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Elimination of the effect of strain gradient from concrete compressive strength test results

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Abstract Poor strength test results are sometimes not an indication of low concrete quality, but rather inferior testing quality. In a compression test, the strain distribution over the ends of the specimen is a critical factor for the test results. Non-uniform straining of a concrete specimen leads to locally different compressive stresses on the cross-section, and eventual premature breaking of the specimen. Its effect on a specimen can be quantified by comparing the compressive strength results of two specimens, one subjected to uniform strain and another to a specified strain gradient. This can be done with the help of a function that relates two parameters, the strain ratio and the test efficiency. Such a function depends on the concrete strength and cross-sectional shape of the specimen. In this study, theoretical relationships between the strain ratio and test efficiency are developed using a concrete stress-strain model. The results show that for the same strain ratio, the test efficiency is larger for normal strength concrete than for high strength concrete. Further, the effect of the strain gradient on the test result depends on the cross-sectional shape of the specimen. Implementation of the results is demonstrated with the aid of two examples.

Keywords: compressive strength; concrete; strain; stress; testing.

1. Introduction

The compressive strength of concrete is an important variable used by engineers in designing buildings, bridges and other structures. From the compressive strength, one can deduce the tensile strength, shear strength, bond characteristics and modulus of elasticity of the concrete. It is normally measured by breaking concrete specimens in a compression test machine at the age of 28 days (ASTM 2001). In North America, the standard concrete specimen is 150 mm by 300 mm cylinder, while in Europe it is 100 or 150 mm cube. Smaller specimens are sometimes used for high strength concrete due to the limitation of the test machine. For a given concrete mix, cylinder test strength is 17-40% less than cube test strength, with the percentage difference decreasing with an increase in the concrete strength (Mansur and Islam 2001).

The compressive strength is calculated by dividing the failure load obtained from the test by the cross-sectional area of the specimen. Concrete compressive strength requirements vary based on the structural application. It is normally between 20 and 35 MPa for residential structures, and can be higher in commercial structures and prestressed concrete. Higher concrete strengths up to and exceeding 70 MPa are specified for special applications, such as the columns of high-rise buildings.

Compressive strength test results are mainly used to ensure that the concrete mixture as delivered

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conforms to the requirements of the target strength in the job specification. The measured results are dependent upon adhering strictly to standardized procedures. The measured concrete strength is not an intrinsic property of the material because it is greatly affected by how the specimens are prepared, handled, transported, cured and tested. There are two major reasons as to why the compressive strength of concrete may be low. First, improper specimen preparation, curing and testing can contribute to low strength results. Second, error in the mix design proportions, use of excessive water or high air content may lead to reduced concrete strength. Experience has shown that most of the errors in the testing of a concrete sample produce lower-than-apparent strength results than it should have.

In a compression test, the strain distribution over the ends of the specimen is a critical factor for the test results. Unevenness in the compression surfaces due to roughness, grooves and impurities, leads to locally different compressive stresses and to a premature breaking of the specimen. Flexibility of the compression platens influences the stress distribution on the ends of the specimen

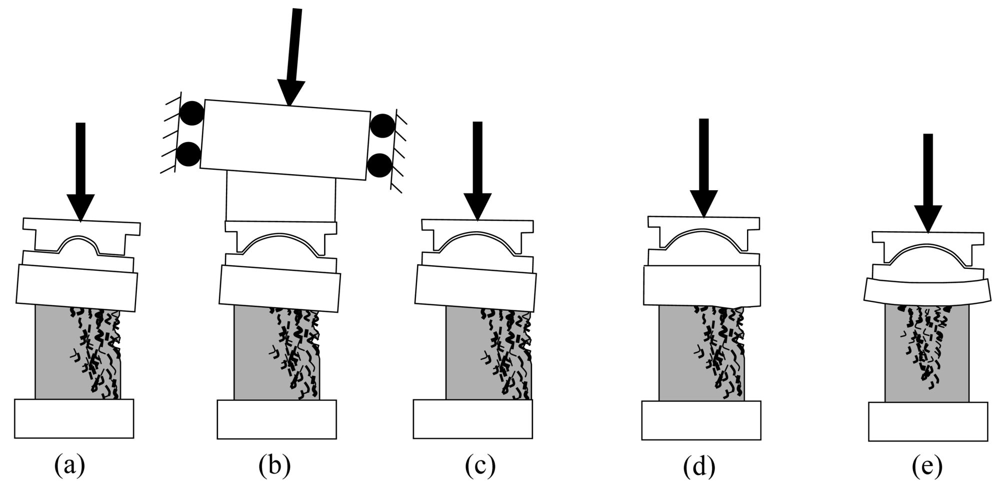


Fig. 1 Sources of non-uniform straining: (a) small radius of spherical head; (b) unsymmetrical deformation of loading frame; (c) mispositionning of specimen; (d) uneven platen surface; and (e) flexible platen

