

P-value significance level test for high-performance steel fiber concrete (HPSFC)

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Abstract. Statistical analysis has found useful application in the design of experiments (DOE) especially optimization of concrete ingredients however, to be able to apply the concept properly using computer aided applications there has to be an upper and lower limits of responses fed to the system. In this study, the production of high-performance steel fiber concrete (HPSFC) at five different fiber addition levels by volume with two aspect ratios of 60 and 83 were studied under two curing methods completely dry cured (DC) and moist cured (MC) conditions. In other words, this study was carried out for those limits based on material properties available in North Cyprus. Specimens utilized were cubes 100 mm size casted and cured for 28 days and tested for compressive strength. Minitab 18 statistical software was utilized for the analysis of results at a 5 per cent level of significance. Experimentally, it was observed that, there was fluctuation in compressive strength results for the two aspect ratios and curing regimes. On the other hand P-value hypothesis evaluation of the response showed that at the stated level of significance, there was a statistically significant difference between dry and moist curing conditions. Upper and lower limit values were proposed for the response to be utilized in DOE for future studies based on these material properties. It was also suggested that for a narrow confidence interval and accuracy of the system, future study should increase the sample size.

Keywords: high performance steel fiber concrete; statistical significance; compressive strength; dry and moist curing; confidence interval

1. Introduction

1.1 Background

In the field of statistics, hypothesis is an “inference” about something or someone projecting a certain statement that as a result of the evidence for or against it is true or false. However, this inference has to be stated in a scientific manner as either as *null*, H_0 (hypothesis to be tested) or *alternative*, H_1 hypothesis (rejecting the null statement in favour of the substitute). Accepting the latter doesn't imply that the former is wrong, however, it only implies that there is no enough evidence in its favour or the evidence provided in favour of the latter was more convincing. The latter also helps the experimenter in reaching a conclusive decision.

There are two types of errors associated with conventional hypothesis testing techniques which are the possibility of committing a Type I error, this occur when the null hypothesis is rejected when it is in fact true. Type II error on the other hand is the failure to reject the null hypothesis when it is actually not true; these types of errors have the tendency to occur during experimentation especially when trying to reduce the possibility of one occurring, however, to curtail this scenario, increasing the sample size is one way of ensuring a better result. The choice of H_0 is very important because it defines

experiment, H_1 defines if it is a one sided or two sided hypothesis and a preselected level of significance, α (the level of risk willing to be taken by the experimenter) upon which “acceptance” or “rejection” is based.

The shortcoming of the approach highlighted above is that the α -value is the decision making parameter, and value above and beyond are discounted from the decision making process where ability of the experimenter to make informed and engineering decision is limited. This is because, in some certain situations, the expected difference is very negligible and as such if only the experimenter had a different decision making process so that all avenues could be exploit. One of such technique is the P-value level of significance approach which gives the experimenter the ability to analyze results far and wider than the acceptance or rejection region. Walpole *et al.* (2012) enumerated the steps used in this approach:

1. State null and alternative hypothesis.
2. Choose an appropriate test statistic.
3. Compute the P-value based on the computed value of the test statistic.
4. Use judgment based on the P-value and knowledge of the scientific system.

Optimization of concrete mix constituents is now beginning to feature prominently in civil engineering application. Simon *et al.* (1997) used typical volume fractions from Kosmatka and Panarese (1988) to optimize the best mixtures for high-performance concretes; Abdullahi *et al.* (2009) modeled lightweight concrete mixtures using a central composite design (CCD) and the statistical modeling at 95% confidence interval to predict compressive strength,

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Table 1 Mix design utilized for HPSFC

Material	Cement	Water	Coarse (0-10 mm)	Fine (0-5 mm)	Silica Fume	HRWR*
Quantity (kg/m ³)	470	165	1050	700	47	14

*HRWR - high range water reducer

air-dry density and slump at a significant level of less than 0.05. A complete description of model analysis and transformation with Minitab software can also be found in Md Diah *et al.* (2012) detailing the efficiency of the software in statistical methodology in calibration and validation of model for research purposes. Ahmad and Alghamdi (2014) used full factorial design with three factors and three levels to optimize concrete mixture; Tirkolaei and Bilsel (2015) investigated urea hydrolysis for biocement production by response surface methodology (RSM) with central composite face-centered (CCF). Mosaberpanah and Eren (2016) utilized central composite design in RSM to model flexural toughness of steel fiber concrete with the assumption that the sum of volume fractions sum to unity. In DOE method of mixture optimization, the design space which is the set of values over which the factor (variable capable of affecting the response) is to be varied over some design points which can be one factor with two (2) levels positive (high level +1) and negative (low level -1). Sometimes for stability as well as observing the curvature, a centre point (zero or midpoint) can be added.

