Fast classification of fibres for concrete based on multivariate statistics

Paweł K. Zarzycki^{1a}, Jacek Katzer^{*2} and Jacek Domski^{3b}

¹Department of Environmental Technologies and Bioanalytics, Faculty of Civil Engineering, Environmental and Geodetic Sciences, Koszalin University of Technology, Śniadeckich 2, 75-453 Koszalin, Poland

²Department of Construction and Building Materials, Faculty of Civil Engineering, Environmental and Geodetic Sciences,

Koszalin University of Technology, Śniadeckich 2, 75-453 Koszalin, Poland

³Department of Concrete Structures and Technology of Concrete, Faculty of Civil Engineering, Environmental and Geodetic Sciences, Koszalin University of Technology, Śniadeckich 2, 75-453 Koszalin, Poland

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Abstract. In this study engineered steel fibres used as reinforcement for concrete were characterized by number of key mechanical and spatial parameters, which are easy to measure and quantify. Such commonly used parameters as length, diameter, fibre intrinsic efficiency ratio (FIER), hook geometry, tensile strength and ductility were considered. Effective classification of various fibres was demonstrated using simple multivariate computations involving principal component analysis (PCA). Contrary to univariate data mining approach, the proposed analysis can be efficiently adapted for fast, robust and direct classification of engineered steel fibres. The results have revealed that in case of particular spatial/geometrical conditions of steel fibres investigated the FIER parameter can be efficiently replaced by a simple aspect ratio. There is also a need of finding new parameters describing properties of steel fibre more precisely.

Keywords: steel fibres; concrete; reinforcement; univariate measurements; multivariate classification; principal component analysis

1. Introduction

Engineered steel fibres are currently the most popular unconventional type of concrete reinforcement. The first modern fibre reinforced concrete was patented in 1874 (Maidl 1995). Its concept was based on ancient ideas (Egyptian and Babylonian) of modifying brittle materials by fibre addition. For the first decades after patenting fibre reinforced concrete was not a popular construction material. This neglect was caused by two main factors: lack of proper engineered steel fibre and problems with workability of steel fibre reinforced concrete (SFRC) fresh mixes. In the late 1950s and early 1960s first engineered steel fibre started to be produced on industrial scale (Maidl 1995). At the same time plasticizers became popular in concrete industry. The above facts enabled production of commercially feasible SFRCs. Along the production of SFRC extensive research programmes were conducted. It was proven that the addition of fibre mainly influences tensile strength, flexural strength and all dynamic properties of concrete (Nawy 1996, Maidl 1995). It has been also demonstrated that substitution of traditional bar and stirrup reinforcement is possible (Katzer and Domski 2013). The

* Corresponding author, Assistant Professor

E-mail: domski@inbox.com

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Fig. 1 Examples of geometric shapes of commonly used steel fibres

current global consumption of steel fibre for concrete reinforcement is equal to 300,000 tons and is growing every year by 20%. In over 90% it consists of engineered (shaped) fibre. There are produced steel fibres with deformed ends (coned, spaded, with end paddles or buttons etc.), surfacetextured, twisted, crimped and hooked. All types of commercially available fibres are varied by length, geometry of a cross-section and area of a cross-section. Examples of different geometric shapes of commonly used types of steel fibre are presented in Fig. 1.

Among engineered steel fibres, the hooked fibres represent over 65% of the market. Half of all types of hooked steel fibre available on the global market are

E-mail: katzer@inbox.com

^aAssistant Professor

E-mail: pkzarz@wp.pl

^bPh.D.

Steel fibre ID	Producer (10)	Nominal Length (mm)	Nominal Diameter (mm)	Code name (10)
	(10)	Bengui (iiiii)	Bruneter (min)	(10)
1	В	60	1.00	B/1.00
2	D	30	0.55	D/0.55
3	D	60	0.75	D/0.75
4	Е	50	0.65	E/0.65
5	Е	50	0.80	E/0.80
6	Е	50	1.00	E/1.00
7	S	50	0.80	S/0.80
8	S	50	1.00	S/1.00

