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The fractal analysis of the fracture surface of concretes made from different coarse aggregates

Grzegorz Prokopski† and Janusz Konkol‡

Rzeszów University of Technology, Powstańców Warszawy 6, 35-959 Rzeszów, Poland

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Abstract. The article presents the results of examination of the fractal dimension D of concrete specimen fracture surfaces obtained in fracture toughness tests. The concretes were made from three different types of coarse aggregate: gravel, dolomite and basalt aggregate. Ordinary concretes (C40) and high-performance concretes (HPC) were subjected to testing after 7, 14, 28 and 90 days of curing, respectively. In fracture toughness and compressive tests, different behaviours of concretes were found, depending on the type of aggregate and class of concrete (C40, HPC). A significant increase in the strength parameters tested occurred also after a period of 28 days (up to the 90th day of curing) and was particularly large for concretes C40. Fractal examinations performed on fracture replicas showed that the fractal dimension D was diverse, depending on the coarse aggregate type and concrete class being, however, statistically constant after 7 and 14 days for respective concretes during curing. The fractal dimension D was the greater, the worse strength properties were possessed by the concrete. A cross-grain crack propagation occurred in that case, due to weak cohesion forces at the coarse aggregate/mortar interface. A similar effect was observed for C40 and HPC made from the same aggregate. A greater dimension D was exhibited by concretes C40, in which case the fracture was easier to form compared with high-performance concretes, where, as a result of high aggregate/mortar cohesion forces, the crack propagation was of inter-granular type, and the resulted fracture was flatter.

Keywords: concrete curing; interfacial transition zone; mechanical properties; silica fume; concrete.

1. Introduction

The properties of coarse aggregate have a crucial effect on the behaviour of concretes in the process of their service-see, among others, (Jones and Kaplan 1957, Kuczyński 1958, Bochenek and Prokopski 1989, Prokopski 1990, 1993), Studies on the effect of coarse aggregate on the strength parameters of concretes, characterized by fracture mechanics parameters, have already had a history of several decades. Early works on the effect of coarse aggregate on the properties of concretes were published in the fifties of the 20th century (e.g. Jones, Kaplan 1957, Kuczyński 1958). These works dealt with the problems of the morphology of shape and surface of coarse aggregate grains and their effect on aggregate adhesion to the cement paste and the process of failure at compression. Later studies by Bochenek and Prokopski (1989) and Prokopski (1990, 1993) covered also the examination of the effect of coarse aggregate on fracture mechanics parameters, as determined in Modes I (tension at bending) and II (shearing) of fracture. These studies found a considerable influence of the type, size

† Associate Professor, grzeprok@prz.rzeszow.pl
‡ Ph. D.

and content of coarse aggregate grains and the coarse aggregate shape and surface topography on the failure process and fracture toughness of concretes.

The use of additives (such as silica fume) and admixtures in the form of various plasticizers facilitating the workability of concrete mixtures in contemporary concretes has resulted in obtaining high-performance concretes (HPC) with a much greater dynamic of strength increment in time (in the curing process), while using ordinary aggregates (Prokopski 2003).

Prokopski and Langier (2000) analyzed the effect of water/cement (*W/C*) ratio and silica fume addition on the morphology of surface fractures and the fracture toughness of gravel concrete. With increasing *W/C* value, a decrease in the stress intensity factor, K_{Ic}^{S}, occurred, while the degree of profile line development, R_L, and the fractal dimension D of fracture surfaces increased. This was due to cohesion forces at the aggregate–cement paste interface decreasing with the increase in the *W/C* and the presence of a cross-grain fracture of high roughness.

A many works concerning the fractal examination of cement-based materials (e.g. Winsolow 1985, Carpinteri, *et al.* 1999, Carpinteri and Chiaia 1997, Chiaia, *et al.* 1998, Dougan and Addison 2001, Heinemann, *et al.* 1999, Saouma and Barton 1990, Saouma, *et al.* 1990, Peng, *et al.* 1997, Yan, *et al.* 2001, 2003, Issa, *et al.* 1993, 2003) are examples of obtaining relationships of the fractal dimension with fracture toughness expressed by the fracture energy, G_F, critical stress intensity factor, K_{Ic}, or compressive strength, f_c, of concretes. However, obtained relationships are sometimes contradictory, the example of which are works (Yan, *et al.* 2001 and Saouma and Barton 1990). Yan, *et al.* (2001) reported that with increasing fractal dimension fracture energy increased, while work (Saouma and Barton 1994) drew an opposite conclusion.

