

A performance-based design method for chloride-induced cover cracking of RC structures

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Abstract. Chloride-induced cover cracking will aggravate the performance deterioration for RC structures under the chloride-laden environment, which may endanger the safety of structures and occupants. Traditional design method cannot ensure that a definite performance is satisfied. To overcome the defects, a study on the performance-based design method was carried out in this paper. Firstly, the limit state functions were established for the corrosion initiation and cover cracking. Thereafter, the uncertainty analysis was performed to study the effects of random factors on the time-dependent performances. Partial factor formulae were deduced through the first-order reliability method for performance verification. Finally, an illustrative example was presented and the sensitivity of cover depth to other parameters was carried out. It is found that the uncertainties of the random variables have great effects on the required cover depth. It is demonstrated that the performance-based design method can ensure that the target performance can be satisfied and support to formulate a rational maintenance and repair strategy for RC structures under the chloride environment.

Keywords: reliability; performance deterioration; partial factor formulae; sensitivity analysis; performance verification

1. Introduction

The structural long-term performance during service life receives great attentions of the researchers from a wide research field (Li *et al.* 2014, Li *et al.* 2014, Nagarajaiah 2014, Yi *et al.* 2013, Zhou *et al.* 2016). Chloride penetration is one of the most dominant factors leading to the performance deterioration of RC structures when they are located in the marine environment or exposed to the de-icing salts in winter. The chloride-induced deterioration process of RC structures consists of three stages, namely the reinforcement corrosion initiation stage, cover cracking stage and cover spalling stage (Liang *et al.* 1999, Nogueira and Leonel 2013, Saassouh and Lounis 2012, Tuutti 1982). The first stage of corrosion initiation starts with the first exposure of structures to the chloride-laden environment and ends with the reinforcement corrosion initiating. Thereafter, the second stage begins and the steel corrosion propagates with the corrosion rust accumulating at the surface of steel bars. The rust of corroded steel occupies a larger volume than the sound steel, which will cause the cracking of concrete cover. At the final stage, the width of chloride-induced crack grows gradually until a certain threshold value is reached, which will finally cause concrete

cover spalling. Before the reinforcement corrosion initiates, no substantive damage occurs and the rising chloride concentration destroys the alkaline environment near the rebar and depassivates the steel bars. By contrast, the reinforcement corrosion has occurred in the cover cracking stage, which will cause loss of steel cross section. Moreover, the accumulated rust finally results in cover cracking, which will make the corrosion media easy to penetrate into the concrete and aggravate the deterioration. At the cover spalling stage, the structural performance usually has been seriously damaged, which may endanger the safety of structures and the occupants. Therefore, the chloride-induced cover cracking is a turning point of performance deterioration, and the performance design for the limit state of cover cracking is significant for the normal use and maintenance in the life cycle of structures.

The current research about the performance deterioration of RC structures under the chloride environment has concentrated on the corrosion initiation and corrosion propagation mechanism (Cao and Cheung 2014, Chen and Mahadevan 2008, Faustino *et al.* 2015, Mohammed *et al.* 2003, Otieno 2017). Chen and Mahadevan (2008) carried out investigations on utilizing the FEM to simulate the whole process of chloride penetration, steel corrosion, rust accumulation and concrete cover cracking. Mohammed *et al.* (2003) performed an experimental research about corrosion mechanism of reinforcement in cracked concrete structures, and the water-cement ratio was considered to be dominant parameter affecting steel corrosion. Otieno (2017) carried out a theoretical analysis about the sensitivity of steel corrosion rate to the diffusion coefficient of concrete, cover depth and

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crack width. It was concluded that concrete quality is the dominant factor affecting steel corrosion rate and the effects of all the parameters above on the steel corrosion need to be considered in design. Faustino *et al.* (2015) analyzed the effects of corrosion inhibitor on the service life of RC structures and Monte Carlo simulation was applied to consider the uncertainties of random factors. Cao and Cheung (2014) simulated the accumulating process of corrosion rust through FEM and found that the pitting corrosion is mainly induced by macro-cell corrosion. The concrete cover cracking which is due to the corrosion propagation of reinforcement has drawn the attentions of some researchers. Dai *et al.* (2016) established a whole model for corrosion-induced cracking of prestressed concrete structures, and an experimental research has been carried out to analyze the influences of cover cracking on the failure of concrete beams. It was found that prestressing had an adverse effect on protecting concrete cover from corrosion-induced cracking. Li *et al.* (2006, 2005) proposed a time-dependent reliability method for analyzing the chloride-induced reinforcement corrosion. Parameter research was carried out to find that the corrosion rate of steel bars was the most dominant factor affecting the corrosion-induced cover cracking.

To ensure that the required performance of RC structures under the chloride attack is satisfied during the design stage, a rational design method is dispensable in which the time-dependency and uncertainties of structural performance can be properly considered. The prescriptive design method is currently widely adopted by most of the current codes and standards for the durability design of concrete structures. With respect to the prescriptive method, a determinate value of cover depth is prescribed, which cannot reflect the deterioration mechanism of performance and the target performance to be satisfied is not specific (Marques and Costa 2010). Ann *et al.* (2010) researched the safety factor design method for the concrete piers under carbonation and a safety formula was deduced based on the in-suit data and theoretical analysis, which can consider the effects of random factors on the performance deterioration and provide a safety margin to structures. However, the safety factors are usually derived through engineering experience which cannot reflect the exact probability of performance failure. To overcome the defects of safety factor design method, some researchers have studied the fully probabilistic design methods. The Monte Carlo simulation was commonly used as the fully probabilistic method to calculate the reliability of the target performance (Saassouh and Lounis 2012). Although the principle of the Monte Carlo method is simple, the computation cost is high which forms an obstacle for practice use. Saassouh and Lounis (2012) utilized the first- and second-order reliability methods to predict the probability of reinforcement corrosion for existing structures. DuraCrete (2000) proposed a partial factor formula for the limit state of chloride-induced cover cracking; however, a definite reliability has not been prescribed for the limit state. Yang *et al.* (2016) proposed a performance-based service life design method for the chloride-induced reinforcement corrosion. The FROM has been applied to deduce a partial factor formula according to the predefined target performance, which can consider the uncertainties of performance.

