

## Reliability assessment of RC shear wall-frame buildings subjected to seismic loading

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(Received May 22, 2016, Revised August 9, 2017, Accepted August 10, 2017)

**Abstract.** A considerable research is available on the seismic response of Reinforced Concrete (RC) shear wall-frame buildings, but the studies on the reliability of such buildings, with the consideration of human error, are limited. In the present study, a detailed procedure for reliability assessment of RC shear wall-frame building subjected to earthquake loading against serviceability limit state is presented. Monte Carlo simulation was used for the reliability assessment. The procedure was implemented on a 10-story RC building to demonstrate that the shear walls improve the reliability substantially. The annual and life-time failure probabilities of the studied building were estimated by employing the information of the annual probability of earthquake occurrence and the design life of the building. A simple risk-based cost assessment procedure that relates both the structural life-time failure probability and the target reliability with the total cost of the building was then presented. The structural failure probability (i.e., the probability of exceeding the allowable drift) considering human errors was also studied. It was observed that human error in the estimation of total load and/or concrete strength changes the reliability sharply.

**Keywords:** serviceability limit state; RC shear wall-frame building; reliability; story drift; earthquake loading

### 1. Introduction

Numerous reinforced concrete (RC) shear wall-frame buildings exist almost in all the big cities of the world including the cities of Saudi Arabia. Many of these cities lie in moderate to high seismic regions. The reliability assessment of such shear wall-frame buildings against earthquake, likely to occur in the life-time of the building, is necessary to assure a minimum safety level for the occupants' lives and the property (Douglas *et al.* 2013, Goulet *et al.* 2007, Haselton *et al.* 2011, Ulrich *et al.* 2014). In the past, researchers have studied the influence of shear walls (or cantilever walls) on controlling the lateral story drift of RC buildings subjected to earthquake forces (e.g., Paulay 1999, Priestley *et al.* 2007). Sezen *et al.* (2003) highlighted the structural deficiencies observed in the damaged structures during 1999 Kocaeli, Turkey earthquake. The deficiencies which were identified include insufficient transverse reinforcement in RC columns, strong-beam and weak-columns, soft and weak stories, poor quality construction, poor detailing in beam-column joint regions etc. They concluded that the buildings having shear walls perform considerably well during the earthquake. Tuken (2004) proposed an analytical procedure to estimate the lateral displacement of a mixed (frame+shear wall) structure subject to earthquake forces. The analytical

procedure was then applied to a 3-D building with different heights. The analytical lateral displacements matched reasonably well with the SAP2000 results. Tuken and Siddiqui (2011) proposed a simple-to-apply analytical method based on "dual system" concept to determine the amount of shear walls which can satisfy the strength, stiffness and ductility requirements imposed by the seismic codes on RC moment resisting frame buildings. The proposed methodology was then applied to a 10-storey RC building containing shear walls. It was shown that the amount of shear walls which is enough to satisfy the strength requirements also fulfills the stiffness criteria (i.e., story drift limitation) required by the seismic codes.

Burak and Comlekoglu (2013) evaluated the effect of shear wall plan area to floor plan area ratio (shear wall ratio) on the seismic response of RC buildings. They carried out nonlinear time-history analysis for 24 mid-rise building models having shear wall ratios between 0.51 and 2.17 percent in both directions. In the analyses, seven different earthquake records were used in the evaluation of the seismic performance of these buildings. They recommended that in order to control the lateral story drift, a minimum of 1.0% shear wall ratio has to be used in the design of mid-rise buildings. They also observed that when the shear wall ratio is more than 1.5%, the effect of the shear wall on the performance was insignificant. Lee and Haldar (2003a) developed an efficient and accurate algorithm to study the reliability of a steel frame and RC shear wall structural system subject to static loading. The algorithm was then extended to consider dynamic loading (including seismic loading) in another companion paper (2003b). The concept integrates the first-order reliability method and the finite-element method, resulting in a stochastic finite-element

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based approach. The reliability of a steel frame with and without the presence of RC shear walls was evaluated for both serviceability and strength performance functions. The results were justified by using Monte Carlo simulations. The algorithm confirms quantitatively that the shear walls are needed, particularly when the steel frame is weak, in satisfying the serviceability requirement of lateral deflection. Wyadtowski *et al.* (2015) outlined a procedure to estimate the reliability index for a laterally loaded diaphragm wall against serviceability limit state. The reliability indices were obtained by creating two response surfaces. One based on the maximum lateral top displacement of the wall and the other one using the maximum values of the bending moment. They also obtained the global reliability index by using the concept of system reliability. Jeong *et al.* (2012) carried out the fragility analyses to evaluate the relative seismic safety margins of multi-story RC buildings under varying ductility level, input motion intensity, and configuration. The studied buildings were designed based on a seismic code. They observed that the damage state probabilities of wall-frame structures designed to high peak ground acceleration and ductility levels do not achieve the most favorable safety objectives. A relationship was also proposed to quantify the damage state probabilities of mid-rise RC buildings. Martins *et al.* (2016) derived a vulnerability model in terms of the ratio of repair cost to replacement cost for a given intensity level of ground shaking. They used the model to estimate economic losses due to seismic action. The results of this study highlight important issues in the derivation of vulnerability functions, which are a fundamental component for an adequate seismic risk assessment.

Monteiro (2016) studied the seismic reliability of RC bridge structures using Latin Hypercube sampling algorithm. He demonstrated the robustness and effectiveness of the Latin Hypercube algorithm in estimating the probability of failure for both regular and irregular bridge configurations. Monteiro *et al.* (2016a) computed the failure probability of existing bridges using the results of a nonlinear dynamic analysis. They statistically characterized the different variables generally considered in the seismic assessment procedure such as geometry, material properties, records of the earthquake, the level of intensity etc. Failure probability was obtained through the probabilistic analysis of a safety indicator. The safety indicator was defined as the difference between capacity and demand. A case study of seven bridge configurations, with different (ir)regularity levels was considered along with many sets of earthquake records. The simulation process was carried out using the Latin Hypercube sampling algorithm. Through this study, they identified the vulnerable configurations and shown the importance of the variable detail level. In the other companion paper, Monteiro *et al.* (2016b) made the use of the simplified procedures and addressed the performance of a nonlinear static procedure by direct comparison with nonlinear dynamic results. They also investigated the use of different static analysis versions corresponding to different types of pushover load distributions. A comparison of the static and dynamic approaches was then carried out on a parametric basis.