Table 1 ID labels of investigated engineered steel fibres used for PCA calculations

characterized by the aspect ratio ranging from 45 to 63.5 (Katzer and Domski 2012). These fibres are used for composition of a wide variety of different types of fibre reinforced concretes including ordinary concretes (Domski 2016, Havlikova et al. 2015), self-compacting concretes (Pajak and Ponikiewski 2013, Ponikiewski et al. 2014), roller compacted concretes (Neocleous et al. 2011) and concretes based on waste aggregates (Ghorpade and Sudarsana 2010, Xie et al. 2015). So far classification of different types of fibres have been done using basic geometrical and mechanical parameters such as length, aspect ratio and tensile strength. Harnessing fibre reinforced concretes for more and more demanding applications (Colajanni et al. 2012, Spinella 2013) creates a need for more complex but at the same time fast and reliable method of their classification. In author's opinion a method based on multivariate statistics involving multiple geometric and mechanical parameters of a given fibre would be the best solution of the fibre classification problem.

Exploration and analysis of large data sets combining a number of objects (or observations), which are characterized by many parameters and variables, is difficult for humans (Chen et al. 2015). In spite of the fact that a human brain may record and process huge amount of signals from different sources, its ability to efficiently analyse of more than three or four factors presented simultaneously, is strongly limited. It should be highlighted that univariate characterization of complex data sets is usually restricted and difficult to perform due to the multifactor nature of the samples and experimental setup performed (Zarzycki and Portka 2015). For that reason, the first approach to large or very large data sets exploration should be based on multivariate techniques such as principal component analysis (PCA), projection to latent structures (PLS) and/or related calculations (Soroush et al. 2015, Tutmez 2014). The concept of data dimensionality reduction as well as determination of the latent information that may exist in the initial data sets was invented at the beginning of the 20th century by Pearson (Pearson 1901). The main idea was that the small set of uncorrelated variables (derived by specific computations from raw large data set) is much easier to interpret and particularly to use for the further analysis than dealing with initial large data containing huge number of correlated variables (Perez and Escandar 2016). Particular data handling (e.g., PCA) allows



Fig. 2 Scheme of a geometric shape of a hooked steel fibre

to extract, with minimum loss of information, the principal information that exist in a number of original variables and subsequently transform a large table into a much smaller set of new latent factors. Since the original set of variables is linearly transformed into a small set of uncorrelated new "virtual" variables, the relationships between investigated objects can be visualized on simple two- or three dimensional plots. Such plots can be easily analysed and interpreted by researchers. Particularly, PCA that is a special case of factor analysis (FA), allows effective data reduction and to determine the latent information from initial large data sets. Presently, PCA concept is considered as one of the successful implementation of linear algebra and has numerous applications in data analysis and signal processing. For many years this methodology was frequently used in chemistry, separation, environmental and material science (Pereira et al. 2016, Pereira et al. 2010, Zarzycki et al. 2010). Popularity and flexibility of the methodology is based on the convenient processing and interpretation of large and multivariate data sets.

In this paper the authors proposed a new approach (from the construction and building materials point of view) for classification of engineered steel fibres. Quantitative mechanical parameters and geometrical properties of different hooked steel fibres were inspected using multivariate analysis based on principal component analysis. In contrary to an univariate approach, which is commonly applied for assessing the engineered steel fibres, the described methodology allows fast and robust objects classifications as well as a selection of a minimum number of key parameters (variables reduction), which should be used for accurate steel fibres classification, with a minimum experimental setup.

2. Experimental setup

All analysed steel fibres (investigated objects) were commercially available on the European market during the research programme. Their specification data, measurement methodology and applied testing protocols as well as experimentally determined mechanical properties were thoroughly described and discussed in our previous paper (Katzer and Domski 2012). Briefly, eight individual hooked fibre types were investigated originating from four producers (Table 1).

Such properties as: length L (EN 14889-1 2009), diameter d (EN 14889-1 2009), aspect ratio (l/d), fibre intrinsic efficiency ratio (*FIER*) (Soulioti *et al.* 2011), hook geometry (Soetensa *et al.* 2013), tensile strength (EN ISO 6892-1 2009) and ductility (EN 10218-1 1994) were taken into consideration as quantitative data. A scheme of a



Fig. 3 Steel fibre prepared for tensile strength test



Fig. 4 Steel fibre prepared for ductility test

geometric shape of a hooked steel fibre with all the considered dimensions annotated is presented in Fig. 2.