Based on the literature review above, the research about the deterioration mechanism of RC structures under the chloride environment focused on the chloride-induced reinforcement corrosion. However, the chloride-induced cracking may be more detrimental for the structure safety. The prescriptive design method cannot ensure a definite target performance to be realized for concrete structures under the chloride attack and the maintenance and repair strategy during service cannot be determined. Considering the defects of current research, the paper presents a study on the performance-based design for the limit state of chloride-induced cover cracking to achieve the following objectives: (i) the target performance can be predefined according to the stakeholders' requirement; (ii) the target performance can be verified by the limit state formulae; (iii) the formulae for the performance verification can reflect the performance deterioration mechanism and deal with the effects of random factors; (iv) the formulae can be easily used for engineering practice. The following parts constitute the paper:

- Quantitatively analyzing the deterioration mechanism of RC structures under chloride and establishing the limit state functions for the chloride-induced cover cracking;
- Carrying out the uncertainty analysis of the limit state of cover cracking;
- Deducing the partial factor formulae for the performance verification based on the target performance; and
- Presenting a case study to illustrate the rationality of the performance-based design method.

2. Limit state function of chloride-induced cover cracking

The chloride-induced deterioration process of RC structures can be divided into three stages, namely the reinforcement corrosion initiation stage, the concrete cover cracking stage and the concrete cover delamination or spalling stage. Before the chloride-induced cracking initiates, the reinforcement corrosion damage develops slowly. Once the chloride-induced cracking occurs, by contrast, the reinforcement steel corrosion rates increase rapidly, which will obviously aggravate the durability performance of RC structures. Consequently, the concrete cover cracking is a turning point of performance deterioration, which should be considered as a limit state for the performance design under the chloride environment.

2.1 Chloride-induced reinforcement corrosion initiation

The chloride-induced cover cracking is mainly caused by the accumulation of rebar corrosion debris. Thus, a certain value of the rebar corrosion rate indicates that the concrete cover cracking initiates due to the reinforcement corrosion. In order to acquire the time-dependent reinforcement corrosion rate under chloride environment, the mechanism and process of chloride-induced reinforcement corrosion should be investigated first.

The process of reinforcement corrosion due to chloride

attack consists of two periods, namely the corrosion initiation stage and the propagation stage. With the chloride ions penetrate into the concrete gradually, the chloride concentration in the concrete increases. When the chloride concentration in the immediate vicinity of the reinforcement reaches a certain threshold value, chloride-induced reinforcement corrosion initiates. The penetration of chloride ions into concrete can be attributed to several mechanisms, which includes absorption, diffusion, permeation and migration. The different physical processes mentioned above have different mechanisms. For example, the absorption process is due to a capillary action; the diffusion is owing to the concentration gradients; the migration is due to the presence of electric potential gradients; and the permeation is attributed to the pressure gradients. Actually, the different transportation processes of chloride into concrete usually exist simultaneously, which make it difficult to exactly depict the process of chloride ingress into concrete. For the RC bridges under the chloride-laden marine environment or de-icing salts in winter, the bridge decks can be considered to be saturated, in which the diffusion process can be taken as the dominant mechanism of chloride penetration into concrete. It is generally accepted that the diffusion process of chloride ingress into concrete obeys the Fick's second law, the effectiveness of which has been validated by many researchers. The partial differential formula of the Fick's second law depicting the chloride diffusion is shown in Eq. (1)

$$\frac{\partial C(x,t)}{\partial t} = \frac{\partial}{\partial x} \left(-D \frac{\partial C(x,t)}{\partial x} \right) \quad (1)$$

where t =exposure time of concrete structures to chloride environment; x =depth from the concrete surface; $C(x,t)$ =chloride concentration at depth x after time t ; and D =“effective” or “apparent” diffusion coefficient. The “effective” or “apparent” diffusion coefficient is the average of actual ones, which is used to take account of the errors induced by assuming solely a diffusion mechanism based on the Fick's second law.

It is assumed that the diffusion coefficient and surface chloride concentration are time-independent constants respectively, and the chloride transportation is considered to be one-dimensional diffusion process. According to the initial condition that $C(x,t)=0$ when $t=0$, and $x>0$, the analytic solution of Eq. (1) can be obtained as Eq. (2)

$$E = C_{th} - C(d, t_i) = C_{th} - C_s \left[1 - \operatorname{erf} \left(\frac{d}{2\sqrt{Dt_i}} \right) \right] = 0 \quad (2)$$

where $\operatorname{erf}(\cdot)$ is the error function; and C_s =surface chloride concentration.

When the chloride concentration in the immediate vicinity of reinforcing steel reaches a certain threshold value, the passive film on the surface of rebars is disrupted, which will depassivate the rebars and cause corrosion. Assuming that the threshold value of chloride concentration for reinforcement corrosion initiation is C_{th} , the limit state of reinforcement corrosion initiation can be represented as Eq. (3)

$$E = C_{th} - C(d, t_i) = C_{th} - C_s \left[1 - \operatorname{erf} \left(\frac{d}{2\sqrt{Dt_i}} \right) \right] = 0 \quad (3)$$

where E is the limit state function of reinforcement corrosion initiation; d =cover depth; and t_i =the time of reinforcement corrosion initiation.

According to the Eq. (3), the time of reinforcement corrosion initiation can be obtained as Eq. (4)

$$t_i = \left[\frac{d}{2\operatorname{erf}^{-1} \left(1 - \frac{C_{th}}{C_s} \right)} \right]^2 \frac{1}{D} \quad (4)$$

where erf^{-1} is the inverse error function; and t =the time of reinforcement corrosion initiation due to chloride attack.