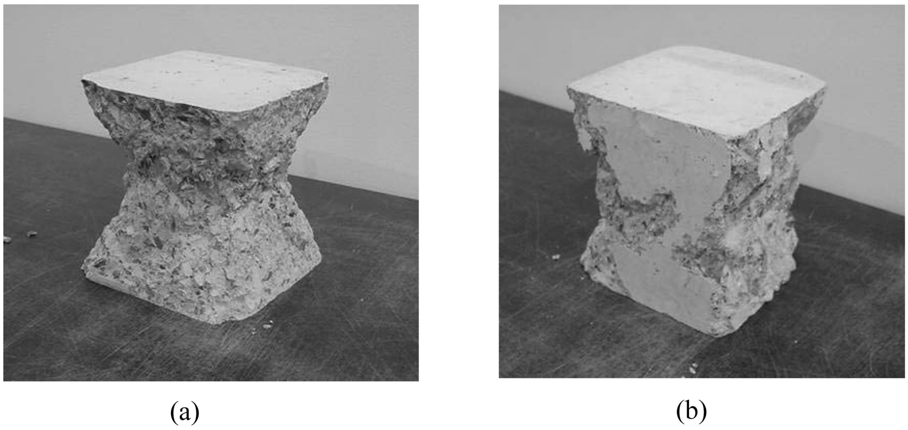


Fig. 2 Concrete cube specimens after failure: (a) uniformly strained; and (b) non-uniformly strained

and unsymmetrical deformation of the machine frame affects the angular position of the compression platens, often resulting in erroneous test results. Fig. 1 shows a summary of the different factors that impact the compression test results of a concrete specimen. Non-uniform straining of a concrete specimen is usually indicated by nonsymmetrical cracking and spalling patterns of the specimen at failure, as shown in Fig. 2.

2. Literature search

Richardson (1991) studied the important variables that influence the accuracy of concrete test results. He concluded that maximum loss in strength of concrete due to convex ends of a specimen is 75%, due concave ends is 30%, and due to rough ends (before capping) is 27%. In addition, the apparent strength of concrete may be lower than the actual strength by up to 53% when rubber capping is used, and by up to 43% when soft capping compound is used. Richardson also found out that the strength of concrete drops by up to 21% when cardboard molds are used, by up to 14% when plastic molds are used, and by up to 22% when plastic molds are reused.

Lessard, Challal, and Aticin (1993) conducted experiments on high-strength concrete (HSC) specimens to quantify the effects of some test variables on the compressive strength. They found out that the compressive strength of concrete when measured on 100×200 mm specimens was greater by 5 percent than on 150×300 mm specimens. Moreover, the coefficient of variation for 100×200 mm specimens was smaller than that for 150×300 mm specimens. A small eccentricity of less than 4 mm between the testing machine and specimen axis did not significantly affect the compressive strength values of concrete.

Detwiler and Bickley (1993) analyzed comparative compressive strength results on similar cylinders but tested in different laboratories to determine the precision of the test method. The study showed that the standard deviation of the concrete strength increases with an increase in the mean concrete strength. Also, it appeared from the data that when blind testing is carried out, the variability of the test results was higher than economically desirable. These findings were verified later by Kennedy, *et al.* (1995), who recommended a precision statement be included in the test standard.

An experimental study to investigate the effects of testing variables on the strength of HSC cylinders was conducted by Carino, *et al*. (1994). The variables included: nominal stress loading rate, end of specimen conditions, and capacity of testing machine. Statistical analyses indicated that all the factors had effects on the measured compressive strength. On average, the faster stress rate produced about 2.6% greater strength than the slower rate, the ground cylinders were 2.1% stronger than the capped ones, and the smaller-capacity testing machine produced about 2.3% greater strength than the lower-capacity machine.

Analen, Lupin, and Ohlsson (1998) investigated the use of the strain-cylinder test (or “Foote” meter) to check the accuracy of concrete testing machines. The strain-cylinder consists of an accurately machined steel cylinder of appropriate capacity fitted with four balanced strain gauge bridges. They concluded that the strain-cylinder test can detect defects such as misalignment of the upper platen and inherent eccentricity in the machine. According to the eccentricity test results, significant eccentricity was found in some machines that were calibrated following the applicable standards.

Burg, *et al*. (1999) conducted an inter-laboratory test program to determine the effects of some testing variables on the compressive strength of HSC. They found out that the current requirements for test platens are not sufficient for concrete with compressive strength in excess of 70 MPa because they cause nonuniform load transfer, resulting in more than 10% reduction in the compressive strength. They recommended revision of the spherical bearing blocks in current test machines to assure uniform load transfer into HSC specimens.

Luker (2003) suggested that recent improvements in instrumentation and computer power had made it possible for checking the performance and quality of a concrete test machine. He proposed strain measurements on a specimen during a test to determine the amount of strain nonuniformity on the specimen. From strain data, he was able to develop empirical relationships between the strain gradient on a specimen and reduction in compressive strength.

3. Significance of study

Poor strength test results are sometimes not an indication of low concrete quality, but rather inferior testing quality. The consequences of obtaining falsely low results can be unnecessary delays, costly follow up testing, needless conservatism, and possible rejection of a good material. Uniform transfer of load from a test machine to a concrete test specimen is necessary if test results are to be reliable. Load transfer is a function of many variables such as platen design and condition, test machine characteristics, rate of applied load, and specimen end conditions. Unfortunately, transfer of load into test specimens is difficult to measure since most techniques require extensive instrumentation that frequently interferes with the load transfer. Therefore, there is a need for a procedure that can correct the compressive strength results obtained from a test machine by filtering out the effect of the strain gradient on the test specimen. In the context of this paper, the true strength is that obtained when the concrete specimen is uniformly strained in compression up to its maximum load resistance.