FIER has been defined as the ratio of bonded lateral surface area of fibre, to its cross sectional area (Naaman 2003). It is calculated for the length L (see Fig. 2) of a given fibre according to the Eq. (1).

$$FIER = (\Psi \cdot L)/A \tag{1}$$

Where: Ψ -perimeter of the fibre; *A*-cross sectional area of the fibre; *L*-see Fig. 2.

Tensile strength test was conducted with increase of force at constant rate (see Fig. 3).

Ductility test was performed on the end diameter before deformation. The steel was bent over a cylindrical support. The radius of the support was ranging from 1.75 to 2.5 mm depending on fibre diameter. A photo of a fibre mounted on the apparatus and prepared for bending over a cylindrical support is presented in Fig. 4.

Geometric characteristics of the hooked ends was the second geometric parameter analysed during this research study. It is commonly known that mechanical clamping of the hook in concrete matrix significantly increases the pullout energy (Kim *et al.* 2008). In majority of cases the hook of the fibre is straightened out during the pull-out process without any matrix failure. A hook contribution to pull-out resistance is directly associated with its geometry. Parameter $l+(a^2+h^2)^{0.5}$ was chosen to describe hook geometry. The parameter was successfully used by other



Fig. 5 Univariate statistical characterization (mean and standard error values; n=28) of selected parameters measured experimentally with represented confidence intervals estimated at 95%

researchers (Kim *et al.* 2008) and fibres described by the highest value of the parameter are recognized as the most efficient.

The research programme was executed on the populations of 28 randomly chosen fibres for each test. Fibre sampling was performed with the help of table of random numbers to guarantee full impartiality of the test. Outliners from all fibre populations were identified and rejected using Dixon's Q test (Ellison 2009). Kolmogorov-Smirnov Test, also known as K-S Test (Corder and Foreman 2009), was used to assess the normal (Gaussian) distribution of the achieved populations of results. The described properties represent all parameters commonly used to characterize steel fibre. Usually, only two or three of these parameters are given simultaneously for developing fibre reinforced concrete mix. Quantitative data concerning investigated steel fibres were inspected with PCA procedure using XLSTAT-Pro/3DPlot statistical and visualization package (version 2008.2.01) provided by Addinsoft (Paris, France) and working with Microsoft Excel 2002. The appropriateness of multivariate calculations for our data was assessed by performing the Bartlett's sphericity test.

3. Results and discussion

The results of statistical study concerning mechanical and spatial properties of fibres in question were presented and thoroughly discussed by Katzer and Domski (2012). For the purpose of this investigation 28 individual fibres (objects) from each steel fibre type were randomly selected and measured. Statistical characteristics of selected fibres populations concerning key geometrical and mechanical parameters (variables) including *FIER*, hook geometry, tensile strength and ductility was summarized on the graphs presented in Fig. 5.

So far, classification of engineered steel fibres was very basic and limited to fibre length and aspect ratio. Usually, factors associated with mechanical and physical parameters reflecting properties of a given material at nanoscale level were omitted (Katzer and Domski 2012, Soetensa *et al.* 2013, Soulioti *et al.* 2011). Such a situation was caused by used equipment for fibre concrete production and utilized mix designing methods. For concrete producers fibre length is a crucial factor influencing feasibility of fibre reinforced concrete production. If fibres are short enough it is possible to harness ordinary mixers, conveyer belts, vibrators etc. which are used for production of ordinary concrete. On the other hand all existing designing methods of fibre concrete mix are based on fibre aspect ratio in one way or another.

The main advantage of using those two parameters is the fact that they are very easy to measure. They are also intuitively understood by all engineers and technicians involved in creating of a fresh fibre mix and erecting a fibre reinforced concrete structure. For the last 60 years, this limited approach to fibre classification has influenced applications of fibre reinforced concretes and development of mix designing methods. Multiple researchers have been aware of this problem and have proposed other parameters (Soetensa et al. 2013, Soulioti et al. 2011) to characterize steel fibre like FIER, hook geometry, tensile strength and ductility. Surprisingly, new characteristics of fibre has not improved fibre classification and mix designing. As it can be seen in Fig. 5, the univariate approach applied for analysis of a particular set of parameters is ineffective in terms of the objects classification for a given type of engineered steel fibres. In addition, evident fibre classification may be still difficult even considering additional geometrical parameters such as fibre length, diameter and aspect ratio. It should be highlighted that parameters listed above combine nanoscale (e.g., elements composition, crystals shape and size) and macroscale (e.g., fibre geometry) properties of fibres. Therefore, for effective steel fibre classification, multivariate approach involving principal component analysis (PCA) should be applied.