2.2 Chloride-induced cover cracking

To obtain the limit state of chloride-induced cover cracking, the corrosion process of reinforcement and the mechanism of chloride-induced cover cracking should be further investigated. The reinforcement corrosion initiation indicates the beginning of the propagation stage of rebar deterioration due to chloride attack. Once the propagation stage begins, the reinforcement corrosion leads to the formation of corrosion products, the volume of which is larger than the original steel bars. With continuous accumulation of the corrosion debris, the radial forces due to the inflation of corroded steel will cause tensile stress in the concrete cover, which finally leads to concrete cover cracking and spalling (DuraCrete 2000).

According to the mechanism of the chloride-induced cover cracking above, the acquisition of time-varying steel corrosion degree is the prerequisite to establish the limit state of cover cracking. As prescribed in DuraCrete (2000), the corrosion penetration after corrosion initiation can be presented as Eq. (5)

$$P = Vw_t (t - t_i) \quad (5)$$

where P =reinforcement corrosion penetration; V =reinforcement corrosion rate; w_t =relative time of wetness; t =the exposure time of RC structures to chloride-laden environment; and t_i =the time initiation of reinforcement corrosion initiation.

When the volume of the corrosion products reaches a certain threshold value, the tensile stress in the concrete cover will lead to cover cracking. According to DuraCrete (2010), the limit state of chloride-induced cover cracking can be represented by the relationship between reinforcement corrosion rate and the corresponding threshold value, as shown in Eq. (6). The threshold value of corrosion penetration is presented as Eq. (7).

$$P = P_0 \quad (6)$$

$$P_0 = a_1 + a_2 \frac{d}{d_{\text{rebar}}} + a_3 f_{c,sp} \quad (7)$$

where P_0 =the threshold value of corrosion penetration necessary to produce a crack in concrete cover; a_1, a_2, a_3 are the regression parameters; d_{rebar} =the diameter of rebar; and $f_{c,\text{sp}}$ =the splitting tensile strength of concrete.

According to Eqs. (5)-(7), the limit state function of chloride-induced cover cracking can be shown as Eq. (8)

$$F = \left(a_1 + a_2 \frac{d}{d_{\text{rebar}}} + a_3 f_{c,\text{sp}} \right) - V w_t (t - t_i) = 0 \quad (8)$$

where F is the limit state function of chloride-induced cover cracking.

3. Uncertainty analysis of chloride-induced cover cracking

Because the deterioration process of concrete structures under chloride-laden environment is affected by many random factors, the chloride-induced concrete cover cracking is actually a random event. To rationally design and predict the performance of concrete structures subjected to chloride attack during their service life, it is necessary to consider the reliability of target performance. To calculate the reliability for the limit state of cover cracking, the uncertainties of the limit state function of chloride-induced cover cracking need to be analysed.

3.1 Uncertainties of random variables in limit state function

For the limit state of chloride-induced cover cracking, the uncertainties can be attributed to many factors which mainly include the inherent uncertainty of random variables in the limit state function, the uncertainty in the physical models. In this paper, the influences of random variable uncertainties on the limit state of chloride-induced cover cracking are focused on.

According to Eq. (8), the random variables in the limit state function of chloride-induced cover cracking can be classified into three categories, namely the structural variables, the material variables and environmental variables. The structural parameters include concrete cover depth (d) and the diameter of the rebar (d_{rebar}), which are the geometric features of the concrete structures. The material variables are the parameters related to material properties of concrete or steel, which include the diffusion coefficient (D), the chloride concentration threshold value (C_{th}), and the splitting tensile stress of concrete cover ($f_{c,\text{sp}}$). The environmental variables consist of the equivalent period of wetting (w_t), and the surface chloride concentration (C_s). Based on the discussion above, there are up to 7 random variables besides several other regression factors in the limit state function of chloride-induced cover cracking, which will create an obstacle to calculate the reliability of the corresponding performance efficiently.

3.2 Reliability analysis of cover cracking using first order reliability method

To calculate the reliability of the limit state function of

the cover cracking, several typical methods can be adopted in which the simplest method is the Monte Carlo simulation (MCS). The basic procedure of MCS is to carry out a large amount of random sampling and simulate the outcomes of the limit state according to the samples. Assuming that n is the total number of the simulations results, the frequency of the limit state being reached, i.e., the ratio of the failure number to the total simulations, is the failure probability. With the increasing of n , the reliability corresponding to the failure probability obtained tends to the accurate theoretical value. It means that a large quantity of simulations is necessary to acquire a satisfactory solution of reliability, which makes MSC not an efficient method. By contrast, another typical method, i.e., the first-order reliability method (FORM), can estimate the failure probability at a relative low computational cost compared to MCS. Compared with Monte Carlo simulation method, it is the prerequisite that there are explicit function equations of the limit state for using FORM to calculate the reliability. Besides, the distribution type and numerical characteristics, i.e. mean and variance, of all the random variables in the limit state function should be known before using FORM. Therefore, FORM is applied to analyze the reliability of the limit state of cover cracking in this paper.