The need for high strength in certain construction activities which is an advantage on one hand and the resulting increase in brittleness on the other hand has necessitated the need to incorporate fibers (steel, polymer, natural etc.) to reduce the brittleness. The addition of fiber is necessitated due to the decrement in material ductility which is a serious hindrance to the application of high strength concrete. Incorporation of the fibers reduces microcracking and crack propagation with an improvement in ductility and strength (Khaloo and Kim 1996).

It is known that in the production of high strength concrete, crushed rock aggregates are most preferred, and ASTM C117-13 allows precision estimates of fine aggregates with material finer than 75 μm between 1.0-3.0%. BS 882: Part 2: 1983 allows up to 15% by mass of crushed rock fine aggregates, however, the crushed rock fine aggregates in North Cyprus can have a percentage up to 25% dust (Celik and Marar 1996). It has therefore become imperative to verify the properties of these materials when it comes to utilization especially regarding national codes produced elsewhere. There has been a significant amount of studies regarding the high strength concrete in North Cyprus with encouraging results; Eren and Celik (1997) reported an increase in compressive strength of 28.27% at 1% fiber volume, V_f and 10% silica fume addition and also a splitting strength increase of 129.91% at 2% V_f at the silica fume percentages when compared with plain concretes. The work of Eren *et al.* (1999) measured some mechanical properties of high-strength fiber-reinforced concrete and reported that there was an increase in

Table 2 Aggregate properties

Properties	ASTM C	Fine aggregate (0-5 mm)	Coarse aggregate (10 mm)
Relative Density (SSD)	127;128	2.68	2.65
Absorption (%)	“	3.0	0.7
Bulk Density (kg/m ³)	29	2082.85	1203.22
Voids in Aggregate (%)	“	24.66	49.55
Material Finer than 75 μm (%)	117	12.73	-

SSD: Saturated Surface Dry

resistance to impact with increase in V_f , plain concrete saw an increase in surface abrasion resistance with fiber and silica fume addition, and poor compaction may likely result in a decrease in compressive strength.

A follow up study (Marar *et al.* 2011) measured the compression specific toughness of normal and high strength fiber concrete at different aspect ratios and concluded that it was significantly increased in both strength classes at V_f of 2.0% and provided equations that can be used to estimate in both cases. The influence of aspect ratio on shear behavior was also studied (Marar *et al.* 2016) and it was observed that shear strength increases with an increase in V_f , and also noted that there was no significant effect on shear strength which they attributed to possibly the same length of fiber utilized. The work of Aydin (2017a, b) utilizing staple wire as reinforcement in high-volume fly ash cement paste composite reported a regression coefficient (R^2) of above 0.8 at the age of 28 days but with a decline in compressive strength at volume fraction above 3%.

1.2 Significance of the study

The significance of this study is based on the fact that compressive strength is to be modeled as a response based on two curing regimes (one moist the other completely deprived of moisture beyond the initial mixing water). Based on the selection process for factor levels, previous studies relied on literature from elsewhere for the maximum and minimum values. In this study, a mix design procedure was developed based on the methodology outlined in Aitcin (1998), specimens prepared with different fiber content, and the results analyzed statistically based on the average results of the responses (strength) that in future will serve as a guide for modeling responses, based on experimental results obtained from material properties domiciled in North Cyprus rather than elsewhere. It is this levels that are needed in defining the range of the response to be modeled and therefore in our opinion this study is relevant.

2. Experimental procedure

Cement utilized was Blast-furnace Slag Cement CEM II/B-S 42.5 N in conformity with ASTM C 595-17 with a specific gravity of 3.15. Silica fume had a 95% content of SiO_2 and utilized at 10% of the cement content. Tap water was utilized which conform to BS EN 1008-02. High range water reducer was utilized conforming with ASTM C 494-

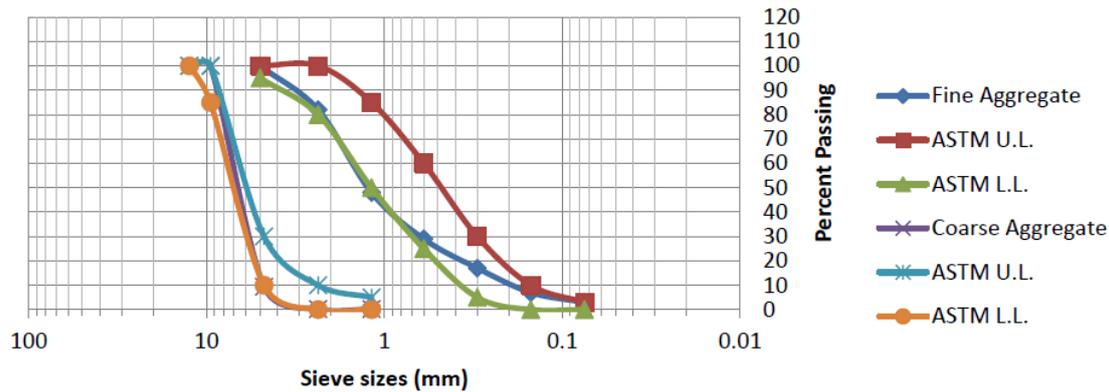


Fig. 1 Sieve analysis of aggregates

Table 3 Properties of steel fibers

Length (mm)	Diameter (mm)	l/d Ratio	Tensile Strength (MPa)
30	0.50	60	1250
50	0.60	83	1200