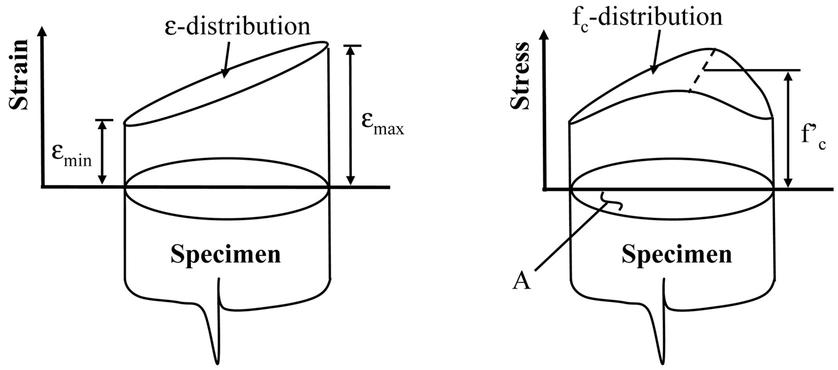
4. Background information

In order to determine the effect of non-uniform straining on a concrete specimen, one needs to compare the compressive strength results of two specimens, one subjected to uniform strain and another to a specified strain gradient. The process must be repeated for a range of strain gradients since the relationship between the correction factor of the compressive strength and the strain gradient is not linear. Further, a wide range of nominal concrete strengths must be considered because the shape of the stress-strain curve of concrete is not the same for low, medium and high-strength concrete. Theoretical relationships between the strength result and strain gradient can be derived if the stress-strain behavior of concrete is modeled by a mathematical function. However, for such relationships to be useful, they should be verified against experimental test results.

The effect of a strain gradient on the compressive strength of a concrete specimen can be quantified with the help of a function that relates the strain ratio ρ to the test efficiency η, defined below by Luker (2003):

$$\rho = \frac{(\varepsilon_{\max} - \varepsilon_{\min})/2}{(\varepsilon_{\max} + \varepsilon_{\min})/2} \tag{1}$$

$$\eta = \frac{\int_A f_c dA}{f'_c A} \tag{2}$$



(a) (b)

Fig. 3 Effect of non-uniform straining: (a) strain gradient; and (b) corresponding stress distribution

where ε_{max} and ε_{min} are respectively the maximum and minimum strains on opposite sides of the specimen, f_c is a function representing the distribution of stress over the strain range ε_{min} to ε_{max}, A is the cross-sectional area of the specimen, and $f_c^{'}$ is the peak stress of the stress-strain relationship for the concrete. Note that there are infinite combinations of ε_{max} and ε_{min} for a given strain ratio, however, the combination that must be used in Eq. (2) is the one that causes the numerator to be maximized since it will result in the failure load. Uniform straining is indicated by $\rho = 0$, and $\rho > 1$ means that the specimen is subjected to some tensile strain, which is a rare event. Fig. 3 shows a strain gradient on a concrete cylindrical specimen with the corresponding induced stress profile. Numerical integration with the help of a computer program is normally employed in the calculation of Eq. (2).

5. Theory

A mathematical stress-strain relationship for the concrete in a specimen is needed to theoretically develop the relationship between the strain ratio ρ and test efficiency η. There is a variety of models for representing the stress-strain relationship of concrete. In this study, Thornfeldt's model (1987) is utilized because it represents well the experimental stress-strain relationships available in the literature, is applicable to a wide range of concrete strengths, and it is relatively easy to implement. The model is a function of only the peak strength of the concrete $f_c^{'}$, which is particularly useful in this study because when interpreting a value of ρ from a test in practice into a value of η, the only other information available about the concrete is the measured strength.

In the model developed by Thornfeldt, *et al.*, the concrete compressive stress, f_c (MPa), is related to the strain, ε, by the following function:

$$f_c = \left[\frac{(f_c^{'}/17 + 0.8)(\varepsilon/\varepsilon_o)}{(f_c^{'}/17 - 0.2) + (\varepsilon/\varepsilon_o)^{(f_c^{'}/17+0.8)k}}\right] f_c^{'} \tag{3}$$

where ε_o is the strain when $f_c^{'}$ (MPa) is reached:

$$\varepsilon_o = \left(\frac{f_c^{'}}{E}\right)\left(\frac{f_c^{'}/17 + 0.8}{f_c^{'}/17 - 0.2}\right) \tag{4}$$

