+
!           !           !           !
!           !           !           !
!           !           !           !
!           !           !           !
O           O           O           O
+-----------+-----------+-----------+
O           O           O           O
!           !           !           !
!           !           !           !
!           !           !           !
O           O           O           O


NOTATION:

-  =  BEAM
!  =  COLUMN          x  =  CRACK
W  =  SHEAR WALL      O  =  YIELD
I  =  EDGE COLUMN     FOR EDGE COLS: C: COMPRESSION
                                     T: TENSION
                                     O: TENSILE YIELD
```

Fig. 13b Hinge and damage location for weak storey frame

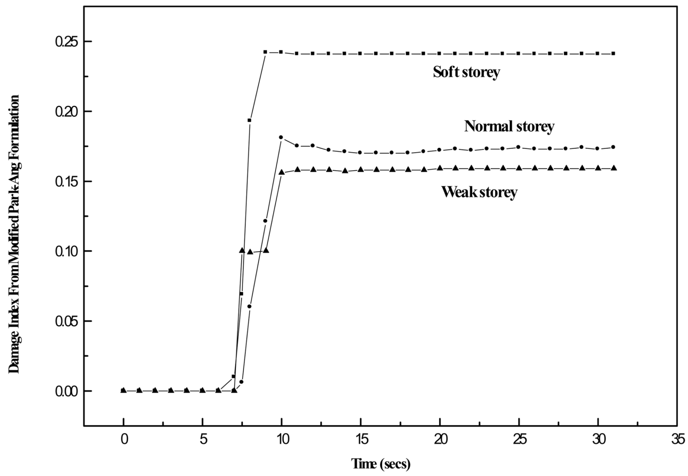

Fig. 14 Variation of damage index from modified Park-Ang model

the Moment curvature variation of a typical beam, a column on the top floor and columns in the ground floor of the soft storey frame. Hardly any energy contribution is available from elements other than ground floor columns. In the case of a weak storied structure, some of the beams become in-elastic and act as partial energy dissipaters. Columns have large energy dissipation. Fig. 13 shows the hinge pattern for all the types of frames.