Considering that the limit state function of chloride-induced cover cracking is too complicated for reliability analysis which includes up to 7 random variables, the target reliability should be analyzed for the limit state of corrosion initiation and cover cracking individually: (i) according to a predefined target service life for the limit state of corrosion initiation, the reliability of chloride-induced reinforcement corrosion is calculated based on Eq. (3); (ii) according to the time of reinforcement corrosion initiation obtained in the first analysis step, the reliability analysis is carried out for the limit state of chloride-induced cover cracking. For the limit state of chloride-induced reinforcement corrosion initiation (as represented in Eq. (3)), FORM is applied to estimate the reliability and the Eqs. (9)-(16) can be obtained (Yang *et al.* 2016)

$$d^* = \mu_d + \alpha_d \beta_1 \sigma_d \quad (9)$$

$$C_s^* = \mu_{C_s} + \alpha_s \beta_1 \sigma_s \quad (10)$$

$$D^* = \mu_D + \alpha_D \beta_1 \sigma_D \quad (11)$$

$$C_{\text{th}}^* = \mu_{C_{\text{th}}} + \alpha_{C_{\text{th}}} \beta_1 \sigma_{C_{\text{th}}} \quad (12)$$

$$\alpha_d = \frac{-\left. \frac{\partial E}{\partial d} \right|_{x^*} \sigma_d}{\sqrt{\left(\left. \frac{\partial E}{\partial d} \right|_{x^*} \right)^2 \sigma_d^2 + \left(\left. \frac{\partial E}{\partial C_s} \right|_{x^*} \right)^2 \sigma_{C_s}^2 + \left(\left. \frac{\partial E}{\partial D} \right|_{x^*} \right)^2 \sigma_D^2 + \left(\left. \frac{\partial E}{\partial C_{\text{th}}} \right|_{x^*} \right)^2 \sigma_{C_{\text{th}}}^2}} \quad (13)$$

$$\alpha_{C_s} = \frac{-\left. \frac{\partial E}{\partial C_s} \right|_{x^*} \sigma_{C_s}}{\sqrt{\left(\left. \frac{\partial E}{\partial d} \right|_{x^*} \right)^2 \sigma_d^2 + \left(\left. \frac{\partial E}{\partial C_s} \right|_{x^*} \right)^2 \sigma_{C_s}^2 + \left(\left. \frac{\partial E}{\partial D} \right|_{x^*} \right)^2 \sigma_D^2 + \left(\left. \frac{\partial E}{\partial C_{\text{th}}} \right|_{x^*} \right)^2 \sigma_{C_{\text{th}}}^2}} \quad (14)$$

$$\alpha_D = \frac{-\frac{\partial E}{\partial D}|_{X^*} \sigma_D}{\sqrt{\left(\frac{\partial E}{\partial d}|_{X^*}\right)^2 \sigma_d^2 + \left(\frac{\partial E}{\partial C_s}|_{X^*}\right)^2 \sigma_{C_s}^2 + \left(\frac{\partial E}{\partial D}|_{X^*}\right)^2 \sigma_D^2 + \left(\frac{\partial E}{\partial C_{th}}|_{X^*}\right)^2 \sigma_{C_{th}}^2}} \quad (15)$$

$$\alpha_{C_{th}} = \frac{-\frac{\partial E}{\partial C_{th}}|_{X^*} \sigma_{C_{th}}}{\sqrt{\left(\frac{\partial E}{\partial d}|_{X^*}\right)^2 \sigma_d^2 + \left(\frac{\partial E}{\partial C_s}|_{X^*}\right)^2 \sigma_{C_s}^2 + \left(\frac{\partial E}{\partial D}|_{X^*}\right)^2 \sigma_D^2 + \left(\frac{\partial E}{\partial C_{th}}|_{X^*}\right)^2 \sigma_{C_{th}}^2}} \quad (16)$$

where μ_X , σ_X are the mean value and standard deviation of the random variables respectively; β_i is the reliability for the limit state of reinforcement corrosion initiation; C_s^* , d^* , D^* and C_{th}^* are the values of the design point in the physical space; α_d , α_{C_s} , α_D and $\alpha_{C_{th}}$ are the direction cosines of the design point in the standard Gaussian space transformed from the physical space.

Considering that the design point is on the failure surface for the limit state of reinforcement corrosion initiation, Eq. (17) is satisfied. According to the simultaneous Eqs. (9)-(17) and the predefined time of reinforcement corrosion initiation (t_i), the corresponding reliability (β_i) can be acquired through FORM.

$$E(C_{th}^*, C_s^*, d^*, D^*) = 0 \quad (17)$$

Because α_1 , α_2 , α_3 are the regression constant parameters and w_i is also a constant value correlated with environmental conditions, the random variables for the limit state function consist of the cover depth (d), rebar diameter (d_{rebar}), splitting tensile strength of concrete ($f_{c,sp}$) and reinforcement corrosion rate (V). When FORM is applied to calculate the reliability for the limit state of cover cracking, Eqs. (18)-(25) can be obtained.

$$d^* = \mu_d + \alpha_d \beta_c \sigma_d \quad (18)$$

$$d_{rebar}^* = \mu_{d_{rebar}} + \alpha_{d_{rebar}} \beta_c \sigma_{d_{rebar}} \quad (19)$$

$$f_{c,sp}^* = \mu_{f_{c,sp}} + \alpha_{f_{c,sp}} \beta_c \sigma_{f_{c,sp}} \quad (20)$$

$$V^* = \mu_V + \alpha_V \beta_c \sigma_V \quad (21)$$

$$\alpha_d = \frac{-\frac{\partial F}{\partial d}|_{X^*} \sigma_d}{\sqrt{\left(\frac{\partial F}{\partial d}|_{X^*}\right)^2 \sigma_d^2 + \left(\frac{\partial F}{\partial d_{rebar}}|_{X^*}\right)^2 \sigma_{d_{rebar}}^2 + \left(\frac{\partial F}{\partial f_{c,sp}}|_{X^*}\right)^2 \sigma_{f_{c,sp}}^2 + \left(\frac{\partial F}{\partial V}|_{X^*}\right)^2 \sigma_V^2}} \quad (22)$$