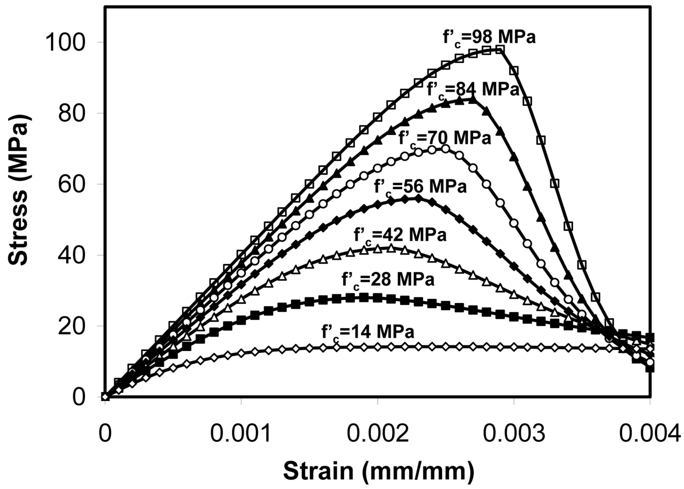


Fig. 4 Theoretical stress-strain relationships for various concrete strengths

and k is a curve-fitting parameter that depends on f_c' and ε_o:

$$k = \begin{cases} 1.0 & \text{for} \quad \varepsilon \le \varepsilon_o \\ 0.67 + \dfrac{f_c'}{61} & \text{for} \quad \varepsilon > \varepsilon_o \end{cases} \tag{5}$$

and E is the modulus of elasticity of the concrete (MPa). Typical theoretical stress-strain curves for a wide range of f_c' based on Thornfeldt's model are shown in Fig. 4.

The method for determining the theoretical relationship between strain ratio ρ and efficiency η for a particular concrete strength starts with determining Thornfeldt's function of f_c–ε relationship. A small strain ratio ρ is selected and a trial-and-error approach is used to produce the combination of $\varepsilon_{\max}$ and $\varepsilon_{\min}$ that gives the maximum value of the load that can be supported by the concrete specimen ($\int_A f_c dA$). The efficiency η for the selected value of ρ is then recorded. The strain ratio is incremented and the procedure is repeated for the new value of ρ. The process is concluded when the value of ρ is equal to unity.

The theory presented above has been verified in an earlier study by Luker and Tabsh (2004) through experimental testing of several batches of cubes and cylinders having a wide range of concrete strengths and subjected to various strain gradients. Good agreement was observed between the theory and experiments. Both the theoretical results and experimental findings showed that non-uniform straining of a concrete specimen gives lower-than-actual compressive strength values. Further, the influence of a strain gradient on the test efficiency was very significant for high strength concrete, especially when the strain ratio was large.

6. Results

Theoretically produced ρ–η curves can be used to correct the apparent concrete strength obtained from a test machine that imposes non-uniform straining on a specimen. This can be done in a

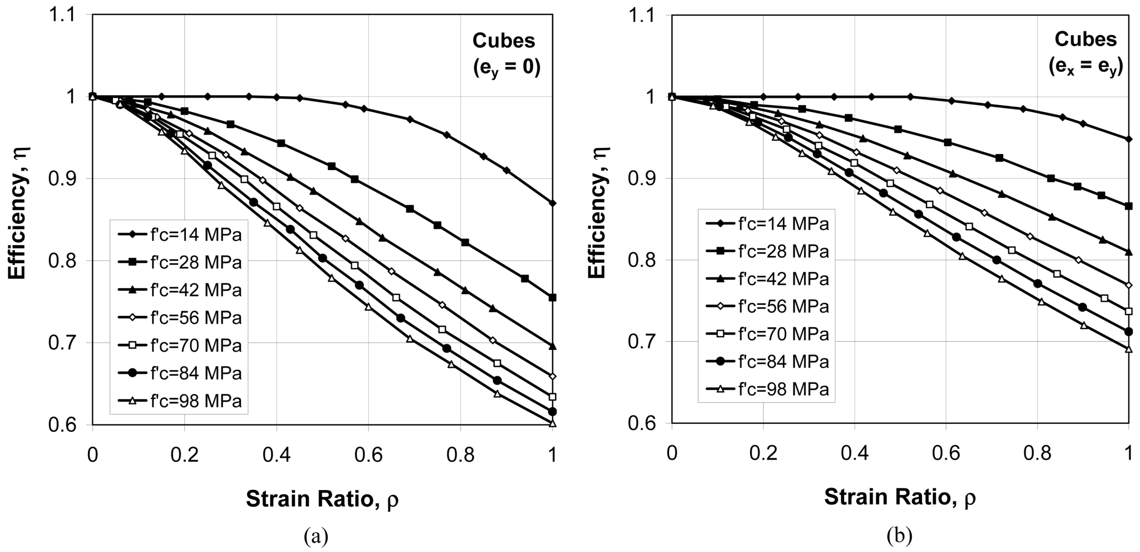


Fig. 5 Efficiency-strain ratio relationships for cubical specimens: (a) uniaxial eccentricity; and (b) biaxial eccentricity