l/d: Length-to-diameter ratio

16 ether brown in colour with a density of 1,023-1,063 kg/lt colour content <0.1% and alkali content <3%.

Aggregates used were crushed limestone rock conforming to the specification of ASTM C 33-16 and sieve analysis was conducted in accordance with ASTM C 136-14 and the result presented.

Steel fibers used in this study were hooked-end bundled type conforming to ASTM A820 with two different aspect ratios (l/d: length-diameter ratio) of 60 and 83. The lengths and diameters of the two types of fibers were 30 and 50 mm and 0.5 and 0.6 mm, respectively. Five different percentages were added to the high-performance steel fiber concrete as 0.5, 0.75, 1.0, 1.25 and 1.5 % by volume of concrete (39.25, 58.88, 78.50, 98.125 and 117.75 kg/m³, respectively).

Mixing operation was done based on ACI 544.3R with the fibers which are stacked in a fibrillated bundles of water soluble glue placed last by distribution in small amount to avoid balling. Immediately upon contact with moisture they are dispersed but the mixing time was relatively longer for volume percentages from 1.25%. Curing was based on the specifications of ASTM C192-16a in the curing room and after 24 hours, the moist-cured (MC) series were demoulded and placed in the curing tank at laboratory conditions while the dried-cured (DC) series were left in the curing room after demoulding until testing date. Concrete compressive strength was determined at 28 days in accordance with BS EN 12390 - 3 for an average of three to five cubes with a loading rate of 0.5 MPa/s. This investigation involved casting 60 cubes of 100 mm size for both MC and DC specimens.

On designing the experiment, the response variable was the compressive strength at two curing regimes with five (5) treatment levels of each factor level (four - MC 60; DC 60; MC 83; DC 83) making 20 runs multiplied by an average of 3 samples equals 60. The results obtained were analyzed by Minitab 18 statistical software at significance level of 0.05.

3. Results and discussion

3.1 Observed compressive strength

Compressive strength test results presented in Table 4, indicates that at aspect ratio 60 and volume fractions less than one per cent, higher strength values were obtained for both curing medium. It was also observed at volume fraction of 1.0% in both moist and dry cured specimens for aspect ratio of 83, there was a decline in the strength values before increasing again. In most high-strength concretes with steel fiber addition there is either a slight increment or decrease in strength, and the peak stress is mostly reached at 1%-1.5%. In this study, there was no reasonable change in compressive strength because it is mostly affected by the strength of the coarse aggregate which in this case the matrix behavior is similar to the aggregate. Contribution of the steel fiber is mostly in post-cracking ductility which prevented the else while explosive failure associated with high-strength concrete. Therefore, there was no noticeable trend in these results. Marginal increase or decrease in strength has been reported in some studies in ACI 544.4R.

3.2 Basic statistics

Results from Table 4 were used to generate Table 5 for statistical analysis.

From the results presented in Table 5 and graphically depicted by boxplot in Fig. 2, the indicator for the centre and spread showed that the median value for the dry cured specimens was midway in the quartile group 3 (third 25%), the boxplot also seems to be missing an upper whisker. This indicates that the whisker length in this case is zero because the maximum value in the range is 102.30 MPa, and the median value (centre) is 100.60 MPa which is supposed to be at the centre but instead has a difference of just about 2 MPa compared with 95.00 MPa which is the minimum value and a strength of 5 MPa difference with the median. It can therefore be observed that the third quartile (Q3) strength is equal to the maximum strength. Whiskers do not normally extend beyond the middle quartile. In the moist cured specimens for aspect ratio of 60, the minimum and maximum strength were 99.60 MPa and 108.30 MPa respectively with the median towards the upper quartile, in

Table 4 Compressive strength results

<i>l/d</i>	Fiber Volume (V_f)	Series Name	Compressive Strength (MPa)
60 DC	0.50	BDC	95.00
	0.75	CDC	102.30
	1.00	DDC	98.60
	1.25	EDC	100.60
	1.50	FDC	102.30
60 MC	0.50	BMC	108.30
	0.75	CMC	105.30
	1.00	DMC	99.60
	1.25	EMC	106.10
	1.50	FMC	105.70
83 DC	0.50	BDC	89.60
	0.75	CDC	94.10
	1.00	DDC	95.40
	1.25	EDC	91.20
	1.50	FDC	93.90
83 MC	0.50	BMC	98.30
	0.75	CMC	98.10
	1.00	DMC	97.80
	1.25	EMC	95.00
	1.50	FMC	95.30