Marzban *et al.* (2014) carried out a case study on the

seismic response of RC shear wall frames to assess the soil-foundation-structure interaction effects. They concluded that the fixed-base assumption overestimates the design of the wall element and underestimates the design of the connected moment frame. Soleimani-Abiat *et al.* (2015) investigated the effects of seismic load combinations on the response behavior of slabs at their connection zones with the shear walls. The results of the study revealed that layout of shear walls substantially affects the magnification of forces at the shear wall-floor slab connections. Taleb *et al.* (2012) studied the influence of opening ratios on the cracks distribution and shear strength of RC structural walls. They tested four RC single span structural walls having various opening sizes and locations under lateral reversed cyclic loading. The results of the study showed that the shear strength changes, depending on the loading direction, due to opening locations. Dahish *et al.* (2015) studied the influence of shear walls in controlling the lateral response of the RC frame building by varying the shear wall thicknesses, height, configuration and opening locations. The earthquake was considered from one direction only while studying the effect of the first two parameters (i.e., thickness and height) as the building and shear wall arrangements were symmetric along the two orthogonal directions. However, in the case of third and fourth parameters (i.e., shear wall configuration and opening location), the earthquake was considered from the two directions separately as the shear wall configuration and opening location was not symmetric in the two orthogonal directions. The results of the study were useful for obtaining the optimum amount and arrangement of shear walls for a given RC frame building against a specified seismic loading.

The above literature review shows that even though a considerable research is available on the static and dynamic analyses of shear walls and shear-wall frame buildings, but the studies on reliability analysis of shear wall-frame buildings are still limited. Such reliability analysis of shear wall-frame buildings is necessary because many times (i) the pure RC frames under seismic loading do not satisfy the target safety (or reliability) requirements and (ii) designer may be interested in knowing how much safety level has been improved by using a certain quantity of shear walls. Furthermore, how to incorporate the human errors in the structural reliability of shear wall-frame structure was not seen in the approachable references. A simple risk-based cost assessment procedure is also needed to relate both the life-time failure probability and the target reliability with the total cost of the building. In the present study, a detailed procedure for carrying out the reliability analysis of RC shear wall-frame buildings subjected to earthquake forces against lateral drift is provided. The influence of human errors in the two governing parameters, *viz.* concrete strength and story weight, on the reliability of shear wall-frame buildings is also presented. The concrete strength was considered as one of the governing parameters where human errors are very common because of the involvement of human beings during mixing, curing or testing of the concrete. Similarly, story weight may be affected due to the errors committed during the selection or calculation of loads (out of the available guidelines e.g., codes). Finally, a simple risk-based cost assessment procedure that relates both the life-time failure probability and the desired

reliability with the total cost of the building is proposed. A few sensitivity analyses were also carried out to obtain the results of practical interest.

## 2. Formulation for reliability analysis

In order to carry out the reliability analysis of RC building, with and without shear walls, a limit state function which describes the failure criterion is needed. This function is negative or zero at the failure and it is positive when the structure is safe. Thus, the probability of failure (i.e., probability of limit state violation) can be defined as

$$P_f = P[G(\mathbf{X}) \leq 0] \quad (1)$$

where  $G(\mathbf{X})$  is the limit state function and  $\mathbf{X}$  is the vector of random variables. Thus, for the serviceability criterion of story drift, the limit state function can be expressed as

$$G(\mathbf{X}) = \delta_{lim} - C_d \delta_{cal} \quad (2)$$

where  $\delta_{cal}$ =calculated design story drift and  $\delta_{lim}$ =allowable story drift prescribed in Clause 10.12 of SBC 301 (2007),  $C_d$ =the deflection amplification factor, which is a random variable and accounts for the inelastic behavior.

The calculated design story drift ( $\delta_{cal}$ ) can be obtained by the difference of the lateral deflections at the top and bottom of the story under consideration (i.e.,  $\delta_i = y_i - y_{i-1}$ ). In the present study, the lateral deflections ( $y_i$ ) for shear wall-frame structure were estimated using the equation derived by Tuken and Siddiqui (2013) as given below

$$K_y(x) = A_1 s^2 \cosh \phi + A_2 s^2 \sinh \phi + \left(1 - \frac{1}{v^2}\right) p H^4 \left(\frac{k^2}{6} - \frac{k^3}{12} + \frac{k^5}{120}\right) - \frac{s^2 p k}{6 v^2} x^2 + A_3 x + A_4 \quad (3)$$

where,  $K$ =total stiffness of all shear walls and columns within the story along the axis considered= $K$  (shear walls)+ $\Sigma K$  (columns);  $H$ =height of the building;  $y$ =lateral deflection of the building at a height of  $x$  from the base;  $k=x/H$ ;  $p$ =top intensity of triangular distributed lateral load;  $s^2=K/(v^2 GA)$ ;  $GA$ =the shear rigidity of the frame per unit height (i.e., equivalent shear stiffness of the building);  $v^2=1+K/K_0$ ;  $K_0$ =flexural rigidity of the structure in the horizontal plane;  $\phi=x/s$ . In the above equation, the coefficients  $A_1$ ,  $A_2$ ,  $A_3$  and  $A_4$  are defined as

$$A_1 = \frac{ps^2}{v^2 \cosh \lambda} \left(1 + \left(\frac{\lambda}{2} - \frac{1}{\lambda}\right) \sinh \lambda\right) \quad (4)$$

$$A_2 = -\frac{ps^2}{v^2} \left(\frac{\lambda}{2} - \frac{1}{\lambda}\right) \quad (5)$$

$$A_3 = -A_2 s \text{ and } A_4 = -A_1 s^2 \quad (6)$$

where  $\lambda=H/s$ .