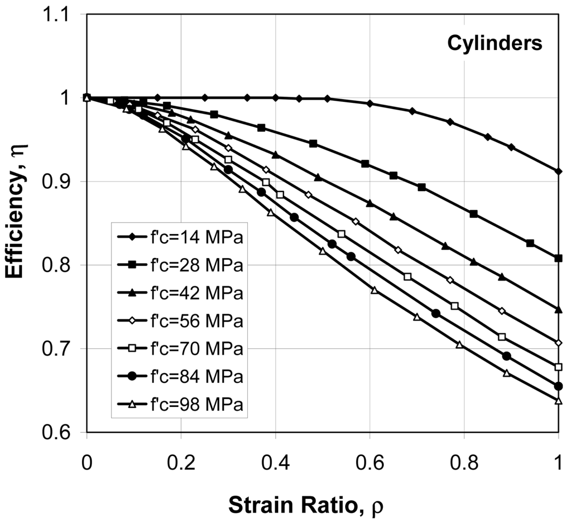


Fig. 6 Efficiency-strain ratio relationships for cylindrical specimens

simple way if simultaneous measurements of load and displacement of the platens are made on the compression test machine at the onset of failure. For rigid platens, a minimum of three displacement measurements are needed between the upper and lower platens of a compression machine.

Figs. 5 and 6 show theoretically generated ρ–η relationships for cubical and cylindrical concrete specimens, respectively. Two plots are presented for the cubical specimens because the eccentricity of the applied load from the test machine with respect to the center of the cross-section can be either uniaxial or biaxial. The results are presented for a wide range of concrete strengths, f_c' = 14–98 MPa.

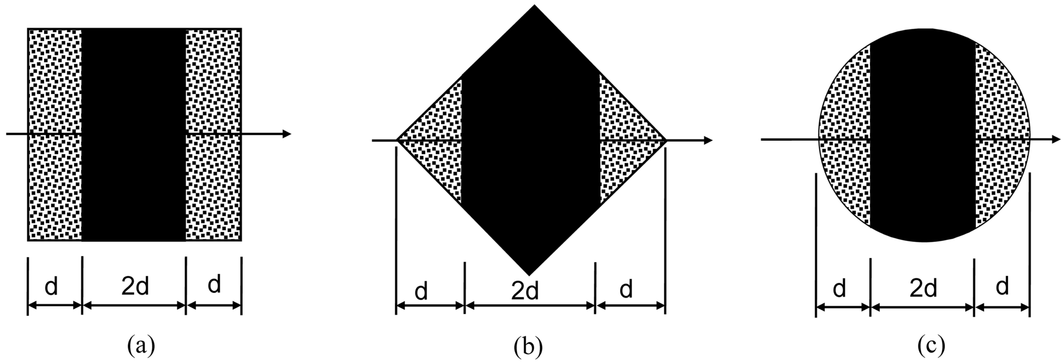


Fig. 7 Effect of cross-sectional shape of specimen on the test efficiency: (a) uniaxially strained cube; (b) biaxially strained cube; and (c) cylinder

As expected, the results show reduced efficiency with an increase in the strain ratio. The reason for such trend is because at the instant of maximum load, a part of the nonuniformly-strained specimen in the high strain region will have passed the peak of its stress-strain relationship, while other part of the specimen in the low strain region lags behind the peak stress. The results also indicate that the relationship between the test efficiency and strain ratio depends on the cross-sectional shape of the specimen and nature of the eccentricity of the applied load with respect to the center of the specimen. For the same strain ratio, the test efficiency is highest for a biaxially-strained cube and lowest for a uniaxially-strained cube, with the results for a cylinder lying in between these limits. Thus, test results of biaxially strained cubes are not affected by nonuniform straining as much as uniaxially strained cubes. This is because the test efficiency, as presented in Eq. (2), is directly related to the percentage of the area that lies within the central region to the total area of the specimen's cross-section. To illustrate, three shapes are considered in Fig. 7: (1) cube, (2) diamond (45^o-rotated cube), and (3) cylinder. In reference to Fig. 7, the central area of the cube occupies 50% of the total area, the central area of the diamond represents 75% of the total area, and central area of the cylinder is equal to 60.9% of the total area. This confirms that biaxially strained cubes are more efficient than cylinders, and cylinders are more efficient than uniaxially strained cubes.

Note that this finding does not mean that the apparent strength of a biaxially strained cube is higher than that of a uniaxially strained cube. Rather, it indicates that the measured strength of a biaxially strained cube is closer to the actual strength than for a uniaxially strained cube. Additional theoretical investigations have shown that linear interpolation of the results between Fig. 5(a) and Fig. 5(b) is valid for cases of e_y / e_x between 0 and 1, where e_y is designated as the smaller eccentricity of a biaxially strained cube.

7. Application of results

The procedure to filter out the effect of a strain gradient from test results is iterative because in order to use the theoretical ρ–η relationships one needs to know the correct value of f'_c, which is initially unknown. The basic steps for correcting the concrete compressive strength test results are:

