Computers and Concrete, Vol. 3, No. 5 (2006) 313-334
DOI: http://dx.doi.org/10.12989/cac.2006.3.5.313

Design optimization of reinforced concrete structures

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(Received April 18, 2006, Accepted August 25, 2006)

Abstract. A novel formulation aiming to achieve optimal design of reinforced concrete (RC) structures is presented here. Optimal sizing and reinforcing for beam and column members in multi-bay and multi-story RC structures incorporates optimal stiffness correlation among all structural members and results in cost savings over typical-practice design solutions. A Nonlinear Programming algorithm searches for a minimum cost solution that satisfies ACI 2005 code requirements for axial and flexural loads. Material and labor costs for forming and placing concrete and steel are incorporated as a function of member size using RS Means 2005 cost data. Successful implementation demonstrates the abilities and performance of MATLAB's (The Mathworks, Inc.) Sequential Quadratic Programming algorithm for the design optimization of RC structures. A number of examples are presented that demonstrate the ability of this formulation to achieve optimal designs.

Keywords: sequential quadratic programming; cost savings; reinforced concrete; optimal stiffness distribution; optimal member sizing; RS means; nonlinear programming; design optimization.

1. Introduction

This paper presents a novel optimization approach for the design of reinforced concrete (RC) structures. Optimal sizing and reinforcing for beam and column members in multi-bay and multi-story RC structures incorporates optimal stiffness correlation among structural members and results in cost savings over typical state-of-the-practice design solutions. The design procedures for RC structures that are typically adapted in practice begin by assuming initial stiffness for the structural skeleton elements. This is necessary to calculate the internal forces of a statically indeterminate structure. The final member dimensions are then designed to resist the internal forces that are the result of the assumed stiffness distribution. This creates a situation where the internal forces used for design may be inconsistent with the internal forces that correspond to the final design dimensions. The redistribution of forces in statically indeterminate structures at incipient failure, however, results in the structural performance that is consistent with the design strength of each member. Although this common practice typically produces safe structural designs, it includes an inconsistency between the elastic tendencies and the ultimate strength of the structure. In some cases this can cause unsafe structural performance under overloads (e.g. earthquakes) as well as unwanted cracking under normal building operations when factored design loads are close to service loads, (e.g. dead load dominated structures). This inconsistency also implies that such designs are unnecessarily expensive as they do not optimize the structural resistance and often result in

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members with dimensions and reinforcement decided by minimum code requirements rather than ultimate strength of allowable deflections.

Because of its significance in the industry, optimization of concrete structures has been the subject of multiple earlier studies. Whereas an exhaustive literature review on the subject is outside the scope of this paper, some notable optimization studies are briefly noted here. For example, Balling and Yao (1997), and Moharrami and Grierson (1993) employed nonlinear programming (NLP) techniques for RC frames that search for continuous-valued solutions for beam, column, and shear wall members, which at the end are rounded to realistic magnitudes. In more recent studies, Lee and Ahn (2003), and Camp, *et al.* (2003) implemented Genetic Algorithms (GA) that search for discrete-valued solutions of beam and column members in RC frames. The search for discrete-valued solutions in GA is difficult because of the large number of combinations of possible member dimensions in the design of RC structures. The difficulties in NLP techniques arise from the need to round continuous-valued solutions to constructible solutions. Also, NLP techniques can be computationally expensive for large models.

In general, most studies on optimization of RC structures, whether based on discrete- or continuous-valued searches, have found success with small RC structures using reduced structural models and rather simple cost functions. Issues such as the dependence of material and labor costs on member sizes have been mostly ignored. Also, in an effort to reduce the size of the problems, simplifying assumptions about the number of distinct member sizes have often been made based on past practices. While economical solutions in RC structures typically require designs where groups of structural elements with similar functionality have similar dimensions, the optimal characteristics and population of these groups should be determined using optimization techniques rather than predefined restrictions. These issues are addressed, although not exhaustively, in this paper, by incorporating more realistic costs and relaxed restrictions on member geometries.

This study implements an algorithm that is capable of producing cost-optimum designs of RC structures based on realistic cost data for materials, forming, and labor, while, at the same time, meeting all ACI 318-05 code and design performance requirements. The optimization formulation of the RC structure is developed so that it can be solved using commercial mathematical software such as MATLAB by Mathworks, Inc. More specifically, a sequential quadratic programming (SQP) algorithm is employed, which searches for continuous valued optimal solutions, which are rounded to discrete, constructible design values. Whereas the algorithm is inherently based on continuous variables, discrete adaptations relating the width and reinforcement of each element are imposed during the search.

This optimization formulation is demonstrated with the use of design examples that study the stiffness distribution effects on optimal span lengths of portal frames, optimal number of supports for a given span, and optimal sizing in multi-story structures. RS Means Concrete and Masonry Cost data (2005) are incorporated to capture realistic, member size dependent costs.

2. Optimization

2.1. RC structure optimization

The goal of optimization is to find the best solution among a set of candidate solutions using efficient quantitative methods. In this framework, decision variables represent the quantities to be

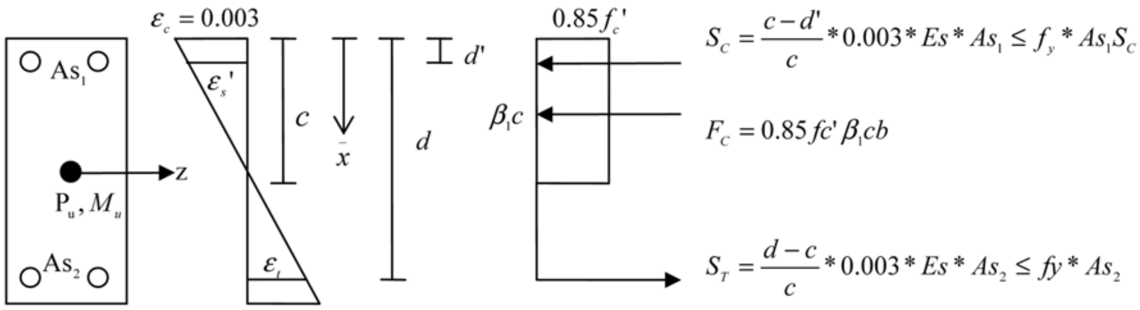

Fig. 1 Reinforced concrete cross section and resistive forces

determined, and a set of decision variable values constitutes a candidate solution. An objective function, which is either maximized or minimized, expresses the goal, or performance criterion, in terms of the decision variables. The set of allowable solutions, and hence, the objective function value, is restricted by constraints that govern the system.

Consider a two dimensional reinforced concrete frame with i members of length L_i. Each member has a rectangular cross section with width b_i and depth h_i, which is reinforced with compressive and tensile steel reinforcing bars, $As_{1,i}$ and $As_{2,i}$ respectively (Fig. 1). The set of b_i, h_i, $As_{1,i}$, and $As_{2,i}$ constitute the decision variables. The overall cost attributed to concrete materials, reinforcing steel, formwork, and labor is the objective function. The ACI-318-05 code requirements for safety and serviceability, as well as other performance requirements set by the owner, constitute the constraints. The formulation of the problem and the associated notation follow:

Indices:

i : RC structural member; beam or column.
m : steel reinforcing bar sizes.

Sets:

Columns : set of all members that are columns.
Beams : set of all members that are beams.
Sym : set of pairs of column members that are horizontally symmetrically located on the same story level.
Horiz : same as *Columns*, but activated only when the structure is subjected to horizontal loading.
et : set of member types; either *Columns* or *Beams*.