$$\alpha_{d_{rebar}} = \frac{-\frac{\partial F}{\partial d_{rebar}}|_{X^*} \sigma_{d_{rebar}}}{\sqrt{\left(\frac{\partial F}{\partial d}|_{X^*}\right)^2 \sigma_d^2 + \left(\frac{\partial F}{\partial d_{rebar}}|_{X^*}\right)^2 \sigma_{d_{rebar}}^2 + \left(\frac{\partial F}{\partial f_{c,sp}}|_{X^*}\right)^2 \sigma_{f_{c,sp}}^2 + \left(\frac{\partial F}{\partial V}|_{X^*}\right)^2 \sigma_V^2}} \quad (23)$$

$$\alpha_{f_{c,sp}} = \frac{-\frac{\partial F}{\partial f_{c,sp}}|_{X^*} \sigma_{f_{c,sp}}}{\sqrt{\left(\frac{\partial F}{\partial d}|_{X^*}\right)^2 \sigma_d^2 + \left(\frac{\partial F}{\partial d_{rebar}}|_{X^*}\right)^2 \sigma_{d_{rebar}}^2 + \left(\frac{\partial F}{\partial f_{c,sp}}|_{X^*}\right)^2 \sigma_{f_{c,sp}}^2 + \left(\frac{\partial F}{\partial V}|_{X^*}\right)^2 \sigma_V^2}} \quad (24)$$

$$\alpha_V = \frac{-\frac{\partial F}{\partial V}|_{X^*} \sigma_V}{\sqrt{\left(\frac{\partial F}{\partial d}|_{X^*}\right)^2 \sigma_d^2 + \left(\frac{\partial F}{\partial d_{rebar}}|_{X^*}\right)^2 \sigma_{d_{rebar}}^2 + \left(\frac{\partial F}{\partial f_{c,sp}}|_{X^*}\right)^2 \sigma_{f_{c,sp}}^2 + \left(\frac{\partial F}{\partial V}|_{X^*}\right)^2 \sigma_V^2}} \quad (25)$$

where β_c is the reliability for the limit state of cover cracking; d^* , d_{rebar}^* , $f_{c,sp}^*$ and V^* are the values of design point in the physical space; α_d , $\alpha_{d_{rebar}}$, $\alpha_{f_{c,sp}}$ and α_V are the direction cosines of the design point in the standard Gaussian space transformed from the physical space.

It is worth noting that if non-normal variables exist in the limit state function, the mean (μ_X) and standard deviation (σ_X) of non-normal variables in need to be replaced by the counterparts of the equivalent normal variables at the design point. Considering that the design point is on the failure surface for the limit state of cover cracking, Eq. (26) should be satisfied. By solving the simultaneous equations (Eq. (18)-(26)) for the limit state of cover cracking, β_c can be obtained for a certain service life time.

$$F(d^*, d_{rebar}^*, f_{c,sp}^*, V^*) = 0 \quad (26)$$

It is worth noting that if the random variables are not normal variables, the mean values (μ_X) and standard deviations (σ_X) need to be transformed to the counterparts of equivalent normal variables at the design point.

4. Partial factor formulae for performance-based design

In Section 3, two-step probabilistic analysis has been carried out for RC components under chloride environment, in which FORM was applied to calculate the reliability of corrosion initiation and chloride-induced cover cracking individually. However, the analyzing process of FORM is suitable for predicting reliability for existing structures rather than ensure the target reliability during design. It is necessary to propose a simplified formula to take account of the uncertainties and verify whether the target performance is satisfied. Consequently, on the basis of the fully probabilistic analysis process, the partial factor formulae are deduced for verifying the limit states of chloride-induced corrosion initiation and cover cracking in this section.

Before deducing the partial factor formulae, the target performance for the limit state of reinforcement corrosion and cover cracking need to be predefined according to the stakeholders' requirements. The target performance of RC structures usually consists of three ingredients, namely deterioration level, reliability and service life. For the limit state of reinforcement corrosion, the target performance deterioration level is reinforcement corrosion initiation, and the target service life and reliability are t_i and β_i respectively. With respect to the limit state of chloride-induced cover cracking, the target performance consists of the target deterioration level of cover cracking, the target service life of t_c and the target reliability of β_c . The target performance levels for the two limit states are shown in Table 1.

A typical partial factor formula is constituted of the

Table 1 Target performance for the two-step design for the limit state of cover cracking

	Target deterioration level	Target service life	Target reliability
Step I	Corrosion initiation	t_i	β_i
Step II	Cover cracking	t_c	β_c

representative values of random variables and corresponding factors, of which the partial factors imply the requirement about the reliability of the limit state. Consequently, the partial factor formulae can consider the effects of random parameters in the limit state function on the performance. Besides, the partial factor formula has a simple form as the determinate formula, which makes it easily applied in engineering practice. The deduction of the partial factor formulae is actually a reverse process of FORM to calculate reliability. In the process of reliability calculation through FORM, the numerical characteristics of all the random variables in Eq. (3) are known, and the reliability is to be solved. By contrast, for the deduction process of partial factor formulae through FORM, the reliability and the numerical characteristics are known. The mean of cover depth is usually considered as the design parameter to be solved. For the limit state of chloride-induce reinforcement corrosion initiation, the general form of partial factor formula is presented as Eq. (27).

$$E\left(\frac{C_{th}^r}{\gamma_{C_{th}}}, \gamma_{C_s} C_s^r, \frac{d^r}{\gamma_d}, \gamma_D D^r\right) = 0 \quad (27)$$

where C_{th}^r , C_s^r , d^r and D^r are the representative values of the random variables in Eq. (3); and $\gamma_{C_{th}}$, γ_{C_s} , γ_d and γ_D are the partial factors.

By solving the simultaneous Eqs. (9)-(17) through the iteration method, the values of design point, the directional cosines of the random variables and the mean of the cover depth can be derived. If the mean of random variables are usually taken as representative values, the partial factors can be obtained through Eq. (28) or (29).

$$\gamma_X = \frac{X^*}{\mu_X} \quad (28)$$

$$\gamma_X = \frac{\mu_X}{X^*} \quad (29)$$

where γ_X =partial factor for random variables; Eq. (28) is for of γ_{C_s} and γ_D ; and Eq. (29) is for $\gamma_{C_{th}}$ and γ_d .