Generally, PCA can be efficiently used for reducing the raw data matrix to a smaller number of uncorrelated variables and to determine possible latent information, which is difficult to recognize considering the initial data set. This technique may also capture the essential data patterns that are specific to the initial large data set. It also allows investigated objects classification. In this study the starting point for principal component analysis was data matrix consisting of 1578 measurements. The matrix consisted 224 objects (8 fibre types×28 individual fibres) characterized by 7 variables including: length (variable 1), diameter (variable 2), aspect ratio (variable 3), FIER (variable 4), hook geometry (variable 5), tensile strength (variable 6) and ductility (variable 7). The number of principal components characterizing this data set was determined by considering eigenvalues that were calculated as follows: 3.002, 1.816, 1.135 0.773, 0.254, 0.015 and 0.004. An optimal number of factors to retain was selected using the Kaiser criterion, in which only factors with eigenvalues greater than 1 should be retained. According to this criterion, the first three factors (F1, F2 and F3) were selected and they "explain" over 85% of total variability (factors F1, F2 and F3 account for 42.8, 25.9 and 16.2% of the variance, respectively). To investigate, which studied parameters are responsible for the steel fibres group clustering, the factor loadings data was analysed and the



Fig. 6 Principal component analysis-projection of variables set in 2D factor loading space (variables labelling described in the text and in Tables 2A-D)

Table 2A Factor loadings

Variable ID	F1	F2	F3
Var1 (Length)	0.182	0.969	-0.078
Var2 (Diameter)	-0.656	0.721	0.079
Var3 (Aspect ratio)	0.928	0.240	-0.201
Var4 (FIER)	0.931	0.269	-0.134
Var5 (Hook geometry)	-0.482	0.276	-0.681
Var6 (Tensile strength)	0.699	-0.204	-0.246
Var7 (Ductility)	0.300	0.328	0.735

results were presented in form of a graph in Fig. 6 (F1 vs F2) as well as in Table 2A consisting numerical data for F1, F2 and F3.

Computed data revealed that all variables impart the most important information and may significantly contribute to the further objects classification. It has also been revealed that aspect ratio and FIER parameters (variables 3 and 4) carry approximately equal information. In case of analysed fibre types, which were all characterized by circular cross-section the value of FIER is proportionally associated with aspect ratio. We may hypothesize that if the population of analysed fibre types was differentiated by shape of a cross-section, this parameter would carry different information than aspect ratio. It is also worth noticing that diameter (variable 2) and strength (variable 3) are almost contrariwise. Considering values of the factor loadings and contribution of the variables (listed in Tables 2A and 2B, respectively) the most contributing variables for main factor F1 are aspect ratio, FIER and tensile strength.

Those three variables contribute together 73.8%. In case of factor F2 length and diameter are the most influential and they contribute together 80.4%. Hook geometry and ductility mainly contribute to factor F3 (together 88.4%). The detailed contribution of all variables to the main PCA factors is presented in Table 2B (most important according to contribution of the variables criterion were marked as bold and labelled with asterisk).

Variable ID	F1	F2	F3
Var1 (Length)	1.097	51.761*	0.541
Var2 (Diameter)	14.321	28.621*	0.550
Var3 (Aspect ratio)	28.718*	3.182	3.559
Var4 (FIER)	28.861*	4.000	1.576
Var5 (Hook geometry)	7.741	4.208	40.825*
Var6 (Tensile strength)	16.255*	2.297	5.333
Var7 (Ductility)	3.006	5.932	47.616*

Table 2B Contribution of the variables (%)