Parameters:

$C_{conc,\ mat'l}$ = 121.00 \$/m^3 - Material Cost of Concrete
C_{steel}(et) = 2420 \$/metric ton for beam members and 2340 \$/metric ton for column members
L_i - Length of member i, meters (typically 4 to 10 meters)
d' = 7 cm = Concrete Cover to the centroid of the compressive steel - same as the cover to the centroid of thee tensile steel.
f_c' = 28 MPa - Concrete Compressive Strength
β_1 = 0.85 - Reduction Factor f_c' = 28 MPa
Ec = 24,900 MPa - Concrete Modulus of Elasticity

Es = 200,000 MPa - Reinforcing Bar Modulus of Elasticity
f_y = 420 MPa - Steel Yield Stress
f_s' = Stress in Tensile Steel $\leq f_y$
bar_numbering = Metric equivalent bar sizes = [#13, #16, #19, #22, #25]
bar_diam$_m$ = Rebar diameters for m = 1:5. i.e., [12.7, 15.9, 19.1, 22.2, 25.4] mm
bar_area$_m$ = Rebar areas for m = 1:5. i.e., [129, 199, 284, 387, 510] mm^2
$\rho^c_{\min}$ = 0.01 - Minimum ratio of steel to concrete cross - sectional area in all column members
$\rho^c_{\max}$ = 0.08 - Maximum ratio of steel to concrete cross - sectional area in all column members
$\rho^b_{\min}$ = 0.0033 - Minimum ratio of steel to concrete cross - sectional area in all beam members

Decision Variables:
Primary Variables:
b_i - width of member i (cm)
h_i - depth of member i (cm)
$As_{1,i}$ - Compressive steel area of member i (cm^2)
$As_{2,i}$ - Tensile steel area of member i (cm^2)

Auxiliary Variables:
p_i - Perimeter of member i, 2*($b_i + h_i$) for columns, and (b_i + 2*h_i) for beams
$C_{forming}(b_i, h_i)$ - Cost of forms in placce (\$/SMCA) as a function of cross-sectional area as described in Fig. 2.1
$C_{conc,\ place}$ (b_i, h_i) - Cost of placing concrete (\$/m^3) as a function of corss-sectional area as described in Fig. 2.2
P_{ui} - Factored Internal Axial Force of member i determined via FEA (kN)
M_{ui} - Factored Internal Moment Force of member i determined via FEA (kN · m)
c_i - Distance from most compressive concrete fiber to the neutral axis for member i (cm)
$\bar{x}_i$ - Location of the plastic centroid of member i (cm) from the most compressive fiber.

Formulation:

$$(C)\text{: } \min \sum_{i=1}^{n} \begin{bmatrix} p_i \cdot L_i \cdot C_{forming}(b_i, h_i) + \\ (b_i \cdot h_i - As_{1,i} \cdot As_{2,i}) \cdot L_i \cdot (C_{conc,mat'l} + C_{conc,place}(b_i, h_i)) + \\ (As_{1,i} + As_{2,i}) \cdot L_i \cdot C_{steel}(et) \end{bmatrix} \tag{1}$$

subject to:

$$b_i = h_i \quad \forall i \in Columns \tag{2}$$

$$b_i = b_j \quad \forall (i \neq j) \in Sym \tag{3}$$

$$h_i = h_j \quad \forall (i \neq j) \in Sym \tag{4}$$

$$As_{1,i} = As_{1,j} \quad \forall (i \neq j) \in Sym \tag{5}$$

$$As_{2,i} = As_{2,j} \quad \forall (i \neq j) \in Sym \tag{6}$$

$$As_{1,i} = As_{2,i} \quad \forall i \in Horiz \tag{7}$$

$$\bar{x}_i = \frac{0.85 \cdot b_i \cdot h_i \cdot f'_c \cdot \dfrac{h_i}{2} \cdot As_{1,i} \cdot f_y \cdot d' \cdot As_{2,i} \cdot f_y \cdot (h_i - d')}{0.85 \cdot b_i \cdot h_i \cdot f'_c \cdot As_{1,i} \cdot f_y + As_{2,i} \cdot f_y} \forall i \tag{8}$$

$$c_i = 0.003 \frac{h_i - d'}{0.003 + f_y/Es} \forall i \tag{9}$$

$$P_{ui} \le 0.8 \cdot \phi_i \cdot 0.85 f'_c \cdot (b_i \cdot h_i - (As_{1,i} + As_{2,i})) + f_y \cdot (As_{1,i} + As_{2,i}) \forall i \tag{10}$$

$$As_{1,i} \le As_{2,i} \quad \forall i \tag{11}$$

$$b_i \le h_i \quad \forall i \tag{12}$$

$$h_i \le 5 \cdot b_i \quad \forall i \tag{13}$$

$$b_i \le 3 + 6/8 + bar\_diam_M \cdot (As_{2i}/bar\_area_M)/2 +$$

$$((As_{2,i}/bar\_area_M)/2 - 1) \cdot \max(1.0, 1.0 \cdot bar\_diam_M) \forall i \tag{14}$$

$$\rho^b_{\min,i} \le \frac{As_{1,i}}{b_i \cdot h_i} + \frac{As_{2,i}}{b_i \cdot h_i} \quad \forall i \in Beams \tag{15}$$

$$\frac{As_{2,i}}{b_i \cdot h_i} \le 0.0206 + \frac{f'_s}{f_y} \frac{As_{1,i}}{b_i \cdot h_i} \quad \forall i \in Beams \tag{16}$$

$$\rho^c_{\min,i} \le \frac{As_{1,i}}{b_i \cdot h_i} + \frac{As_{2,i}}{b_i \cdot h_i} \quad \forall i \in Columns \tag{17}$$

$$\frac{As_{1,i}}{b_i \cdot h_i} + \frac{As_{2,i}}{b_i \cdot h_i} \le \rho^c_{\max,i} \quad \forall i \in Columns \tag{18}$$

$$M_{ui}/\phi - a_o - a_1 P_{ui}/\phi - a_2 (P_{ui}/\phi)^2 - a_3 (P_{ui}/\phi)^3 - a_4 (P_{ui}/\phi)^4 - a_5 (P_{ui}/\phi)^5 \le 0 \tag{19}$$

$$b_i \ge 16 \text{ cm } \forall i,\ h_i \ge 16 \text{ cm } \forall i,\ As_{1,i} \ge 258 \text{ mm}^2\ \forall i,\ As_{2,i} \ge 258 \text{ mm}^2\ \forall i,$$

$$b_i \le 500 \text{ cm } \forall i,\ h_i \le 500 \text{ mm } \forall i,\ As_{1,i} \le 130{,}000 \text{ mm}^2\ \forall i,\ As_{2,i} \le 130{,}000 \text{ mm}^2\ \forall i \tag{20}$$

The objective function C, in Eq. (1), describes the cost of a reinforced concrete structure and includes, in order of appearance, forms in place cost, concrete materials cost, concrete placement and vibrating including labor and equipment cost, and reinforcement in place using A615 Grade 60 steel including accessories and labor cost. The costs of forming and placing concrete are a function of the cross-sectional dimensions b and h of the structural elements. These costs are detailed in Table 1 and in Figs. 2, 3, and 4. As shown in Figs. 2 through 4, linear interpolation between points is used to calculate cost of forming and the cost of placing concrete. Note that RS Means provides only the discrete points. The assumption of linear interpolation between these points is made by the authors due to lack of better estimates.

The constraints in Eqs. (2) through (7) define relative geometries for members in one of the specified sets: *Columns*, *Sym*, and *Horiz*. Eq. (8) defines the location of the plastic centroid of element i as a function of the decision variables. Eq. (9), defines the location of the neutral axis. Eq.

Table 1 RS Means 2005 concrete cost data all data in english units “from means concrete & masonry cost data 2005. Copyright Reed Construction Data, Kingston, MA 781-585-7880; All rights reserved.”