1. Calculate the "apparent" strength of the concrete, $(f_c')_{app}$, by dividing the failure load by the cross-sectional area of the specimen.
2. From the displacement readings at (at least) three locations on the moving platen, find the maximum and minimum strains imposed on the specimen.
3. Calculate the strain ratio ρ using Eq. (2).
4. Use the theoretically produced ρ–η relationships in Fig. 5 or Fig. 6 to find an initial value of the test efficiency η_{ini} for the value of ρ calculated in step 2. When using the graph, assume as a first approximation that $f_c' = (f_c')_{app}$. For a biaxially strained cube, interpolate between Figs. 5(a) and 5(b), based on the value of e_y/e_x.
5. Correct the value of the concrete strength using: $(f_c')_{cor} = (f_c')_{app}/\eta_{ini}$.
6. The above value of f_c' needs to be corrected once again since η_{ini} was based on the $(f_c')_{app}$, not $(f_c')_{cor}$. This is done by re-entering the ρ–η relationships in Fig. 5 or Fig. 6 with $(f_c')_{cor}$ and obtaining a more accurate value of the test efficiency, $(\eta)_{new}$.
7. The correct value of the concrete strength is now obtained from: $f_c' = (f_c')_{app}/(\eta)_{new}$. Iterate again if $(\eta)_{ini}$ is much different from $(\eta)_{new}$, otherwise stop. In most cases, one iteration is needed to obtain a sufficiently accurate value of f_c'.

The above procedure is illustrated later in the paper with the help of two examples.

8. Practical considerations

In order to derive the equations of the strain ratio ρ for concrete cylindrical and cubic specimens subjected to strain gradient, strain measurements between the upper and lower platens at 3 points on a square grid of dimension L are considered, as shown in Fig. 8. The coordinates of the 3 points at which the strain measuring devices are located are: Point #1 (L/2, L/2), Point #2 (-L/2, L/2), and Point #3 (-L/2, -L/2). If the platen is assumed to be infinitely rigid, then the general equation of the strain at any point on the platen is:

$$\varepsilon = a\,x + b\,y + c$$

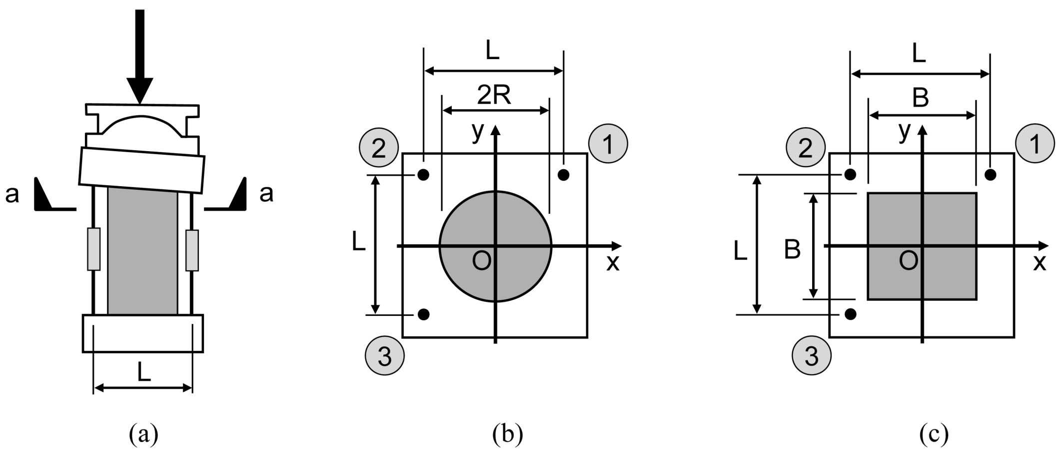


Fig. 8 Location of strain measurement devices for cylindrical and cubic specimens: (a) elevation; (b) section a-a for a cylinder; and (c) section a-a for a cube

where *a, b* and *c* are constants that can be obtained by applying the above expression of ε at points # 1, 2 and 3:

$$\varepsilon_1 = a\,(L/2) + b\,(L/2) + c$$
$$\varepsilon_2 = a\,(-L/2) + b\,(L/2) + c$$
$$\varepsilon_3 = a\,(-L/2) + b\,(-L/2) + c$$

Solving the above 3 equations for the 3 constants and substituting back into the general equation of ε leads to:

$$\varepsilon = \left(\frac{\varepsilon_1 - \varepsilon_2}{L}\right)x + \left(\frac{\varepsilon_2 - \varepsilon_3}{L}\right)y + \left(\frac{\varepsilon_1 + \varepsilon_3}{2}\right) \tag{6}$$

8.1. Cylindrical specimens

To get the strain at the perimeter of the circular cross-section of a cylinder, the expression of x from the equation of a circle ($x^2 + y^2 = R^2$) is substituted into Eq. (6):

$$\varepsilon = \left(\frac{\varepsilon_1 - \varepsilon_2}{L}\right)\sqrt{R^2 - y^2} + \left(\frac{\varepsilon_2 - \varepsilon_3}{L}\right)y + \left(\frac{\varepsilon_1 + \varepsilon_3}{2}\right)$$

The y-coordinate of the point at which ε is maximum is determined by taking the derivative of ε in the expression above with respect to y, and setting it equal to zero:

$$y = \frac{(\varepsilon_2 - \varepsilon_3)R}{\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2}}$$

Substituting the above expression of y in the equation of the circle, one can get the corresponding x-coordinate of the point at which ε_{max} occurs:

$$x = \frac{(\varepsilon_1 - \varepsilon_2)R}{\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2}}$$

Now, the x- and y-coordinates above are substituted into Eq. (6) to get ε_{max}:

$$\varepsilon_{max} = (R/L)\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2} + (\varepsilon_1 + \varepsilon_3)/2$$