As the fracture surface, described by the fractal dimension, reflects a large number of structural factors affecting the formation of a particular fracture, the fractal dimension, too, contains information on some parameters that influence the fracture toughness of the material in question. It is, however, hard to formulate analytically the relationship of fracture toughness with the structure; hence the differences in reports, as those provided by the above-mentioned works, are likely to originate.

Formulating general relationships concerning the properties of concretes in the curing process is a complex problem, since this is a function of numerous factors, the main of which include: the type of aggregate and cement, the water–cement ratio and concrete care conditions.

In this article, the authors use the chord method to estimate the fractal dimensions of profiles cut perpendicularly to the fracture surface of concrete. In many studies, the analysis of fracture surfaces is reduced to the analysis of vertical sections. Although the amount of information related to the spatial structure of the fracture is much lesser in the case of profile line analysis than when the whole fracture surface is subjected to analysis, the use of profiles, despite the laboriousness of their preparation (Prokopski 2003, Prokopski and Langier 2000, Konkol and Prokopski 2004), makes it possible to obtain a highly accurate shape of profile line of the whole fracture length. Commonly used direct methods of fractal dimension measurement usually limit the size of inspected area of fracture surface. This is because using a comparable discretisation step on the whole surface would require much longer time and much more computer processing power to analyse such big amount of data.

The investigation discussed in the present study aimed at obtaining data determining the changes in the fractal dimension D of the surfaces of specimen fractures formed during fracture toughness test, between the 7^{th} and 90^{th} days of curing.

2. Experiment

2.1. Strength tests

The subjects of testing were concretes made with the use of three types of coarse aggregate: cobble (washed gravel) aggregate, chemically active dolomite aggregate, and basalt aggregate. Ordinary concretes (C40) and high-performance concretes (HPC) after 7, 14, 28 and 90 days of curing, respectively, were tested.

Compression tests were performed on prisms of an edge of 150 mm, while fracture toughness tests according to Mode I (tension at bending) were carried out on 80 × 150 × 700 mm specimens, each with one primary crack, according to the RILEM recommendations (Determination of fracture parameters (K_{Ic}^{S} and $CTOD_c$) of plain concrete using three-point bend test 1990). In the fracture toughness tests, critical values were determined for: stress intensity factor, K_{Ic}^{S}; crack tip opening displacement, $CTOD_c$; crack length, a_c; and elastic modulus, E. During the tests, a diagram of the loading force as a function of crack mouth opening displacement (CMOD) was recorded for each specimen. An example of the diagram of load vs. CMOD is shown in Fig. 1.

The results of the strength tests of concretes, i.e., f_c and E, with standard deviations ∂ are given in Table 1.

2.2. Fractal examinations

Fractal examinations were carried out on 120 specimen fractures obtained from the concrete fracture toughness tests. Replicas for examination were prepared in the following manner: white gypsum was poured onto the fracture surface coated with water-glass; after drying, the replica was taken off and then a stained gypsum layer was applied on it, which clearly defined the line of separation between both layers. So obtained replicas were mechanically cut perpendicularly to the fracture surface to obtain 8-11 layers which were then subjected to analysis (each of the layers