Product Description	Total Cost Incl. Overhead and Profit	Units
REINFORCING IN PLACE A615 Grade 60, including access. Labor		
Beams and Girders, #3 to #7	2420 (2200)	\$/metric ton (\$/ton)
Columns, #3 to #7	2340 (2125)	\$/metric ton (\$/ton)
CONCRETE READY MIX Normal weight		
4000 psi	121.0 (92.5)	$\$/m^3$ ($\$/Yd^3$)
PLACING CONCRETE and Vibrating, including labor and equipment.		
Beams, elevated, small beams, pumped. (small =< 929 cm2 (144 in^2))	79.8 (61.0)	$\$/m^3$ ($\$/Yd^3$)
Beams, elevated, large beams, pumped. (large =>929 cm2 (144 in^2))	53.0 (40.5)	$\$/m^3$ ($\$/Yd^3$)
Columns, square or round, 30.5 cm (12") thick, pumped	79.8 (61.0)	$\$/m^3$ ($\$/Yd^3$)
45.7 cm (18") thick, pumped	53.0 (40.5)	$\$/m^3$ ($\$/Yd^3$)
70.0 cm (24") thick, pumped	51.7 (39.5)	$\$/m^3$ ($\$/Yd^3$)
91.4 cm (36") thick, pumped	34.0 (26.0)	$\$/m^3$ ($\$/Yd^3$)
FORMS IN PLACE, BEAMS AND GIRDERS		
Interior beam, job-built plywood, 30.5 cm (12") wide, 1 use	41.0 (12.5)	\$/SMCA* (\$/SFCA)
70.0 cm (24") wide, 1 use	35.8 (10.9)	\$/SMCA (\$/SFCA)
Job-built plywood, 20.3 × 20.3 cm (8" × 8") columns, 1 use	41.0 (12.5)	\$/SMCA (\$/SFCA)
30.5 × 30.5 cm (12" × 12") columns, 1 use	37.1 (11.3)	\$/SMCA (\$/SFCA)
40.6 × 40.6 cm (16" × 16") columns, 1 use	36.3 (11.05)	\$/SMCA (\$/SFCA)
70.0 × 70.0 cm (24" × 24") columns, 1 use	36.7 (11.2)	\$/SMCA (\$/SFCA)
91.4 × 91.4 cm (36" × 36") columns, 1 use	34.3 (10.45)	\$/SMCA (\$/SFCA)

*Square Meter Contact Area and Square Foot Contact Area

(10) ensures that the applied factored axial load P_u is less than ϕP_n for the minimum required eccentricity, as defined by ACI 318-05 Eq. (10-2). Eq. (11) maintains that the tensile steel area is greater than the compressive steel area. The intent of this restriction is to facilitate the algorithmic search. Eqs. (12) and (13) are problem specific restrictions related to the width, b_i, and depth, h_i, of all members. Whereas these restrictions are common practice in low seismicity areas, they are by no means general requirements for all construction. While Eq. (12) ensures that the width is less than the depth, Eq. (13) prevents the creation of large shear walls and maintains mostly frame action for the design convenience of this study. Eq. (14) ensures that the tensile steel can be placed in element i with appropriate spacing and concrete cover as specified by ACI 318-05. In Eq. (14), the subscript M on *bar_diam* and *bar_area* corresponds to the discrete bar area that is closest to and not less than the continuous value of $As_{2,i}$. The constraints listed in Eqs. (15) through (18) ensure that the amount of reinforcing steel is between code specified minimum and maximum values. And finally, Eq. (19) ensures that the applied axial and bending forces of element i, P_{ui} and M_{ui}, determined with a Finite Element Analysis (FEA), are within the bounds of the factored P-M interaction diagram which is

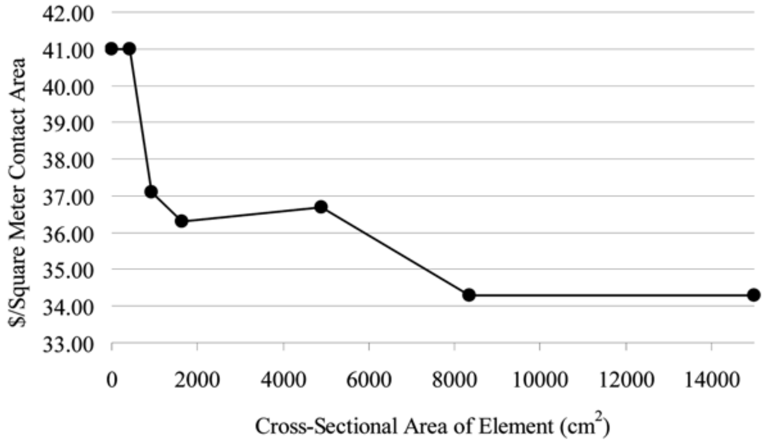

Fig. 2 Cost of FORMS IN PLACE for columns

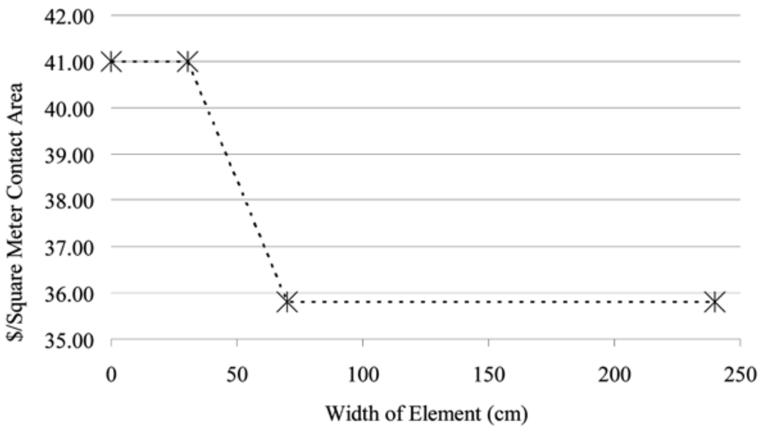

Fig. 3 Cost of FORMS IN PLACE for beams

modeled as a spline interpolation of five strategically selected (M_n, P_n) pairs. The lower and upper bounds designate the range of permissible values for the decision variables. The lower bounds on the width and depth are formulated from the code required minimum amount of steel and the minimum cover and spacing. Upper bounds decrease the range of feasible solutions by excluding excessively large members.

2.2. Optimization technique for RC structures

Various optimization algorithms can be used depending on the mathematical structure of the problem. MathWork's MATLAB is used to apply an SQP optimization algorithm to the described problem through MATLAB's intrinsic function "*fmincon*", which is designed to solve problems of the form:

Find a minimum of a constrained nonlinear multivariable function, $f(\mathbf{x})$,