B, C, D, E, F-series (average of at least 3 specimens) **M** or **D** - moist or dry **C**-cubes

Table 5 Basic statistics

Variable	Dry Cured <i>l/d</i> =60	Moist Cured <i>l/d</i> =60	Dry Cured <i>l/d</i> =83	Moist Cured <i>l/d</i> =83
Samples (N)	5	5	5	5
Mean	99.76	105.00	92.84	96.90
SE Mean	1.37	1.45	1.06	0.720
StDev	3.07	3.23	2.37	1.611
Variance	9.40	10.23	5.613	2.595
Sum of Squares	49797.9	55166.84	43118.78	46958.43
Minimum	95.00	99.60	89.60	95.000
Q1	96.80	102.45	90.40	95.15
Median	100.60	105.70	93.90	97.800
Q3	102.30	107.20	94.75	98.200
Maximum	102.30	108.30	95.40	98.300
Skewedness	-1.10	-1.48	-0.58	-0.57

this case both whiskers were present. The range is spread over an 8 MPa strength which is considerably longer. In the case of the aspect ratio of 83 dry cured specimens, the range was from 86.6 MPa to 95.4 MPa and the median value was also having the same trend as that of aspect ratio 60 samples in Q3. On the other hand the range for moist cured samples in this category was relatively short a little over 3 MPa long with the median as seen in Fig. 2.

In general, the boxplot displayed the range and spread of the strength values and how closer (agreement) or wider (different) the range is. The result also indicates all the data are skewed to the left with the skewedness higher in the specimens with aspect ratio of 60. This indicates that the bulk of the strength is in the right side of the frequency distribution which corresponds with the strength gain of

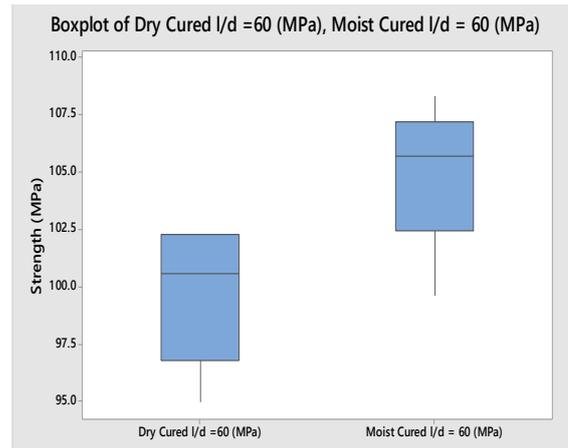


Fig. 2(a) Boxplots of strength at aspect ratio 60 and different curing medium

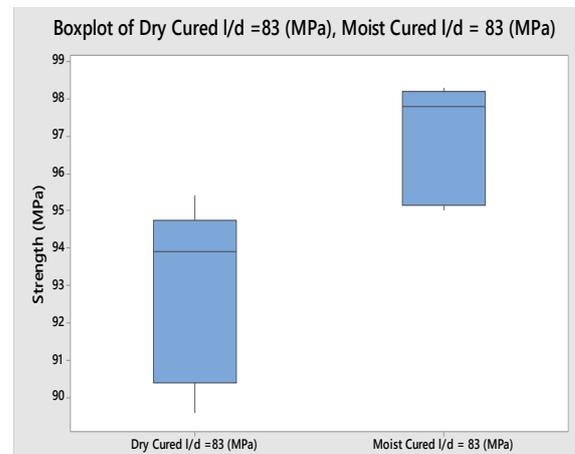


Fig. 2(b) Boxplots of strength at aspect ratio 83 and different curing medium

steel fiber reinforced concrete until above 1 per cent when it starts to decrease. Also, the mean is less than the median as in most cases when data is skewed to the left.

Standard error of the mean that shows the degree of variability of the sample mean indicated that the precision increased with aspect ratio of 83 due to the lower value of the range recorded in Table 5. Also, the standard deviation which measured variability in the single sample followed the same trend observed in standard error of mean and was found to decrease significantly in the aspect ratio of 83 specimens than their counterparts in aspect ratio of 60. This variation could be attributed to the range of values, a larger value of standard deviation is an indication that the spread is widely around the mean of the data.