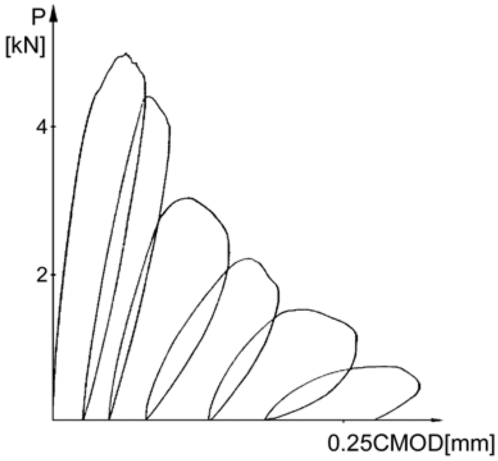

Fig. 1 Example diagram of load-CMOD

Table 1 Results of concrete strength investigation

Parameter	Concrete class	Curing period (days)			
		7	14	28	90
		Gravel concrete			
$K_{Ic}^{S} \pm \partial$ [MN/m$^{3/2}$]	C40	1,50 ± 0,39	2,08 ± 0,23	2,28 ± 0,26	3,22 ± 0,14
	HPC	2,37 ± 0,47	2,59 ± 0,39	2,86 ± 0,52	3,22 ± 0,53
$f_c \pm \partial$ [MPa]	C40	35,4 ± 0,5	38,1 ± 0,5	44,4 ± 1,0	59,7 ± 1,0
	HPC	78,6 ± 1,2	82,5 ± 1,4	88,0 ± 0,8	92,5 ± 1,8
$E \pm \partial$ [GPa]	C40	21,9 ± 3,4	30,0 ± 2,3	33,3 ± 2,7	34,1 ± 2,4
	HPC	42,7 ± 9,3	43,1 ± 3,7	44,5 ± 10,5	49,5 ± 5,7
Parameter	Concrete class	Curing period (days)			
		7	14	28	90
		Dolomite concrete			
$K_{Ic}^{S} \pm \partial$ [MN/m$^{3/2}$]	C40	1,35 ± 0,11	2,34 ± 0,25	2,94 ± 0,19	3,60 ± 0,33
	HPC	2,70 ± 0,18	2,90 ± 0,42	3,38 ± 0,20	4,22 ± 0,34
$f_c \pm \partial$ [MPa]	C40	25,7 ± 1,6	43,0 ± 1,2	49,4 ± 1,2	69,6 ± 3,1
	HPC	81,9 ± 3,0	95,2 ± 1,6	98,5 ± 1,2	102,3 ± 3,1
$E \pm \partial$ [GPa]	C40	24,6 ± 1,0	28,9 ± 0,7	38,8 ± 2,3	40,7 ± 3,0
	HPC	41,0 ± 1,3	46,3 ± 2,8	52,6 ± 1,8	54,2 ± 1,8
Parameter	Concrete class	Curing period (days)			
		7	14	28	90
		Basalt concrete			
$K_{Ic}^{S} \pm \partial$ [MN/m$^{3/2}$]	C40	1,49 ± 0,19	1,90 ± 0,20	2,19 ± 0,07	2,98 ± 0,45
	HPC	1,70 ± 0,15	2,17 ± 0,31	3,27 ± 0,17	3,87 ± 0,47
$f_c \pm \partial$ [MPa]	C40	31,1 ± 1,6	41,2 ± 1,9	54,6 ± 1,6	62,5 ± 1,0
	HPC	55,7 ± 1,8	63,0 ± 2,2	78,0 ± 1,9	84,7 ± 1,8
$E \pm \partial$ [GPa]	C40	24,5 ± 0,9	27,5 ± 1,5	31,8 ± 2,1	34,9 ± 1,4
	HPC	27,4 ± 1,3	32,1 ± 4,5	37,7 ± 1,3	41,6 ± 1,6

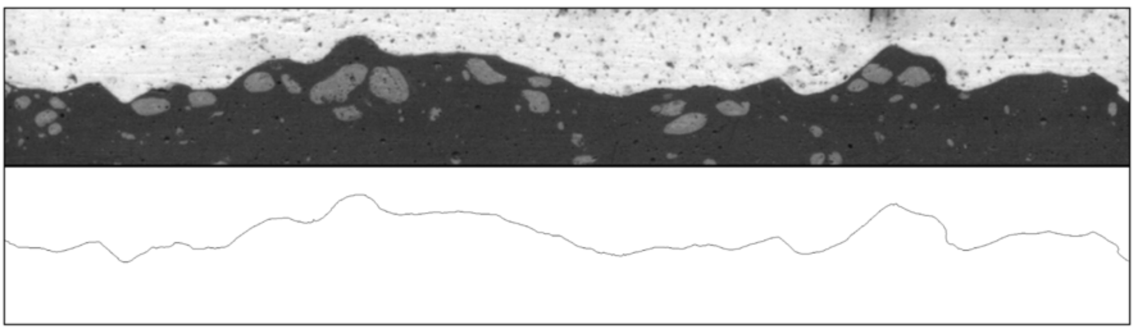

Fig. 2 Image of a scanned profile (at the top) and the result of digitalization (at the bottom)