According to the target performance prescribed in Table 1 for the second design step, the reliability and service life are known. The mean of cover depth is also taken as the design parameter to be solved. Based on the limit state function of cover cracking as shown in Eq. (8), the general form of the partial factor formula for the limit state of cover cracking is presented as Eq. (30).

$$F\left(\frac{d^r}{\gamma_d}, \gamma_{d_{rebar}} d_{rebar}^r, \gamma_{f_{c,sp}} f_{c,sp}^r, \gamma_V V^r\right) = 0 \quad (30)$$

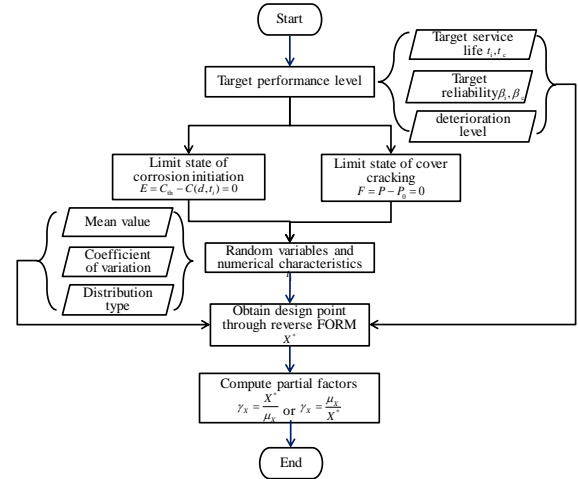


Fig. 1 Flow chart of performance-based design process for the limit state of cover cracking

where d^r , d_{rebar}^r , $f_{c,sp}^r$ and V^r are the representative values of the random variables in Eq. (8); and γ_d , $\gamma_{d_{rebar}}$, $\gamma_{f_{c,sp}}$ and γ_V are the partial factors of the representative values.

By solving Eqs. (18)-(26) through the iteration method, the values of design point, the directional cosines of the random variables and the mean value of the cover depth can be obtained. If the means of the random variables in Eq. (8) are taken as the representative values, the partial factors can be calculated through Eq. (28) or (29).

Through the two-step design process, the partial factor formulae to verify the performance related to cover cracking are achieved finally. A flow chart is presented in Fig. 1 to illustrate the application process of the performance-based design.

5. Illustrative examples

5.1 Description of the illustrative model

The reinforced concrete bridge deck model under chloride-laden environment used by Yang *et al.* (2016) is adopted to illustrate the performance-based design process for the limit state of cover cracking. The bridge deck is reinforced concrete component, and the splitting tensile strength and the characteristic cylinder compressive strength are 2.61 MPa and 40.00 MPa respectively. According to the literature research (Hoffman and Weyers 1994, Matsui *et al.* 1998, McGee 2000, Papadakis *et al.* 1996, Srubar III 2015, Suzuki *et al.* 1989, Vu and Stewart 2000, Zhang and Lounis 2006), the numerical characteristics and distribution type of all the random variables in the limit state functions of corrosion initiation and cover cracking are presented in Table 2. Considering that the steel bars are produced in factories, and the construction quality of concrete in field is well controlled, the relative low coefficients of variation (CV) are applied for d_{rebar} and $f_{c,sp}$ (as shown in Table 2). The CV of corrosion rate is assumed to be 0.3. The distribution of all the random variable is assumed to be fitted to normal distribution to simplify the illustration. Besides the random variables

Table 2 Numerical characteristics of random variables

Design parameter	Mean	Coefficient of variation	Distribution
C_s	1.15 kg/m ³	0.50	Normal
d	To be solved	0.30	Normal
D	0.63 cm ² /year	0.45	Normal
C_{th}	0.90 kg/m ³	0.20	Normal
d_{rebar}	6.0 mm	0.10	Normal
$f_{c,sp}$	2.61 MPa	0.20	Normal
V	24.4 μ m/year	0.30	Normal

Table 3 Determinate parameters in the limit state functions

Determinate parameter	a_1	a_2	a_3	w_t
	74.4 μ m	7.3 μ m	-17.4 μ m mm ² /N	1.0

Table 4 Target performance for two-step design of cover cracking limit state

	Target deterioration level	Target service life	Target reliability
Step I	Corrosion initiation	20 years	1.5
Step II	Cover cracking	23 years	1.5

discussed above, the other parameters in the limit state functions can be taken as determinate ones. As proposed in DuraCrate (2010), the specific values of the other determinate parameters are listed in Table 3.

5.2 Target performance and partial factor formulae

As discussed before, the target performance is the prerequisite of performance-based design, which includes the target deterioration level of performance, the target service life and reliability. For the two-step design of the limit state of chloride-induced cover cracking, the target performance should be predefined for both the limit states of corrosion initiation and cover cracking. Considering that the requirement of preventing the concrete cover from cracking and reinforcing steel from corrosion is relative stricter than that of other serviceability, the target service lives for the limit states of corrosion initiation and cover cracking are determined as 20 years and 30 years respectively according to Model Code 2010 (Model-Code 2010) and Eurocode 1990 (Eurocode 2002). With respect to the target reliability, it is recommended by Model Code 2010 that the reliability index of 1.5 is for irreversible damage, which includes reinforcement corrosion and concrete cover cracking. Therefore, the reliability index of 1.5 is adopted for the two limit states above. Moreover, the target deterioration levels for two limit states are reinforcement corrosion initiation and cover cracking. The predefined target performances for the two-step design are presented in Table 4.