For pure frame, the above lateral deflection formula simplifies into

$$y(x) = \frac{pH^2}{2GA} \left(k - \frac{k^3}{3}\right) \quad (7)$$

### 2.1 Probability of limit state violation

The probability of limit state violation can be obtained by considering the probability models and statistics of all the basic variables involved in the limit state function. There are several methods for estimating the probability of failure such as First Order Reliability Method (FORM), Second Order Reliability Method (SORM), Monte Carlo Simulation Technique, Latin Hypercube Sampling Method, LHSM (Monteiro 2016, Monteiro et al. 2016a, 2016b) etc. In the present study, Monte Carlo Simulation technique was preferred as this method, although computationally expansive, is considered as the most accurate method (Nowak and Collins 2012). As the authors had access to a powerful workstation, they could perform a large number of simulations, required in Monte Carlo Simulation, in a reasonable time.

### 2.2 Annual probability of failure

In order to obtain the annual probability of failure of the shear wall-frame structure, annual probability of occurrence of the earthquake is required. For this purpose, the mean occurrence rate of an earthquake in the selected site class is required. Assuming that the number of earthquakes that occur within a certain time interval follow a Poisson distribution, the annual probability of occurrence of an earthquake (or the annual probability of at least one earthquake) can be estimated. Having known the annual probability of occurrence of the earthquake, the absolute annual failure probability of the building  $P_{fa}$  can be estimated by

$$P_{fa} = P_{fn} \times P_{ann\_earthqke} \quad (8)$$

where  $P_{fn}$ =probability of failure when the building is subjected to the earthquake;  $P_{ann\_earthqke}$ =annual probability of occurrence of the earthquake. The corresponding value of the annual reliability index  $\beta_a$  can then be calculated as

$$\beta_a = -\Phi^{-1}(P_{fa}) \quad (9)$$

### 2.3 Life-time probability of failure

Assuming that the design life of the structure is  $N_d$  years and the probability of failure in each year remains constant and independent during lifetime, the probability of failure in the entire life of the structure can be derived using Binomial distribution as given below.

$$P_{fl} = \binom{N_d}{1} P_{fa} (1 - P_{fa})^{N_d - 1} \quad (10)$$

### 2.4 Failure probability considering human errors

If the probability of occurrence and non-occurrence of human error are  $P(E_1)$  and  $P(E_2)$  respectively and the probability of structural failure (i.e., probability of exceeding the allowable drift) with and without this error are  $P_f|E_1$  and  $P_f|E_2$  respectively, then the total failure probability  $P_f$  considering human error can be obtained by

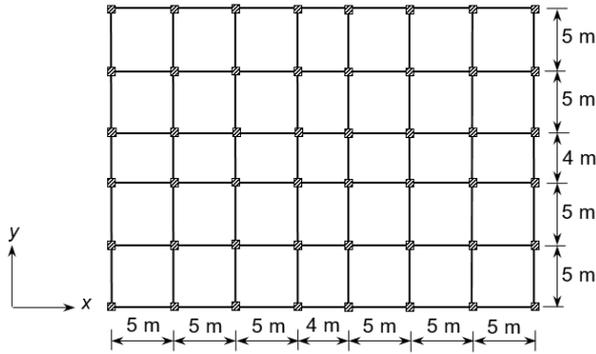


Fig. 1 Plan of the building without shear walls

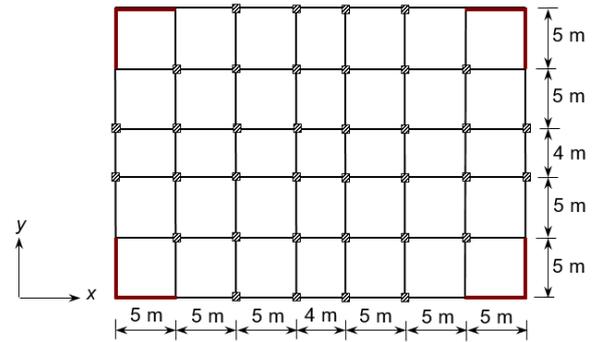


Fig. 2 Plan of the building with shear walls

using the theorem of total probability as below

$$P_f = P_f|E_1 \times P(E_1) + P_f|E_2 \times P(E_2) \quad (11)$$

where,  $E_1, E_2$  are events that denote the occurrence and non-occurrence of human error and  $P(E_1), P(E_2)$  represent the corresponding probabilities;  $P_f|E_1, P_f|E_2$  are probabilities of failure of the building under the condition of occurrence and nonoccurrence of human errors respectively. Since  $E_1, E_2$  are mutually exclusive and collectively exhaustive events then

$$P(E_1) + P(E_2) = 1 \quad (12)$$

Substituting,  $P_f|E_1 = P_{f1}$  ;  $P_f|E_2 = P_{f0}$  ;  $P(E_1) = P(E)$  and  $P(E_2) = 1 - P(E)$ , we have

$$P_f = (1 - P_E)P_{f0} + P_E P_{f1} \quad (13)$$

### 2.5 Risk-based cost assessment

The total cost of the building (for the purpose of the risk assessment or insurance) can be divided into two parts: initial cost and failure cost. In a probabilistic sense, the initial cost depends on the target reliability index (or target probability of failure) of the building in its life-time. In the present study, the following equation was proposed to show the dependence of initial cost over the target reliability index ( $\beta_T$ ) of the building.

$$C_I = (\beta_T^{\alpha_1} + \alpha_2 \beta_T) C_{I_0} \quad (14)$$

where  $\alpha_1$  and  $\alpha_2$  are factors that allow expected variation in the initial cost (with target reliability index  $\beta_T$ ). The values of  $\alpha_1$  and  $\alpha_2$  are very much dependent on the type, importance, and location of the building. In the above equation,  $C_{I_0}$  is the initial cost of the building corresponding to a fixed target reliability index value (e.g.,  $\beta_T=3.5$ ). In order to derive expression for the total cost of the building, the two extreme cases were considered: (i) if likelihood of the building failure is zero, the failure cost will be zero and thus failure cost will not be a part of the total cost; (ii) if failure is 'certain' in the design life, the

failure cost should be added 'as is' to the initial cost to get the total cost. Thus, for all practical purposes, the failure cost must be multiplied by the failure probability of the building to obtain the total cost. That is

$$C_T = C_I + P_{fL} C_F \quad (15)$$

where,  $C_T$ =the total cost of the building;  $C_I$ =initial cost;  $C_F$ =failure cost;  $P_{fL}$ =life-time failure probability. It is worth mentioning that  $C_I$  is a function of target (or desired) reliability index as mentioned above and shown by Eq. (14). The failure cost can be expressed as a multiple of initial cost, that is

$$C_F = \gamma C_I \quad (16)$$

where  $\gamma$  is a multiplying factor. Substituting  $C_I$  and  $C_F$  from Eqs. (14) and (16) respectively into Eq. (15), yield the equation for the total cost as follows

$$C_T = [1 + \gamma P_{fL}] (\beta_T^{\alpha_1} + \alpha_2 \beta_T) C_{I_0} \quad (17)$$

The above equation shows the dependence of the building's total cost on failure probabilities and the selected target reliability index  $\beta_T$ . The above equation will be employed to carry out the simple risk assessment of the building.