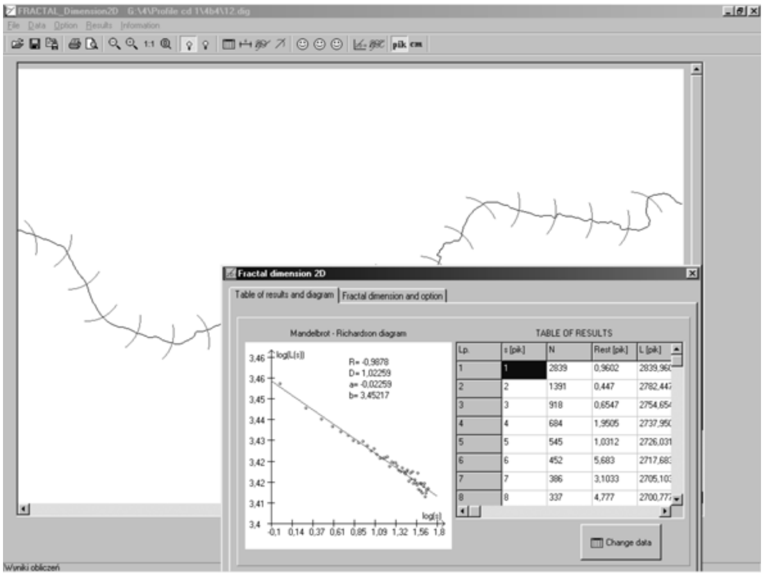

Fig. 3 Example of Mandelbrot-Richardson diagram

Table 2 Results of fractal dimension D investigation

Concrete class	Curing period (days)			
	7	14	28	90
		Gravel concrete		
C40	1,0216	1,0182	1,0174	1,0223
HPC	1,0150	1,0142	1,0173	1,0120
		Dolomite concrete		
C40	1,0210	1,0202	1,0190	1,0174
HPC	1,0162	1,0106	1,0144	1,0143
		Basalt concrete		
C40	1,0288	1,0294	1,0298	1,0301
HPC	1,0226	1,0203	1,0211	1,0242

allowed the analysis of 2 profile lines). The profile lines were scanned and stored in the computer as bitmaps (Fig. 2), and subsequently analyzed using a suitable computer program.

The calculation of the fractal dimension was made by the chord method, while using measuring steps in the range from 2 to 50 pixels (every 1 pixel), at a resolution of 600 dpi. The fractal dimension was determined as the tangent of the slope of a double-logarithmic diagram (Fig. 3) of the relationship of the logarithm of profile line length measured with the step s to the logarithm of the length of this step. The length of the profile line projection was 100 mm.

The obtained results of fractal dimension examinations are summarized in Table 2.

3. Test results and their analysis

On the basis of the obtained results of strength tests (Table 1), the effect of the fractal dimension on the values of measured parameters was determined. In the case of the critical stress intensity factor, K_{Ic}^S and elastic modulus, E, the correlation analysis was performed for all testing results, whereas for the compressive strength, f_c, this was done based on mean values.

Linear relationships were obtained in the following form:

$$\begin{aligned} K_{Ic}^S &= a + b \cdot D \\ f_c &= c + d \cdot D \\ E &= e + f \cdot D \end{aligned} \tag{4}$$

for which the value of linear correlation coefficient was determined, and in the case of compressive strength – also the significant level.

The values of the coefficients $a \div f$ for Eqs. (4), depending on concrete age, are given in Table 3, and the obtained function graphs are shown in Figs. 4-6.

Analysis of the influence of concrete age on the fractal dimension and the mechanical properties of concretes was also determined (Figs. 7-9).