subject to

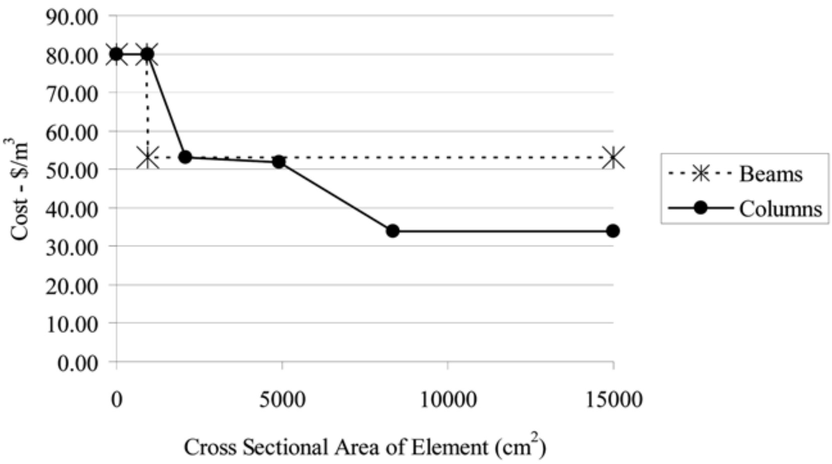

Fig. 4 Cost of PLACING CONCRETE and vibrating, including labor and equipment

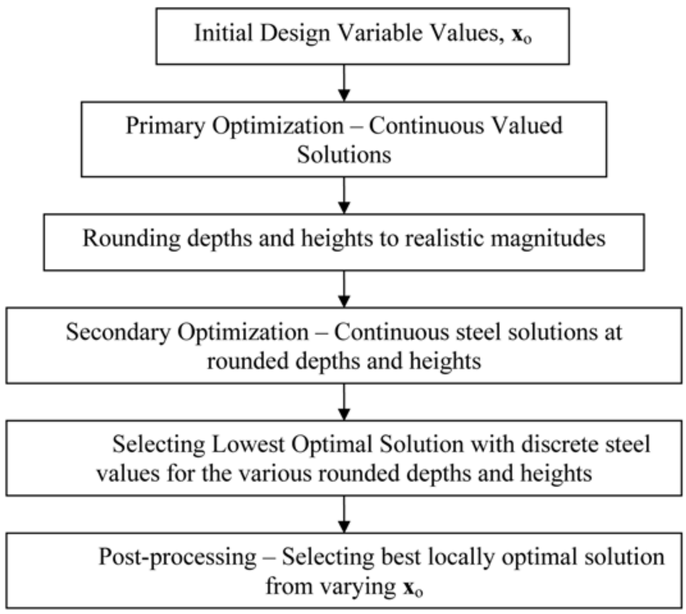

Fig. 5 Optimization routine flow chart

$g(\boldsymbol{x}) = 0;$

$h(\boldsymbol{x}) \leq 0;$

$\mathbf{Ib} \leq \mathbf{x} \leq \mathbf{ub};$

where $\mathbf{x}$ are the decision variables, $g(\mathbf{x})$ and $h(\mathbf{x})$ are constraint functions, $f(\mathbf{x})$ is a nonlinear objective function that returns a scalar (cost), and **Ib** and **ub** are the lower and upper bounds on the decision variables. All variables in the optimization model must be continuous.

The SQP method approximates the problem as a quadratic function with linear constraints within each iteration, in order to determine the search direction and distance to travel (Edgar and Himmelblau 1998).

The flow chart in Fig. 5 demonstrates the entire optimization procedure from generating initial decision variable values, $\mathbf{x}_o$, to selecting the best locally optimal solution from a set of optimal solutions found by varying $\mathbf{x}_o$. Initial decision variable values are found by solving the described optimization formulation for each individual element subjected to internal forces of an assumed stiffness distribution. At least ten different assumed stiffness distributions are utilized; each leads to a local optimal solution. Comparison of all local optimal solutions, not all of which are different, provides a reasonable estimation of the global optimum solution. Whereas the initial decision is based on an element-by-element optimization approach, the final optimization (Eqs. 1-20) is global and allows all element dimensions to vary simultaneously and independently in order to achieve the optimal solution. As such, the final design is achieved at an optimal internal stiffness configuration. This corresponds to the internal force distribution that ultimately results in the most economical design.

3. Design requirements

3.1. Cross-section resistive strength

Consider a concrete cross section reinforced as shown in Fig. 1, subjected to axial loading and bending about the z-axis. The resistive forces of the RC cross-section include the compressive strength of concrete and the compressive and tensile forces of steel, and are calculated in terms of the design variables (b, h, $As_{1,i}$, $As_{2,i}$), the location of the neutral axis c, and the concrete and steel material properties. It is assumed that concrete crushes in compression at ε_c=0.003 and that the strains associated with axial loading and bending very linearly along the depth of the cross-section. The bending resistive capacity M_n for a given compressive load P_n is calculated iteratively by assuming ε_c=0.003 at the most compressive fiber of the cross-section, and by varying c until force equilibrium is achieved. The strength reduction factor is calculated based on

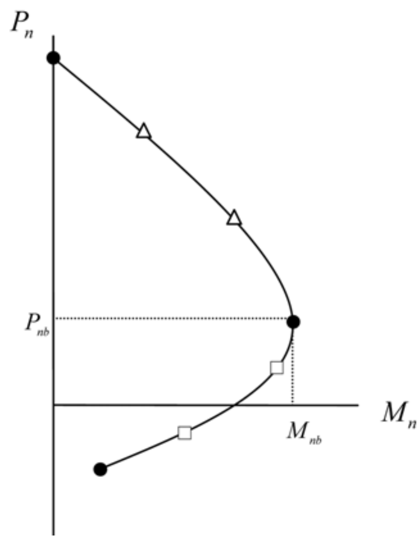

Fig. 6 Interaction diagram at failure state

the strain in the tensile steel. At this state, the resulting moment is evaluated, and the pair (M_n, P_n) at failure is obtained. The locus of all (M_n, P_n) failure pairs is known as the *M-P* interaction diagram for a member (Fig. 6).

3.2. M-P interaction diagram

Safety of any element i requires that the factored pairs of applied bending moment and axial compression $(M_{ui}/\phi, P_{ui}/\phi)$ fall within the *M-P* interaction diagram. The strength reduction factor, ϕ, is evaluated based on the strain of the most tensile reinforcement and is 0.65 for tensile strain less than 0.002, 0.9 for tensile strains greater than 0.005, and is linearly interpolated between 0.65 and 0.9 for strains between 0.002 and 0.005, as defined in ACI-318-05, Section 9.3. Finally, an axial compression cutoff for the cases of small eccentricity was placed equal to $P_u/\phi = 0.8[0.85f'_c(A_g - A_{st}) + f_y A_{st}]$ as per ACI-318-05 Eq. (10-2). Mathematically, if $F(M_n, P_n) = 0$ is a function that describes the interaction diagram, safety requires that $F(M_{ui}/\phi, P_{ui}/\phi) \leq 0$ for all i members. For a given cross-section, the interaction diagram is typically obtained point-wise by finding numerous combinations (M_n, P_n) that describe failure. For the purpose of this study, the interaction diagram is modeled as a cubic spline based on five points (Fig. 6), three of which are the balance failure point (M_{nb}, P_{nb}), the point of zero moment, and the point that corresponds to a neutral axis location at the level of the compressive steel axis. The remaining two points are located either above or below the balance failure point (M_{nb}, P_{nb}) depending on whether the applied axial compression load is greater or smaller than P_{nb}, respectively. Fig. 6 shows the three fixed points as solid circles and the two conditional points which are located below or above the balance failure point as open circles and open triangles, respectively.

It is assumed that the design for shear loads does not alter the optimal design decision variables b_i, h_i, $As_{1,i}$, and $As_{2,i}$. This assumption is typically acceptable for long slender elements where the combinations of flexural and axial loads commonly control the element dimensions. It is also assumed that the optimal solution is not sensitive to connection detailing. For structures in Seismic Design Category A, B, and C as classified in the ASCE 7 Standard (SEI/ASCE 7-98) this assumption is acceptable.

3.3. Rounding the continuous solution

Concrete design is ultimately a discrete design problem, where typical element dimensions are multiples of 50 mm and steel reinforcement consists of a finite number of commonly available reinforcing bars. Rounding b_i and h_i to discrete values is incorporated through the use of a secondary optimization process that finds the optimal reinforcing steel amounts for fixed b_i and h_i. Various combinations of rounding b_i and h_i either up or down to 5 cm multiples are examined to find the discrete solution with the lowest total cost.

Selecting a discrete number and size of longitudinal reinforcing steel from continuous-valued solutions is accomplished by finding the discrete number and size that is closest to and not less than the continuous-valued optimal solution. This is implemented into the optimization model so that the minimum width $b_{\min}$ that is required to fit the selected reinforcing steel becomes a lower bound that ensures proper cover and spacing of the longitudinal reinforcing steel. This process begins by calculating the discrete number of bars that are necessary to provide at least the steel area of the continuous solution. The minimum width required for proper cover and spacing for each set of reinforcing steel bars is calculated. Next, the combined cost of each bar set and the corresponding minimum required width is evaluated. The required width that is associated with the lowest combined cost is used as the minimum width requirement.