## 3. Numerical study and discussion of results

### 3.1 Description of the selected building

A 10 story RC frame building was selected for the present study. The detailing of the RC frame without shear walls satisfy the requirements of special RC moment frames, and with shear walls it satisfies the requirements of the dual system (with special moment frames) as per Clause 10.2 of SBC 301 (2007). The building is rectangular in plan having a total height of 40 m with 7-bays in  $x$ -direction and 5-bays in the  $y$ -direction and a constant floor plan area of 816 m<sup>2</sup> at each story. The height of every story is equal to 4 m. The building is assumed to be fixed at the base. The floors of the building are considered to act as rigid diaphragms. All the columns, beams and slabs were considered to be of the same sizes (columns: 500×500 mm; beams: 200×500 mm; slab thickness: 150 mm). Fig. 1

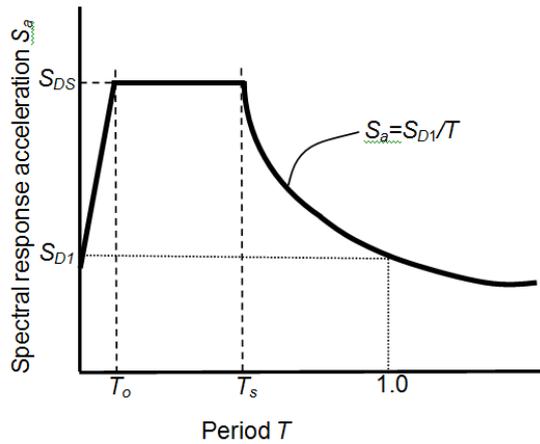


Fig. 3 Acceleration response spectrum (SBC 301)

shows the plan of the building. The frame is subjected to dead, live, and horizontal seismic loads and it was stiffened with shear walls in  $x$ - and  $y$ -directions as shown in Fig. 2. The thickness of all the shear walls was 300 mm and the ratio of horizontal web reinforcement of the wall to the gross area of the wall,  $\rho_n$ , was 0.025 (minimum reinforcement as per SBC 304).

The reliability of the above RC building containing shear walls was studied in the present paper. The statistical data and probability distributions of the selected random parameters, required for the reliability analysis, are given in Table 1. The probability distribution of different random variables was taken from the various sources e.g., Lee and Haldar (2003), Nowak and Collins (2012) and Dahesh *et al.* (2014), Dahesh (2015). The variables shown in Table 1 were considered random as they have considerable uncertainty in their values. All the other values were assumed to be deterministic.

The total base shear was estimated using an acceleration response spectrum, defined in Saudi Building Code (SBC 301). This spectrum is shown in Fig. 3. The acceleration response spectrum reveals that the effective ground acceleration is magnified by a factor of  $S_{DS}$ , for natural periods of  $T_o$ – $T_s$  seconds.  $S_{DS}$  and  $S_{D1}$ , shown in Fig. 3, are the nominal values of design spectral response accelerations at short periods and at 1-sec period respectively. Their typical values for Haql City, a seismically active region of the Kingdom of Saudi Arabia, are given in Table 1.

The nominal value of the story weight is taken to be 8.0 kN/m<sup>2</sup>. This weight includes self-weight of the slab, weight of floor finishes and superimposed dead and live loads.

The probability of failure of the RC building frames against strength limit state is, in general, very small as these frames (with or without shear walls) are substantially strong against the maximum expected base shear (Lee and Haldar 2003a, 2003b). It is due to this reason, in the present study; the reliability analysis has been carried out only against serviceability limit state of lateral drift. The results of the numerical studies have been presented and discussed in the following sections. In the reliability discussion, desired reliability was taken as 3.5. This is a typical value of the reliability index which is generally used to assure a desirable safety level in buildings and structures (Dahesh *et*

Table 1 Basic random variables and their statistical values

| Random Variable   | Nominal value         | Bias factor | COV  | Distribution   |
|---|-----------------------|-------------|------|----------------|
| Story weights, $w_i$  | 8.0 kN/m <sup>2</sup> | 1.05        | 0.15 | Extreme Type I |
| Design spectral response acceleration at short period, $S_{DS}$ | 0.664                 | 0.78        | 0.16 | Normal         |
| Design spectral response acceleration at 1-sec period, $S_{D1}$ | 0.344                 | 0.78        | 0.16 | Normal         |
| Modulus of elasticity of concrete, $E$                          | 23500 MPa             | 1.00        | 0.18 | Lognormal      |
| Beam width  | 200 mm                | 1.00        | 0.05 | Lognormal      |
| Beam depth  | 500 mm                | 1.00        | 0.05 | Lognormal      |
| Column width  | 500 mm                | 1.00        | 0.05 | Lognormal      |
| Column depth  | 500 mm                | 1.00        | 0.05 | Lognormal      |
| Shear wall thickness  | 300 mm                | 1.00        | 0.05 | Lognormal      |
| Compressive strength of concrete, $f_c'$                        | 25 MPa                | 1.00        | 0.10 | Normal         |
| Yield strength of steel, $f_y$                                  | 420 MPa               | 1.00        | 0.08 | Normal         |
| Response modification factor, $R$ (for pure frame)              | 6.5                   | 1.00        | 0.05 | Normal         |
| Response modification factor, $R$ (for dual system)             | 6.5                   | 1.00        | 0.05 | Normal         |
| Occupancy importance factor, $I$                                | 1.0                   | 1.00        | 0.05 | Normal         |
| Deflection amplification factor, $C_d$ (for pure frame)         | 5.5                   | 1.00        | 0.05 | Normal         |
| Deflection amplification factor, $C_d$ (for dual system)        | 6.5                   | 1.00        | 0.05 | Normal         |