Since the platen is assumed rigid, ε_{min} occurs at a location diametrically opposite to the location of ε_{max}. Hence, the x- and y-coordinates at the location of ε_{min} have the same magnitude, but opposite sign, to those corresponding to ε_{max}. Substitution of the coordinates of the location of ε_{min} into Eq. (6) results in:

$$\varepsilon_{min} = -(R/L)\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2} + (\varepsilon_1 + \varepsilon_3)/2$$

Finally, the strain ratio can be obtained from Eq. (1), as follows:

$$\rho = \frac{2R\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2}}{L(\varepsilon_1 + \varepsilon_3)} \tag{7}$$

8.2. Cubical specimens

Unlike cylindrical specimens, cubical specimens may be subjected to biaxial straining. To derive the expressions of e_y/e_x and ρ for a cube, the equation of stress at any point within the cross-section of a biaxially loaded square area is considered:

$$\sigma = \frac{P}{B^2} + \frac{12Pe_x x}{B^4} + \frac{12Pe_y y}{B^4}$$

where P is the applied load, B is the side of the square, e_x is the eccentricity along the x-axis, and e_y is the eccentricity along the y-axis. Comparing the above equation after dividing it by the modulus of elasticity with Eq. (6) leads to:

$$e_x = \left(\frac{B^2}{6L}\right)\left(\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_3}\right) \text{ and } e_y = \left(\frac{B^2}{6L}\right)\left(\frac{\varepsilon_2 - \varepsilon_3}{\varepsilon_1 + \varepsilon_3}\right)$$

Hence,

$$\frac{e_y}{e_x} = \frac{|\varepsilon_2 - \varepsilon_3|}{|\varepsilon_1 - \varepsilon_2|}$$

In order to utilize the test efficiency-strain ratio plots in Fig. 5, the above expression needs to be positive and less than 1. Hence, it is modified as follow to allow for linear interpolation between Figs. 5(a) and 5(b):

$$\frac{e_y}{e_x} = \text{Min}\left(\frac{|\varepsilon_2 - \varepsilon_3|}{|\varepsilon_1 - \varepsilon_2|}, \frac{|\varepsilon_1 - \varepsilon_2|}{|\varepsilon_2 - \varepsilon_3|}\right) \tag{8}$$

The minimum and maximum strains, located at two diagonally opposite corners of the cube, are computed from Eq. (6), after substituting the coordinates of the corners:

$$\varepsilon_{\text{min}} = \frac{\varepsilon_1 + \varepsilon_3}{2} - \left[\frac{|\varepsilon_1 - \varepsilon_2|}{L}\right]\left(\frac{B}{2}\right) - \left[\frac{|\varepsilon_2 - \varepsilon_3|}{L}\right]\left(\frac{B}{2}\right)$$

$$\varepsilon_{\text{max}} = \frac{\varepsilon_1 + \varepsilon_3}{2} + \left[\frac{|\varepsilon_1 - \varepsilon_2|}{L}\right]\left(\frac{B}{2}\right) + \left[\frac{|\varepsilon_2 - \varepsilon_3|}{L}\right]\left(\frac{B}{2}\right)$$

Finally, the above expressions are substituted in Eq. (1) to find the strain ratio ρ:

$$\rho = \left(\frac{B}{L}\right)\left[\frac{|\varepsilon_1 - \varepsilon_2| + |\varepsilon_2 - \varepsilon_3|}{\varepsilon_1 + \varepsilon_3}\right] \tag{9}$$

9. Example 1

9.1. Problem statement

Correct the compressive strength of a 150 mm by 300 mm concrete cylinder having a failure load P = 1,100 kN, with a strain gradient defined by a plane having displacements equal to Δ_1 = 1.08 mm, Δ_2 = 0.84 mm, and Δ_3 = 0.33 mm measured, on a 200 mm-grid. Note that Δ_1 is measured at a location diametrically opposite to Δ_3, as shown in Fig. 8(b).

9.2. Solution:

First, calculate the observed compressive strength for the concrete:

$$(f'_c)_{app} = \frac{P}{A} = \frac{1,100\times 10^3}{\pi(75)^2} = 62.2 \text{ MPa}$$

Second, find the strain at points 1, 2 and 3:

$$\varepsilon_1 = \Delta_1/l = 1.08/300 = 0.0036 \text{ mm/mm}$$

$$\varepsilon_2 = \Delta_2/l = 0.84/300 = 0.0028 \text{ mm/mm}$$

$$\varepsilon_3 = \Delta_3/l = 0.33/300 = 0.0011 \text{ mm/mm}$$

Third, determine the strain ratio using Eq. (7):

$$\rho = \frac{2R\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2}}{L(\varepsilon_1+\varepsilon_3)} = \frac{2(75)\sqrt{(0.0036-0.0028)^2+(0.0028-0.0011)^2}}{200(0.0036+0.0011)} = 0.300$$

In Fig. 6, the initial value of η corresponding to $\rho = 0.3$ and apparent strength $(f'_c)_{app} = 62.2$ MPa is equal to $\eta_{ini} = 0.94$. Therefore, the corrected value of the concrete strength is:

$$(f'_c)_{cor} = \frac{(f'_c)_{app}}{\eta_{ini}} = \frac{62.2}{0.94} = 66.2 \text{ MPa}$$