It is noted that when a discrete solution requires more than five reinforcing bars, the minimum required width is calculated based on bundles of two bars placed side-by-side. Also, for beam members, the compression and tensile reinforcements are designed to use the same size bars for construction convenience, as long as the strength requirements are still met. Finally, stirrups consist of #13 reinforcing bars.

4. Design examples

4.1. Optimization design examples

Three examples of optimal design for multi-story and multi-bay reinforced concrete frames are presented here to demonstrate the method. The first example studies the optimal design of a one-story portal frame with varying span length. The second example studies the optimal design of a multi-bay one-story frame with varying number of bays but with a constant over-all girder length of 24 meters. The third example creates optimal designs of multi-story, single-bay RC structures with and without horizontal seismic forces.

Designs based on standard approaches are created to compare optimal and typical-practice designs. While the same cost function used in the optimization formulation is implemented to calculate the *Typical Design Costs*, the design dimensions for the *Typical Design Costs* are based on a simplified state-of-the-practice design method. This method initially assumes that all columns will be 25 cm by 25 cm and that the width of all beams is 25 cm. Additionally, for beams the depth is equal to the width times a factor of one-third the beam length in meters (McCormac 2001). At this point an internal stiffness distribution has been defined and internal forces can be calculated. The amount of reinforcement is then designed to meet strength and code requirements for each member. The width of each beam member, or width and depth of each column member, is increased in increments of 5 cm if the assumed size of the member is not sufficient to hold the needed steel to maintain strength requirements. Note that the internal forces are based on the initially assumed stiffness distribution and not on the design dimensions.

For all examples presented here, the length of columns is four meters, the compressive strength of concrete is 28 MPa, the yield stress of steel is 420 MPa, the cover of compressive and tensile reinforcement is 7 centimeters, the unit weight of steel reinforced concrete is 24 kN/m^3, and the associated materials and construction costs are listed in Table 1.

4.2. Loading conditions

All frames examined here are loaded by their self weight w_G, an additional gravity dead load, w_D = 30 kN/m, and a gravity live load, w_L = 30 kN/m.

Seismic horizontal forces, wherever they are applied, are determined using the ASCE 7 (SEI/ASCE7-98) equivalent lateral load procedure for a structure in Denver, Colorado that is classified as a substantial public hazard due to occupancy and use, and is founded on site class C soil (SEI/ASCE7-98). A base shear force V is calculated using gravity loads equal to $1.0(w_D) + 0.25w_L$, and is

Table 2 Calculated equivalent lateral loads for multi-story, one-bay structures

Number of Stories:	Lateral load at:				
	First floor	Second Floor	Third Floor	Fourth Floor	Fifth Floor
	(kN)	(kN)	(kN)	(kN)	(kN)
One Story	83.23	--	--	--	--
Two Story	54.99	109.98	--	--	--
Three Story	41.24	82.48	123.73	--	--
Four Story	32.99	65.986	98.981	131.97	--
Five Story	27.494	54.99	82.484	109.98	137.47

then distributed appropriately to each frame story. Table 2 details the equivalent seismic horizontal forces for the multi-story design examples.

The frames that are subjected to gravity loads only are designed for factored loads of 1.2 w_G +1.2 w_D + 1.6 w_L = 1.2 w_G + 84 kN/m. The frames that are subjected to gravity and seismic loads are designed for the worse combination of the following factored load combinations (ACI 318-05):

1.2 w_G + 1.2 w_D + 1.6 w_L = 1.2 w_G +84 kN/m
1.2 w_G + 1.2 w_D + 1.0 w_L + 1.0 E = 1.2 w_G + 66 kN/m + 1.0 E
0.9 w_G + 0.9 w_D + 1.0 E = 0.9 w_G + 27 kN/m + 1.0 E

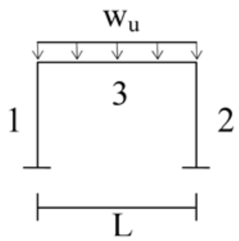

Fig. 7 Portal frame

Table 3 Optimal portal frame costs for various span lengths

Span Length (L)	Typical Design Costs	Optimal Design Cost	Cost Savings Over Typ. Design Cost	Optimal Cost per Foot of Structure	Optimal Cost per Foot of Beam
meters	Dollars	Dollars	Percent	Dollars	Dollars
4	2204.7	1913	13.2	159	478
6	3203.8	2979	7.0	213	497
8	4547	4504.7	0.9	282	563
10	6402.5	6336.8	1.0	352	634
12	8559.1	8407.9	1.8	420	701
14	11629	10702	8.0	486	764
16	14533	13192	9.2	550	825
24	33787	27975	17.2	874	1166

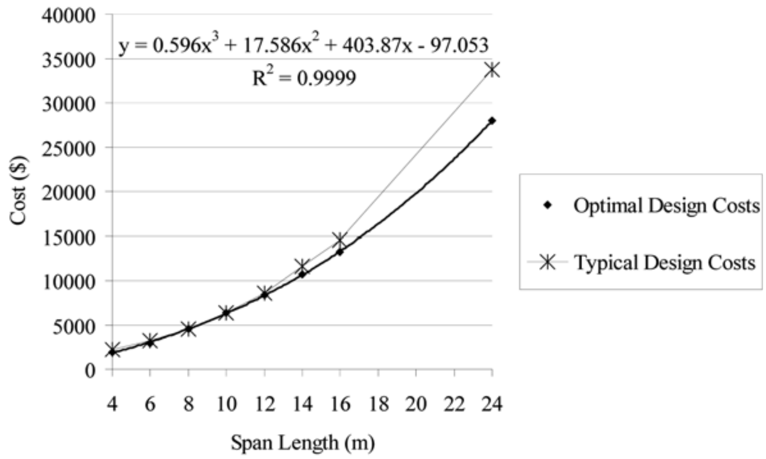

Fig. 8 Optimal costs for varying span lengths in a one-story structure

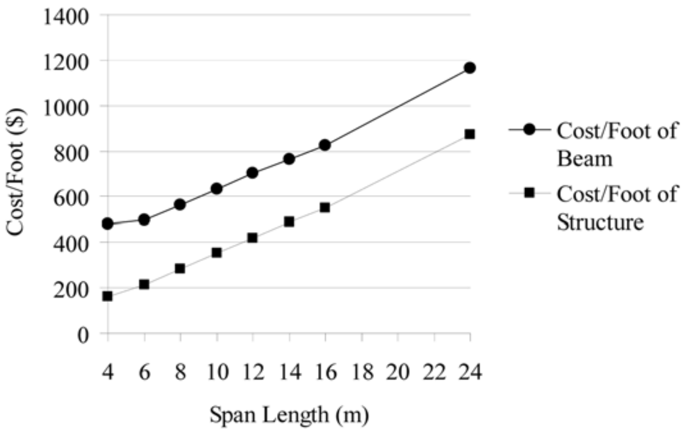

Fig. 9 Normalized optimal cost for varying span lengths in a one-story structure

4.3.1. One-story portal frame with varying span length

Consider a single-story portal frame subjected to gravity factored live and dead loads, w_U, totaling 84 kN/m. The structural model is shown in Fig. 7. The height of the structure is 4 meters, and the span, L, varies in order to study the effect of the span length on the optimal solution. Comparison of the costs at the specified span lengths is presented in Table 3, while a graphical presentation of the same data is shown in Fig. 8. The normalized cost per foot is presented in Fig. 9 to demonstrate the pure cost burden per foot associated with the larger span lengths. It should be noted that every point in the cost calculation of Figs. 8 and 9, with the exception of the typical design points, represents an optimal solution for the specific structure. Although the cost increases smoothly as a function of span length, the associated element cross-sectional dimensions do not follow an equally smooth change pattern. For spans of length up to 12 meters optimal solutions result in small columns and a large girder. The girders under such design develop moment diagrams that have small negative end-moments, large positive mid-span moments, and behave almost as simply supported beams. For span lengths of 14 m or larger, the pattern changes abruptly to one where columns become