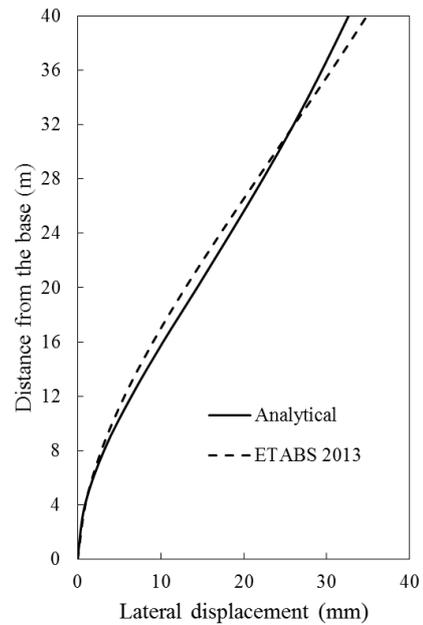


Fig. 4 A comparison of analytical and ETABS 2013 results

*al.* 2014, Siddiqui *et al.* 2009, Siddiqui 2011, Siddiqui *et al.* 2014).

### 3.2 Validation of the structural drift formulation

To validate the formulation, presented in Section 2, for computing the lateral deflections ( $y_i$ ) the response of analytical formulation for the shear wall-frame building was compared with ETABS 2013 results and shown in Fig. 4. The other data required for obtaining the structural drift response are given in the second column of Table 1. It can be observed from Fig. 4 that the two responses are reasonably close to each other. This gives a confidence on the analytical formulation which was employed in the

Table 2 Probability of structural failure when earthquake hits the structure

| Building Frame               | Building frame without shear wall |           | Building frame with shear wall |           |
|------------------------------|-----------------------------------|-----------|--------------------------------|-----------|
|                              | $P_{fn}$                          | $\beta_n$ | $P_{fn}$                       | $\beta_n$ |
| Earthquake in $x$ -direction | $2.35 \times 10^{-2}$             | 1.986     | $5.00 \times 10^{-6}$          | 4.417     |
| Earthquake in $y$ -direction | $2.89 \times 10^{-2}$             | 1.897     | $6.60 \times 10^{-6}$          | 4.357     |

derivation of the limit state function and subsequently in the present reliability analysis.

### 3.3 Reliability of the frame building with and without shear walls

Table 2 shows the results of the Monte Carlo simulation-based reliability analysis performed on the building with no shear walls (Fig. 1) and building with shear walls (Fig. 2). Two million simulations were used for carrying out the reliability analysis of building frame containing no shear walls and about 15 million simulations were employed for carrying out the reliability analysis of building frames with shear walls. A substantially higher number of simulations were used for the reliability analysis of the shear wall-frame building because the expected probability of failure of the shear wall-frame building was of the order of  $10^{-6}$ .

The accuracy of Monte Carlo Simulation was evaluated by computing the variance of the estimated probability of failure  $P_f$ . The variance was computed by assuming (i) each simulation cycle constitutes a Bernoulli trial and (ii) the number of failures in  $n$  trials follow a binomial distribution. The variance of the probability of failure was then approximately computed as

$$\text{Var}(P_f) = \frac{(1 - P_f)P_f}{n} \quad (18)$$

The statistical accuracy of the calculated failure probability was estimated by computing its coefficient of variation (COV) as

$$\text{COV}(P_f) \cong \frac{\sqrt{\frac{(1 - P_f)P_f}{n}}}{P_f} \quad (19)$$

The smaller the COV, the better the accuracy of the estimated probability of failure is.

However, for all practical purposes, the number of simulation cycles for which  $\text{COV}(P_f)$  approaches less than 5% may be considered as an appropriate number of simulation cycles (Nowak and Collins, 2012). In the present study, around 1 to 15 million of Monte Carlo simulations were required to achieve the desired accuracy in the estimation of the failure probabilities.

Table 2 clearly illustrates the advantage of using shear walls in improving the serviceability-reliability of the building. When there were no shear walls the reliability index of the building was substantially less than the desired

Table 3 Annual probability of structural failure

| Building Frame               | Building frame without shear wall |           | Building frame with shear wall |           |
|------------------------------|-----------------------------------|-----------|--------------------------------|-----------|
|                              | $P_{fa}$                          | $\beta_a$ | $P_{fa}$                       | $\beta_a$ |
| Earthquake in $x$ -direction | $1.15 \times 10^{-3}$             | 3.049     | $2.44 \times 10^{-7}$          | 5.031     |
| Earthquake in $y$ -direction | $1.41 \times 10^{-3}$             | 2.987     | $3.22 \times 10^{-7}$          | 4.977     |

reliability index of 3.5 which improves to above 4.0 due to the presence of shear walls. The improvement of the reliability due to shear walls can be attributed to the increased lateral stiffness of the building. The shear walls reduced the lateral drift of the building considerably and thus improved the reliability substantially. Table 2 also shows that the probability of failure of the building in the  $x$ -direction is lesser than the probability of failure in the  $y$ -direction. This can be attributed to the higher stiffness of the building in the  $x$ -direction compared to the  $y$ -direction. This study thus clearly illustrates the beneficial effects of shear walls in improving the reliability by substantially reducing the lateral displacement at the top of the building frame. This is worth mentioning that the trend obtained may substantially modify if the building is irregular instead of regular (as considered in the present study). The irregularity in the building can be owing to plan irregularity or vertical irregularity. Plan irregularity is due to the difference between the center of mass and the center of resistance of the building, whereas vertical irregularity is due to abrupt changes in the geometry, strength, or stiffness of the structure from floor to floor.