The results in Table 6 shows the Grubb's outlier test (Eq. (1)) that was conducted at a significance level of 0.05 usually represented in boxplots by an asterisks (*) appearing far away from the group especially if the level of skewedness in the data is very high. These can have a significant influence on the mean of the distribution. Minitab test the difference between the mean and either the largest or smallest value divided by the standard deviation.

$$Grubb's\ test = \frac{Mean - (largest\ or\ smallest\ value)}{standard\ deviation} \quad (1)$$

Table 6 Grubbs' outlier test

Variable	Dry Cured l/d=60	Moist Cured l/d=60	Dry Cured l/d=83	Moist Cured l/d=83
Mean	99.76	105.00	95.84	96.900
StDev	3.07	3.23	2.37	1.611
Min	95	99.60	89.60	95.000
Max	102.30	108.30	95.40	98.300
G	1.55	1.67	1.37	1.18
P	0.283	0.102	0.661	1.000

No outlier was detected at the 5% level of significance for l/d= 60; No outlier at 5% level of significance for l/d=83

Based on the hypothesis that $P\text{-value} \leq \alpha$ (significance level), an outlier exist, if the $P\text{-value} > \alpha\text{-value}$ the conclusion than an outlier exist cannot be made and the null hypothesis remains valid. In the results of the Grubb's test statistic in Table 6, all the values were well above the P-value, and we concluded that based on the range of values available for aspect ratio of 60 and 83 at a significance level of 0.05, outliers were not detected.

3.3 Probability plots

The probability plots presented in Fig. 3 for the 95 per cent confidence interval for moist and dry cured specimens at aspect ratios of 60 and 83 indicated that all the samples were in the confidence band and the straight line between the band shows how fitted the results are by their distribution along the line. The figure also shows the mean value of each series, standard deviation as well as P-value which were all above 0.05 in all cases and the assumption of normality is valid. Anderson-Darling statistic in all cases was lower but the lowest result came from dry cured specimen with aspect ratio of 83. It is quite understandable because it was the series with the highest P-value (0.449) which is an indication of the fitted nature of the data in this case.

3.4 Confidence interval (CI)

3.4.1 Confidence interval of the mean

Two sample t-test was conducted on both aspect ratios and curing regimes with the assumption that the variances were not equal (degree of freedom determined using Welch-Satterthwaite method) and subsequently the confidence interval obtained. The hypotheses were in both cases:

Null hypothesis $H_0: \mu_1 - \mu_2 = 0$

Alternative hypothesis $H_1: \mu_1 - \mu_2 \neq 0$

Where:

μ_1 : mean of Dry Cured Samples

μ_2 : mean of Moist Cured Samples

The 95 per cent CI for the difference in the mean compressive strength for dry and moist cured samples at the given aspect ratios were -9.95 and -0.53. It is also worthy of note that the specimens at aspect ratio of 60 had a difference in mean of 5.24 while that of 83 was 4.06 with a CI of -7.09 and -1.03. The standard error of the mean presented in Table 7 is an approximation of the one

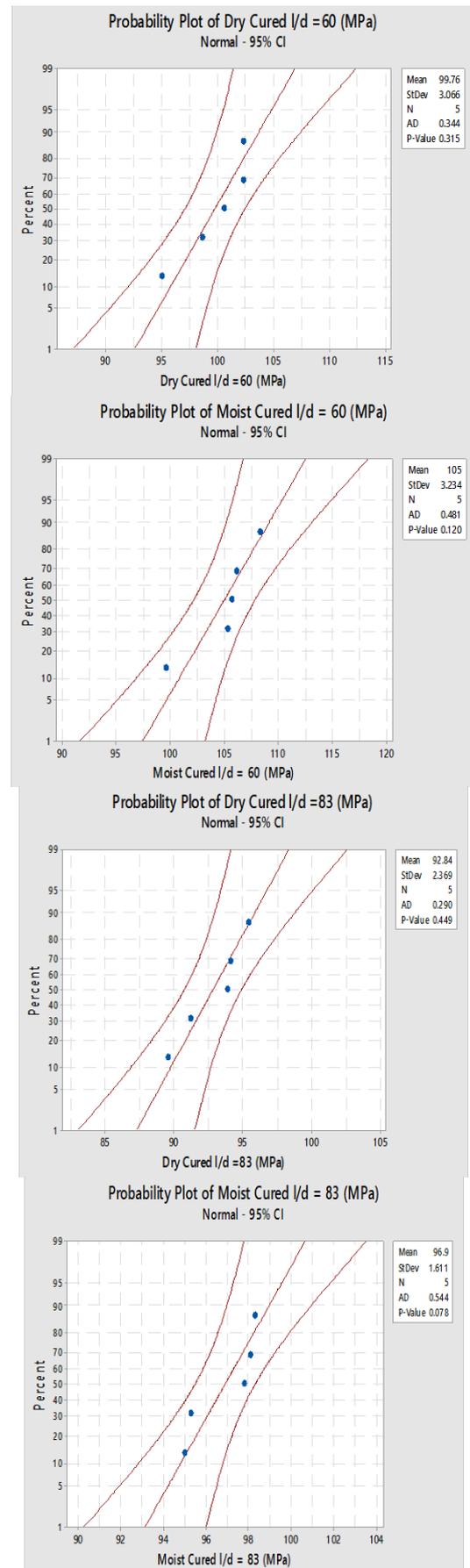


Fig. 3 Probability plots of dry and moist cured specimens at different aspect ratios