Modified version of the Park-Ang Damage index, obtained as output of IDARC is plotted in Fig. 14. It is seen that excepting for the duration between 7-9 seconds, damaging effect of the earthquake is not seen. Also, the cyclic damaging factor (second term of Park-Ang expression) is also not there. Similarly Di Pasquale-Cakmak Damage index is plotted in Fig. 15. It is seen that this index over-estimates the damage and needs to be suitably modified. For this purpose, Di Pasquale-Cakmak Damage index is calibrated with modified Park-Ang index in the following fashion.

Re-writing the expression,

$$DI_m = 1 - \left(\frac{T_{und}}{T_d}\right)^{\gamma} \tag{15}$$

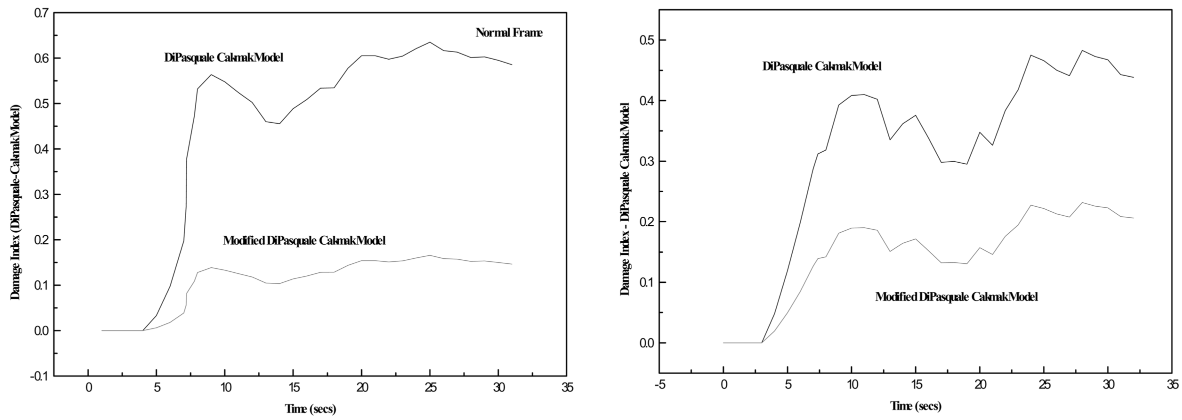

Fig. 15a Variation of damage index based on original and modified Di Pasquale-Cakmak model for normal and soft storey frames

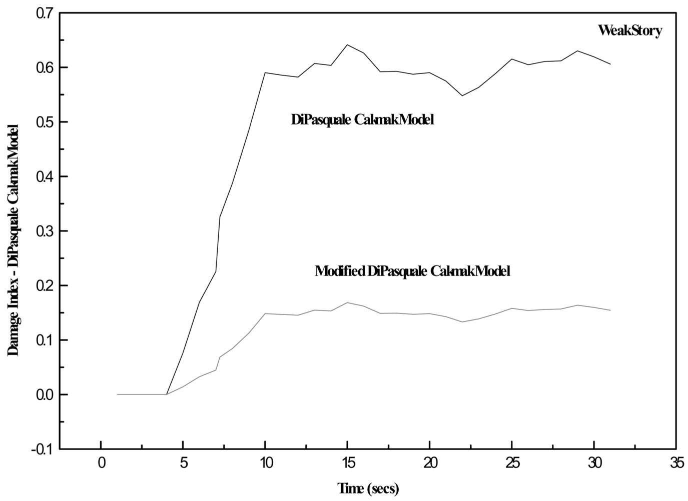

Fig. 15b Variation of damage index based on original and modified Di Pasquale-Cakmak model for weak storey frame

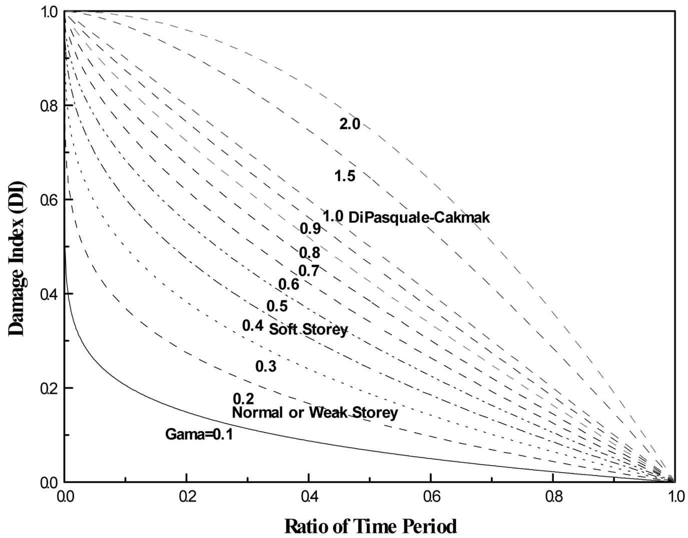

Fig. 16 Modified Di Pasquale-Cakmak formulation with varying values of exponent

Where, ' γ' is the exponent which needs to be evaluated. Taking logarithm on both sides and simplifying,

$$\gamma = \frac{\log(1 - D_m)}{\log\left(\frac{T_{und}}{T_d}\right)} \tag{16}$$

After suitable curve fitting the value of ' γ' is evaluated as 0.18 and 0.17 in the case of normal and weak storey frames and 0.4 in the case of soft-storey frame. Fig. 15 also shows the resulting variation of modified DiPasquale-Cakmak model and this closely coincides with the Park-Ang Model. Fig. 16 shows the modified DiPasquale-Cakmak model with varying exponent values.