### 3.4 Annual probability of failure of the RC shear wall-frame building

In order to obtain the annual probability of failure of the shear wall-frame building the mean occurrence rate of the earthquake having the nominal values of design spectral response accelerations at short periods,  $S_{DS}$ , and at 1-sec period  $S_{D1}$ , as given in Table 1, is required. In the present study, the mean occurrence rate was selected to be 0.05 which corresponds to a return period of 20 years (Al-Amri 2014). Assuming that the number of earthquakes that occur within a certain time interval follow a Poisson distribution (Nowak and Collins 2012), the annual probability of occurrence of an earthquake (or the annual probability of at least one earthquake) was estimated as

$$\begin{aligned} P_{ann\_earthquake} &= P(n \geq 1) = 1 - P(n = 0) = 1 - \frac{(\lambda t)^n}{n!} e^{-\lambda t} \\ &= 1 - \frac{(0.05 \times 1)^0}{0!} e^{-0.05 \times 1} = 0.0488 \end{aligned}$$

Here,  $t$ =time in years;  $\lambda$ =mean occurrence rate (=1/return period);  $n$ =number of earthquakes; and  $P_{ann\_earthquake}$ =annual probability of occurrence of the earthquake. Having known the annual probability of occurrence of the earthquake and the nominal probability of failure of the building (probability of failure when the building is subjected to the earthquake,  $P_{fn}$ ), the absolute annual failure probability of the building was estimated using Eq. (8). The corresponding value of the annual

Table 4 Life-time failure probability of the building

| Building Frame               | Building frame without shear wall |           | Building frame with shear wall |           |
|------------------------------|-----------------------------------|-----------|--------------------------------|-----------|
|                              | $P_{fL}$                          | $\beta_L$ | $P_{fL}$                       | $\beta_L$ |
| Earthquake in $x$ -direction | $5.42 \times 10^{-2}$             | 1.605     | $1.22 \times 10^{-5}$          | 4.220     |
| Earthquake in $y$ -direction | $6.58 \times 10^{-2}$             | 1.508     | $1.61 \times 10^{-5}$          | 4.157     |

reliability index  $\beta_a$  was then calculated using Eq. (9). The results are presented in Table 3 which clearly illustrates that due to the small annual probability of earthquake occurrence, the annual failure probability of the building is one order (i.e., one tenth) lesser than the nominal probability of failure. The reliability indices have also improved significantly. In fact, even frame building which had no shear walls has achieved reliability index up to 3.0.

### 3.5 Life-time failure probability of the RC shear wall-frame building

Assuming that the design life of the structure is 50 years and the probability of failure in each year remains constant and independent during lifetime, the probability of failure in the entire life of the structure was estimated using Eq. (10). The results of the analysis are presented in Table 4 which shows that the life-time failure probability is substantially higher than the annual probability of failure. This is due to the fact that the life-time failure probability can be treated as the failure probability of a series system, and in a series system, if any element fails, the system fails. Due to this reason system failure probability (i.e., life-time failure probability) is much higher than the individual element's probability of failure (i.e., annual failure probability).

Table 4 clearly shows that the life-time reliability indices (obtained from life-time probabilities of failure using Eq. (9)) for a frame building without shear walls are much less than the desired reliability index value. However, with shear walls, the life-time reliability indices are higher than the target reliability index value of 3.5.

As the influence of earthquake on reliability is similar in  $x$ - and  $y$ -directions, in the following studies, the results will be presented for earthquake acting in  $x$ -direction only.

### 3.6 Influence of human error on reliability of RC shear wall-frame building

The human error was thought to cause the deviation in nominal values from their error-free values by a certain amount. The nominal values presented in Table 1 are assumed to be the values which are free from human error. Due to the human error, the nominal values deviate. This deviation could be either positive or negative. The positive deviation indicates an increase in the nominal value while negative deviation shows a decrease.

There is a possibility of human error at many stages from design to construction (Epaarachchi and Stewart 2004, De Haan 2012). Human error involvement is very much expected in the determination of concrete strength, steel yield strength, loads, and workmanship etc. However, in the

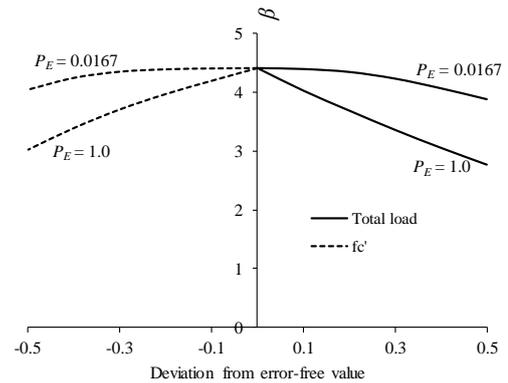


Fig. 5 Effect of human error in some of the governing parameters on shear wall-frame building (considering probability of human error occurrence,  $P_E=1.0$  and  $P_E=0.0167$ )

present study, the effect of the human error was considered only in the two major parameters—one in the determination of concrete strength and the other in the selection of load value as the human error in these two parameters are very probable and common.

The positive deviation was considered for the load and negative for the concrete strength to study the adverse effects of human error. Fig. 5 clearly illustrates that there is a sharp decrease in the reliability index with increasing magnitude of deviation from error-free value. The results presented in this figure are for the earthquake acting in the  $x$ -direction. A similar graph is expected if the earthquake hits the building in the  $y$ -direction. The probability of human error,  $P_E=1.0$ , mentioned in the figure indicates that how the reliability index will be affected when (certainly) there is a human error in the estimation of the total load and/or concrete strength.

Fig. 5 also shows the reliability index variation of the structure when the human error is not certain but has a probability of occurrence of 0.0167 (Melchers 2002). In this case, the total reliability index of the structure is almost constant with the variation of deviation from error-free value. It is due to the low probability of occurrence of the human error. Mathematically, if the probability of occurrence of human error is  $P_E$ , and the probability of structural failure without and with this error is  $P_{f0}$  and  $P_{f1}$  respectively, then the total failure probability can be obtained by Eq. (13). As an extreme case if  $P_E=0$ , then  $P_f=P_{f0}$  and if  $P_E=1$ ,  $P_f=P_{f1}$ . Thus, if there is a small probability of human error occurrence, there will be a very little change in the overall probability of failure of the building. It is due to this reason, in Fig. 5, there is no significant fall in the reliability index when nominal values deviate from the error-free value with a probability of occurrence of human error as 0.0167.