Table 7 Confidence interval of the difference

Variables	Dry Cured l/d=60	Moist Cured l/d=60	Dry Cured l/d=83	Moist Cured l/d=83
Mean	99.76	105.00	92.84	96.90
StDev	3.07	3.23	2.37	1.61
SE Mean	1.4	1.4	1.1	0.72
Difference in Mean	-5.24		-4.06	
95% CI for Difference	(-9.95, -0.53)		(-7.09, -1.03)	
T-Value	-2.63		-3.17	
Degree of Freedom	7		7	
P-Value	0.034		0.016	

Table 8 CI for two variances using Bonett and Levene's test

Variables	Dry Cured l/d=60	Moist Cured l/d=60	Dry Cured l/d=83	Moist Cured l/d=83
StDev	3.066	3.234	2.369	1.611
Variance	9.403	10.460	5.613	2.595
95 % CI for σ	(1.368, 11.305)	(1.199, 14.351)	(1.244, 7.422)	(0.999, 4.274)
Estimated Ratio	0.948129		1.47072	
95 % CI for Ratio using Bonett	(0.272, 4.213)		(0.528, 3.325)	
95 % CI for Ratio using Levene	(0.040, *)		(*,*)	

*The interval for ratio using Levene was not calculated because one or both confidence limits cannot be calculated.

presented earlier in Table 5. The significance of the mean difference is that if no difference exists between the two curing regimes at the aspect ratios of 60 and 83 then the difference of the mean ($\mu_1 - \mu_2 = 0$) will be equal to zero (null hypothesis), then there is no statistically significant difference in each case. On the other hand, if the null value is not included, then the conclusion is that there is a statistically significant difference. In both cases, from Table 7, it can be seen that the value of the null hypothesis is not included, leading to conclusion that there is a statistically significant difference in the mean difference and a very wide difference exist.

To correlate the CI with the P -value, if P -value < alpha value (0.05), the result is statistically significant and the CI will not include value of the null hypothesis. The results showed there is a statistically significant difference in the means because zero is not included in the confidence interval.

Aspect ratio 60: P -value (0.034) < alpha (0.05) within CI (-9.95, -0.53) Zero not included; Aspect ratio 83: P -value (0.016) < alpha (0.05) within CI (-7.09, -1.03) Zero not included.

3.4.2 Confidence interval of two variances

A test for CI of two variances using Bonett and Levene

Table 9 Bonett and Levene test for standard deviation or variances

σ_1/σ_2 for l/d=60				
Method	Test Statistic	DF1	DF2	P-Value
Bonett	0.01	1		0.917
Levene	0.04	1	8	0.843
σ_1/σ_2 for l/d=83				
Method	Test Statistic	DF1	DF2	P-Value
Bonett	0.87	1		0.350
Levene	0.27	1	8	0.616

was conducted for the two curing regimes and aspect ratios and presented in Table 8 however, Bonett result is mostly more reliable than Levene when the result is not heavily skewed and the result shown graphically in Fig. 4.

To ascertain if the ratio of two variances is statistically significant, it was compared with the P -value as presented in Table 9 at alpha value of 0.05. The guideline is that if the ratio of either standard deviation or variance when compared with P -value ≤ 0.05 it is an indication that the result is statistically significant and the null hypothesis should be rejected.

σ_1 = standard deviation of dry cured samples

σ_2 = standard deviation of moist cured samples

$$\text{Null hypothesis } H_0: \sigma_1/\sigma_2 = 1 \quad (2)$$

$$\text{Alternative hypothesis } H_1: \sigma_1/\sigma_2 \neq 1 \quad (3)$$

From Table 9 it can be seen that the P -values were all above the alpha value of 0.05 leading to the conclusion that the standard deviation of the moist cured are not equal to that of the dry cured specimens and not statistically significant. In Fig. 5 graphical representations of the equal variances test is shown at different aspect ratios, and if there is no overlapping in the standard deviations, then the variances are significantly different. As can be seen, there is overlapping in both aspect ratios and when correlated with Table 8 the dry and moist cured aspect ratio of 60 overlapped at 95 per cent CI of standard deviation of 1.368 (dry cured) and 14.351 (moist cured), the end point of dry cured and start of moist cured are within this particular range. That of aspect ratio of 83 boundary conditions is within 0.999 (moist cured) and 7.422 (dry cured) while the starting point of dry cured specimen and end point of moist cured is in this range.