8. Spatial identification of damage using wavelet multi–resolution analysis

A theoretical study of the wavelet analysis of the lateral deflected shape of typical reinforced concrete column which form part of the lower most bay of the soft storey frame is carried out. The moment variation across the column length is obtained through IDARC output. The moment variation is such that it changes sign over the column length and there is a contra-flexure point in between. Neglecting the mass of the column, the moment variation is assumed as linear as shown in Fig. 17 and Fig. 18. The moment curvature relationship of the column is developed through a computer program and the tri-linear relationship between the two parameters is established. The three distinct points on the *M-ϕ* curve are the initial cracking moment, steel-yielding moment and the moment due to the ultimate compressive strain of concrete. Tangent slope of the tri-linearly approxi-mated *M-ϕ* curve give the effective flexural rigidity (EI) of the column span at different sections, depending on the bending moment applied at that section as shown in Fig. 18. However, effective EI increases mid way between

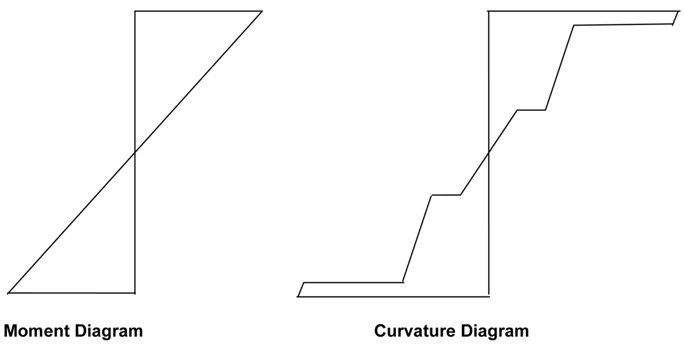

Fig. 17 Moment- curvature variation of a typical soft storey column

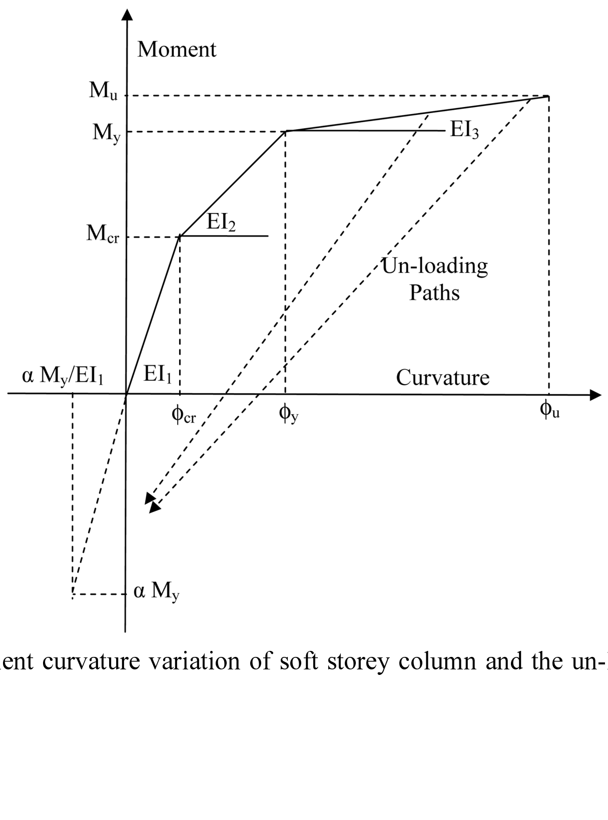

Fig. 18 Moment curvature variation of soft storey column and the un-loading paths

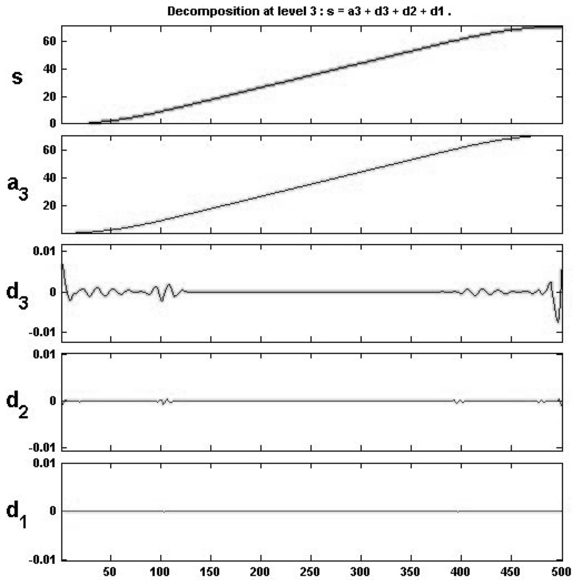

Fig. 19 Multi-resolution analysis of the displacement profile of soft storey column - load stage-I (BIOR-6.8)

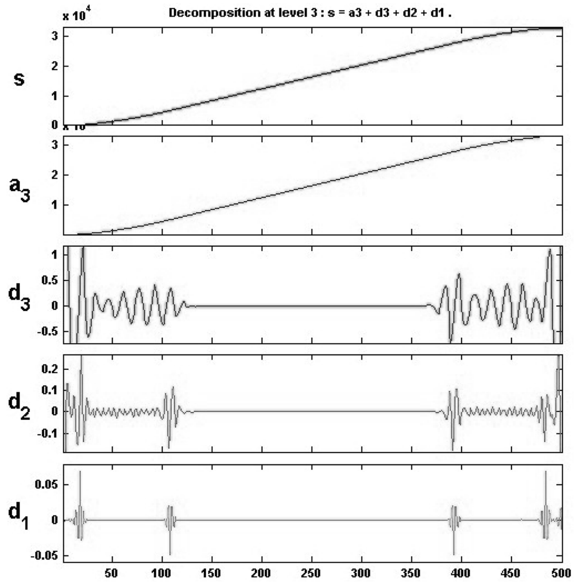

Fig. 20 Multi-resolution analysis of the displacement profile of soft storey column-load stage-II (BIOR 6.8)

cracks but decreases near crack tips due to tension stiffening of the un-cracked regions. This is approximately modeled with a sinusoidal profile of EI variation, over and above the mean EI profile, once the crack spacing and the fluctuation in EI are given as input. The cosine variation of the effective EI, over and above the EI, as predicted through the tri-linear moment curvature relationship is assumed as 10% of the background EI value. Also, the crack spacing in the column is assumed as $d/2$. These are approximate values and are obtained through experimental results. Another computer program is written, which picks up the appropriate value of EI from the tri-linear moment curvature relationship, adds up the local fluctuation in EI due to tension stiffening and uses that to construct the curvature variation (ϕ) along the length. The integrated value of the curvature gives the deflected shape of the column element. Fig. 19 shows the multi-resolution analysis result of the column when the loading stage (Stage-II) is between cracking and yield moment. The detail function clearly shows three distinctive zones, with a central un-cracked zone and the end cracked zones. Fig. 20 shows the multi-resolution analysis result of the column when the loading stage (Stage-II) is between yield and ultimate moment. Here, the detail function clearly shows five distinctive zones, with a central un-cracked zone, two yield zones at the end and two sandwiched zones between cracked and yield stages. Local variation of the displacement due to tension stiffening between the cracks is also seen.

9. Conclusions

The paper has presented a three level damage screening procedure based on,

(a) Fundamental period alone

(b) regenerated system matrices, which may be obtained exciting the structure after an earthquake or through on-line monitoring of the seismic response of the structure and