Designations of concretes:

C40	**HPC**
G – gravel concrete	G_{HPC} – high-performance gravel concrete
D – dolomite concrete	D_{HPC} – high-performance dolomite concrete
B – basalt concrete	B_{HPC} – high-performance basalt concrete

In the case of the C40 basalt concrete, the analysis confirmed the statistical equality of mean values at a significant level of 0.05 in the whole curing period from 7 to 90 days. In the case of the HPC gravel concrete and the C40 and HPC dolomite concretes, this equality occurred for 7- and

Table 3 Values of the coefficients for Eq. (4)

Parameter	Coefficient	Concrete age (days)			
		7	14	28	90
K_{Ic}^S	*a*	65,65	51,47	61,69	44,65
	b	-62,44	-48,15	-57,59	-40,25
	Correlation coefficient *R* (number of samples)	0,599 (30)	0,632 (27)	0,720 (24)	0,451 (33)
f_c	*c*	3967,5	2965,1	1947,6	1881,7
	d	-3836,1	-2848,2	-1842,2	-1767,7
	Correlation coefficient *R* (significant level)	0,774 (0,07)	0,621 (0,19)	0,441 (0,38)	0,692 (0,13)
E	*e*	1171,49	1044,85	970,44	966,58
	f	-1117,4	-990,14	-912,08	-905,71
	Correlation coefficient *R* (number of samples)	0,689 (30)	0,687 (27)	0,713 (24)	0,719 (33)

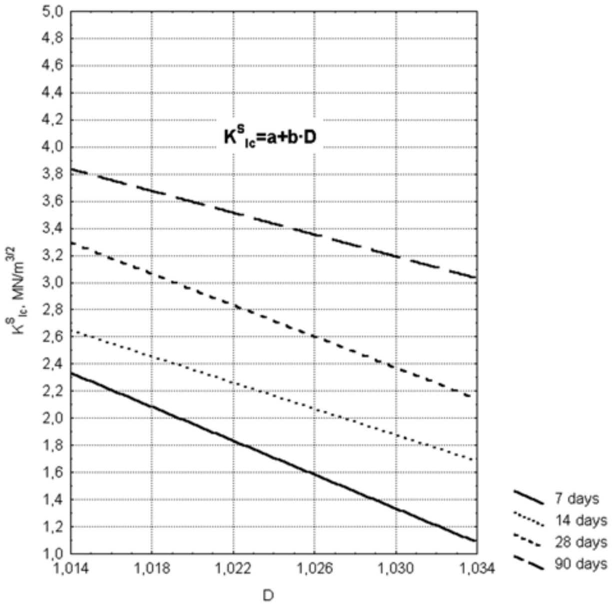

Fig. 4 Dependence of the stress intensity factor K_{Ic}^{S} on the fractal dimension D and concrete age

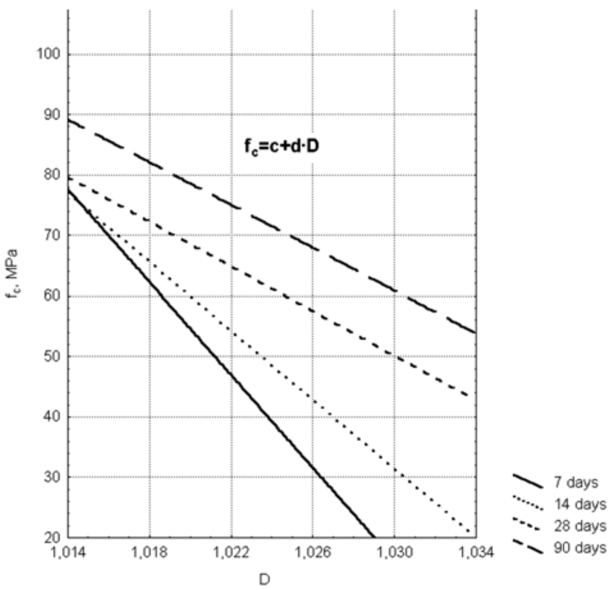

Fig. 5 Dependence of the compressive strength f_c on the fractal dimension D and concrete age

14-day concretes, and in the high-performance basalt concrete and the C40 gravel concrete – after 14 and 28 days of curing.