3.4.3 Confidence interval for the median

Man-Whitney test was conducted to estimate the difference in population medians for both aspect ratios and were found to be 5.1 and 4 the result presented in Table 10 and graphically depicted in Fig. 6 with an achieved confidence interval of 96.33 %. To test the level of statistical significance at 0.05 it was assumed that if: P -value ≤ 0.05 : the difference is statistically significant P -value > 0.05: the difference is not statistically significant and we may have to reject the null hypothesis at that level of significance.

η_1 : median of Dry Cured η_2 : median of Moist Cured

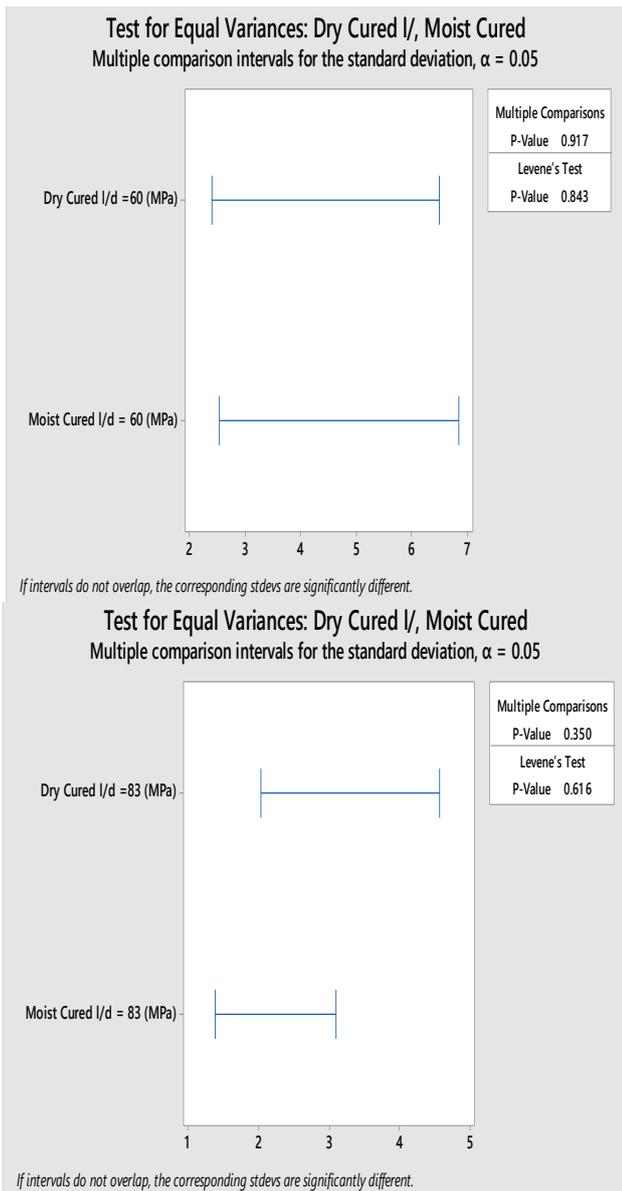


Fig. 5 Equal variances test

Difference: $\eta_1 - \eta_2$.

$$\text{Null hypotheses } H_0: \eta_1 - \eta_2 = 0 \quad (4)$$

$$\text{Alternative hypotheses } H_1: \eta_1 - \eta_2 \neq 0 \quad (5)$$

From Table 10 the P-values when adjusted for ties and when not adjusted were all above the alpha value an indication that the difference is not statistically significant. W-value is the “Wilcoxon test statistic” which is used to correct ties using the rank test however, the number of ties should be large enough to make an impact on the results. On the other hand, for aspect ratio of 83, only P-value for not adjusted ties was displayed by the software and it was below the alpha value leading to the conclusion that the difference in the median was statistically significant.

3.5 Coefficient of determination

Results presented in Table 11 is for testing the difference

Table 10 Man-Whitney test for median

Variables	0.46	Moist Cured l/d=60	Dry Cured l/d=83	Moist Cured l/d=83
Median	100.6	105.7	93.9	97.8
Difference		-5.1		-4
CI for Difference		(-10.7, 1)		(-8.2, -0.9)
Achieved Confidence		96.33%		96.33%
Not adjusted for ties (W-Value)		18.00		17.00*
Not adjusted for ties (P-Value)		0.060		0.037*
Adjusted for ties (W-Value)		18.00		
Adjusted for ties (P-Value)		0.059		

*The output did not distinguish between adjusted or otherwise in the case of ties.