A numerical program for iteration has been composed by the authors to solve the simultaneous equations obtained through FORM. Based on the data of Tables 2 and 3 and the predefined target performance of Step-I in Table 4, Eqs. (9)-(17) can be solved and the unknown parameters can be

Table 5 Calculation results of the iteration and partial factors for the two limit state functions

	Target deterioration level	Target service life	Target reliability
$\gamma_{C_{th}}$	1.13	$\gamma_{f_{c,sp}}$	1.07
γ_{C_s}	1.44	$\gamma_{d_{rebar}}$	1.02
γ_d	1.42	γ'_d	1.54
γ_D	1.20	γ_V	1.25
μ_d	54.7 mm	μ'_d	85.3 mm
α^2_d	0.34	α^2_d	0.60
$\alpha^2_{C_s}$	0.43	$\alpha^2_{d_{rebar}}$	0.03
α^2_D	0.08	$\alpha^2_{f_{c,sp}}$	0.05
$\alpha^2_{C_{th}}$	0.15	α^2_V	0.32

obtained. Similarly, the simultaneous Eqs. (18)-(26) for the target performance of Step-II in Table 4 can be solved based on the data in Tables 2 and 3. The direction cosines of the random variables and the mean values of required cover depth can be obtained during the calculation process through FORM, which are listed in Table 5. The mean values of the random parameters are taken as the representative values in Eqs. (27) and (30); therefore, the corresponding partial factors for the two limit states can be finally derived through Eqs. (28) and (29), which are presented in Table 5. Consequently, the partial factor formulae are shown as Eqs. (31) and (32) for verifying the target performance in Table 4.

$$E\left(\frac{C_{th}}{\gamma_{C_{th}}}, \gamma_{C_s}, \frac{d^*}{\gamma_d}, \gamma_D D^*\right) = \frac{\mu_{C_{th}}}{1.13} - 1.44\mu_{C_s} \left(1 - \operatorname{erf}\left(\frac{\frac{\mu_d}{1.42}}{2\sqrt{1.20}\mu_{Df}}\right)\right) = 0 \quad (31)$$

$$F\left(\frac{d^*}{\gamma_d}, \gamma_{d_{rebar}} d^*, \gamma_{f_{c,sp}} f^*, \gamma_V V^*\right) = a_1 + a_2 \frac{\mu_d}{1.02\mu_{d_{rebar}}} + 1.07a_3\mu_{f_{c,sp}} - 1.25\mu_V w_t (t - t_1) = 0 \quad (32)$$

According to the calculation results in Table 5, the mean value of required cover depth for the cover cracking limit state is greater than the limit state of corrosion initiation, which means that the cover depth with the mean value of 85.3 mm is required to satisfy the target performances in Table 4 simultaneously. Besides, the importance factors of the random variables can be represented as the squares of the direction cosines (as shown in Table 5) which can reflect the influences of random variables on the target performance. For the limit state of corrosion initiation, the cover depth and the surface chloride concentration are the dominant parameters. With respect to the limit state of cover cracking, the cover depth and the corrosion rate have a great influence on target performance.

5.3 Sensitivity analysis of cover depth for chloride-induced cover cracking

According to the calculation results during the deduction process of partial factor formulae above, the cover depth satisfying the target service life for chloride-induced corrosion initiation and cover cracking are obtained respectively (Table 5). It can be found that the cover depth of 54.7 mm is necessary for ensuring a service life of 20

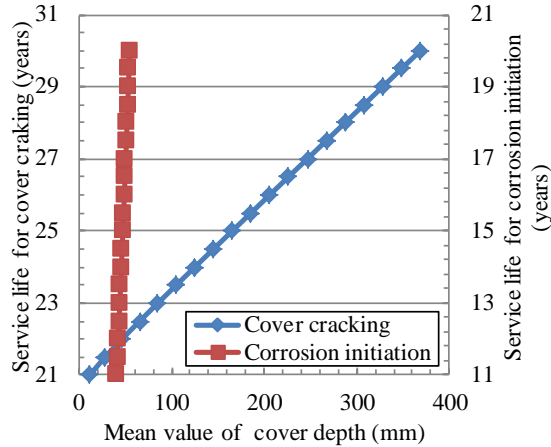


Fig. 2 The sensitivity of cover depth to the service life for the two limit states

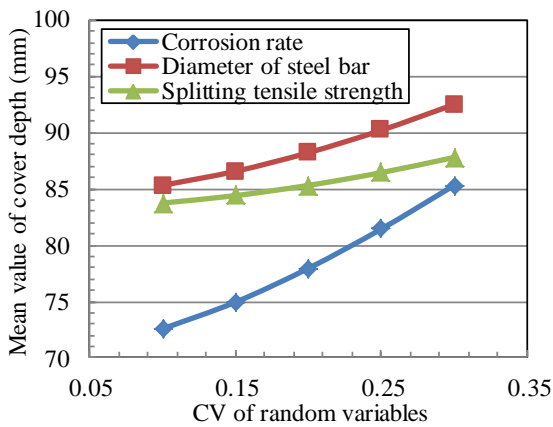


Fig. 3 The sensitivity of cover depth to the CVs of random variable for cover cracking

years without reinforcement corrosion. By contrast, a larger depth of concrete cover is required to guarantee an obviously shorter service life of 3 years without cracking after reinforcement corrosion initiation. To further study the sensitivity of cover depth to the service life of the two limit states, the required cover depth for different target service life of reinforcement corrosion initiation and cover cracking are presented in Fig. 2. As shown in Fig. 2, the service life of the limit state of reinforcement corrosion initiation increases sharply with a slight increase of concrete cover depth (from 40.6 mm to 54.7 mm). With respect to the limit state of cover cracking, the required cover depth increases nearly by 350 mm for the same increment of service life. Therefore, it can be concluded that the concrete cover depth is more sensitive to the service life for the limit state of reinforcement corrosion initiation than the limit state of cover cracking. Moreover, a linear relationship between the required cover depth and the service life can be found both for the limit states of corrosion initiation and cover cracking. To extend the service life RC concrete structures without cover cracking, it is more efficient to postpone the time of reinforcement corrosion than extend the time increment between corrosion initiation and cover cracking.