(c) wavelet based multi-resolution analysis of individual structural elements.

Vulnerability of soft storied structure is more compared to a normal structure and hence such a structure is taken for the illustration. Weak storied structures show a behaviour in between a normal frame and a soft storied frame and hence also taken for comparative investigation. All the sustained damages fall under the category of "minor" and repairable. Fundamental time period based damage evaluation models based on DiPasquale-Cakmak's expression is simple to use, but needs fine tuning and for all the three structures, such a model over-estimates the damage. The present paper goes one step ahead of the DiPasquale-Cakmak model and attempts to refine the equation with a new exponent value other than 1.0. Exponent of the time period ratio is fitted through curve fitting procedure and this value is more for soft storied structure. For the same time period ratio, the damage undergone by the soft storied structure is more or in other words sensitivity to frequency variation of the soft storied structure is less. In this example for a damage of 16-18%, (DI : 0.16-0.18) the frequency change in the case of normal and weak storied structures are 65-70% whereas for a damage of 24% (DI : 0.24) in the case of a soft storied structure, frequency change of 50% is noted. This may be due to the distributed nature of damage in normal frame, whereas in soft storied case the damage is concentrated and more on vulnerable column elements, whose weighting factors are more. This is an important observation and needs further study with experimental corroboration.

It is possible to re-generate the system characteristics from the modal parameters and this can also be obtained on-line during the progression of an earthquake. Conventional FFT analysis could be used for linear systems by taking the entire earthquake duration but a damaging system being non-linear, short term Fourier transform (STFT) may have to be employed. In this paper, FFT of a short duration of the earthquake response and input prior and after a major damaging event is used to establish the transfer function relationship. This input-output relationship is used to compute frequencies and mode shapes. The changes in the frequencies and mode shapes prior to and after the earthquake are used to quantify the damage suffered by the structure. A normal frame, which fails in a beam side sway mode, undergoes reduction in all the frequencies and stiffness values of all the floors get reduced. However the stiffness reduction is more for the bottom storey. A soft storied frame, which fails in a column side sway mode, undergoes reduction mostly in its first mode frequency. Also, the stiffness of the bottom storey alone undergoes changes. A weak storied frame, starts as a normal frame but ends up as a soft-storied one, fails in a column side sway mode and undergoes reduction in all three modes. The stiffness of bottom story is eroded to the maximum. The local damages can also be computed using the multi-resolution analysis techniques of wavelet transforms. An effective EI obtained from the moment-curvature relationship with superimposed fluctuations, representing local cracking and tension stiffening is modeled to show distinct regions of EI change for seismically damaged soft storied columns. Wavelet is an excellent tool for time frequency analysis of strong non-linear systems. The wavelet coefficients could be used to track the frequency as well as amplitude changes. It can also be found that BIOR 6.8 class of wavelet mother functions has given good results.

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