Table 11 R- squared results

Variables	Dry Cured l/d=60	Moist Cured l/d = 60	Dry Cured l/d = 83	Moist Cured l/d = 83
Mean	99.76	105.00	92.84	96.900
StDev	3.07	3.23	2.37	1.611
95 % CI	(95.95, 103.57)	(100.98, 109.02)	(89.90, 95.78)	(94.900, 98.900)
R-sq		46.35 %		55.66 %

Table 12 Welch test for variances

Welch's Test l/d = 60				
Source	DF Num	DF Den	F-Value	P-Value
Factor	1	7.97741	6.91	0.031
Welch's Test l/d = 83				
Source	DF Num	DF Den	F-Value	P-Value
Factor	1	7.04724	10.04	0.016

between equality of the means based on a 95 per cent CI working on the assumption that if P-value is less than or equal alpha value, the difference between the means are statistically significant. On the other hand when the P-value is greater than alpha, it is an indication that the means are equal. Another important results presented in the table is R² or coefficient of determination which has a range of 0 ≤ R² ≤ 1 with a perfect fit when R²=1. According to Walpole *et al.* (2012), reliability of R² depends on the regression data size and the applicability; they also noted that too much premium should not be placed on the use of R² as the only consideration. Aspect ratio of 60 presented a percentage of variation of 46.35 % and 53.66 % in 83.

Welch test which is a robust test for equality of means usually conducted for unequal variances was done for the two aspect ratio and the result as seen in Table 12 showed that the P-value is less than alpha value because the F-statistic exceeds the critical value $f_{\alpha}(1, n-2)$ where the P-values were 0.031 and 0.016 for aspect ratios of 60 and 83 respectively. This leads us to conclude that there is significant variation in the means.

3.6 Effect of sample size

In this study the sample size was limited to five samples per series for each of the curing regimes at the given aspect ratios and it was due to the amount of fiber percentage that was included in each batch. This has resulted in a wider confidence interval (Fig. 2) at the selected significance level of 0.05 as opposed to a narrow confidence interval that would have been obtained when using a large sample size. According to Walpole *et al.* (2012) on normal approximation regarding sample size, that if the sample size is less than 30 and the population is approximately normal or known to be normal, the sampling distribution of the mean will follow the normal distribution even if the sample size is small, but the distribution becomes better when the sample size grows bigger.

Lloyd (2014) examined the influence of Satterthwaite error on sample size by simulating 2.5 percentile for T using $2 \leq m \leq 11$ and $2 \leq n \leq 11$ for a 95 per cent CI where m and n are sample size from two independent samples. It was found that the errors for $n=3$ were significantly higher than those for $n=4$, and boxplots for the error difference showed for sample size less than five the results presented too many outliers and was skewed to the right with a symmetric plot observed with sample size greater than six.

4. Conclusions

A study was conducted to test the level of statistical significance using P -value on HPSFC at two different curing regimes in modeling upper and lower level of responses in design of experiments, and the following conclusions have been drawn:

- The additions of steel fiber resulted in either an increase or decrease in compressive strength with no consistency, however, the higher the aspect ratio, the lower the compressive strength under both curing regimes. Compressive strength values were higher for specimens subjected to moist curing, and fluctuations in the results were much more under dry curing for both aspect ratios. However, according to statistical analysis moist cured specimens with aspect ratio 83 seems to be the best among the others in terms of compressive strength.
- According to the experimental results, the upper and lower limits for statistical analysis of the examined response were determined as 99.76 MPa and 105 MPa for dry and moist cured specimens at aspect ratio of 60. While 95.84 MPa and 96.90 MPa for aspect ratio of 83, respectively.
- A P -value statistical evaluation was conducted at 95 % confidence interval on the mean difference, ratio of the variances, and median values, findings revealed that the means and medians of the moist and dry cured samples were significantly different. On the other hand, the ratio of the variance was found to be above the α -value of 0.05. Having considered all these evidences for and against the hypothesis, we can only conclude that there is a significant difference in moist and dry curing

methods in HPSFC based on P -value analysis.

The following recommendation is suggested for further study:

- It could be seen that the coefficients of determination were 46.35% and 53.66% for aspect ratio of 60 and 83, respectively, these could be as a result of some errors or that there is no direct relationship between the combination of moist and dry curing technique. To improve the efficiency of the system, there is a need to increase the sample size from five (5) to a higher number so that the probability plots and confidence interval could become narrower giving a more accurate results.
- In Minitab 18, from Power and Sample Size under Statistics, sample size estimation could be done with the knowledge of the mean difference, standard deviation, and significance level. However, in our study which is a one-factor-at-a-time experiment which possess a challenge on how much increment in fiber addition level could be obtained, it is hereby suggested that fiber reinforcement index (FRI) which is a product of l/d and V_f be used to increase the sample size and data points for analysis of results.

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