### 3.7 Risk-based cost assessment of RC shear wall-frame building

In the present study employing Eq. (14) through Eq. (17) with their selected coefficient values  $\alpha_1$ ,  $\alpha_2$  and  $\gamma$  as 2.0, -0.5 and 1.5 respectively, the initial cost and the total cost of

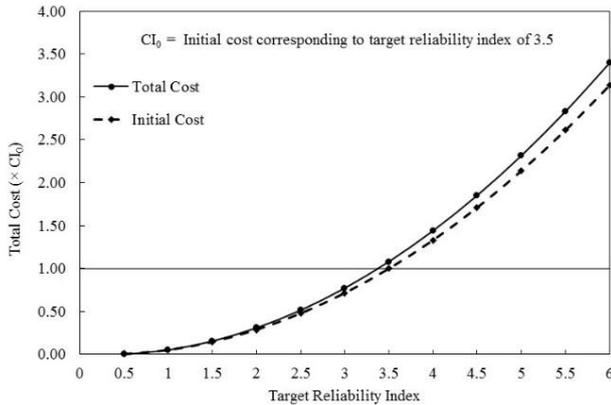


Fig. 6 Variation of initial and total costs of shear-wall frame building with target reliability index (a measure of risk)

the shear wall-frame building were estimated for different values of target reliability index and the results are shown in Fig. 6. This figure illustrates that as the target reliability index increases, the initial cost, as well as the total cost of the building, rises sharply. The difference between the initial cost and final cost is not much due to small life-time failure probability of the building. The two curves match exactly when  $P_{fL}$  is zero. This is very well expected because when life-time failure probability is zero, the failure cost becomes zero which makes the total cost same as the initial cost. The curve clearly illustrates that when the selected level of risk or probability of failure is high (i.e., when target reliability index is small), the cost is substantially small, but when the selected level of risk or probability of failure is small (i.e., when target reliability index is high) cost is substantially high. The curve also shows that when target reliability index is 3.5, the initial cost is  $C_{I_0}$  and the total cost is little higher than  $C_{I_0}$ . It is due to the fact that  $C_{I_0}$  is the initial cost corresponding to the target reliability index=3.5. It is worth mentioning that the values of coefficients  $\alpha_1$  and  $\alpha_2$  depend on how the initial cost was assumed to vary with target reliability index. On the other hand, the value of coefficient  $\gamma$  depends on how the initial and the failure costs are related to each other. The selected values of  $\alpha_1$ ,  $\alpha_2$  and  $\gamma$  may change as the above-mentioned dependence change.

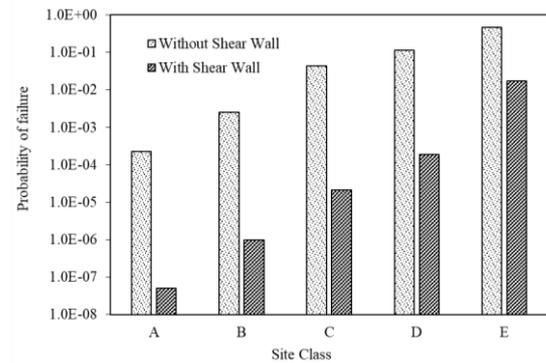
### 3.8 Sensitivity analysis

#### 3.8.1 Effect of seismic design category on structural reliability

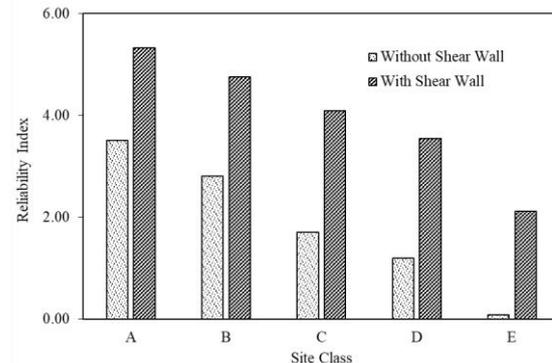
In this study, the same building was assumed to exist in different sites i.e., site class A through E (SBC 301) and its reliability index and the probability of failure were obtained. In this analysis, the earthquake parameters related to different site classes are tabulated in Table 5 and using these data the reliability analysis was carried out for building frame without and with shear walls and the results obtained are shown in Fig. 7 which shows that building without the shear wall is having a substantially high probability of failure for the site classes C through E due to

Table 5 Earthquake parameters considered for sensitivity analysis

| Site Class | $S_s$<br>(g) | $S_1$<br>(g) | $F_a$ | $F_v$ | $S_{MS}$<br>(g) | $S_{DS}$<br>(g) | $S_{M1}$<br>(g) | $S_{D1}$<br>(g) |
|------------|--------------|--------------|-------|-------|-----------------|-----------------|-----------------|-----------------|
| A          | 1.0          | 0.4          | 0.8   | 0.8   | 0.800           | 0.533           | 0.320           | 0.213           |
| B          | 1.0          | 0.4          | 1.0   | 1.0   | 1.000           | 0.667           | 0.400           | 0.267           |
| C          | 1.0          | 0.4          | 1.0   | 1.4   | 1.000           | 0.667           | 0.560           | 0.373           |
| D          | 1.0          | 0.4          | 1.1   | 1.6   | 1.100           | 0.733           | 0.640           | 0.427           |
| E          | 1.0          | 0.4          | 0.9   | 2.4   | 0.900           | 0.600           | 0.960           | 0.640           |



(a)



(b)

Fig. 7 Effect of site class on probability of failure and reliability of building frame

its poor lateral stiffness. However, when the shear walls are provided there is a dramatic decrease in the failure probability due to increased lateral stiffness of the building. Fig. 7 shows that for the site class A to D the decrease in the failure probability due to provided amount of shear walls makes the building as reliable as desired (i.e. reliability index becomes greater than 3.5). However, for the site class E the present quantity of shear wall is not sufficient as reliability index is less than the target reliability index. This indicates that more shear walls are required to increase the reliability of the building to the desired level.