The chloride-induced concrete cover cracking is considered to be a random event, which means that the

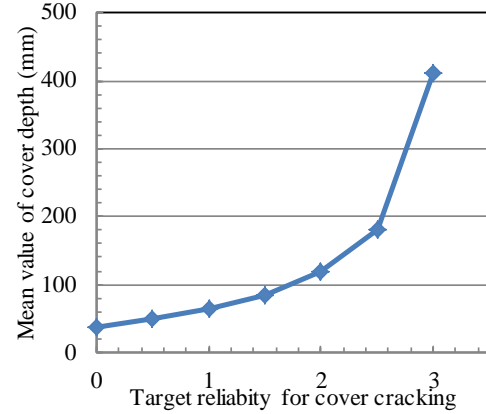


Fig. 4 Required cover depth under different target reliability for the cover cracking limit state

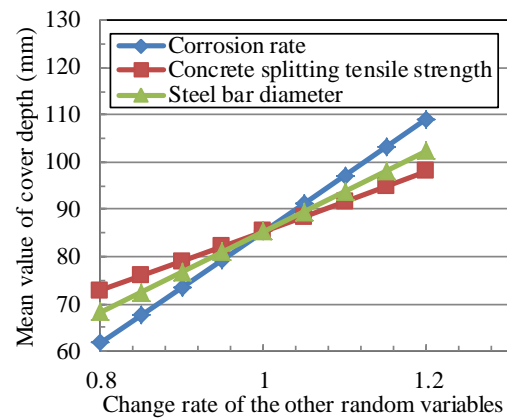


Fig. 5 Required cover depth under different mean values of other random variables for the cover cracking limit state

corresponding performance is affected by the randomness of parameters in the limit state function. Therefore, it is necessary to study the sensitivity of required cover depth to the uncertainties of the random variables in Eq. (8). For the CVs of V , d_{rebar} and $f_{c,sp}$ varying from 0.1 to 0.3, the mean values of cover depth required for the target performance are calculated respectively, which are presented in Fig. 3. According to the results shown in Fig. 3, it can be found that the mean values of required cover depth increases by 17.5%, 8.4% and 4.8% respectively for the CV of V , d_{rebar} and $f_{c,sp}$ varying from 0.1 to 0.3. The mean value of required cover depth has the largest increasing rate with the CV of V varying, which means that the required cover depth is more sensitive to the variation of corrosion rate than the variation of the other parameters and the variation of $f_{c,sp}$ has the slightest influence on the required cover depth. It can be concluded that the accurate estimation of the CVs of corrosion rates is important for the limit state design to resist chloride-induced cover cracking.

To research the effects of target reliability on the required cover depth, the mean values of cover depth under different target reliability are calculated through FORM. The relationship between the mean values of required cover depth for different target reliability can be presented in Fig. 4. It can be found that there is a linear relationship between the mean value of cover depth and the target reliability

when the target reliability is less than 1.5. When the target reliability is set to be greater than 1.5, an obvious nonlinear increase of required cover depth with target reliability appears. For the target reliability of 3.0, a cover depth of 411.6 mm is required which increases by 382.5% compared with the counterpart for the target reliability of 1.5. Consequently, it can be concluded that excessive high target reliability will lead to a disproportional increase of the design cost for resisting chloride-induced cover cracking.

To instruct the design for the limit state of cover cracking, it is necessary to consider the effects of different values of $\mu_{d_{rebar}}$, $\mu_{f_{c,sp}}$ and μ_V on the required cover depth. When the mean values of $\mu_{d_{rebar}}$, $\mu_{f_{c,sp}}$ and μ_V vary from 80% to 120% of the original values in Table 2, the corresponding mean values of required cover depth are computed through FORM. As presented in Fig. 5, a linear relationship can be observed between the mean values of required cover depth and all the other three variables. The slope of the line “corrosion rate” is larger than the other lines in Fig. 5, which indicates that the corrosion rate has a greater influence on the required cover depth than the concrete splitting tensile strength and steel bar diameter. Therefore, it is the most efficient measure to control the corrosion rate of reinforcement for reducing the required concrete cover depth for limit state of cover cracking.

6. Conclusions

An investigation was carried out to study the performance-based design method of chloride-induced cover cracking in this paper. The limit state functions were established to consider the performance deterioration mechanism of RC structures under the chloride-laden environment. Additionally, the partial factor formulae were deduced based on the uncertainty analysis of random factors. Finally, a case study of a RC bridge deck model was presented and parameter research was performed. The following conclusions can be drawn:

- Compared with the prescriptive design method, a quantitative analysis is adopted in the performance-based design method which can take account of the performance deterioration mechanism. Besides, the effects of random factors on the chloride-induced cover cracking can also be considered properly.
- The partial factor formulae deduced through the FORM implies the target reliability, which can be used to verify whether the predefined target performance is satisfied. Compared with the fully probabilistic design method, the partial factor formulae consist of determinate variables and partial factors, which make the proposed method convenient for practice use.
- Increasing the cover depth is an effective measure to delay the chloride-induced reinforcement corrosion initiation. By contrast, the duration between the corrosion initiation and cover cracking cannot obviously be extended by increasing the cover depth.
- A nonlinear relationship exists between the required cover depth and the target reliability for the limit state of cover cracking. When the target reliability over 2.0 is specified, increasing the cover depth is not economic design

strategy to resist the cover cracking.

- The numerical characteristics of the random variables, namely the mean and CVs, have significant effects on the required cover depth for the target performance. Especially the corrosion rate has the greatest influences on the required cover depth to resist the chloride-induced cover cracking.

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