Table 2C Squared cosines of the variables

Variable ID	F1	F2	F3
Var1 (Length)	0.033	0.940**	0.006
Var2 (Diameter)	0.430	0.520**	0.006
Var3 (Aspect ratio)	0.862**	0.058	0.040
Var4 (FIER)	0.866**	0.073	0.018
Var5 (Hook geometry)	0.232	0.076	0.463
Var6 (Tensile strength)	0.488	0.042	0.061
Var7 (Ductility)	0.090	0.108	0.541**

Table 2D Scaled contributions (relative variancecontributions) (%)

Variable ID	F1	F2	F3
Var1 (Length)	0.033	0.940**	0.006
Var2 (Diameter)	0.430	0.520**	0.006
Var3 (Aspect ratio)	0.862**	0.058	0.040
Var4 (FIER)	0.866**	0.073	0.018
Var5 (Hook geometry)	0.232	0.076	0.463
Var6 (Tensile strength)	0.488	0.042	0.061
Var7 (Ductility)	0.090	0.108	0.541**

To inspect more strictly the PCA factors in terms of the relevant variables the squared cosines of the variables and scaled contributions (relative variance contributions) were calculated and presented in Tables 2C and 2D, respectively.

For variables 4 and 5 (F1 column) the squared cosines are greater than 0.5. According to such criterion (David and Jacobs 2014), the real correlation may exist and these variables strongly contribute for PCA factor F1 (appropriate numbers are marked as bold and labelled as double asterisks within Table 2C). Considering scaled contributions values (Table 2D) related to the first PCA factor, the variables 3 and 4 are responsible for 75.4% of information included in F1 (sum of data bolded and labelled as triple asterisks within F1 column). In case of PCA factor F2 the squared cosines and scaled contributions data confirm the analysis based on the factor loadings (Table 2A) and contribution of the variables (Table 2B) criteria. Basically, the fibres length (variable 1) and diameters (variable 2) are strongly contributing to F2. Nevertheless, considering data included in Tables 2C and 2D the ductility (variable 7) should be considered as the main variable contributing to PCA factor F3.



Fig. 7 Principal component plot showing objects clustering (all individual steel fibres) in two dimensional space (F1 and F2 factors scores)-numbers from 1 to 8 are related to particular types of engineered steel fibres (see Table 1)



Fig. 8 Principal component plot showing relationships between objects investigated (all individual steel fibres) in three dimensional space (F1, F2 and F3 factors scores) numbers from 1 to 8 are related to particular types of engineered steel fibres (see Table 1)

The principal component plot showing objects (individual fibres) clustering is presented in Fig. 7. According to data presented in the principal component plot, information that persists in studied mechanical and spatial parameters is sufficient for accurate fibres classification using the multivariate approach. As it may be observed, this technique allows precise classification of individual fibres taking into account two dimensional space, involving factors F1 and F2. Complete clusters separation (including fibre types 6, 8 and particularly 5 and 7) can be

performed considering three factors simultaneously, due to their location in three dimensional factor scores space (Fig. 8).

Observed on these PCA graphs individual steel fibres clustering and significant separation between clusters related to particular types of the fibres, strongly supports the hypothesis that parameters selected in this study contain key and almost complete information that may reveal the important differences between steel fibre types. Nevertheless, the calculations should be repeated using additional parameters such as pull-out strength etc.

There is also a need to find new parameters describing, more precisely, properties of steel fibre that may improve accurate classification of such objects using multivariate computations. PCA factors, proposed in this study, may occur very valuable for modelling fibre reinforced concrete and identification of its properties (Sucharda *et al.* 2015).

4. Conclusions

Presented research has revealed that direct comparison of engineered steel fibres used as reinforcement for concrete via univariate approach may be strongly limited. It became clear that properties of tested fibre populations were significantly differentiated. Due to the nature of parameters commonly used for their characterization, resulting quantitative data may be treated as multivariate data set. The applied multivariate approach enables easy comparison of the parameters (variables) and to classify individual fibres for given type of engineered steel fibre. Utilized PCA calculations indicated that length/diameter ratio and FIER parameters may carry equal information in case of the fibres that are characterized by similar cross-section shape. The calculations should be repeated using additional parameters such as pull-out strength etc. The achieved results proved that there is a need of finding new parameters describing, more precisely, properties of steel fibre. New parameters may improve accurate classification of steel fibre using multivariate computations.

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