Finally, the above value of f'_c needs to be corrected again since it was based on an efficiency corresponding to $f'_c = 62.2$ MPa, not the more accurate value of 66.2 MPa obtained above. Hence, re-enter Fig. 6 with $\rho = 0.3$ and $(f'_c)_{cor} = 66.2$ MPa to get a new efficiency $(\eta)_{new} = 0.93$. The strength of the concrete after filtering out the effect of the strain gradient is:

$$f'_c = \frac{(f'_c)_{app}}{\eta_{cor}} = \frac{62.2}{0.93} = 66.9 \text{ MPa}$$

The above value represents 7% increase from the observed strength.

10. Example 2

10.1. Problem statement

Correct the compressive strength of a 150 mm-cube having a failure load $P = 1{,}100$ kN, with a strain gradient defined by a plane having displacements equal to $\Delta_1 = 0.54$ mm, $\Delta_2 = 0.42$ mm, and $\Delta_3 = 0.16$ mm, measured on a 200 mm-grid. Note that Δ_1 is measured at a location diametrically opposite to Δ_3, as shown in Fig. 8(c).

10.2. Solution:

First, calculate the observed compressive strength for the concrete:

$$(f'_c)_{app} = \frac{P}{A} = \frac{11,00\times10^3}{(150)^2} = 49 \text{ MPa}$$

Second, find the strain at points 1, 2 and 3:

$$\varepsilon_1 = \Delta_1/l = 0.54/150 = 0.0036 \text{ mm/mm}$$

$$\varepsilon_2 = \Delta_2/l = 0.42/150 = 0.0028 \text{ mm/mm}$$

$$\varepsilon_3 = \Delta_3/l = 0.16/150 = 0.0011 \text{ mm/mm}$$

Third, determine the strain ratio using Eq. (9):

$$\rho = \left(\frac{B}{L}\right)\left[\frac{|\varepsilon_1-\varepsilon_2|+|\varepsilon_2-\varepsilon_3|}{\varepsilon_1+\varepsilon_2}\right] = \left(\frac{150}{200}\right)\left[\frac{|0.0036-0.0028|+|0.0028-0.0011|}{0.0036+0.0011}\right] = 0.53$$

The ratio of the strains about the cube's two axes is obtained from Eq. (8):

$$\frac{e_x}{e_y} = \text{Min}\left(\frac{|\varepsilon_2-\varepsilon_3|}{|\varepsilon_1-\varepsilon_2|},\frac{|\varepsilon_1-\varepsilon_2|}{|\varepsilon_2-\varepsilon_3|}\right) = \text{Min}\left(\frac{|0.0028-0.0011|}{|0.0036-0.0028|},\frac{|0.0036-0.0028|}{|0.0028-0.0011|}\right) = 0.47$$

For the case of $e_y = 0$, the initial value of η in Fig. 5(a) corresponding to $\rho = 0.53$ and apparent strength $(f'_c)_{app} = 49$ MPa is equal to $\eta_{ini} = 0.85$. For the case of $e_y/e_x = 1$, the initial value of η in Fig. 5(b) corresponding to $\rho = 0.53$ and apparent strength $(f'_c)_{app} = 49$ MPa is equal to $\eta_{ini} = 0.91$. Using linear interpolation, one can get $\eta_{ini} = 0.88$ for $e_y/e_x = 0.47$. Therefore, the corrected value of the concrete strength is:

$$(f'_c)_{cor} = \frac{(f'_c)_{app}}{\eta_{ini}} = \frac{49}{0.88} = 56 \text{ MPa}$$

Finally, the above value of f'_c needs to be corrected again since it was based on an efficiency corresponding to $f'_c = 49$ MPa, not the more accurate value of 56 MPa obtained later. Hence, re-enter Figs. 5(a) and 5(b) with $\rho = 0.53$ and $(f'_c)_{cor} = 56$ MPa to get a new efficiency $(\eta)_{\text{new}} = 0.87$, after interpolation. Therefore, the strength of the concrete after filtering out the effect of the strain gradient is:

$$f'_c = \frac{(f'_c)_{app}}{\eta_{ini}} = \frac{49}{0.87} = 56 \text{ MPa}$$

The above value is 14% higher than the observed strength.

11. Conclusions

The results of this study lead to the following conclusions:

1. Measurements of displacement between the upper and lower platens of a test machine can be used to quantify the effect of a strain gradient on the compressive strength result of concrete specimen and correct it.
2. For the same strain ratio, the test efficiency is larger for normal strength concrete than for high strength concrete. This is because the ascending and descending portions of the stress-strain curve for HSC are steeper than for NSC.
3. The effect of a strain gradient on the compressive strength test result depends on the cross-

sectional shape of the specimen. This is because the test efficiency is directly related to the percentage of the area that lies within the central region to the total area of the specimen's cross-section.

4. For the same strain ratio, the test efficiency is highest for a biaxially-strained cube and lowest for a uniaxially-strained cube, with the results for a cylinder lying in between these limits.

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