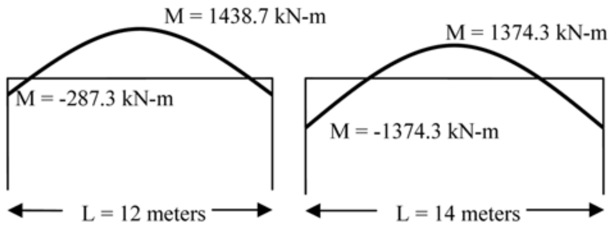

Fig. 10 Girder moments at optimal solution values

Table 4 Portal frame, L = 12 meters, cost = $8408

t = 3.4 minutes Element Number	b cm	h cm	As_1 Comp.	As_2 Tens.	P_u kN	M_u kN-m	ϕ
1	30	35	6#25	6#25	575.3	287.3	0.89
2	30	35	6#25	6#25	575.3	287.3	0.89
3	40	105	2#25	9#25	107.7	1438.6	0.90

Table 5 Portal frame, L = 14 meters, cost = $10702

t = 1.7 minutes Element Number	b cm	h cm	As_1 Comp.	As_2 Tens.	P_u kN	M_u kN-m	ϕ
1	60	65	4#25	13#25	668.2	1374.3	0.90
2	60	65	4#25	13#25	668.2	1374.3	0.90
3	40	90	2#25	9#25	511.0	1374.3	0.90

comparable in size to the girder, and are associated to bending moment distributions where the girders have equal negative and positive moment magnitudes. Fig. 10 illustrates the girder moment diagrams at the optimal solution values for the 12 and 14 meter span lengths. The characteristics and performance of the one-story portal frame for the 12 and 14 meter span lengths are presented in Tables 4 and 5. P_u and M_u are the critical internal forces of each element at the optimal solution and t is the total computation time in minutes using a Pentium 4 2.20 GHz laptop with a Windows XP Pro operating system.

Optimal designs result in cost savings of just under 1% for the 8-meter span to 17% for the 24-meter span (See Fig. 8 and Table 3). It is very interesting to note that for certain span lengths the typical design costs are relatively close to the optimal design costs. This demonstrates the regions where the typical practice assumptions result in efficient structures and where they do not. A beam length of eight meters for a portal frame with four meter long columns appears to be the most efficient structure for the typical practice assumptions as it results in a cost closest to the optimal cost.

4.3.2. Increasing the number of bays in a constant 24-meter span

In this example, a series of multi-bay one story frames are designed with a total span of 24 meters (Fig. 11). The frames range from a one bay structure supported by two columns (bay length of 24.0 m) to a twenty-four bay structure supported by 25 equally spaced columns (bay length of 1.0 m). A factored gravity load of 1.2 w_G + 84 kN/m is placed on the entire 24-meter girder. The cost of each

Table 6 Optimal cost for increasing number of bays in a 24 meter span

Number of Bays	Typical Design Cost ($)	Optimal Design Cost ($)	Cost Savings (%)
1	--	27975	--
2	--	14743	--
3	11432	10789	5.6
4	10226	9072	11.3
5	--	8443	--
6	9692.1	8161	15.8
7	10048	8117	19.2
8	--	8312	--
9	10875	8378	23.0
10	--	8773	--
12	--	9464	--
16	--	10852	--
20	--	12682	--
21	--	13050	--
22	--	13536	--
24	--	14508	--

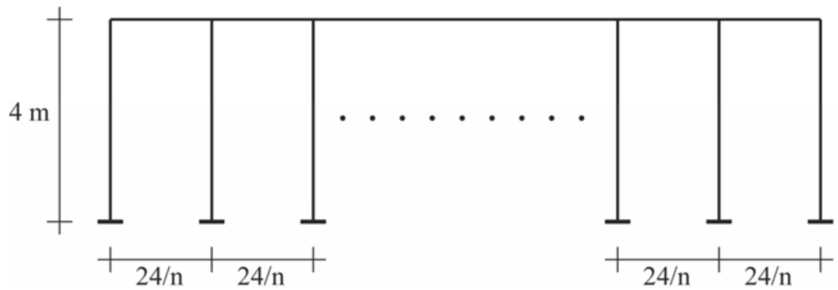

Fig. 11 Frame of length 24 m with n bays

typical and optimal design is presented in Table 6 and is plotted against the number of bays in Fig. 12. This example covers the entire range of $(M_u/\phi, P_u/\phi)$ combinations in the interaction diagram. The inner girders carry virtually no axial load (tension controlled), while the outer girders are loaded at high eccentricity (tension controlled or transition). The inner columns carry their loads with very small eccentricity and are controlled by the code cap on axial compression for columns of small eccentricities. Finally, the outer columns are compression controlled with a significant bending moment component.

Note in Fig. 12, the linear relationship between cost values for frames with 21, 22, and 24 bays. These correspond to solutions where all members are controlled by minimum code requirements for member dimensions and reinforcing steel: 20 cm by 20 cm members with a total of 4#13 reinforcing steel bars. For 20 bays or less, minimum code requirements gradually stop controlling the problem, starting with the outer beams. The lowest cost corresponds to seven spans of 3.4 meters each. It is noted again that every design presented in Fig. 12 is optimal for the specific geometry, i.e., span length.

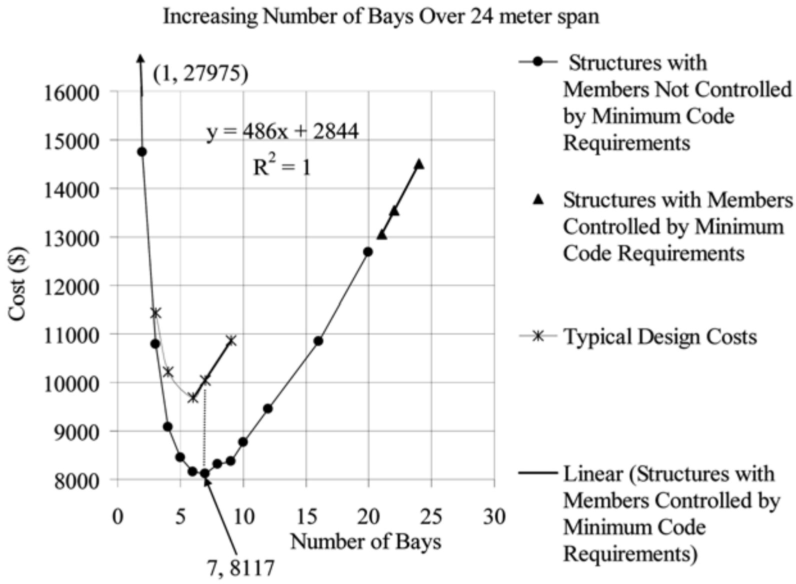

Fig. 12 Increasing number of bays in a 24 meter span

In all cases with 20 bays and less, the model is such that beams increase in size to keep the columns as small as possible. Only the two-span and the one-span frames have columns with cross-sectional dimensions larger than 25 cm. In all multi-span structures, the outermost beams carry larger load and, in most cases, the inner beams are close to minimum values.

Substantial costs savings over typical design is demonstrated for multi-bay structures. Note in Fig. 12 that after reaching the lowest typical design cost of approximately \$9700 for the six-bay structure, the costs increase linearly with each additional bay. The linear increase indicates that minimum dimensions have been reached for the typical design assumptions. Thus, each additional bay increases the total cost by the cost of one column and one beam.