The statistical equality of the mean values of the fractal dimension *D* within the same concrete, particularly in the initial curing period (up to 28 days), provides evidence for a similarity of occurring fracture surfaces, which can be the result of their "pre-programmed" surface.

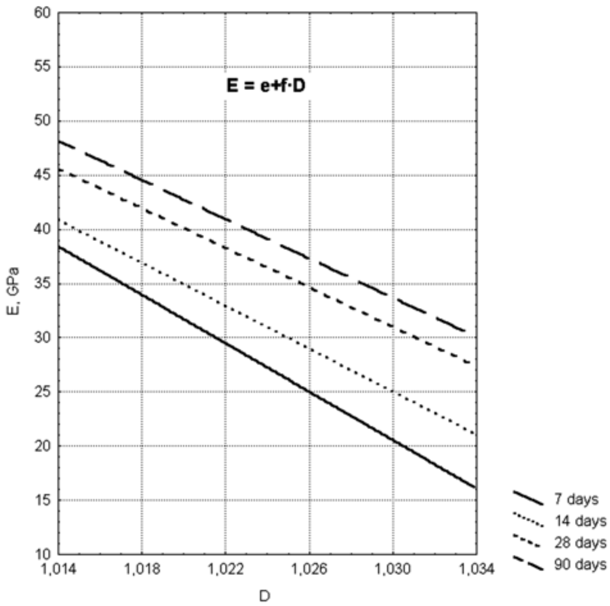

Fig. 6 Dependence of the elastic modulus E on the fractal dimension D and concrete age

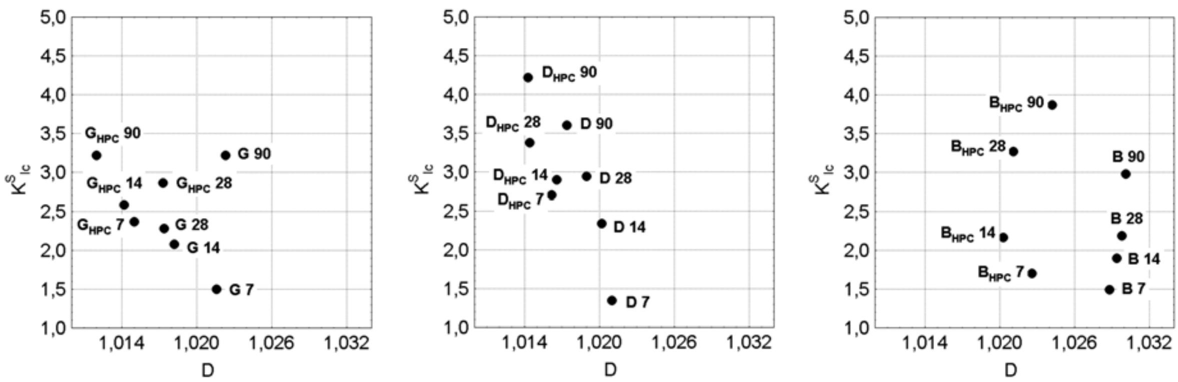

Fig. 7 Variation of the fractal dimension D and stress intensity factor K^S_{Ic} as a function of the class and age of concrete and the type of aggregate

It can be observed that as the age of concrete increases, the values of all strength parameters tested (K^S_{Ic}, f_c, E) also increase, with the fractal dimension value being similar for the same type of concrete (regardless its age), which may suggest the crack propagating in a "preprogrammed" manner in the structure. High-performance concretes (HPC) exhibit lower values of the fractal dimension D than ordinary concretes (C40), owing to a lower level of complexity of the fracture surfaces of these concretes compared with the fracture surfaces of ordinary concretes.