#### 3.8.2 Effect of story weight on reliability of RC shear wall-frame building

The effect of story weight on reliability index of shear-wall frame building was studied to obtain the results of design interest. For this purpose, the parameters were selected in such a way that  $(\beta - \beta_T)^2 \approx 0$ . Here  $\beta$  and  $\beta_T$  are

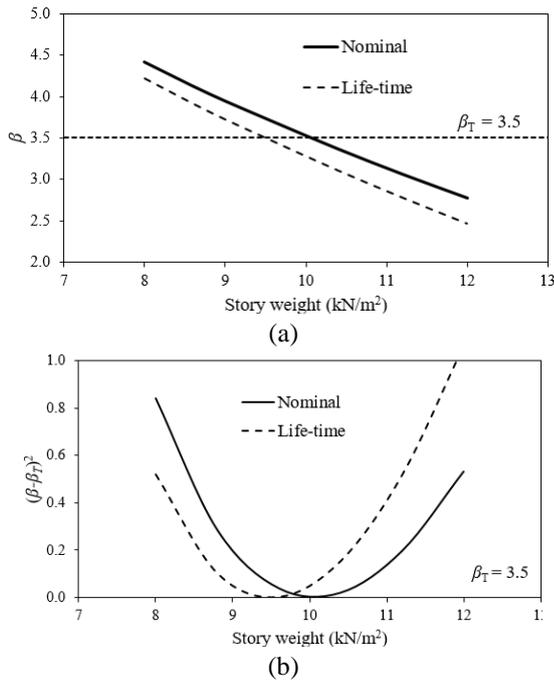


Fig. 8 Variation of reliability index  $\beta$  with story weight

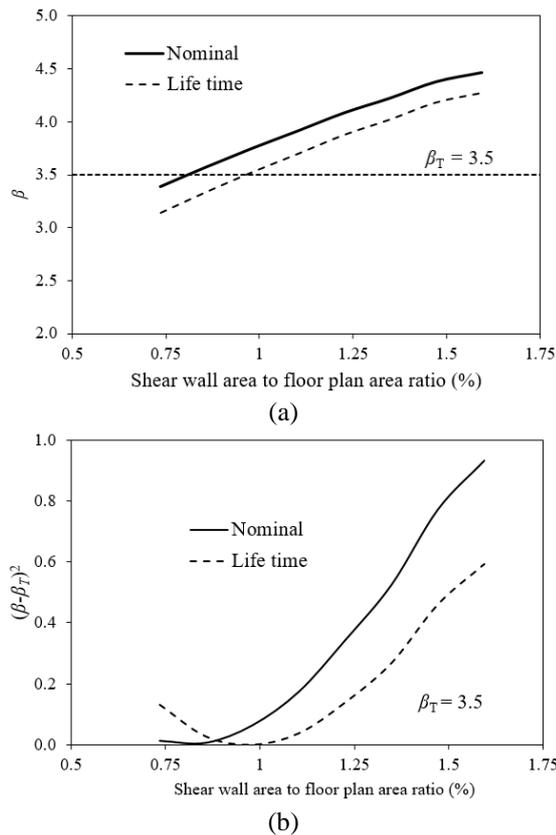


Fig. 9 Variation of reliability index  $\beta$  with shear wall ratio (%)

the actual and target reliability index values.  $(\beta - \beta_T)^2 \approx 0$  indicates that the reliability of the shear wall-frame building is almost equal to the target reliability value. Fig. 8(a) shows that as the story weight is increasing reliability is continuously decreasing. This is due to the fact that with the

increase of story weight, base shear increases which in turn increases the story drift. Consequently, the probability of reaching to the limiting drift value increases; thus, reliability decreases or probability of serviceability failure increases. Fig. 8(b) shows that the present building is reliable to the desired extent provided the story weight is approximately less than or equal to 10 kN/m<sup>2</sup>. Beyond this story weight, reliability will sharply decrease to a value less than 3.0. However, this optimum value is approximately 9.5 kN/m<sup>2</sup> for achieving the life-time reliability index of 3.5.

### 3.8.3 Effect of shear wall ratio

The effect of shear wall ratio i.e., shear wall plan area to floor plan area ratio on the reliability of shear wall-frame building was studied by varying the nominal thickness of the shear walls. Fig. 9(a) shows that as the shear wall ratio is increasing, reliability is continuously increasing. This is because with the increase of shear wall ratio, building stiffness increases which in turn decreases the story drift. Consequently, the probability of reaching to the limiting drift value decreases; thus, reliability increases or probability of serviceability failure decreases. Fig. 9(b) shows that the present building is reliable to the desired extent provided the shear wall ratio is approximately more than or equal to 0.8%. Beyond this shear wall ratio, the reliability of the building will be more than 3.5. However, this value is approximately 1.0% for the life-time reliability index of 3.5.

## 4. Conclusions

The following conclusions can be drawn from the present reliability study of a RC shear wall-frame building against serviceability limit state.

- i. Shear walls play an important role in reducing the risk or improving the reliability of the building under seismic excitation. Even a small quantity of shear walls (1% or less of floor plan area) can improve the reliability of RC frame building dramatically. It was observed that an RC frame building (without shear walls) whose life-time reliability is substantially smaller than the desired/target reliability index value of 3.5, achieves the desired reliability level in the presence of a small quantity of shear walls.
- ii. The annual failure probability of the studied shear wall-frame building is approximately one order (i.e., one tenth) lesser than the nominal probability of failure (i.e., the probability of failure when the building is subjected to the earthquake) of the building.
- iii. There is a sharp change in the reliability of the studied shear wall-frame building due to human error involvement in the estimation of total load and/or concrete strength. However, when the probability of occurrence of the human error is small, the reliability of the building is almost unaffected.
- iv. As the target reliability index increases, the initial cost, as well as the total cost of the building, rises sharply. The curves of initial and failure cost match exactly when the life-time probability of failure is assumed to be zero.
- v. For site class A to D the decrease in the failure probability due to shear walls is to an extent that the studied

RC building (with shear walls) becomes as reliable as desired. However, for site class E the present quantity of shear wall is not sufficient and thus more shear walls are required. On the other hand, the provided quantity of shear walls is enough for the studied building if it is located in site class A through D.

vi. The studied building (with the shear walls) is reliable to the desired extent provided that the total story weight is approximately less than or equal to 10 kN/m<sup>2</sup>. However, this optimum value is approximately 9.5 kN/m<sup>2</sup> for achieving the life-time reliability index of 3.5.

vii. The studied building is reliable to the desired extent provided the shear wall ratio is approximately more than or equal to 0.8%. Beyond this shear wall ratio, the reliability of the building is more than 3.5. However, this value is approximately 1.0 % for the life-time reliability index of 3.5.

## Acknowledgments

The work presented in this paper was funded by the Deanship of Scientific Research, Research Centre, College of Engineering, King Saud University, Riyadh, Saudi Arabia through research grant number 3/437.

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