4.3.3. Multi-story design examples

The designs presented in this section address three groups of multi-story, single bay frames. The first group consists of frames that have a span length of 4 m, one to six stories and are subjected to

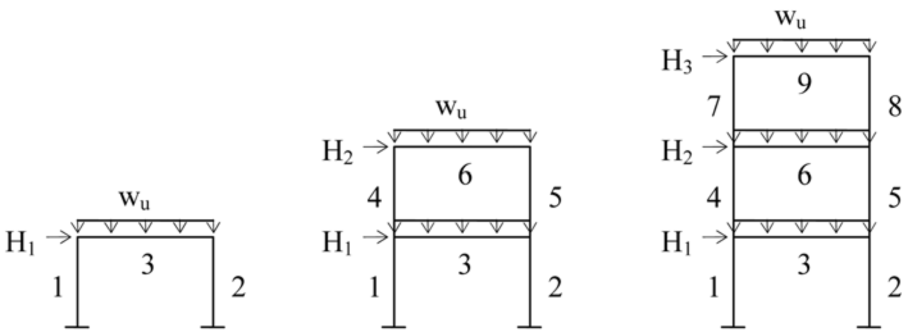

Fig. 13 Multi-story single-bay structures

gravity loads only. The second group is similar to the first group. However, the frames have a span length of 10 m. The third group consists of frames that have a span length of 10 m, one to five stories, and are subjected to gravity and seismic loads.

Fig. 13 displays the one, two, and three story frames subjected to gravity and seismic forces. Element numbering and loading for the four-, five- and six-story structures follows the same pattern as in the frames presented in Fig. 13. The magnitudes of the seismic forces at each floor are listed in Table 2.

4.3.3.1. Multi-story - vertical load only

Given the effects of girder length on the optimal design patterns of portal frames, *short-span* and *long-span*, (4m and 10 m respectively) multistory frame designs are examined here. The optimal costs of multi-story frames subjected to gravity loads only are presented in Fig. 14, where circular marks represent the long span frames and rectangular marks represent the short span frames. Note that the *long-span* results are presented in two groups: open circles for the frames that have an odd number of stories (1, 3, or 5), and solid circles for the frames that have an even number of stories (2, 4, or 6). No such distinction is necessary for the short span frames.

In general, it can be seen from Fig. 14 that the cost increases linearly with the number of stories for both the *long*- and *short-span* frames. The linear relation between the number of stories and cost is almost exact in the case of *short-span* frames. On the other hand, a closer examination of the *long-span* frames (see data points and their least-square-fit lines) indicates that the odd-story frames are slightly more expensive than the even-story frames.

The practical significance of this observation is not clear. Nevertheless, from the theoretical standpoint, this is an interesting, and rather unexpected finding that merits further analysis and explanation. Let us consider the n-story frame of Fig. 15. Note that the end-rotational tendencies of each girder are resisted by one column above and one column below at each end, with the

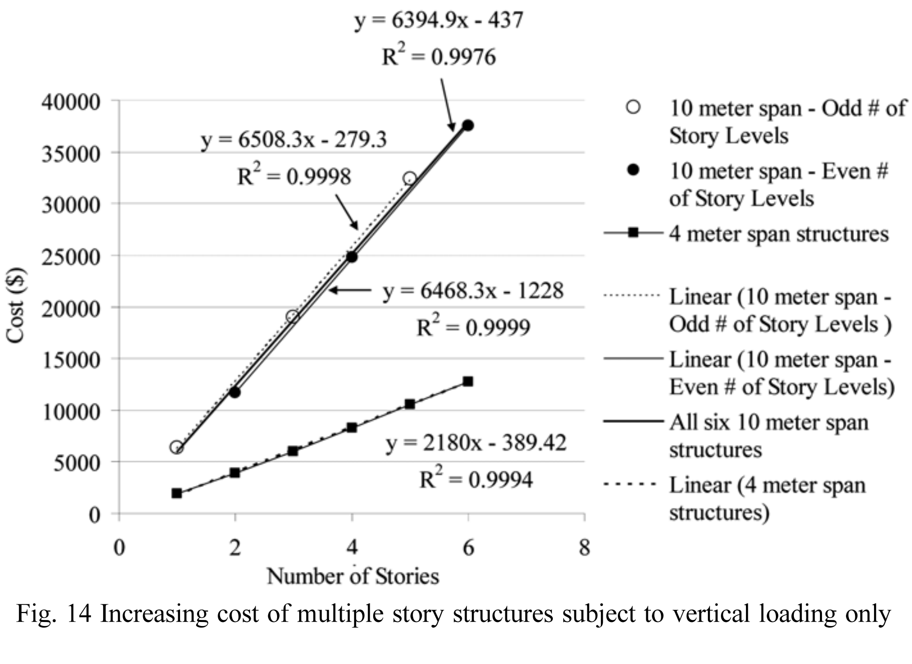

Fig. 14 Increasing cost of multiple story structures subject to vertical loading only

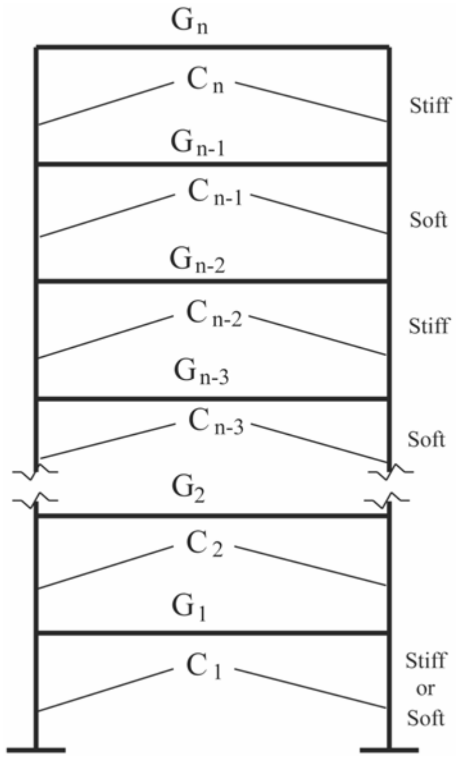

Fig. 15 Column stiffness tendencies for *Long-Span* multistory frames under gravity loads

exception of the roof girder, where only one column below the girder provides the rotational resistance at each end of the girder. To assist each of the long girders carry their large moments in a cost-effective way, columns tend to be stiff in order to restrict rotation and develop sufficient negative end moment, which in turn reduces the mid-span positive moment. Thus, considering the n-story frame of Fig. 15, the columns C_n underneath the top girder G_n, tend to be stiff. The bottom-ends of these columns (C_n) also provide large rotational resistance to the next girder $G_{n\text{-}1}$. As a result, the next set of columns $C_{n\text{-}1}$ does not need to be as stiff. This is indicated in Fig. 15 by the label “soft” next to columns $C_{n\text{-}1}$. Since the bottom-end of these columns do not provide sufficient rotational stiffness to the next girder $G_{n\text{-}2}$, the next set of columns $C_{n\text{-}2}$ must be stiff (see label in Fig. 15). Thus, an alternating pattern of stiff-soft columns is created. In frames of even number of stories, this pattern results in an equal number of stiff and soft sets of columns. On the other hand, in frames of odd number of stories, the same pattern results in one extra story of stiff columns. Thus, the odd-story frames are relatively more expensive than the even-story frames. It should be pointed out however, that a soft column at a lower story tends to be stiffer than a soft column at a higher story, since it carries larger axial load. Tables 7 and 8 detail the optimal solution results for the four- and five-story frame without lateral loads.