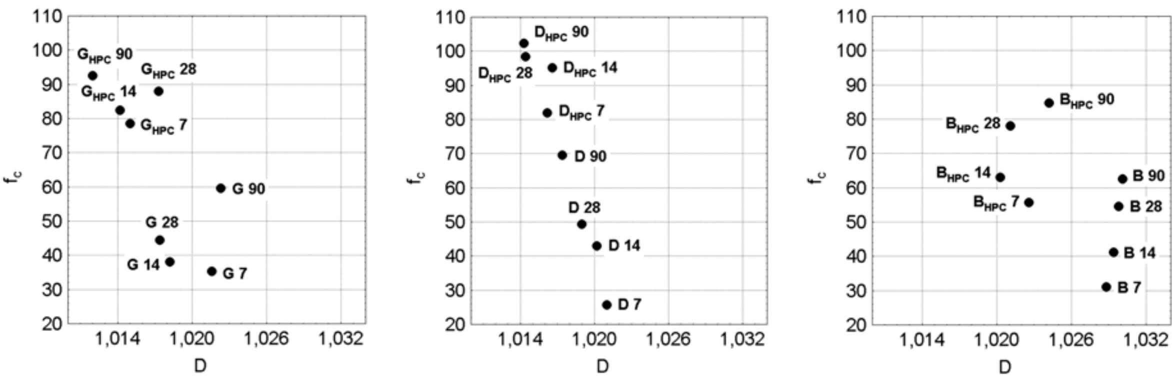

Fig. 8 Variation of the fractal dimension D and compressive strength f_c as a function of the class and age of concrete and the type of aggregate

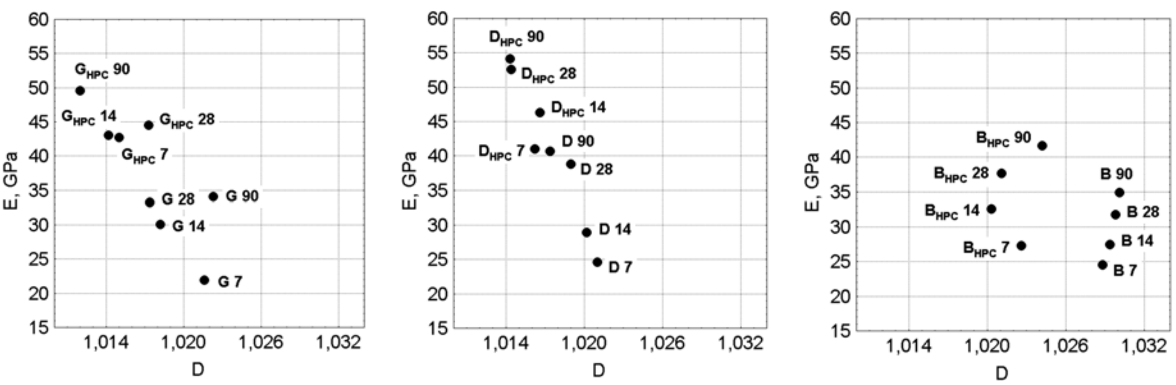

Fig. 9 Variation of the fractal dimension D and elastic modulus E as a function of the class and age of concrete and the type of aggregate.

4. Conclusions

With increasing concrete age, the values of all strength parameters (K_{Ic}^{S}, f_c and E) increase.

A dependence of the strength parameters of concretes on the fractal dimension, D, characterizing the surface of the fracture occurred in the fracture process, has been found. Obtained linear correlations for K_{Ic}^{S} and E are significant, whereas for f_c, owing to the fact that the analysis was carried out on mean values only (a small number of results), the linear correlation is insignificant.

Fractal examinations showed that the fractal dimension D was diverse, depending on the coarse aggregate type and concrete class being, however, statistically constant for respective concretes during curing.

The fracture surfaces of concretes made from basal aggregate, whose grains had the most irregular shape and the highest roughness, were characterized by the highest D value. Less complex were the fracture surfaces of concretes from chemically active dolomite aggregate with good adhesion to the mortar (being the sum of the mechanical component – depending on the aggregate grain roughness and shape – and the chemical component resulting from the activity of this aggregate towards the cement paste). This caused fracturing of dolomite grains throughout and the formation of a relatively smooth (low-roughness) fracture surface.

The fracture surfaces were less complex in the case of high-performance concretes than ordinary concretes. This is due to the fact that, in the case of these concretes, the aggregate/mortar adhesion forces exceeded the strength of coarse aggregate grains, which caused the aggregate grains to fracture throughout and a smoother-surface fracture to form. This mode of fracture resulted also in obtaining greater values of K^S_{Ic}, f_c and E.

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