For *short-span* structures, creation of stiff columns to help distribute the moment more efficiently in the girder is not cost effective, given that girders and columns have a similar length, which makes two small columns and one large girder less expensive. Thus in the case of *short-span* frames, the stiff-soft pattern described earlier is not efficient, and there is no distinction between

Table 7 Four-story, single bay long span structure, vertical load only

t = 17.7 minutes Element Number	b cm	h cm	As_1 Comp.	As_2 Tens.	P_u kN	M_u kN-m	ϕ
1	40	40	4#16	4#16	1834.2	91.3	0.65
2	40	40	4#16	4#16	1834.2	91.3	0.65
3	55	55	2#22	14#22	-291.7	739.7	0.90
4	60	60	4#19	5#25	1371.4	656.9	0.90
5	60	60	4#19	5#25	1371.4	656.9	0.90
6	30	75	2#25	6#25	299.6	707.7	0.90
7	30	30	4#13	4#13	919.6	55.9	0.65
8	30	30	4#13	4#13	919.6	55.9	0.65
9	55	55	2#22	14#22	-316.3	727.0	0.90
10	55	55	2#19	14#19	456.8	701.0	0.90
11	55	55	2#19	14#19	456.8	701.0	0.90
12	40	65	2#19	12#19	343.0	701.0	0.90

Table 8 Five-story, single bay long span structure, vertical load only

t = 18.5 minutes Element Number	b cm	h cm	As_1 Comp.	As_2 Tens.	P_u kN	M_u kN-m	ϕ
1	50	55	2#25	5#25	2374.0	431.5	0.65
2	50	55	2#25	5#25	2374.0	431.5	0.65
3	35	70	2#25	7#25	32.7	701.2	0.90
4	40	40	11#16	11#16	1914.4	269.7	0.65
5	40	40	11#16	11#16	1914.4	269.7	0.65
6	50	55	2#22	13#22	-116.1	735.2	0.90
7	50	50	5#22	15#13	1447.7	491.8	0.89
8	50	50	5#22	15#13	1447.7	491.8	0.89
9	50	55	2#22	12#22	117.6	737.9	0.90
10	40	40	3#19	10#19	985.0	264.0	0.65
11	40	40	3#19	10#19	985.0	264.0	0.65
12	45	60	2#22	11#22	-105.2	714.4	0.90
13	45	45	5#19	6#25	519.0	480.5	0.67
14	45	45	5#19	6#25	519.0	480.5	0.67
15	65	95	2#22	6#22	232.7	817.1	0.90

odd- and even-story frames.

The structural tendencies described above were also observed in the one-story portal frame discussed in section 4.2.1. It was found there that smaller span frames favored small end moments (small columns-larger girder), while the larger span frames were more economical when larger end moments were developed (large columns-smaller girder). In the one story example of section 4.2.1 the transition between "small" and "large" span occurred between 12 m and 14 m. In the multistory frames of this section the transition occurred at less than 10 m, due to the different end conditions of the girders.

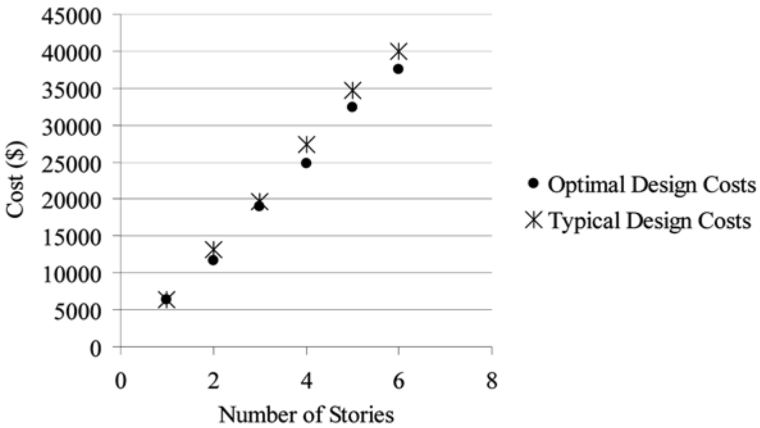

Fig. 16 Comparison of optimal and typical design costs in multiple story structures subject to vertical loading only

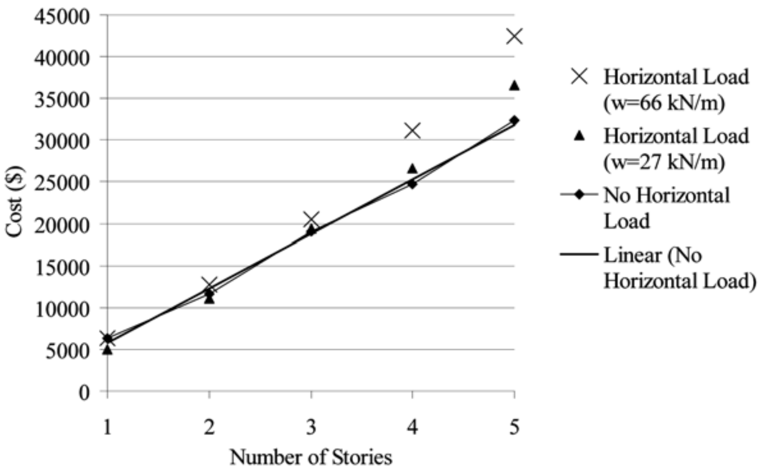

Fig. 17 Increasing cost of multiple story structures subject to horizontal loading

Fig. 16 illustrates the same optimal costs for the *long-span* structures along with typical design costs. The typical design assumptions resulted in efficient stiffness distributions for the one-story structure. The two-through six-story structures showed cost savings of 11.6%, 3.0%, 9.4%, 6.9%, and 6.2%, respectively.

4.3.3.2. Multi-story frames with lateral loads

The long-span multi-story frames of the previous section are designed here for gravity and seismic loads, as described in Table 2. Following ACI 318-05 requirements, both *heavy* (66 kN/m) and *light* (27 kN/m) gravity loads are considered as calculated in the Section 4.2, on Loading Conditions. The increased stiffness requirements due to the lateral loads eliminate the stiff-soft patterns observed in the gravity-only examples of the previous section. The cost of each optimized design is presented in Fig. 17, along with the costs of the gravity only frame designs to indicate the cost increase due to lateral loading. It can be seen that the seismic loads do not cause significant cost burdens for frames with three stories or less, but they become fairly costly for taller structures. This observation is site dependent, and can be different for seismic loads that correspond to a different site.

5. Conclusions

This paper presents a novel approach for optimal sizing and reinforcing multi-bay and multi-story RC structures incorporating optimal stiffness correlation among structural members. This study incorporates realistic materials, forming, and labor costs that are based on member dimensions, and implements a structural model with distinct design variables for each member. The resulting optimal designs show costs savings of up to 23% over a typical design method. Comparison between optimal costs and typical design method costs demonstrates instances where typical design assumptions resulted in efficient structures and where they did not. The formulation, including the structural FEA, the ACI-318-05 member sizing and the cost evaluation, was programmed in MATLAB (Mathworks, Inc.) and was solved to obtain the minimum cost design using the SQP algorithm implemented in MATLAB's intrinsic optimization function *fmincon*.

A number of fairly simple structural optimization problems were solved to demonstrate the use of the method to achieve optimal designs, as well as to identify characteristics of optimal geometric spacing for these structures.

It was found that optimal portal frame designs follow different patterns for small and large bay lengths. More specifically it was found that *short-span* portal frames are optimized with girders that are stiff compared to the columns, thus ensuring girder *simple supported action*, while *long-span* portal frames are optimized with girders that are approximately as stiff as the columns, thus splitting the overall girder moment to approximately equal negative and positive parts.

It was also found that girders that are supported by multiple supports, as in the case of multi-bay frames, have an optimal span length, below which the design becomes uneconomical because some members are controlled by code imposed minimum sizes, and above which the design also becomes uneconomical as the member sizes tend to become excessive.

Finally it was found that optimal design multi-story frames present similar characteristics to one-story portal frames where short-bay designs are optimal with girders that are stiff compared to the columns, and *long-span* designs are optimal with girders that are approximately as stiff as the columns. For gravity dominated *long-span* multi-story frames, optimal designs tend to have alternating stiff and soft columns. It is not however clear that this pattern exists in optimal designs of multi-bay multistory frames, considering that the interior columns typically do not carry significant moments due to gravity. Finally, the alternating stiff/soft column pattern was not observed when the horizontal seismic loads had a significant influence on the design.

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